



Stochastic Exchange Rate Dynamics, Intervention Dynamics and the Market Efficiency Hypothesis

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Abstract

Since currency price fluctuations hinder economic activity, exchange rate dynamics have an effect on national economies. To have a proper exchange rate policy in place, these dynamics are essential for nations with a trade economy. This study presents and examines a distinctive stochastic dynamics exchange rate model (ESI) in order to address the challenges associated with predicting the behavior of participants in some complex economic systems, which might lead to the system's collapse. To address the issue of ESI stability by central bank interventions (managed currency) in a specified target value, a target value technique is also provided and tested. Last but not least, we examine the noise traders' role as a major source of market uncertainty as we look at the market efficiency hypothesis for the foreign exchange market (FX).

Keywords Stochastic processes · Exchange rates · Central bank interventions · Market efficiency hypothesis

1 Introduction

Exchange rates, like all prices, are determined by the law of supply and demand, as seen in the foreign exchange market. There have been many approaches created over the years to explain the excess demand and price adjustment of exchange rate dynamics in stylized financial markets over time, but the chartist-fundamentalist method (Day & Huang, 1990; Athanasiou & Kotsios, 2008; Brianzoni & Campisi, 2020; Wieland & Westerhoff, 2005; Franke & Westerhoff, 2016) is the most capable of explaining the exchange rate dynamics. We choose to develop the

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proposed model of (Wieland and Westerhoff, 2005) because a lot of studies create false data that is difficult to distinguish from true data. According to their paper price dynamics are at least partially owing to an endogenous nonlinear rule of motion, while being buffeted by dynamic noise. The fact that traders utilize nonlinear trading strategies to determine their investment position may be the source of such non linearity.

The COVID-19 shock is an excellent example of how significant is the stabilization of the exchange rates. More specifically, an increase in the number of confirmed COVID-19 instances intensifies exchange rate volatility, and the continuous spread of COVID-19 greatly raises exchange rate volatility (Feng et al., 2021). Furthermore, when a government implements various COVID-19 intervention strategies, it should thoroughly evaluate the resulting implications and endeavor to minimize any negative ones on the general economy, and central banks must reduce exchange rate volatility, because exchange rates play a critical role in shaping a domestic economic growth (Bashir et al., 2019). Another significant shock is the United Kingdom's vote (Brexit), where the drop in the real value of the British pound following the Brexit referendum may be traced entirely to UK economic policy uncertainty shocks (Nilavongse et al., 2020; Jawad, 2019).

The central bank have an important role as an institutional market maker in intervening in exchange rate dynamics to stabilize them. There are a lot of empirical papers that contribute to the understanding of the central bank intervention. Firstly, according with Hung (1997), authorities may utilize intervention methods that increase volatility as well as those that decrease volatility to regulate the exchange rate level. Some examples of intervention strategies presented by Neely (2000) are the Monetary Base, Spot and Forward Markets, the Options Market, and the mechanics that analyze some significant type of strategy such as the necessity, the horizon, and the effects of the intervention strategy that the central bank is going to follow. Furthermore, the FX intervention is a valuable policy instrument, but the success of these interventions is dependent on conditions (Fratzscher et al., 2019).

From a theoretical point of view there are a lot of literature contributions that we must take into consideration. Firstly, according to Reitz et al. (2006), the targeting zone interventions is an effective tool for the central bank strategy and more specifically they determine the central bank's real per-period intervention volume, and an intervention is carried out when the exchange rate departs from a predetermined range around the fundamental value. Chartists amplify the new exchange rate trend, and fundamentalists are urged to take greater holdings. Furthermore, according to Westerhoff and Franke (2018) an agent-based modeling, in addition to theoretical reasoning, may be a valuable tool for economic policy analysis, and more specifically, agent-based models are an excellent tool for enhancing our understanding of regulatory policies, such as exploring the impact of simple intervention strategies, particularly if they are empirically supported. Secondly, according to Beine et al. (2009) the central bank interventions affect the relative profitability strategies of the fundamentalists and chartists and the exchange rate tending to his fundamental value in the medium-run. There is also a different approach to control chaos in exchange rates through central bank interventions with the use of neuronets, presented by Mourtas et al. (2022).

An endogenous law of motion, which it is not entirely exogenous are drive the exchange rate fluctuations and according to Wieland and Westerhoff (2005), the central bank may be able to reach the stabilization. More specifically, in their paper, they employ control chaos methods such as DFC, as well as the OGY method, which may be used to represent central bank actions caused by an uncertain participant environment.

There have been a lot of studies done about the uncertainty and the role of noise traders in the stock market, and according to Bloomfield et al. (2009), noise traders may choose to trade despite having no informational advantages or exogenous motives to trade. Furthermore, in their work (Peress and Schmidt, 2020), they emphasise the importance of noise traders being diverted from trading, liquidity and volatility decreasing, and prices reversing less. In order to avoid the problem of the prediction of a complex market player like noise traders we will use three different types of stochastic processes: Arithmetic Brownian Motion, Geometric Brownian Motion, and Ornstein-Uhlenbeck (Nelson, 2020; Calin, 2012; Choe, 2016; Hassler et al., 2016).

The ESI is based on the chartist-fundamentalist model described in Wieland and Westerhoff (2005), and it is introduced and examined in this study using the Targets Value Stochastic Algorithm (TVS-Algorithm), an improvement of Athanasiou and Kotsios (2008). More specifically, central banks regulate currency through participating in the market and affecting its value or purchasing power, notably in foreign exchange markets. Furthermore, we make the assumption that the FX market dynamics are equivalent to the stock market, so we prove the market efficiency hypothesis, and more specifically, we prove in an experimental environment that the semi-strong hypothesis (Fama, 1970) is satisfied with the use of three different stochastic processes, which represent external or internal shocks to an economy. In particular, our paper aims to incorporate noise traders into the foreign exchange market models by considering three different types of specification, specifically three different stochastic differential equations, and we propose an algorithm that central banks can use to design their intervention strategy.

2 The Model

There are a lot of models that describe the exchange rate dynamics, but we choose a chartist-fundamentalist approach which has been introduced by Wieland and Westerhoff (2005) and updated by Athanasiou and Kotsios (2008) and Brianzoni and Campisi (2020). The ESI model simulates the currency prices of the exchange rates instance at any time and determined by the next recurrent relation:

$$p_{t+1} = \begin{cases} Y, p_t > Y \\ p_t + c \left(\alpha(p_t) + \beta(p_t) + \gamma(p_t) + \delta(p_t) \right), y < p_t < Y, & t = 1, 2, 3, \dots \\ y, p_t < y \end{cases} \quad (1)$$

$$\alpha(p_t) = \frac{a * (X - p_t)}{(p_t - y + \epsilon)^{d_1} (Y + \epsilon - p_t)^{d_2}}, \quad \text{where } \geq 0, \quad (2)$$

$$\beta(p_t) = b * (p_t - x) \quad (3)$$

$$\gamma(p_t) = u_t \quad (4)$$

$$\delta(p_t) = w_t \quad (5)$$

where the constants $c, y, Y, a, X, d_1, d_2, \epsilon, b$ and x are non negative real numbers.

The quantities $p_t, u_t \in \mathbb{R}, t = 1, 2, 3, \dots$ are real sequences. Whilst, w_t is a random variable determined by the stochastic differential equation:

$$dw_t = f(w_t, \mu, \sigma)dt + g(w_t, \mu, \sigma)dB_t \quad (6)$$

with $\mu \geq 0, \sigma > 0$ and $B_t, t = 1, 2, 3, \dots$ a Brownian motion.

The economical and financial interpretation of the parameters and variables of the ESI model, is as follows:

- t is the time.
- p_t is the present value of exchange rate.
- y and Y are the predicted lower and upper bounds of the exchange rate.
- $\alpha(p_t)$ is the sophisticated traders.
- $\beta(p_t)$ is the technical traders.
- $\gamma(p_t)$ is the central bank intervention.
- $\delta(p_t)$ is the noise traders.
- a and b are metrics for sophisticated and technical traders demand strength.
- X is the fundamental value of sophisticated traders.
- x is the estimated investment value of technical traders, where according to (Wieland and Westerhoff, 2005) is equal with fundamental value.
- d_1, d_2 characterize the slope of (2).
- c is a constant efficiency factor in order to denote the rate at which prices adjust in response to new information.
- u_t is considered as the control sequences with values in \mathbb{R} . It represent the quantity of her currency that the central bank will employ in order to intervene in the exchange rate dynamics.

The ESI model simulates the next transaction procedure. If your currency rate is below (above) its perceived fundamental value X , sophisticated traders place purchase (sell) orders. When the exchange rate approaches its predicted lower bound y or upper bound Y , sophisticated traders enter the market more aggressively. The non-linearity of the above equation derives from the fact that as the spread between the exchange rate and its underlying value X increases, so does the sophisticated trader's reaction. The technical traders frequently enter the market when prices are high, expecting the market to rise and depart when prices are low, expecting the

market to fall. Assuming the semi-strong efficiency hypothesis, the exchange rate adjust rapidly to the release of all new public information for all the market participants. The central bank or institutional market maker u_t represents the intervention strategy that the central bank will employ in the dynamics of the exchange rate, which will lead to the stabilization of the national currency. More specifically, central bank buying or selling assets such as other currencies, sets interest rate on loans and bonds through regulating growth, consumer spending, inflation etc. and capital controls which they are a non-standard monetary policy.

The central bank intervention dynamics could absorb changes in excess demand in ESI model, decreasing the price when they must buy and rising it when they must sell.

Let us now combine the Eqs. (2), (3), (4) and (5) into (1), then, as a result we take the Eq. (7), which is the last type of ESI model:

$$p_{t+1} = \begin{cases} Y, p_t > Y \\ p_t + c \left(\frac{a * (X - p_t)}{(p_t - y + \epsilon)^{d_1} (Y + \epsilon - p_t)^{d_2}} + b * (p_t - x) + u_t + w_t \right), y < p_t < Y \\ y, p_t < y \end{cases} \tag{7}$$

Furthermore, we give to the stochastic differential Eq. (6), three different specific discrete forms, each one corresponding to a concrete economical behaviour. Specifically:

- The ABM (Arithmetic Brownian) with formula:

$$\Delta w_t = \mu \Delta t + \sigma \Delta B_t$$

This discrete equation is an appropriate description for an economic variable such as the exchange rate, which develops in a stable mood and it is characterized by increasing uncertainty, due to day-to-day transactions that take place in the global financial market and in which national currencies are traded.

- The GBM (Geometric Brownian), with formula:

$$\Delta w_t = \mu w \Delta t + \sigma \Delta w \Delta B_t$$

This discrete equation gives always positive values and independent of the process value(exchange rate). Furthermore, this equation matches reality, has paths that are as harsh as real exchange rates, and as a result, it is quite useful in analyzing models such as ESI.

- Finally the OU (Ornstein-Uhlenbeck) discrete equation, with formula:

$$\Delta w_t = \lambda w_t \Delta t + \sigma \Delta B_t$$

This discrete equation is frequently used, in order to forecast future exchange rate variations.

In all the above equations, the parameter μ indicates the equilibrium or average value supported by fundamentals, i.e. the fundamental principles contain the basic qualitative and quantitative information that contribute to economic success and, as

a result, the economic valuation of currencies. The parameter σ is the degree of variability and it is centered on what is induced by shocks, i.e. external factors, such as the demand and supply of the other tradable currencies.

3 The Problem

We want to determine a sequence u_t such that the expected value of the total error function,

$$\mathbb{E}[L] = \mathbb{E} \left[\sum_{t_1}^{t_2} (p_t - p^*)^2 \right] \rightarrow \text{minimum} \quad (8)$$

where p_t is determined by the Eq. (7). The exchange rate p_t is now a stochastic process, and we are attempting to determine which situations call for the central bank to intervene in order to incur the least feasible loss. More specifically, as we evaluate each individual loss function in order to get the anticipated value of the loss function. This loss means that the monetary policies that will follow will have a negative influence on the supply of the national currency as well as the definition of interest rates on loans and bonds. In other words, it is a central bank decision tool that, after identifying the target value of the exchange rate, computationally determines all the u_t that are close to the target value and chooses whether to perform the specific intervention. Due to the difficulties in forecasting the behavior of players in some complex economic systems, which may cause the system to collapse, this work introduces and investigates the noise traders and which are their reflection in an exchange rate dynamics model like ESI. All of the noises in the ESI model are stochastic noises, which were chosen because this form of noise in financial markets may create minor price changes, causing corrections to disturb the trend in both the short and long term. Athanasiou and Kotsios (2008) did the same with the absence of our proposed time - varying noise traders. More specific, we are going to use three different types of stochastic differential equations Arithmetic Brownian Motion (ABM), Geometric Brownian Motion (GBM) and Ornstein-Uhlenbeck (OU).

3.1 The Algorithm

In the Appendix 1 we present the algorithm we use for facing the problem, two theorems and their proofs for our algorithm. In the main algorithm, we execute h predefined Monte Carlo simulations while the two variables under inquiry, the state variable p_t and control variable u_t , change. In order to calculate the level of noise traders, we use the *NOISE* subroutine and we assume that the noise traders are informed at least for the past values of the exchange rate denoted by the variable p_g , with mean value equal to $\mathbb{E}[p_g]$ and the variance equal to $\mathbb{V}[p_g]$. The \mathcal{P} subroutine is then used to calculate the state variable, which is a formal expression of u_t , which changes when the state variable is constant, and to calculate all the u that minimize the central bank's loss function $L(u_t)$ and find the optimized solution that minimizes the

quantity. Last but not least, we replace the control variable solution in the \mathcal{P} subroutine in order to determine the state variable p_t .

4 Simulation Experiments

This section examines the TVS-Algorithm's performance in two stages. The ESI model is initially investigated in a single scenario, or in the absence of noise traders. The ESI model is examined in the second stage using three scenarios where the mean value and variance of the three stochastic differential equations are the same as the mean value and variance of the first 10 historical exchange rate values. The noise traders use technical analysis in the form of the "head and shoulders" chart pattern according to Bender et al. (2013) so we make the previous assumption about the mean value and the variance of this type of traders. More specifically, we assume that, in order to intervene, the central bank considers the ten random past exchange rate values $p_g \sim N(0, 1)$ and that the sequence in which she will act is equivalent to five days. We make the assumption of five days, because from 5 p.m. EST on Sunday to 4 p.m. EST on Friday, the market is open 24 hours a day in various regions of the world. At any one moment, at least one market is open, and there is a few hours of overlap between one region's market shutting and another region's market starting. Because currency trading is so global, there are constantly traders all around the world making and satisfying requests for a certain currency.

- $\mu_1 = \mathbb{E}[p_g]$, is a non negative number with $\mu_1 \geq 0$.
- $\mu_2 = \mathbb{E}[p_g]$, is a non negative number with $\mu_2 \geq 0$.
- $\mu_3 = \mathbb{E}[p_g]$, is a non negative number with $\mu_3 \geq 0$.
- $\sigma_1 = \mathbb{V}[p_g]$, is a non negative number with $\sigma_1 > 0$.
- $\sigma_2 = \mathbb{V}[p_g]$, is a non negative number with $\sigma_2 > 0$.
- $\sigma_3 = \mathbb{V}[p_g]$, is a non negative number with $\sigma_3 > 0$.

which they are different in every case because of the randomness of the 10 past values and these values take into consideration in charts from the noise traders and we assume as the initial value for exchange rate $p_0 = 0.6$.

In the following two subsections, we will present the difference results of exchange rate dynamics p_t , the central bank's interventions u_t , and the noise traders dynamics w_t after 1000 iterations in the non stochastic and the stochastic world in three different target values of exchange rate stabilization set by the central bank, where $p^* = 0.4$, $p^* = 0.6$ and $p^* = 0.8$.

4.1 The ESI Model in $p^* = 0.4$

In this section we present the results of ESI model with the use of TVS-Algorithm in the target value $p^* = 0.4$.

We obtain that in Fig. 1a, the exchange rate dynamics are unstable in the absence of noise traders, and in Fig. 1b, the central bank must intervene with $\mathbb{E}[u_t] = 0.036$, implying that the central bank will incur losses and will be unable to stabilize the exchange rate dynamics at the target value and the national currency is going to appreciate because $\mathbb{E}[p_t] > p^*$. Furthermore, we obtain that in Fig. 1c there is no relationship between exchange rate and intervention dynamics.

We obtain that in Fig. 2a the exchange rate dynamics drive to stabilize in the target value setted by the central bank for $t = 65$ with the presence of noise traders denoted as an ABM, for $t = 92$ with the presence of noise traders denoted as a GBM and for $t = 100$ with the presence of noise traders denoted as an OU. Furthermore, in Fig. 2b when the noise traders denoted by ABM and OU the central bank must intervene with $\mathbb{E}[u_t] = -0.4718$ and $\mathbb{E}[u_t] = -0.4092$ respectively, implying that the central bank will minimize the losses, but the national currency will be appreciated because $\mathbb{E}[p_t] > p^*$. In the case that the noise traders denoted as GBM the central bank must intervene with $\mathbb{E}[u_t] = -0.1995$, implying that the central bank will minimize the losses and the national currency is going to depreciated because $\mathbb{E}[p_t] < p^*$. In addition, we obtain that in Fig. 2c there is relationship between exchange rate and intervention dynamics. More specifically, in the case of ABM the central bank may easily stabilize the behavior of noise traders by selling a small amount of the national currency; in the case of GBM the central bank might straggle to stabilize the behavior of noise traders with aggressive strategic decisions selling quantities of the national currency and in case of OU selling quantities of the national currency in order to stabilize the exchange rate dynamics in the long run.

The noise traders dynamics presented in Fig. 2d where in the case of ABM are evolving with $\mathbb{E}[p_g] = 0.2505$, $\mathbb{V}[p_g] = 0.0479$, and are stable in their market participation for $t = 65$, in the case of GBM are evolving with $\mathbb{E}[p_g] = 0.2232$ and $\mathbb{V}[p_g] = 0.0362$ and tend to remain steady in their market involvement for $t = 92$ and in the case of OU are evolving with $\mathbb{E}[p_g] = 0.5684$, $\mathbb{V}[p_g] = 0.1344$ and are with minimal amount of volatility in their market participation for $t = 14$.

4.2 The ESI Model in $p^* = 0.6$

In this section we present the results of ESI model with the use of TVS-Algorithm in the target value $p^* = 0.6$.

We obtain that in Fig. 3a, the exchange rate dynamics are unstable in the absence of noise traders, and in Fig. 3b, the central bank must intervene with $\mathbb{E}[u_t] = -0.033$, implying that the central bank will incur losses and will be unable to stabilize the exchange rate dynamics at the target value and the national currency is going to depreciated because $\mathbb{E}[p_t] < p^*$. Furthermore, we obtain that in Fig. 3c there is no relationship between exchange rate and intervention dynamics.

We obtain that in Fig. 4a, the exchange rate dynamics tend to stabilize in the target value setted by the central bank for $t = 72$ with the presence of noise traders denoted as an ABM, for $t = 95$ with the presence of noise traders denoted as a GBM and for $t = 93$ with the presence of noise traders denoted as an OU. Furthermore, in Fig. 4b when the noise traders denoted as ABM the central bank must intervene with

$\mathbb{E}[u_t] = -0.575$, implying that the central bank will have losses, but the national currency will not depreciate, but it will not be appreciated either because $\mathbb{E}[p_t] = p^*$. In the case that the noise traders denoted as GBM the central bank must intervene with $\mathbb{E}[u_t] = -0.84$, implying that the national currency is going to appreciate because $\mathbb{E}[p_t] > p^*$. Also, in the case that the noise traders denoted as OU the central bank must intervene with $\mathbb{E}[u_t] = -0.5751$, implying that the central bank will minimize the losses and the national currency is going to depreciate because $\mathbb{E}[p_t] < p^*$. In addition, we obtain that in Fig. 4c there is relationship between exchange rate and intervention dynamics. More specifically, in the cases of ABM, and OU the central bank making aggressive strategic decisions to sell large amounts of the domestic currency in order to control the dynamics of exchange rates over the long term.

The noise traders presented in Fig. 4d, where in the case of ABM are evolving with $\mathbb{E}[p_g] = 0.8262$, $\mathbb{V}[p_g] = 0.0314$) and are stable in their market participation for $t = 72$, in the case of GBM are evolving with $\mathbb{E}[p_g] = 0.7536$, $\mathbb{V}[p_g] = 0.0317$ and tend to be stable in their market participation for $t = 95$ and in the case of OU are evolving with $\mathbb{E}[p_g] = 0.7536$, $\mathbb{V}[p_g] = 0.0317$ and are with minimal amount of volatility in their market participation for $t = 11$.

4.2.1 ESI Model Results in $p^* = 0.8$

In this section we present the results of ESI model with the use of TVS-Algorithm in the target value $p^* = 0.8$.

We obtain that in Fig. 5a, the exchange rate dynamics are unstable in the absence of noise traders, and in Fig. 5b, the central bank must intervene with $\mathbb{E}[u_t] = 0.025$, implying that the central bank will incur losses and will be unable to stabilize the exchange rate dynamics at the target value and the national currency is going to depreciate because $\mathbb{E}[p_t] < p^*$. Furthermore, we obtain that in Fig. 5c there is no relationship between exchange rate and intervention dynamics.

We obtain that in Fig. 6a, the exchange rate dynamics tend to stabilize in the target value setted by the central bank for $t = 75$ with the presence of noise traders denoted as an ABM, for $t = 92$ with the presence of noise traders denoted as a GBM and for $t = 67$ with the presence of noise traders denoted as an OU. Furthermore, in Fig. 6b when the noise traders denoted as ABM the central bank must intervene with $\mathbb{E}[u_t] = 0.049$, when the noise traders denoted as GBM the central bank must intervene with $\mathbb{E}[u_t] = -0.32$ and when the noise traders denoted as OU the central bank must intervene with $\mathbb{E}[u_t] = -0.454$, implying that the central bank will minimize the losses and the national currency is going to depreciate because $\mathbb{E}[p_t] < p^*$ respectively. In addition, we obtain that in Fig. 6c there is relationship between exchange rate and intervention dynamics. More specifically, in the case of ABM the central bank may easily stabilize the exchange rate dynamics with infinitesimally modest strategic decisions by buying quantities of the national currency; and in the cases of GBM and OU the central bank making aggressive strategic decisions to sell large amounts of national currency in order to stabilize the exchange rate dynamics in the long term.

The noise traders dynamics presented in Fig. 6d, where in the case of ABM are evolving with $\mathbb{E}[p_g] = 0.7963$, $\mathbb{V}[p_g] = 0.0271$) and are stable in their market participation for $t = 67$, in the case of GBM are evolving with $\mathbb{E}[p_g] = 0.8146$, $\mathbb{V}[p_g] = 0.0161$ and tend to be stable in their market participation for $t = 87$ and in case of OU are evolving with $\mathbb{E}[p_g] = 0.8128$, $\mathbb{V}[p_g] = 0.0019$ and are with minimal amount of volatility in their market participation for $t = 11$.

Thus, we understand that when the choice of the target value by central bank's is very important for the intervention policy she is going to follow and if the national currency is going to be appreciated, depreciated or stabilized,

- $p^* < \mathbb{E}[p_t]$, where $p^* \in R_0^+$ results in the appreciation of national currency
- $p^* = \mathbb{E}[p_t]$, where $p^* \in R_0^+$ results in the stabilization of national currency
- $p^* > \mathbb{E}[p_t]$, where $p^* \in R_0^+$ results in the depreciation of national currency

According to the data in Table 2, there is a negative correlation between bank interventionist policies and currency appreciation or depreciation. With the aid of shocks that the noise traders input into ESI, we study the central bank's role in maintaining the monetary policy of the country's domestic currency within the complicated framework of the global economy.

5 Conclusion

This paper introduces a novel stochastic exchange rate model, called ESI, along with a Target Value method for stabilizing the uncertainty behavior of ESI through central bank interventions. We consider the fourth market participant, the noise traders, to be a significant factor and we offer three stochastic differential equations to represent their trading behavior. More specifically, we use these three type of stochastic differential equations because the ABM process is characterized by increasing uncertainty due to day-to-day transactions that take place in the FX, the GBM process is always positive, and its predicted returns are independent of the process value (exchange rate), which is compatible with reality and we use OU process because it is a diffusive process, which means it can be used to simulate financial market characteristics such as FX, specifically in order to forecast future exchange rate variations.

We notice that as the stabilization process begins, all of the intervention dynamics figures influenced by ABM, GBM, and OU processes adopt negative values. According to Wieland and Westerhoff (2005), they only evaluate sterilized and secret interventions. Sterilization occurs when central banks balance the acquisition of foreign currencies or securities by selling domestic ones, hence reducing its own money supply. Central banks employ sterilization to insulate or shield their economies from the negative effects of things like currency appreciation or inflation, both of which can impair a country's export competitiveness in the global market. Central banks primary purpose is to keep exchange rate swings under control. Making international commercial and investment decisions becomes significantly more complicated when the currency rate moves fast, up or down. In this event, traders and

Table 1 Parameters values on simulation scenarios

Simulation Scenario	Parameters										
	y	Y	e	d1	d2	c	a	b	w	μ	σ
Sn1	0	1	0.01	0.5	0.5	1	0.2	0.88	0	0	0
Sn2	0	1	0.01	0.5	0.5	1	0.2	0.88	ABM	μ_1	σ_1
Sn3	0	1	0.01	0.5	0.5	1	0.2	0.88	GBM	μ_2	σ_2
Sn4	0	1	0.01	0.5	0.5	1	0.2	0.88	OU	μ_3	σ_3

where,

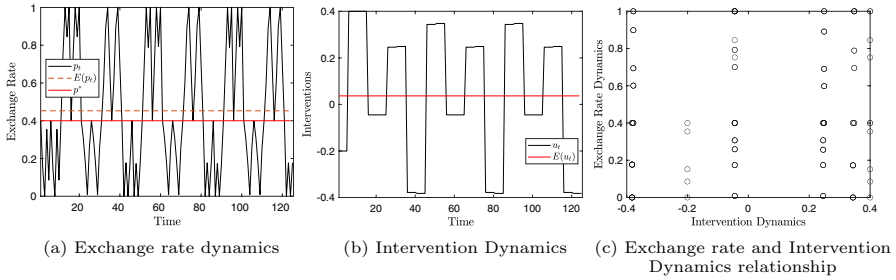


Fig. 1 ESI model results without noise traders

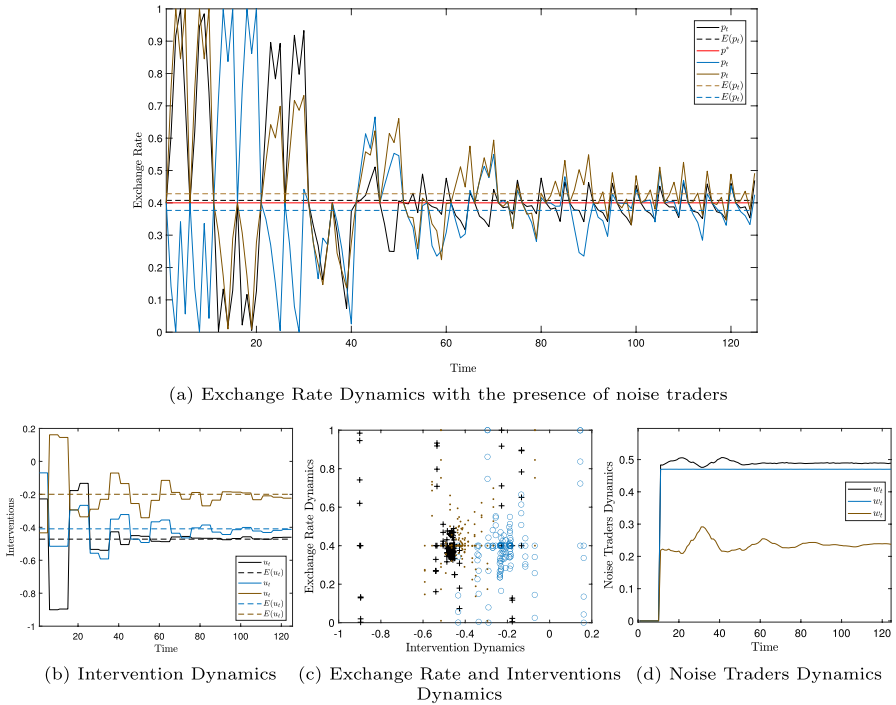


Fig. 2 Exchange rate dynamics affected by ABM (black color), GBM (blue color) and OU (brown color)

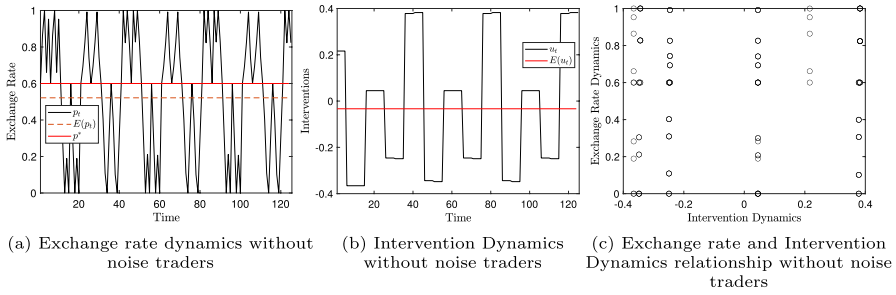


Fig. 3 ESI model results without noise traders

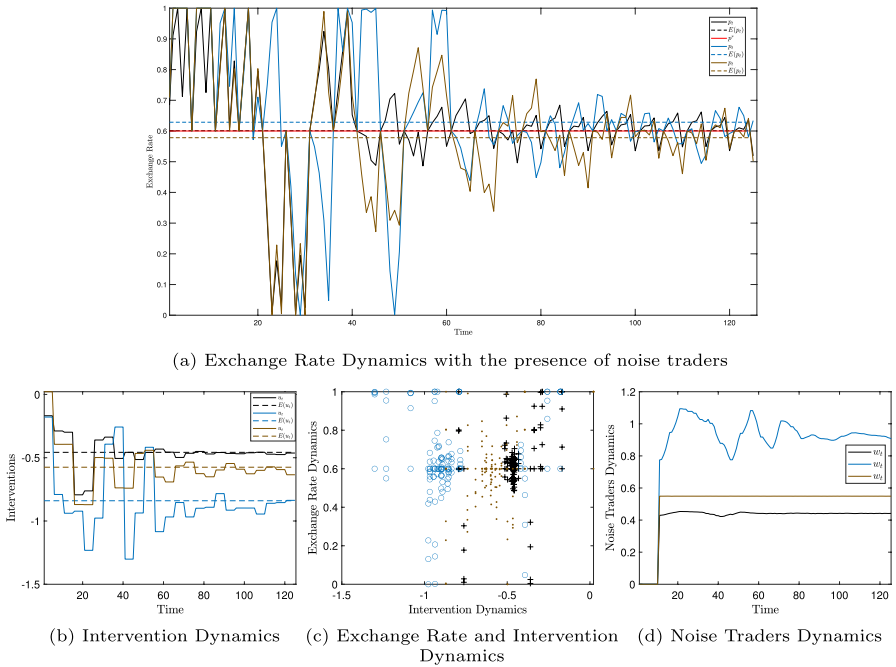


Fig. 4 Exchange rate dynamics affected by ABM (black color), GBM (blue color) and OU (brown color)

investors will be less confident in the success of their trades and investments, and their international activity would likely be limited. As a result, international merchants and investors prefer more stable currency rates, and governments and central banks are regularly pressed to interfere in the foreign exchange market when the rate varies excessively. The second motive is to reduce the country’s growing trade imbalance. Trade imbalances can swiftly increase if a country’s currency gains substantially. Foreign products and services will look to foreigners to be cheaper, increasing imports, while local goods would appear to foreigners to be more costly, reducing exports. As a result, a rising currency value might lead to an increase in

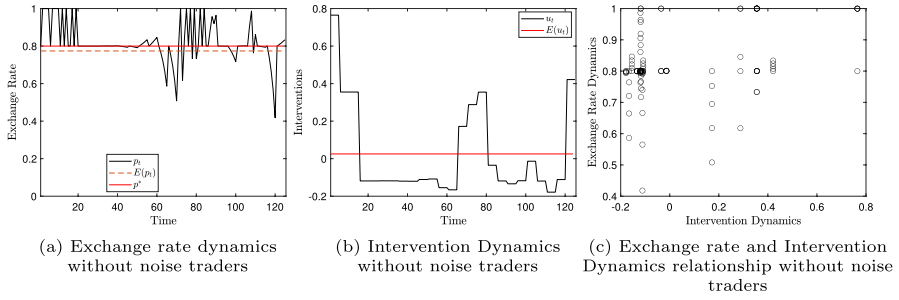


Fig. 5 ESI model results without noise traders

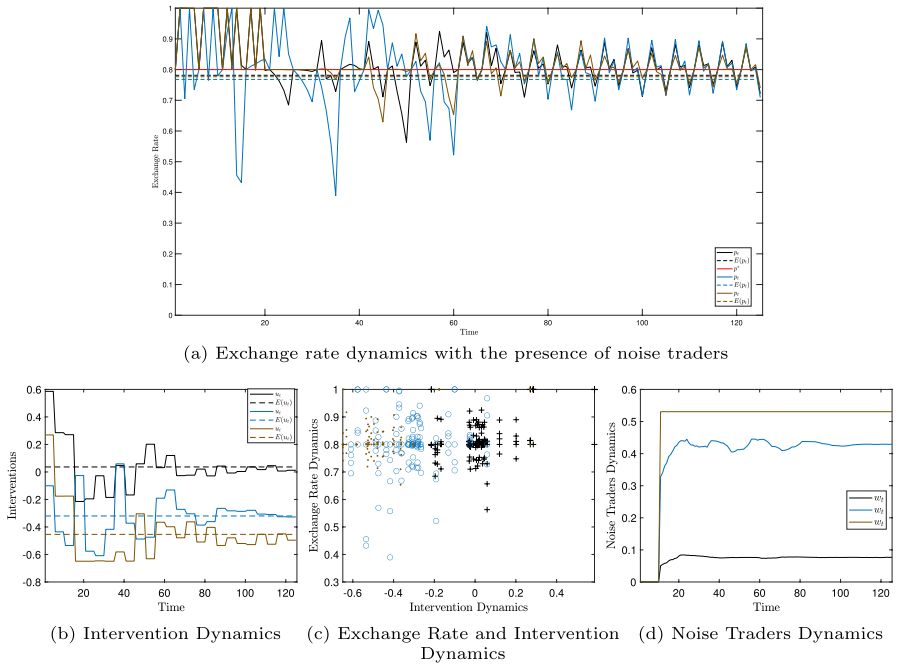


Fig. 6 Exchange rate dynamics affected by ABM (black color), GBM (blue color) and OU (brown color)

the trade imbalance. If the central bank is under pressure to interfere in the foreign exchange market in order to lower the value of the currency and so reverse the growing trade imbalance, it may be obliged to do so.

Furthermore, the constant efficiency factor c in ESI is equivalent to 1 in Table 1, implying that exchange rate prices represent all information and that exchange rates trade at their fair market value. In particular, we can demonstrate that in a non-stochastic environment, the market cannot be efficient due to the lack of noise traders. The market efficiency hypothesis states that the more individuals who participate in a market, the more efficient it becomes as more people compete and bring more

Table 2 Simulation Results

Simulation scenarios	p^*	$\mathbb{E}[u_t]$	$\mathbb{E}[p_t]$	$p^* - \mathbb{E}[p_t]$	National currency
Sn1	0.4	0.0553	0.452	-0.052	<i>Depreciation</i>
	0.6	-0.0379	0.521	0.079	Appreciation
	0.8	0.0214	0.773	0.027	Appreciation
Sn2	0.4	-0.4718	0.4076	-0.0076	<i>Depreciation</i>
	0.6	-0.4575	0.6	0	Stabilization
	0.8	0.0363	0.7814	0.0186	<i>Depreciation</i>
Sn3	0.4	-0.1995	0.3766	0.0234	Appreciation
	0.6	-0.84	0.6285	-0.0285	<i>Depreciation</i>
	0.8	-0.32	0.768	0.032	Appreciation
Sn4	0.4	-0.4092	0.4281	-0.0281	<i>Depreciation</i>
	0.6	-0.5751	0.5781	0.0219	Appreciation
	0.8	-0.4538	0.7768	0.0232	Appreciation

Bold and italics is to highlight the different results

and varied sorts of information to bear on the price. Arbitrageurs will develop when markets grow more active and liquid, benefiting on little inefficiencies that exist and restoring efficiency rapidly. As a result, the central banks major purpose is to maintain market efficiency and stable the exchange rate in order to attract more investors for the currency's benefit.

We prove that adding noise traders to ESI makes the model more realistic and that the noise is depreciated by the market over time, as well as proving the semi-strong market efficiency hypothesis.

Some directions for further research can be identified.

- Evolving stochastic processes in every market participant except the noise traders.
- Brownian Motion, according to Einstein (1905), would gain popularity as a beneficial theory for predicting naturally-occurring random occurrences and developing them into the financial market by the position of the investors decision making.
- A stochastic target zone approach can be used to improve this work (Athanasiou & Kotsios, 2008).
- We assume that there is no extreme shock, such as COVID-19, in our model, and we would like to see further research on this topic in the future.

Appendix A: The Algorithm

Since our algorithmic method is based on certain subroutines we present them first, shortly. The first two are common in literature henceforth we have not described them explicitly.

1. *RANDOM*($N(0, 1)$)

Description: This subroutine generates a random number following the standard normal distribution.

2. *FINDMIN*(F, x)

Description: This subroutine minimizes the function F with respect to the variable x . It returns the value of x^* of x , where the minimum is achieved and in our case we use a subroutine, which is common at all the software systems for instance in MATLAB is called "fminsearch".

3. *ABM*($\mu, \sigma, \Delta t, w$)

Description: This subroutine returns the value w_{next} such the quantity $w_{next} - w$ satisfies the relation $w_{next} - w = \mu\Delta t + \sigma\Delta B$, in other words the set of the quantities $w_{next} - w$ is an Arithmetic Brownian motion.

Inputs: $\mu, \sigma, \Delta t, w$
 $U = RANDOM(N(0, 1))$
 $w_{next} = w + \mu\Delta t + \sigma\sqrt{\Delta t}U$
 Output: w_{next}

4. *GBM*($\mu, \sigma, \Delta t, w$)

Description: This subroutine returns a value w_{next} so that the quantity $w_{next} - w$ satisfies the relation $w_{next} - w = \mu w\Delta t + \sigma w\Delta B$, in other words the set of the quantities $w_{next} - w$ is a Geometric Brownian motion.

Inputs: $\mu, \sigma, \Delta t, w$
 $U = RANDOM(N(0, 1))$
 $w_{next} = w + \mu w\Delta t + \sigma w\sqrt{\Delta t}U$
 Output: w_{next}

5. *OU*($\mu, \sigma, \Delta t, \lambda, w$)

Description: This subroutine it returns a value w_{next} so that the quantity $w_{next} - w$ satisfies the relation $w_{next} - w = -\lambda\Delta t + \sigma\Delta B$, in other words the set of the quantities $w_{next} - w$ is an Ornstein-Uhlenbeck process with the time step equal to Δt , the parameter λ is the long-term mean of the process.

Inputs: $\mu, \sigma, \Delta t, \lambda, w$
 $U = RANDOM(N(0, 1))$
 $w_{next} = w - \lambda w\Delta t + \sigma w\sqrt{\Delta t}U$
 Output: w_{next}

6. *NOISE*($\mu, \sigma, \lambda, \Delta t, w, \tau$)

Description: It combines the three preceding usefulness subroutines and utilizes the variable τ to employ the specific stochastic differential equation upon usefulness. In other words if $\tau = 0$ there are no noise traders in the market, $\tau = 1$ it returns the ABM, if $\tau = 2$ it returns the GBM and if $\tau = 3$ it returns the OU. More specifically:

Inputs: $\mu, \sigma, \lambda, \Delta t, w, \tau$
 $U = RANDOM(N(0, 1))$
 $IF[\tau = 0, w_{next} = 0]$
 $IF[\tau = 1, w_{next} = w + \mu\Delta t + \sigma\sqrt{\Delta t}U]$
 $IF[\tau = 2, w_{next} = w + \mu w\Delta t + \sigma w\sqrt{\Delta t}U]$
 $IF[\tau = 3, w_{next} = w - \lambda w\Delta t + \sigma w\sqrt{\Delta t}U]$
 Output: w_{next}

7. *P*(p, u, w)

Description: This subroutine returns a value p_{next} , with respect to the u we have, where w is a random value and u is either a symbolic variable or a constant number.

Inputs: p, u, w

$$p_{next} = \begin{cases} Y & , p > Y \\ p + c \left(\frac{a * (X - p)}{(p - y + \epsilon)^{d_1} (Y + \epsilon - p)^{d_2}} + b * (p - x) + u + w \right) & , y < p < Y \\ y & , p < y \end{cases}$$

Output: p_{next}

We are now ready to present the main algorithm. The next variables are considered as predetermined and known the algorithm. Therefore they must not be given as inputs.

Variables: $c, \alpha, X, y, \epsilon, d_1, Y, d_2, \beta, \Delta t, \tau, \lambda, p_0, w_0$ and x .

Target Value Stochastic Algorithm (TVS-Algorithm)

INPUTS:

- h is a predetermined amount of times that the experiment is repeated, since we are running Monte Carlo simulations
- n is the end of time
- m are the days that the central bank intervene
- g is the amount of past values of the state variable
- j is the number of times that the control variable u change, when the state variable p is constant
- p^* is the target value setted by the central bank
- p_0 is the initial value of the state variable p_t
- w_0 is the starting point of noise traders

FOR $t = 1$ TO n

FOR $i = 1$ TO h

FOR $r = 1$ TO g

$$p_r = \text{RANDOM}(N(0, 1))$$

NEXT r

FOR $j = (t - 1) * m + 1 + g$ TO $tm + g$

$$p_g = \sum_{r=j-g}^{j-1} p_r$$

$$\mu = \mathbb{E}[p_g]$$

$$\sigma = \mathbb{V}[p_g]$$

IF $\tau = 0$ OR $\tau = 0$ OR $\tau = 2$ OR $\tau = 3$ THEN

$$w_t = \text{NOISE}[\mu, \sigma, \lambda, \Delta t, w_{t-1}, \tau]$$

END

$p_t = \mathcal{P}[p_{t-1}, u_t, w_t]$, where this is a formal expression of u only.

$$L(u) = \sum_{\rho=(t-1)*m+1+g}^{tm+g} (p_\rho - p^*)^2$$

$$u_{t,i} = \text{FINDMIN}[L(u), u]$$

$$p_{t,i} = \mathcal{P}[p_{t-1}, u_{t,i}, w_t]$$

NEXT j

NEXT i

NEXT t

OUTPUTS: $p_t = \mathbb{E}_i[p_{t,i}]$ and $u_t = \mathbb{E}_i[u_{t,i}]$, where $t = 1, \dots, n$.

Theorem 1 *The TVS-Algorithm terminates*

Proof This comes straightforward from the fact that, the times that the state variable and the control variables change are finite, with starting time 1 and stopping time n , where $n > 1$.

Theorem 2 *The output u_t of the TVS-Algorithm minimizes the loss function in Eq. (8).*

Proof Since the loss function in the TVS-Algorithm is calculated between the time instants $t_1 = (t - 1) * m + 1 + g$ and $t_2 = tm + g$, where $t = 1, \dots, n$ and the variables m and g are constant variables, respectively, we will work with the loss function provided only by the formula,

$$L = \sum_{t=t_1}^{t_2} (p_t - p^*)^2 \tag{A1}$$

We shall prove that if p_t is given by the formula Eq. (7) and $u_t = \mathbb{E}_i[u_{t,i}]$ is the output of the TVS-Algorithm, then u_t minimizes the mean value in Eq. (8). Substituting Eq. (7) into Eq. (A1) we get:

$$L = \sum_{t=t_1}^{t_2} \left(f(p_{t-1}) + u + w_t - p^* \right)^2 \tag{A2}$$

where by $f(p_{t-1})$ we denote the quantity: $c(\alpha(p_t) + \beta(p_t))$. Hence, in order to minimize Eq. (A2) with respect to the variable u , we take the first order condition:

$$\frac{\partial L}{\partial u} = 0 \Rightarrow 2 \sum_{t=t_1}^{t_2} \left(f(p_{t-1}) + u + w_t - p^* \right) = 0 \Rightarrow u = \frac{-\sum_{t=t_1}^{t_2} f(p_{t-1}) - \sum_{t=t_1}^{t_2} w_t + p^*k}{k}$$

where $k = t_2 - t_1 + 1$, measures the total additions. This value corresponds to one iteration of the TVS-Algorithm, in other words to one value of i . Repeating the procedure for another value of i we will take another value of u due to the presence of the random variable w_t . Taking, eventually the mean value of all these values we will get:

$$\mathbb{E}[u_i] = \frac{\sum_{i=1}^h u_i}{h} \Rightarrow \frac{\sum_{i=1}^h u_i}{h} = \frac{1}{hk} \left(\sum_i p^*k - \sum_i \sum_{t=t_1}^{t_2} w_i - \sum_i \sum_{t=t_1}^{t_2} f(p_{t-1}) \right) \tag{A3}$$

Now, the loss function in Eq. (8) is re-written as:

$$\mathbb{E}[L] = \frac{1}{h} \sum_i \left(\sum_{t=t_1}^{t_2} \left(f(p_{t-1}) + u + w_i - p^* \right) \right)^2 \tag{A4}$$

where the number of iterations has been taken into consideration. By differentiating Eq. (A4) with respect to u , we get:

$$\frac{\partial \mathbb{E}[L]}{\partial u} = 0 \Rightarrow \frac{1}{h} \left(\sum_i \sum_{t=t_1}^{t_2} f(p_{t-1}) + hku + \sum_i \sum_{t=t_1}^{t_2} w_i - \sum_i (p^*k) \right) = 0 \quad (\text{A5})$$

Solving with respect to u we have:

$$u = \frac{1}{hk} \left(\sum_i p^*k - \sum_i \sum_{t=t_1}^{t_2} w_i - \sum_i \sum_{t=t_1}^{t_2} f(p_{t-1}) \right) \quad (\text{A6})$$

which is identical equal to (A3). The theorem has been proved. \square

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Declarations

Conflict of interest No conflict of interest exists in the submission of this manuscript.

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References

- Athanasίου, G., & Kotsios, S. (2008). An algorithmic approach to exchange rate stabilization. *Economic Modelling*, 25(6), 1246–1260.
- Bashir, U., Zebende, G. F., Yu, Y., Hussain, M., Ali, A., & Abbas, G. (2019). Differential market reactions to pre and post brexit referendum. *Physica A: Statistical Mechanics and its Applications*, 515, 151–158.
- Beine, M., De Grauwe, P., & Grimaldi, M. (2009). The impact of fx central bank intervention in a noise trading framework. *Journal of Banking & Finance*, 33(7), 1187–1195.
- Bender, J. C., Osler, C. L., & Simon, D. (2013). Noise trading and illusory correlations in us equity markets. *Review of Finance*, 17(2), 625–652.
- Bloomfield, R., O'hara, M., & Saar, G. (2009). How noise trading affects markets: An experimental analysis. *The Review of Financial Studies*, 22(6), 2275–2302.
- Brianzoni, S., & Campisi, G. (2020). Dynamical analysis of a financial market with fundamentalists, chartists, and imitators. *Chaos, Solitons & Fractals*, 130, 109434.
- Calin, O. (2012). An introduction to stochastic calculus with applications to finance. *Ann Arbor*.
- Choe, G. H. (2016). *Stochastic processes*. Springer.
- Day, R. H., & Huang, W. (1990). Bulls, bears and market sheep. *Journal of Economic Behavior & Organization*, 14(3), 299–329.

- Einstein, A. (1905). Über die von der molekularkinetischen theorie der wärme geforderte bewegung von in ruhenden flüssigkeiten suspendierten teilchen. *Annalen der Physik*, 4.
- Fama, E. F. (1970). Efficient capital markets: A review of theory and empirical work. *The Journal of Finance*, 25(2), 383–417.
- Feng, G.-F., Yang, H.-C., Gong, Q., & Chang, C.-P. (2021). What is the exchange rate volatility response to covid-19 and government interventions? *Economic Analysis and Policy*, 69, 705–719.
- Franke, R., & Westerhoff, F. (2016). Why a simple herding model may generate the stylized facts of daily returns: Explanation and estimation. *Journal of Economic Interaction and Coordination*, 11(1), 1–34.
- Fratzsch, M., Gloede, O., Menkhoff, L., Sarno, L., & Stöhr, T. (2019). When is foreign exchange intervention effective? Evidence from 33 countries. *American Economic Journal: Macroeconomics*, 11(1), 132–156.
- Hassler, U. et al. (2016). *Stochastic processes and calculus*. Springer.
- Hung, J. H. (1997). Intervention strategies and exchange rate volatility: A noise trading perspective. *Journal of International Money and Finance*, 16(5), 779–793.
- Jawad, M. (2019). Pre and post effects of brexit polling on united kingdom economy: An econometrics analysis of transactional change. *Quality & Quantity*, 53(1), 247–267.
- Mourtas, S. D., Katsikis, V. N., Drakonakis, E., Kotsios, S. (2022). Stabilization of stochastic exchange rate dynamics under central bank intervention using neuronets. *International Journal of Information Technology & Decision Making*, 1–29.
- Neely, C. J. (2000). *The practice of central bank intervention: Looking under the hood*. FRB of St. Louis Working Paper No.
- Nelson, E. (2020). *Dynamical theories of Brownian motion*. Princeton university press
- Nilavongse, R., Michał, R., & Uddin, G. S. (2020). Economic policy uncertainty shocks, economic activity, and exchange rate adjustments. *Economics Letters*, 186, 108765.
- Peress, J., & Schmidt, D. (2020). Glued to the tv: Distracted noise traders and stock market liquidity. *The Journal of Finance*, 75(2), 1083–1133.
- Reitz, S., Westerhoff, F., & Wieland, C. (2006). Target zone interventions and coordination of expectations. *Journal of Optimization Theory and Applications*, 128, 453–467.
- Westerhoff, F., Franke, R. (2018). Agent-based models for economic policy design: Two illustrative examples.
- Wieland, C., & Westerhoff, F. H. (2005). Exchange rate dynamics, central bank interventions and chaos control methods. *Journal of Economic Behavior & Organization*, 58(1), 117–132.