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Comparison of Value at Risk (VaR) Multivariate Forecast Models

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Abstract

We investigate the performance of VaR (Value at Risk) forecasts, considering different multivariate models: HS (Historical Simulation), DCC-GARCH (Dynamic Conditional Correlation-Generalized Autoregressive Conditional Heteroskedasticity) with normal and Student's *t* distribution, GO-GARCH (Generalized Orthogonal-Generalized Autoregressive Conditional Heteroskedasticity), and copulas Vine (C-Vine, D-Vine, and R-Vine). For copula models, we consider that marginal distribution follow normal, Student's *t* and skewed Student's *t* distribution. We assessed the performance of the models using stocks belonging to the Ibovespa index during the period from January 2012 to April 2022. We build portfolios with 6 and 12 stocks considering two strategies to form the portfolio weights. We use a rolling estimation window of 500 and 1000 observations and 1%, 2.5%, and 5% as significance levels for the risk estimation. To evaluate the quality of the risk forecasts, we compute the realized loss and cost. Our results show that the performance of the models is sensitive to the use of different significance levels, rolling windows, and strategies to determine portfolio weights. Furthermore, we find that the model that presents the best trade-off between the costs from risk overestimation and underestimation does not coincide with the model suggested by the realized loss.

Keywords Risk forecasting · Value at Risk (VaR) · Copulas · Multivariate GARCH models

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1 Introduction

The increase in financial collapses and periods of instability have increased attention to the reappraisal of the models used to forecast risk. A risk overestimation generates an opportunity cost, as an unnecessary portion of capital is allocated to investment security. On the other hand, risk underestimation can lead to irreversible losses, as insufficient capital is destined to fulfill the safety purpose.

Value at Risk (VaR) is the standard measure used to forecast market risk in the financial industry. VaR computes the maximum loss that is expected for a given period and significance level. For more details regarding on VaR, we suggest Duffie and Pan (1997) and Jorion (2000). Although VaR has been historically criticized for not being a coherent risk measure[1], recently the literature has paid attention to its statistical properties. Among these properties, we refer to robustness. According to Kou and Peng (2016), a risk measure is robust if it is stable to small deviations in the model and theoretical distribution of the data. Robust risk measures, used for regulatory purposes, allow reducing the problem of regulatory arbitrage (Kou et al., 2013).[2] Another interesting statistical property that VaR satisfies is the elicitability. A risk measure is elicitable when it minimizes the expected value of a score function (Ziegel, 2016; Acerbi & Szekely, 2017). This property is interesting in risk management because it allows determining, among competing models, the model that generates the best forecast.

The most common VaR estimation approach is the non-parametric one, known as Historical Simulation (HS). Pérignon and Smith (2010) verify that approximately 73% of banking institutions that disclose the value of their VaR use this procedure. Other frequently applied methods are the parametric, such as the models of the GARCH family (Generalized Autoregressive Conditional Heteroskedasticity), and the semiparametric, including the Filtered Historical Simulation. However, the VaR results are conditioned to the model and specification. Two models or a model with two different specifications generate different values and consequently compromise decision-making. The uncertainty regarding model choice and specification leads to a risk referred to in the literature as model risk. Although there is no unanimous definition for this risk, regulatory agencies define it as the uncertainty present in the estimation process and the choice of the estimation model (Reserve, 2011).

In order to minimize the impacts of model risk, many studies are carried out comparing different VaR forecasting models. Research developed in this regard can be found in Kuester et al. (2006); Marinelli et al. (2007); Weiß (2013); Telmoudi

[1] A coherent risk measure fulfills Monotonicity, Translation Invariance, Subadditity, and Positive Homogeneity (Artzner et al., 1999). VaR does not respect the Subadditity axiom. By this axiom, the risk of a combined position (portfolio) is less than the individual sum of the risks of the stocks that make up the portfolio. Another criticism of VaR is that it ignores the size and the probabilistic distribution of the losses beyond the α-quantile of interest.

[2] For two or more institutions that maintain the same portfolio, it is required to maintain the same or approximately the same amount of regulatory capital. However, even if institutions employ the same risk measure, they will use different models, resulting in different amounts of capital requirement. This problem can be reduced as long as the risk measures are robust.

et al. (2016); Patton et al. (2019), and Müller et al. (2022). In most of these studies, the main focus has been using empirical data and univariate models. However, in a practical sense, the models used for risk management are multivariate. Furthermore, studies such as Weiß (2013); Righi and Ceretta (2015), among others, use backtesting tools to compare different estimation procedures. This procedure performs the validation of a given estimation procedure for a risk measure using historical data (Ziegel, 2016). For VaR, backtesting is generally used to compare predicted losses, that is, VaR forecasts, against actual losses, for a given time horizon. See Kupiec (1995) and Christoffersen (1998) for examples of backtesting for VaR. However, these tests do not allow direct comparison and ranking of the performance of competing risk forecasting procedures (Gneiting, 2011; Ziegel, 2016). For elicitable measures (including VaR), a suitable procedure for comparing and verifying current forecasting models is to use the scoring rule (Gneiting, 2011; Ziegel, 2016; Acerbi & Szekely, 2017).

In this sense, this work aims to analyze the performance of multivariate models to predict VaR. The estimation procedures considered are HS, copulas (C-Vine, D-Vine, and R-Vine),[3] and multivariate models from the GARCH family, which including GO (Generalized Orthogonal)-GARCH e DCC (Dynamic Conditional Correlation)-GARCH. We consider these models because they are the main estimation methods considered in risk management studies (Pérignon & Smith, 2010; Müller & Righi, 2018; Silahli et al., 2019; Nagler et al., 2019). To assess the quality of the predictions, we consider descriptive statistics, such as mean and standard deviation, realized loss function (elicitable loss function), and a measure of model risk (Müller & Righi, 2020), which allows us to be quantified realized cost. The data set used in the study refers to the main stocks belonging to the Bovespa Index (Brazilian market index) from January 2012 to April 2022, making 2,559 daily observations for each stock. In the analysis, we use two rolling estimation windows and the main significance levels considered in the literature.

This research contributes to the literature investigating what model results in more reliable risk predictions. Previous studies mainly compare univariate risk prediction models. Besides that, the researchers that evaluate multivariate models focus mainly on the bivariate case. An example is the study of Weiß (2013), which investigates multivariate GARCH models and bivariate copulas to predict VaR and ES (Expected Shortfall). Müller and Righi (2018) consider a set of models similar to the one used in our study. However, the authors focus on a numerical assessment. The limitations of numerical analysis are that simulated returns do not have stylized facts so close to real stock returns. Additionally, our study contributes to the studies carried out to identify the model with the lowest model risk to predict VaR. Estimation methods with lower model risk are essential for the financial system's stability. In addition to financial losses, this risk can compromise strategic decision-making and damage the financial institution's reputation (Reserve, 2011).

[3] We consider Vine copulas because they perform well compared to other types of multivariate copulas (Aas et al., 2009).

Regarding structure, the remainder of this paper divides into the following contents: in Sect. 2, we expose the multivariate models, which we use to forecast VaR; in Sect. 3, we exhibit the empirical procedures of VaR forecasts assessing; in Sect. 4, we describe empirical results; in Sect. 5, we summarize and conclude the paper.

2 Background

This section briefly describes the multivariate models that we consider in VaR forecasting. Consider X as a financial random variable, where $X \geq 0$ is a gain and $X < 0$ is a loss. X is defined in a random variable space $\mathcal{X}(\Omega, \mathcal{F}, \mathbb{P})$. F_X represents the cumulative distribution function of X and F_X^{-1} its inverse (left) function. VaR can be defined by:

$$VaR^{\alpha}(X) = -\inf\{x : F_X(x) \geq \alpha\} = -F_X^{-1}(\alpha),$$

where $\alpha \in (0, 1)$ corresponds to the significance level. The negative sign of the measure is to represent a monetary loss. This measure represents the maximum loss for a given period and significance level.

2.1 Multivariate GARCH Models

Consider N series of log-returns, and X_T a returns vector ($N \times 1$). We represent X_t by:

$$X_t = \mu_t + \epsilon_t$$
$$\epsilon_t = \mathbf{H}_{t,m}^{\frac{1}{2}} z_t$$

where $t \in T$ refers to the period, μ_t is the conditional mean (for simplicity, for the definition, we assume that $\mu_t = 0$), ϵ_t is the error process, $\mathbf{H}_{t,m}^{\frac{1}{2}}$ is a matrix $N \times N$ positive definite, which is specific for each m model, and z_t is a white noise process. Besides that, $\mathbf{H}_{t,m} = E[X_t X_T']$ is the conditional covariance matrix of X_t, given the information passed, which can be computed from multivariate GARCH models. The main difference between multivariate GARCH models is the treatment given to obtain the conditional covariance matrix. This study focuses on the DCC-GARCH and GO-GARCH models. See Francq and Zakoian (2019) for a review regarding GARCH models.

2.1.1 DCC-GARCH

The DCC-GARCH model, presented by Engle (2002) and Tse and Tsui (2002), allows jointly modeling of volatility and conditional correlation. Through this model, the conditional covariance matrix is obtained by:

$$\mathbf{H}_t = \mathbf{D}_t \mathbf{R}_t \mathbf{D}_t, \quad \mathbf{D}_t = diag\{\sqrt{\sigma_{i,t}}\},$$

where $\mathbf{H}_t$ is the conditional covariance matrix, $\mathbf{D}_t$ is the diagonal matrix of conditional standard deviations, which is computed with a univariate GARCH model for each asset i and period t. Moreover, $\mathbf{R}_t$ is the correlation matrix containing the conditional correlation coefficients.

In general, $\mathbf{R}_t$ can be defined as follows:

$$\mathbf{R}_t = \mathbf{Q}_t^{*-1}\mathbf{Q}_t\mathbf{Q}_t^{*-1},$$

where $\mathbf{Q}_t$ is given by:

$$\mathbf{Q}_t = (1 - a + b)\bar{\boldsymbol{Q}} + a\epsilon_{t-1}\epsilon_{t-1}^{'} + b\mathbf{Q}_{t-1},$$

where $\mathbf{Q}_t$ is the conditional covariance matrix of residuals with unconditional covariance matrix $\bar{\boldsymbol{Q}}$, which is obtained by means of a univariate GARCH model. a and b are non-negative parameters, which satisfy $a + b < 1$, and $\mathbf{Q}_t^*$ is a diagonal matrix containing the square root of the diagonal elements of $\mathbf{Q}_t$.

2.1.2 GO-GARCH

The GO-GARCH model, proposed by Van der Weide (2002), belongs to the family of factorial GARCH models. This model is an extension of the Orthogonal Factor GARCH Model (Alexander & Chibumba, 1997). The main assumption of the model is that the process X_t is governed by a linear combination of unobserved variables (factors) z_t, independent and with zero mean. For this model, the covariance matrix of X_t is defined by:

$$\mathbf{H}_t = \mathbf{W}\mathbf{H}_t^z\mathbf{W}^{'},$$

where $\mathbf{H}_t^z$ is the conditional variance matrix, which is obtained through the univariate GARCH model. The positivity of $\mathbf{H}_t$ results from the positivity of $\mathbf{H}_t^z$, which is ensured by the positivity of the GARCH model estimators. $\mathbf{W}$ is a non-singular matrix and invertible parameter.

Since $E[X_t, X_T^{'}] = \mathbf{H} = \mathbf{W}\mathbf{W}^{'}$, Van der Weide (2002) use a singular value decomposition to obtain $\mathbf{W}$, as:

$$\mathbf{W} = \mathbf{P}\Lambda^{\frac{1}{2}}\mathbf{U}.$$

The orthonormal eigenvectors of $\mathbf{H}$ form the columns of $\mathbf{P}$, and their respective eigenvalues make up of the diagonal matrix Λ. $\mathbf{U}$ is an orthogonal matrix ($N \times N$) of eigenvectors of $\mathbf{H}$, with determinant equal to 1. The matrices $\mathbf{P}$ and Λ are obtained considering the unconditional information of the matrix $\mathbf{H}$, while $\mathbf{U}$ depends on the conditional information of $\mathbf{H}_t$. For more information regarding approaches that can be considered to compute $\mathbf{W}$, we suggest Boswijk and Van Der Weide (2006) and Lanne and Saikkonen (2007).

2.2 Copulas

Consider $X_1, X_2, \cdots, X_N$ with cumulative distribution functions $F_1, F_2, \cdots, F_N$ and inverse distribution functions given by $F_1^{-1}, F_2^{-1}, \cdots, F_N^{-1}$. A copula function C is a N-dimensional distribution with uniformly distributed marginals $[0, 1]^N$. Using Theorem of Sklar (1959), one can obtain the N-dimensional distribution function F of the copula by:

$$F(x_1, x_2, \cdots, x_N) = C(F_1(x_1), F_2(x_2), \cdots, F_N(x_N)), \tag{1}$$

for all $\mathrm{x} = (x_1, x_2, \cdots, x_N)' \in \mathbb{R}^N$, C is unique if the corresponding distribution functions are continuous.

Given (1), C is obtained in the following way:

$$C(u_1, u_2, \cdots, u_N) = F(F_1^{-1}(u_1), F_2^{-1}(u_2), \cdots, F_N^{-1}(u_N)) \qquad (u_1, u_2, \cdots, u_N) \in [0, 1]^N,$$

where F_i^{-1} is the inverse function of the marginal distribution function of F_i. For a formal definition of multivariate copulas, we suggest Nelsen (2007).

2.3 Vine Copula

Copula Vine is a flexible model to describe multivariate copulas, which are constructed from a cascade of bivariate copulas. We will work with three variations of these copulas, R-Vines (Regular Vines), D-Vines and C-Vines. As presented in Kurowicka and Cooke (2006), the density of the R-Vine copula is given by:

$$f(u_1, u_2, \cdots, u_N) = \prod_{k=1}^{N} f_k(u_k) \prod_{i=1}^{N-1} \prod_{e \in E_i} c_{j(e),k(e)|D(e)} \left\{ \begin{array}{c} F(u_{j(e)}|\mathbf{u}_{D(e)}), \\ F(u_{k(e)}|\mathbf{u}_{D(e)}). \end{array} \right\},$$

where $E_i, i = 1, 2, \cdots, N$, are the edges, D(e) are the nodes that the bivariate copulas share, and $j(e)$ and $k(e)$ are the nodes that do not share. The nodes $j(e)$ and $k(e)$ are called the conditioned nodes and $D(e)$ indicates the conditioning set, and the union between the three nodes is called the constraint set. $\mathbf{u}_{D(e)}$ is a sub-vector of pseudo-observations. f_k is the density function of each asset i, and c is the bivariate copula density, which can be different for each pair-copula.

In the case of D-Vine copula, the density can be represented by (Aas et al., 2009):

$$f(u_1, u_2, \cdots, u_N) = \prod_{k=1}^{N} f_k(u_k) \prod_{j=1}^{N-1} \prod_{i=1}^{N-j} c_{i,i+j|i+1,\cdots,i+j-1} \left\{ \begin{array}{c} F(u_i|u_{i+1}, \cdots, u_{i+j-1}), \\ F(u_{i+j}|u_{i+1}, \cdots, u_{i+j-1}). \end{array} \right\},$$

where $\mathbf{u} = (u_1, u_2, \cdots, u_N)' \in [0, 1]^N$ are pseudo-observations, the index j indicates the trees, and i connects each edge in each tree. Similarly, the density of the C-Vine copula is given by:

$$f(u_1, u_2, \cdots, u_N) = \prod_{k=1}^{N} f_k(u_k) \prod_{j=1}^{N-1} \prod_{i=1}^{N-j} c_{i,i+j|i+1,\cdots,i+j-1} \left\{ \begin{array}{c} F(u_j|u_1, \cdots, u_{j-1}), \\ F(u_{j+i}|u_1, \cdots, u_{j-1}). \end{array} \right\}.$$

The difference between the densities of the D-Vine and C-Vine copulas is that in the former, no node in any tree is connected at more than two ends, and in the case of C-Vine copulas, each tree has a unique node, which is connected with $N-j$ ends. The marginal distribution for cascade-like constructions, such as Vine copulas, can be obtained by (Joe, 1996; Czado et al., 2013):

$$F(u_i|\mathbf{u}) = \frac{\partial Cu_i, u_j|\mathbf{u}_{-j}\{F(u_i|\mathbf{u}_{-j}), F(u_j|\mathbf{u}_{-j})\}}{\partial F(u_j|\mathbf{u}_{-j})},$$

where $i, j \in \mathbb{N}$, $i \neq j$, $Cu_i, u_j|\mathbf{u}_{-j}$ is the dependency structure of u_i and u_j.

We refer the reader to Joe (2014) for a review on vine copulas and related topics.

3 Methodological Procedures

In this section, we describe the methodological procedures used in our empirical analysis. As a dataset to build the portfolios, we used stocks belonging to the Ibovespa index during the period from January 2, 2012 to April 28, 2022, making a total of 2559 observations for each stock.[4] The period from 2012 to April 2022 allows us to analyze periods of calm in the Brazilian market and turbulence, including, for example, the crisis triggered by the COVID-19 outbreak. Besides that, our sample includes the period of the Russian invasion of Ukraine. We consider the Brazilian market because it is an emerging market and the largest stock exchange in Latin America. Besides that, we selected this market because of data availability. Countries in emerging financial markets differ substantially from those in developed markets. In periods of financial instability, emerging markets are more affected than developed economies. Gencay and Selcuk (2004) comment that a significant part of total savings in developed economies is invested in emerging markets, either for the hedge or mutual funds. Thus, the implications of modeling emerging countries' risk investments are not limited to investors residing in the country. We compute the log-returns for each stock in our sample using the adjusted price series. According to the KPSS test (Kwiatkowski et al., 1992), the log-returns are stationary. For the sake of brevity, these results are omitted but are available on request.

To build the portfolios, we use two strategies: equally weighted portfolio (naive diversification) and minimum risk portfolio.[5] To limit transaction and information costs, we limit the number of stocks in the portfolio with a cardinality constraint. In general, we use the simplest model for the minimum risk portfolio with cardinality

[4] Our dataset contains 63 stocks. We verified the composition of the Ibovespa index in April 2022 and maintained this composition throughout our sample. In total, the index presents 91 stocks in April 2022. Due to the lack of data availability for the entire period, some stocks were excluded from the sample. For this reason, our sample is made up of only 63 stocks. The database is available in the package Quandl (Dotson et al., 2021) from Comprehensive R Archive Network.

[5] The equally weighted portfolio was considered in the study because it is common in studies that compare models to predict risk measures; see Müller and Righi (2018). Moreover, DeMiguel et al. (2009) point out that the performance out-of-sample of the naive portfolio is competitive concerning optimal portfolios. The stocks considered in the equally weighted portfolio were those selected by the optimization problem defined in 2.

constraint since we want to check the impact of the weights on the performance of the risk prediction models.[6] This model is given by:

$$
\begin{aligned}
&\min_{W \in R^N,\, z \in \{0,1\}^N} \rho\left(\sum_{i=1}^{N} w_i X_i\right)\\
&\text{st}\\
&\sum_{i=1}^{N} w_i = 1\\
&w_i \geq 0,\ \forall\ i = 1, \cdots, N,\\
&w_i \leq z_i,\ \forall\ i = 1, \cdots, N,\\
&\sum_{i=1}^{N} z_i \leq K,
\end{aligned}
\tag{2}
$$

where $X_i \in \mathcal{X}, i = 1, \cdots, N$, being N the number of stocks, is a vector of returns (stocks) that might compose the portfolio, w_i refers to the weight of the i–th stock, z_i is a binary decision variable that receives value 1 if the i-th stock is included and 0 (zero) otherwise, and K sets the maximum number of stocks to be allowed.[7] Our problem does not allow short selling $\left(w_i \geq 0,\ \forall\ i = 1, \cdots, N\right)$ and requires all capital will be allocated $\left(\sum_{i=1}^{N} w_i = 1\right)$. These restrictions are common in portfolio optimization problems. See, for example, Righi and Borenstein (2018). The weights obtained by the formulation (2) were not rebalanced. $\rho(\cdot)$ refers to the Expected Shortfall. We use this measure in the portfolio optimization problem because it is a subadditive/convex measure. For details see Artzner et al. (1999). Thus, the risk of the combined position is less than or equal to the sum of the individual risks of the stocks. ES is quantified using Historical Simulation. We selected the HS method because our focus is not on selecting the most appropriate model for portfolio optimization. Furthermore, this method is widely used in academic studies and the financial industry (Pérignon & Smith, 2010). We did not determine the portfolio weights using VaR because this does not respect the subadditivity axiom.

The estimation methods used to predict VaR are the traditional HS, Vine copulas, and multivariate GARCH models, being the last two groups of models explained in

[6] We emphasize that the construction of portfolios is not the focus of this study but the comparison of multivariate models to predict VaR. For this reason, we prefer to use a simple model, such as the one performed by Righi and Borenstein (2018).

[7] To compute this portfolio we use the fPortfolio package (Wuertz et al., 2020). In this study, we considered K equal to 6 and 12 stocks. Larger portfolios were not used due to computational difficulty. For both portfolios, we used two sample sizes (500 and 1000 observations) to estimate the weights. For 1000 observations, the weights were obtained considering data from January 2, 2012, to January 20, 2016. For 500 observations, the weights were obtained considering data from January 13, 2014, to January 20, 2016. Thus, considering the risk minimization problem, we have 4 distinct portfolios: 6 stocks considering 500 and 1000 observations for estimating the weights and 12 stocks considering 500 and 1000 observations for estimating the weights.

Sect. 2. For the copula method and multivariate GARCH models, at first, we model the mean ($\mu_{i,t}$) and conditional standard deviation ($\sigma_{i,t}$) of each univariate series, that is, of each stock i. The Ljung-Box test (Ljung & Box, 1978) indicated the presence of significant autocorrelation in the stocks log-returns. Thus, we fit the conditional mean with an autoregressive (AR) model of order 1 with a constant. According to Garcia-Jorcano and Novales (2021), the AR(1) model is sufficient to produce serially uncorrelated innovations. We also applied the Ljung-Box test to the squared standardized residuals. Test values indicate a significant presence of conditional heteroskedasticity.[8] To model the conditional standard deviation, we consider the GARCH model. We use the AR-GARCH because the risk forecasting literature shows that the model presents good results in forecasting risk measures for univariate series. See, for instance, Hartz et al. (2006) and Garcia-Jorcano and Novales (2021). Furthermore, it is a common model to obtain the uniform marginal distributions, which are necessary for estimating the copulas and multivariate GARCH models (Müller & Righi, 2018).

The AR(p)-GARCH(q, s) model is represented by:

$$
\begin{aligned}
X_{i,t} &= \phi_0 + \sum_{l=1}^{p} \phi_l X_{i,t-l} + \epsilon_{i,t} = \mu_{i,t} + \epsilon_{i,t}, \\
\epsilon_{i,t} &= \sigma_{i,t} z_{i,t}, \qquad z_{i,t} \sim i.i.d.\, F(z_{i,t};\boldsymbol{\theta}), \\
\sigma_{i,t}^2 &= a_0 + \sum_{j=1}^{q} a_j \epsilon_{i,t-j}^2 + \sum_{k=1}^{s} b_k \sigma_{i,t-k}^2,
\end{aligned}
\tag{3}
$$

where $X_{i,t}$ for time t and stock i is the return. ϕ_l, for $l = 1, \cdots, p$, being p the autoregressive order, are parameters of the autoregressive model, $\epsilon_{i,t}$ is the error term, $z_{i,t}$ is a white noise process with distribution $F(z_{i,t};\boldsymbol{\theta})$, where $\boldsymbol{\theta}$ is a vector of parameters of distribution, including zero mean, unit variance and some additional parameter that varies with the distribution. $\mu_{i,t}$ and $\sigma_{i,t}^2$ are the mean and conditional variance given past information of each stock i, and a_j, for $j = 1, \cdots, q$, as well as b_k, for $k = 1, \cdots, s$, are parameters of the GARCH model ($a_0 > 0$, $a_j \geq 0$, $b_k \geq 0$), and q and s are its order. For F, we assume the normal (norm), Student's t (std) and skewed Student's t (sstd) distribution. We use the normal distribution as it is the most common and is often used in stock market analyses (Müller & Righi, 2018). The first two moments describe this distribution: mean (μ) and variance (σ^2). We also consider the Student's t distribution because it considers the heavy-tailed behavior of financial stocks. This distribution is described completely by the shape parameter (ν). In addition to the heavy tails, the skewed Student's t distribution allows modeling the asymmetry in financial stocks. This distribution is described by the shape (ν) and skewness (ξ) parameter. For more details, see Fernández and Steel (1998). To see the probability density function of three distributions, we suggest Ghalanos (2020). To choose the lags of the AR-GARCH model, we compare the Akaike Information Criteria (AIC) values. Our results suggest that the most suitable model to

[8] Ljung-Box test values are omitted for brevity but are available on request.

adjust the log-returns is the AR(1)-GARCH(1,1). We obtain the standardized residuals after forecasting the mean and standard deviation values based on the adjusted AR(1)-GARCH(1,1) model, considering normal, Student's t and skewed Student's t distribution for $z_{i,t}$. Then, we transform them into pseudo-observations $\mathbf{u} \in [0, 1]$ by inverting the fitted distribution of each series. This procedure is necessary to predict risk measures using copulas. Given the marginal distribution and the estimated parameters, we use the following algorithm to predict VaR with copulas (Aas and & Berg, 2010; Righi & Ceretta, 2013):

(i) For each stock, we forecast the mean $\mu_{i,t+1}$ and the conditional standard deviation $\sigma_{i,t+1}$ using the AR(1)-GARCH(1,1) model.
(ii) We simulate N samples $u_{i,T}$ with size T, being that each i represents a stock of our sample, using Vine copulas.
(iii) Given $u_{i,T}$, we generated N $z_{i,T}$ through the inversion of marginal probability, as $z_{i,T} = F^{-1}(u_{i,T})$.
(iv) For each stock i, we obtain the returns by $r_{i,T} = \mu_{i,t+1} + \sigma_{i,t+1} z_{i,T}$.
(v) Based $r_{i,T}$ of each i, we calculate the portfolio returns $W\mathbf{r}_T$, where $W = \{w_1, w_2, \cdots, w_N\}$ is a vector with the weights and $\mathbf{r}_T = \{r_{1,T}, r_{2,T}, \cdots, r_{N,T}\}$ are the log-returns of stocks.
(vi) Then, we quantified the VaR forecast for $t + 1$ ($\text{VaR}^{\alpha}_{t+1}$) of portfolio returns using the Historical Simulation, i.e., the empirical data distribution.

The Vine copulas are constructed from a cascade of bivariate copulas. See Sect. 2.3. In this study, we choice of bivariate copulas utilizing AIC. The bivariate copulas considered were: Gaussian, Student's t, Clayton, Gumbel, Frank, Joe, BB1, BB6, BB7, BB8, as well as the rotated copulas Clayton (180 degrees), Gumbel (180 degrees), Joe (180 degrees), BB1 (180 degrees), BB6 (180 degrees), BB7 (180 degrees), BB8 (180 degrees), Clayton (90 degrees), Gumbel (90 degrees), Joe (90 degrees), BB1 (90 degrees), BB6 (90 degrees), BB7 (90 degrees), BB8 (90 degrees), Clayton (270 degrees), Gumbel (270 degrees), Joe (270 degrees), BB1 (270 degrees), BB6 (270 degrees), BB7 (270 degrees) and BB8 (270 degrees). We consider portfolios formed with 6 and 12 stocks, i.e., $K = 6$ and 12; for T, we use the same value of the rolling estimation window, which was 500 and 1000 observations. Thus, we have a total of 1,559 out-of-sample observations, that is, predictions. The out-of-sample period coincides with both rolling estimation windows. For the rolling estimation window of 1000 observations, our first out-of-sample prediction was obtained considering data from January 2, 2012, to January 20, 2016. For the rolling estimation window of 500 observations, our first out-of-sample prediction was obtained considering data from January 13, 2014, to January 20, 2016. We exclude the first 500 observations for this rolling window so that the out-of-sample period matches. Thus, we can compare the performance of the models for both rolling estimation windows. Furthermore, the period used for the first out-of-sample risk forecast was the one used to obtain the weights of the different portfolios. As significance levels, we use 1%, 2.5% and 5% because they are the most common values in the literature, and regulatory agencies recommend 1% for VaR estimation (Basel Committee on

Banking Supervision, 2013; Righi & Ceretta, 2015). For W, we use the vector of weights given by formulation (2). We also consider the equally weighted weights for the stocks selected in the optimization problem, i.e., $\frac{1}{N}$. In the case of multivariate GARCH models, the difference is that $z_{i,T}$ is obtained by employing these models instead of the copula method. For the GO-GARCH model, we performed the estimation of the **W** matrix through independent component analysis (ICA). Regarding the HS approach, the returns $r_{i,T}$ of each stock i is the empirical data distribution.

To describe the risk forecasts, we quantified the mean and standard deviation. The quality of the forecasts was assessed using the VaR score function, which is given by (Gneiting, 2011):

$$\mathcal{L}_{VaR}(X, Y) := \frac{1}{n}\sum_{i=1}^{n}\left[\alpha\left(x_{t+1} - y_{t+1}\right)^{+} + (1-\alpha)\left(x_i - y_i\right)^{-}\right],$$

where $x_{t+1} \in X$ are the observations for the out-of-sample period and $y_{t+1} \in Y$ are the risk forecasts for this period. According to this criterion, the model with the lowest value is the most accurate in predicting VaR. The results of this function are called realized loss.

Besides that, we quantified realized cost, which is obtained in the following way:

$$\text{Cost}(X, Y) := \text{Cost}_{G,L}(X, Y) = \frac{1}{T}\sum_{t=1}^{T}\left[\left(x_{t+1} - y_{t+1}\right)^{+} g_{t+1} + (x_{t+1} - y_{t+1})^{-} l_{t+1}\right] \tag{4}$$

where $x_{t+1} \in X$ and $y_{t+1} \in Y$. Besides, $g_{t+1} \in G$ represents costs from risk overestimation and $l_{t+1} \in L$ costs from risk underestimation. G and L are positive random variables. The formulation (4) is based on the robust risk measurement approach proposed by Righi et al. (2020) and model risk measure discussed in Müller and Righi (2020). The realized cost identifies the model with the best trade-off between the sum of the costs from risk overestimation and underestimation. We use overestimation and underestimation cost Selic interest rate (Selic) and average interest rates for credit operations in Brazil (C.rate), respectively.[9] These rates represent, respectively, a risk-free investment with high liquidity where the surplus over capital requirement could be safely applied and an average interest rate of resources can be obtained when the capital requirement is not enough to cover losses. We convert both yield rates to daily frequency. Details of both series are available under request. The results for the out-of-sample period are presented considering a complete out-of-sample sample (Full sample). The out-of-sample period comprises from January 21, 2016, to April 28, 2022. Furthermore, we divided the out-of-sample period into two intervals to analyze the results: 2016 to 2019 and 2020 to April 2022. From 2016 to 2019, we have an interval that is free from the influence of the COVID-19 outbreak. This period also includes a relatively stable period in the Brazilian market. From 2020 to April 2022 reflects the influence of the COVID-19 pandemic from the

[9] Both rates were collected in the Central Bank of Brazil's Time Series Management System, which can be found at the following link: https://www3.bcb.gov.br/sgspub/localizarseries/localizarSeries.do?method=prepararTelaLocalizarSeries.

beginning of the pandemic until the end of the sample. In this way, we can verify whether the performance of risk prediction models has changed during the COVID-19 outbreak.

4 Results

This section presents and discusses the results obtained. First, we describe the descriptive analysis of the stock data and portfolio returns considering the sample period. We omitted these descriptive results for out-of-sample period intervals for brevity, but the results are available under request. We then analyzed the performance of the competing forecasting models using realized loss and cost. In Table 1, we describe the descriptive statistics, which include mean, minimum, maximum, standard deviation, skewness, and excess kurtosis (E. kurtosis), of stocks that belong to the Brazilian market index and are considered in our investigation. In general, it can be seen that the average of the stocks is close to zero. We note that around 70% of the stocks show negative asymmetry, which indicates a fatter tail on the left side of the distribution. In contrast, the positive skewness of other stocks represents a long tail on the right side. We note that the average asymmetry of the stocks that make up the sample is negative. Excess kurtosis values greater than zero indicate that the data distribution is leptokurtic, that is, the stock returns distribution has heavy tails. The stock's average excess kurtosis during the sample period is 10.4129. However, we found that BRAP4[10] and SULA11[11] have the highest excess kurtosis values (72.8797 and 33.5521, respectively). PRIO3[12] is the stock with the highest standard deviation (4.8952), possibly indicating that this stock is the riskiest among those considered. This stock is a common stock, which is usually riskier compared to preferred stocks. The characteristics observed by stocks are stylized facts observed in financial returns (Cont, 2001).

In Table 2, we describe the mean and standard deviation values of portfolios of minimum risk and equally weighted with 6 and 12 stocks,[13] considering 500 and 1000 observations for weights estimation. Portfolio returns, as well as stock returns, have an average close to zero. The portfolios built with 12 stocks are less volatile than portfolios with 6 stocks, i.e., they have a lower standard deviation. This result indicates that portfolios with 12 stocks are less risky than those with 6. Portfolios with more stocks, that is, more diversified, tend to reduce investment risk. For the portfolios with 6 stocks, we verify that the minimum risk portfolio has a higher standard deviation, indicating that they tend to be more volatile than

[10] BRAP4 refers to an Bradespar SA Preference Shares common stock. Bradespar is an investment company controlled by Bradesco Bank.

[11] SULA11 is referring to common stock of *Sul Amárica*, which is a company that operates in the health and dental insurance sector.

[12] PRIO3 is a common stock of PetroRio. This company is a publicly-traded company focused on oil and gas production.

[13] We present the mean and standard deviation of portfolio returns for brevity. Other descriptive statistics of the portfolios are available on request.

Table 1 Descriptive statistics of the log-returns of the sixty-three (63) stocks belonging to the Ibovespa index

Stocks	Mean	Minimum	Maximum	SD	Skewness	E. Kurtosis
ABEV3	0.0258	−17.1759	10.6991	1.6984	−0.2857	8.8885
ALPA4	0.0492	−23.3993	25.4025	2.4153	−0.0578	13.3310
AMER3	0.0414	−18.5997	34.7927	3.8403	0.6564	5.1329
BBAS3	0.0381	−23.7891	15.8081	2.7099	−0.4287	8.2798
BBDC3	0.0352	−15.4898	15.1240	2.1641	−0.0560	5.1941
BBDC4	0.0366	−15.4019	15.5865	2.2061	−0.0805	5.7417
BEEF3	0.0453	−20.5485	16.7054	2.4403	−0.1634	6.3744
BPAN4	0.0211	−41.0084	38.0464	3.3337	0.3994	20.3695
BRAP4	0.0287	−61.7822	16.7644	2.9897	−3.4495	72.8797
BRFS3	−0.0342	−21.9987	15.0814	2.3405	−0.4928	11.5499
BRKM5	0.0660	−−28.0681	27.5769	2.9988	0.0886	16.5872
BRML3	−0.0049	−27.2886	14.2900	2.4863	−0.5486	9.1939
CCRO3	0.0165	−19.7610	20.0134	2.4182	−0.1013	9.1036
CIEL3	−0.0246	−23.7959	21.0721	2.5930	0.3946	9.3436
CMIG4	0.0232	−26.4318	16.3848	2.7708	−1.1068	11.8986
CPFE3	0.0330	−18.5968	8.4257	1.8123	−0.6836	9.2368
CPLE6	0.0526	−16.8471	15.7509	2.3689	−0.3581	5.0373
CSAN3	0.0607	−20.3794	13.3300	2.2175	−0.3339	5.4177
CSNA3	0.0339	−29.1590	18.9137	3.6820	0.0409	4.4788
CYRE3	0.0125	−28.3029	16.5985	2.7366	−0.7864	10.9875
DXCO3	0.0376	−28.0260	17.1492	2.5833	−0.9101	13.2340
ECOR3	−0.0100	−20.5378	19.6280	2.5964	−0.2072	6.8493
EGIE3	0.0472	−10.1242	9.4605	1.5465	−0.0491	3.1296
ELET3	0.0572	−23.5340	40.0759	3.4369	0.6556	12.1566
ELET6	0.0457	−22.4163	27.8243	3.0791	0.1486	8.7808
EMBR3	0.0101	−30.7091	20.2929	2.6750	−0.3938	14.0867
ENBR3	0.0409	−13.6602	14.4581	1.9341	−0.0279	4.4860
ENEV3	−0.1001	−44.1833	28.7682	3.6737	−1.0344	21.5019
EQTL3	0.0974	−11.4750	8.0581	1.7199	−0.2179	3.7226
EZTC3	0.0290	−21.4783	22.6030	2.6186	−0.1631	7.6248
FLRY3	0.0278	−16.5515	10.5318	1.9840	−0.3523	5.5190
GGBR4	0.0364	−19.7922	16.0867	2.8380	−0.1093	3.7065
GOAU4	−0.0037	−23.9185	18.7627	3.1081	−0.3718	5.7305
GOLL4	0.0072	−45.0890	40.7641	4.4539	0.1369	13.3129
HYPE3	0.0645	−16.4988	19.1800	2.0304	0.1901	9.4921
ITSA4	0.0384	−11.5022	9.7758	1.9344	−0.1239	2.7936
ITUB4	0.0334	−19.8015	11.1275	2.0579	−0.3235	5.9729
JBSS3	0.0834	−37.6051	21.9915	3.0345	−0.3624	14.3154
JHSF3	0.0176	−19.8070	25.4234	3.1146	0.3873	5.0387
LREN3	0.0485	−23.7244	13.9762	2.2988	−0.3972	8.8997
MGLU3	0.1143	−23.6698	31.6925	3.7588	0.5310	8.4714
MRFG3	0.0381	−27.2989	22.4777	2.9648	−0.2807	8.5806
MRVE3	0.0206	−22.5046	19.1808	2.8122	−0.3303	6.2884

Table 1 (continued)

Stocks	Mean	Minimum	Maximum	SD	Skewness	E. Kurtosis
MULT3	0.0325	−25.3473	15.9602	2.1983	−0.4430	12.2198
PETR3	0.0305	−35.2054	20.5024	3.1270	−0.8480	12.0073
PETR4	0.0317	−35.2367	20.0671	3.1163	−0.9180	11.7969
POSI3	0.0165	−42.4211	31.3483	3.6357	0.0555	13.7895
PRIO3	0.0326	−45.4770	60.7989	4.8952	0.5106	19.2333
QUAL3	0.0075	−34.7700	31.2153	2.7210	−0.7889	20.7942
RADL3	0.0848	−13.7365	8.8455	1.9539	−0.0872	2.1917
RENT3	0.0867	−26.6857	23.7608	2.5196	−0.1284	12.0071
SANB11	0.0541	−14.4726	14.6496	2.2565	0.0518	4.3794
SBSP3	0.0501	−19.6380	16.0930	2.3733	−0.2470	5.7825
SLCE3	0.0989	−10.4702	14.2797	2.2785	0.3172	2.0585
SULA11	0.0515	−19.2765	39.4467	2.3823	1.5193	33.5521
TIMS3	0.0234	−15.9900	14.2587	2.1295	−0.0416	4.2140
TOTS3	0.0482	−16.6569	18.0650	2.2711	−0.2115	5.8467
UGPA3	−0.0020	−24.0265	21.0112	2.3183	−0.5707	16.1585
USIM5	0.0096	−23.8675	30.0892	3.6949	0.3484	5.2156
VALE3	0.0500	−28.1822	19.3574	2.7521	−0.2735	7.9994
VIVT3	0.0338	−13.1920	11.6196	1.8311	0.0440	4.9830
WEGE3	0.1013	−23.0921	13.0102	2.0307	−0.4848	11.1561
YDUQ3	0.0509	−29.0010	21.2984	3.0514	−0.1852	7.9072
Average	0.0339	−24.1571	20.4973	2.6668	−0.2117	10.4129

The sample comprises daily data from January 2012 to April 2022. The log-returns are multiplied by 100

Note This table shows the mean, minimum, maximum, SD (standard deviation), skewness, and E. kurtosis (excess kurtosis) of log-returns. ABEV3 refers to Ambev, ALPA4 is Alpargatas, AMER3 is Americanas, ASAI3 is Sendas Distribuidora, AZUL4 is Azul, BBAS3 is Bank of Brazil, BBDC3 and BBDC4 are stocks from Bradesco, BBSE3 Bank of Brazil *Seguridade Participações*, BEEF3 refers to Minerva, BIDI11 is Inter Bank, BPAC11 is BTG Pactual Bank, BPAN4 is Pan Bank, BRAP4 is Bradespar, BRFS3 is Brazil Foods, BRKM5 is Braskem, BRML3 is BR Malls *Participações*, CASH3 is Máliuz, CCRO3 refers to *Companhia de Concessões Rodoviárias*, CIEL3 is Cielo, CMIG4 is *Companhia Energática de Minas Gerais*, CMIN3 is *CSN Mineração*, COGN3 is Cogna *Educação*, CPFE3 is CPFL *Energia*, CPLE6 is *Companhia Paranaense de Energia*, CRFB3 is Atacadão, CSAN3 is Cosan, CSNA3 is *Companhia Siderúrgica Nacional*, CVCB3 is CVC Brazil *Operadora e Agência de Viagens*, CYRE3 is Cyrela Brazil Realty, DXCO3 is Dexco, ECOR3 is EcoRodovias *Infraestrutura e Logística*, EGIE3 is Engie Brazil Energy, ELET3 and ELET6 are stocks from Eletrobras, EMBR3 is Embraer, ENBR3 is *Energias do Brasil*, ENEV3 is Eneva, ENGI11 is Energisa, EQTL3 is *Equatorial Energia*, EZTC3 is Eztec *Empreendimentos e Participações*, FLRY3 is Fleury, GGBR4 is Gerdau, GOAU4 refers to Gerdau, GOLL4 is Gol, HAPV3 is Hapvida *Participações e Investimentos*, HYPE3 is Hypera Farma, IGTI11 is Iguatemi, IRBR3 is Brazil *Resseguros*, ITSA4 is Itaúsa, ITUB4 is Itaú, JBSS3 is JBS, JHSF3 is JHSF, KLBN11 is Klabin, LCAM3 is *Companhia de Locação das Amáricas*, LREN3 is Renner, LWSA3 is Locaweb, MGLU3 is Magazine Luiza, MRFG3 is Marfrig Global Foods, MRVE3 is MRV *Engenharia e Participações*, MULT3 is *Multiplan Empreendimentos Imobiliários*, NTCO3 is Natura & Co Holding, PCAR3 is *Companhia Brasileira de Distribuição*, PETR3 and PETR4 are stocks from Petrobras, PETZ3 is Pet Center *Comárcio e Participações*, POSI3 is Positivo, PRIO3 is Petro Rio, QUAL3 is Qualicorp, RADL3 is Raia Drogasil, RAIL3 is Rumo, RDOR3 is *Rede D'Or São Luiz*, RENT3 is Localiza, RRRP3 is 3R Petroleum *Óleo E Gás*, SANB11 is Santander Bank, SBSP3 is *Companhia de Saneamento Básico do Estado de São Paulo*, SLCE3 is SLC *Agrícola*, SOMA3 is *Moda Soma*, SULA11 is *Sul Amárica*, SUZB3 is Suzano, TAEE11 is *Transmissora Aliança de Energia Elátrica*, TIMS3 is TIM, TOTS3 is TOTVS, UGPA3 is Ultrapar, USIM5 is *Usinas Siderúrgicas de Minas Gerais*, VALE3 is Vale, VBBR3 is *Vibra Energia*, VIIA3 is Via, VIVT3 is *Telefônica Brasil*, WEGE3 is WEG, and YDUQ3 is Yduqs

Table 2 Descriptive statistics [Mean and standard deviation (SD)] of portfolios with 6 and 12 stocks

	Minimum risk		Equally weighted	
	Mean	SD	Mean	SD
Portfolio 6–500 observations	0.0088	2.5210	−0.0080	2.3118
Portfolio 6–1000 observations	0.0096	2.6486	0.0132	2.3370
Portfolio 12–500 observations	0.0117	1.9041	0.0150	2.0020
Portfolio 12–1000 observations	0.0147	1.7089	0.0155	1.8564

For both portfolios, the weights were determined using 500 and 1000 observations to estimate the weights. The sample comprises daily data from January 2012 to April 2022. The portfolio returns are multiplied by 100

Note BPAN4, ENEV3, GOAU4, GOLL4, JHSF3, and USIM5 are the stocks in the portfolio with 6 stocks and 500 observations for weight estimation, with their respective weights equal to 0.4108, 0.0551, 0.0012, 0.1974, 0.1249, and 0.2106. BRAP4, ENEV3, GOAU4, GOLL4, MGLU3, and PRIO3 are the stocks in the portfolio with 6 stocks and 1000 observations for weight estimation, with their respective weights equal to 0.4066, 0.0948, 0.0030, 0.3709, 0.0086, and 0.1161. BPAN4, ECOR3, ENEV3, GOAU4, GOLL4, JHSF3, MGLU3, PRIO3, POSI3, TIMS3, USIM5, and YDUQ3 are the stocks in the portfolio with 12 stocks and 500 observations for weight estimation, with their respective weights equal to 0.0757, 0.0954, 0.0507, 0.1686, 0.0382, 0.0026, 0.0048, 0.0503,0.1979, 0.1839, 0.0007, and 0.1313. BPAN4, BRAP4, ECOR3, ENEV3, FLRY3, GOAU4, GOLL4, JHSF3, MGLU3, PRIO3, POSI3, and VIVT3 are the stocks in the portfolio with 12 stocks and 1000 observations for weight estimation, with their respective weights equal to 0.1485, 0.0573, 0.1777, 0.0001, 0.2332, 0.1390, 0.0018, 0.0192, 0.0023, 0.0011, 0.1341, and 0.0859

equally weighted portfolios. We also point out that the composition of the portfolios changes when we consider different numbers of stocks and observations to estimate the weights, which justifies the differences in the descriptive statistics of the portfolios. For visual analysis of the differences among portfolio returns, we present in Fig. 1 a graphic evolution of the different portfolios. We present portfolio returns for the full period (January 2012 to April 2022). Note that for the risk estimation considering 500 observations, we do not consider the first 500 observations. We did that with the intent that the out-of-sample period coincides with portfolios where we use 500 and 1000 observations for risk prediction. This way, we can compare the results obtained for both rolling estimation windows. The illustration confirms the greater variability and possibly greater risk of the portfolio with 6 stocks compared to the 12 stocks. For all portfolios, we have noticed an increase in volatility around March 2020. The COVID-19 outbreak can explain the increase in variability. Ashraf (2020) and Vasileiou et al. (2021) point out that the decline in returns in financial markets occurred primarily when

Table 3 Descriptive statistics [Mean and standard deviation (SD)], loss ($\mathcal{L}_{VaR}$), and realized cost (Cost) of VaR forecasts, considering portfolios with 6 stocks and 500 observations to estimate the weights

Models	Mean	SD	$\mathcal{L}_{VaR}$	Cost	Mean	SD	$\mathcal{L}_{VaR}$	Cost	Mean	SD	$\mathcal{L}_{VaR}$	Cost	Mean	SD	$\mathcal{L}_{VaR}$	Cost
	Minimum risk				Equally weighted				Minimum risk				Equally weighted			
	$\alpha = 1\%$, Full Sample								$\alpha = 5\%$, Full Sample							
R-Vine$_{norm}$	8.8870	3.4849	0.1112	26.5808	5.6143	1.9947	**0.0906**	16.9278	4.6353	1.7732	0.3064	15.0023	3.8477	1.3965	0.2690	12.5504
R-Vine$_{std}$	13.0225	5.2315	0.1366	38.3835	9.0175	3.3194	0.1087	26.8065	6.1988	2.3413	0.3606	19.4184	5.1932	1.8459	0.3090	16.2191
R-Vine$_{sstd}$	8.1696	3.2386	**0.1091**	24.9305	5.5698	2.2445	0.0932	17.0282	4.0796	1.6436	**0.2959**	**13.6252**	3.3952	1.3245	0.2661	**11.5748**
C-Vine$_{norm}$	8.7832	3.4558	0.1136	26.5309	5.5428	2.0553	0.0918	16.7991	4.6172	1.8344	0.3044	14.9986	3.8021	1.4214	**0.2650**	12.4604
C-Vine$_{std}$	12.7996	4.9705	0.1396	37.7692	8.8361	3.2716	0.1101	26.2606	6.1449	2.3149	0.3577	19.2127	5.1736	1.8648	0.3083	16.1491
C-Vine$_{sstd}$	8.3352	3.4128	0.1099	25.1645	5.6789	2.2580	0.0932	17.1539	4.1534	1.6828	0.2967	13.7431	3.4630	1.3304	0.2685	11.6931
D-Vine$_{norm}$	8.8614	3.4155	0.1146	26.6826	5.6069	2.0218	0.0929	17.0409	4.6243	1.8123	0.3075	14.9997	3.8205	1.4419	0.2705	12.5214
D-Vine$_{std}$	12.9334	5.2212	0.1416	38.2036	8.9963	3.3504	0.1119	26.7864	6.1815	2.3672	0.3585	19.3185	5.1886	1.8549	0.3091	16.2601
D-Vine$_{sstd}$	8.1389	3.3323	0.1134	**24.7663**	5.5729	2.2331	0.0925	16.9699	4.0994	1.6765	0.2996	13.7376	3.4005	1.3267	0.2675	11.6146
HS	9.2574	2.2410	0.1398	27.0354	6.5949	1.9064	0.1229	19.1937	4.2375	0.7390	0.3301	14.2109	3.5492	0.5670	0.3006	12.2808
DCC	8.1647	3.1757	0.1103	24.9266	5.0404	1.8792	0.0932	**15.7718**	4.4063	1.7278	0.2991	14.4074	3.5771	1.3360	0.2669	12.0260
DCC$_{std}$	8.4661	3.1665	0.1125	25.7363	5.1714	1.8700	0.0932	16.1106	4.6291	1.7215	0.3038	14.9352	3.7081	1.3285	0.2679	12.3064
GO-GARCH	9.3545	5.5026	0.1133	29.6458	5.9453	3.5867	0.1007	19.2890	4.8854	2.8053	0.3218	16.5665	3.9972	2.2901	0.2849	13.8353
$\alpha = 1\%$, 2016-2019									$\alpha = 5\%$, 2016-2019							
R-Vine$_{norm}$	8.4434	2.5762	0.0933	32.1236	5.2525	1.3479	0.0681	20.2055	4.5013	1.4746	0.2650	18.0563	3.6597	1.0630	0.2319	14.9400
R-Vine$_{std}$	12.5744	3.9304	0.1324	47.2899	8.7034	2.5169	0.0978	32.9746	6.1198	1.9936	0.3335	23.9923	5.0465	1.4766	0.2807	19.8430
R-Vine$_{sstd}$	7.7491	2.8147	**0.0879**	30.1457	5.1769	1.6360	0.0702	20.3041	3.9428	1.4159	0.2483	**16.2497**	3.2085	1.0516	**0.2246**	**13.6603**
C-Vine$_{norm}$	8.4374	2.7203	0.0963	32.2405	5.1998	1.4124	0.0699	20.0885	4.5214	1.5011	0.2687	18.1684	3.6583	1.0778	0.2303	14.9181
C-Vine$_{std}$	12.3405	3.7877	0.1284	46.3389	8.4935	2.4221	0.0958	32.1522	6.0534	1.9330	0.3304	23.7029	5.0224	1.4498	0.2791	19.7168
C-Vine$_{sstd}$	7.8220	2.6868	0.0885	30.2396	5.2015	1.5545	0.0703	20.2568	3.9733	1.4048	**0.2472**	16.2961	3.2493	1.0222	0.2262	13.7373
D-Vine$_{norm}$	8.5187	2.6846	0.0950	32.4302	5.2920	1.3906	0.0698	20.3964	4.5069	1.4974	0.2654	18.0836	3.6595	1.0744	0.2307	14.9192

Table 3 (continued)

Models	Mean	SD	$\mathcal{L}_{VaR}$	Cost	Mean	SD	$\mathcal{L}_{VaR}$	Cost	Mean	SD	$\mathcal{L}_{VaR}$	Cost	Mean	SD	$\mathcal{L}_{VaR}$	Cost
	Minimum risk				Equally weighted				Minimum risk				Equally weighted			
	$\alpha = 1\%$, Full Sample								$\alpha = 5\%$, Full Sample							
D-Vine$_{std}$	12.4897	3.8752	0.1314	46.9582	8.6999	2.4590	0.0953	32.8823	6.1012	1.9432	0.3300	23.8323	5.0643	1.4774	0.2822	19.9287
D-Vine$_{sstd}$	7.7002	2.6340	0.0899	**29.8249**	5.1599	1.5840	0.0699	20.1580	3.9589	1.3992	0.2519	16.3521	3.2269	1.0617	0.2266	13.7206
HS	8.4706	1.8395	0.0950	31.4447	5.5127	1.1640	0.0725	21.2728	4.2364	0.8426	0.2617	16.7479	3.5592	0.6779	0.2401	14.4959
DCC	7.8945	2.5939	0.0902	30.4043	4.8439	1.4468	**0.0667**	**19.0619**	4.2959	1.4455	0.2581	17.3663	3.4476	1.0550	0.2280	14.3483
DCC$_{std}$	8.1946	2.5834	0.0928	31.4211	4.9739	1.4370	0.0673	19.4928	4.5172	1.4350	0.2651	18.0642	3.5775	1.0459	0.2299	14.7191
GO-GARCH	8.9908	6.0769	0.0997	36.7806	5.6715	3.9605	0.0817	23.7843	4.7459	3.1401	0.2914	20.4326	3.8596	2.5673	0.2521	16.9191
$\alpha = 1\%$, 2020-2022									$\alpha = 5\%$, 2020-2022							
R-Vine$_{norm}$	9.6448	4.5465	0.1418	17.1118	6.2324	2.6599	0.1291	11.3285	4.8641	2.1734	0.3773	9.7848	4.1690	1.7866	0.3324	8.4681
R-Vine$_{std}$	13.7882	6.8457	0.1436	23.1684	9.5540	4.3113	**0.1272**	16.2693	6.3338	2.8352	0.4069	11.6047	5.4440	2.3261	0.3574	10.0283
R-Vine$_{sstd}$	8.8881	3.7515	0.1453	16.0212	6.2411	2.8928	0.1326	11.4318	4.3133	1.9521	0.3771	**9.1416**	3.7140	1.6445	0.3370	**8.0120**
C-Vine$_{norm}$	9.3740	4.3789	0.1433	16.7770	6.1288	2.7381	0.1292	11.1798	4.7808	2.2868	**0.3655**	9.5835	4.0478	1.8429	**0.3242**	8.2617
C-Vine$_{std}$	13.5839	6.4406	0.1588	23.1293	9.4214	4.2949	0.1345	16.1957	6.3012	2.8466	0.4043	11.5420	5.4319	2.3937	0.3582	10.0544
C-Vine$_{sstd}$	9.2119	4.2447	0.1464	16.4945	6.4946	2.9388	0.1322	11.8531	4.4610	2.0382	0.3814	9.3817	3.8281	1.6735	0.3407	8.2010
D-Vine$_{norm}$	9.4468	4.3317	0.1482	16.8637	6.1449	2.7051	0.1324	11.3086	4.8248	2.2379	0.3795	9.7314	4.0955	1.8824	0.3385	8.4252
D-Vine$_{std}$	13.6914	6.8796	0.1589	23.2479	9.5029	4.4374	0.1405	16.3724	6.3186	2.9509	0.4070	11.6075	5.4009	2.3508	0.3550	9.9928
D-Vine$_{sstd}$	8.8884	4.1659	0.1535	16.1244	6.2784	2.9054	0.1311	11.5236	4.3394	2.0450	0.3812	9.2711	3.6972	1.6447	0.3373	8.0169
HS	10.6016	2.2276	0.2163	19.5027	8.4436	1.4523	0.2091	15.6417	4.2393	0.5167	0.4469	9.8768	3.5321	0.2926	0.4039	8.4967
DCC	8.6262	3.9372	0.1447	**15.5687**	5.3761	2.4118	0.1385	**10.1512**	4.5950	2.1132	0.3693	9.3526	3.7984	1.6909	0.3334	8.0589
DCC$_{std}$	8.9299	3.9288	0.1461	16.0246	5.5089	2.4020	0.1376	10.3327	4.8202	2.1112	0.3700	9.5897	3.9311	1.6841	0.3327	8.1848
GO-GARCH	9.9758	4.2837	**0.1364**	17.4573	6.4130	2.7777	0.1332	11.6095	5.1237	2.0951	0.3739	9.9620	4.2322	1.6922	0.3410	8.5672

The statistics were applied under the percentage log-returns

Note Values in bold indicate the models with the best result according to realized loss and realized cost

Table 4 Descriptive statistics [Mean and standard deviation (SD)], loss ($\mathcal{L}_{VaR}$), and realized cost (Cost) of VaR forecasts, considering portfolios with 12 stocks and 1000 observations to estimate the weights

Models	Mean	SD	$\mathcal{L}_{VaR}$	Cost	Mean	SD	$\mathcal{L}_{VaR}$	Cost	Mean	SD	$\mathcal{L}_{VaR}$	Cost	Mean	SD	$\mathcal{L}_{VaR}$	Cost
	Minimum risk				Equally weighted				Minimum risk				Equally weighted			
	$\alpha = 1\%$, Full Sample								$\alpha = 5\%$, Full Sample							
R-Vine$_{norm}$	7.2968	2.5000	0.0846	21.4117	4.2453	1.4449	0.0762	12.8794	3.9912	1.3573	0.2375	12.4394	2.9058	0.9599	**0.2236**	9.7460
R-Vine$_{std}$	10.2685	3.4722	0.1053	29.1482	6.4394	2.2704	0.0857	18.5049	5.0786	1.5986	0.2796	15.2100	3.8369	1.2134	0.2404	11.8635
R-Vine$_{sstd}$	6.8851	2.4895	0.0818	20.2436	4.2693	1.5128	**0.0753**	12.9601	3.5423	1.2453	0.2254	11.2787	2.6326	0.8963	0.2256	**9.1977**
C-Vine$_{norm}$	7.2525	2.5125	0.0824	21.2804	4.2169	1.4840	0.0766	12.8646	3.9549	1.3827	0.2373	12.3232	2.8819	0.9869	0.2240	9.6989
C-Vine$_{std}$	10.1888	3.4159	0.1046	29.1478	6.3310	2.2385	0.0803	18.3057	5.0793	1.6647	0.2803	15.2465	3.8090	1.2462	0.2393	11.7975
C-Vine$_{sstd}$	6.9487	2.3987	0.0842	20.4310	4.3116	1.5078	0.0768	13.0874	3.5708	1.2404	0.2259	11.3282	2.6529	0.8806	0.2254	9.2293
D-Vine$_{norm}$	7.2889	2.5407	0.0856	21.4516	4.2375	1.4281	0.0778	12.9632	3.9510	1.4009	0.2370	12.3366	2.8922	0.9837	0.2244	9.7382
D-Vine$_{std}$	10.2624	3.5576	0.1063	29.3705	6.4636	2.4076	0.0850	18.6904	5.0536	1.6414	0.2786	15.1358	3.8367	1.2855	0.2420	11.8901
D-Vine$_{sstd}$	6.8902	2.4959	**0.0798**	20.2702	4.2513	1.5350	0.0758	12.9661	3.5315	1.2411	**0.2247**	**11.2330**	2.6264	0.8879	0.2288	9.2229
HS	7.2099	0.7517	0.1010	20.7885	4.6577	0.5133	0.1022	13.8982	3.5075	0.3490	0.2495	11.2893	2.6667	0.1022	0.2487	9.3885
DCC	6.6774	2.3052	0.0811	**19.8086**	3.8010	1.2891	0.0806	**11.9183**	3.7541	1.3142	0.2292	11.7188	2.6746	0.9131	0.2266	9.2902
DCC$_{std}$	6.8560	2.2922	0.0827	20.2606	3.8796	1.2819	0.0798	12.0979	3.9091	1.3022	0.2345	12.0841	2.7532	0.9063	0.2260	9.4325
GO-GARCH	8.0889	4.6443	0.0905	24.1028	4.7817	2.9836	0.0810	14.7093	4.3722	1.6676	0.2608	13.8325	3.1872	1.5626	0.2390	10.7197
$\alpha = 1\%$, 2016-2019									$\alpha = 5\%$, 2016-2019							
R-Vine$_{norm}$	6.7953	1.5257	0.0741	25.3767	3.8803	0.7962	**0.0513**	15.0169	3.8376	0.9704	0.2122	14.9515	2.7281	0.6172	0.1803	11.3241
R-Vine$_{std}$	9.5249	1.9301	0.0975	34.5744	5.8032	1.0523	0.0671	21.5180	4.8916	1.0371	0.2607	18.4255	3.6080	0.6947	0.2051	14.0244
R-Vine$_{sstd}$	6.3885	1.5489	0.0711	23.9730	3.8860	0.8734	0.0523	15.0969	3.4318	0.9045	0.1968	13.5452	2.4996	0.5931	0.1789	**10.6531**
C-Vine$_{norm}$	6.7206	1.5624	0.0743	25.1587	3.8410	0.8277	0.0520	14.9590	3.7877	0.9410	0.2097	14.7407	2.7080	0.6146	0.1788	11.2459
C-Vine$_{std}$	9.4670	2.0189	0.0978	34.5291	5.7222	1.1600	0.0664	21.3651	4.8816	1.0705	0.2618	18.4557	3.5733	0.7103	0.2029	13.9111
C-Vine$_{sstd}$	6.4417	1.5648	0.0716	24.1787	3.9133	0.8916	0.0517	15.2118	3.4449	0.8820	0.1993	13.6004	2.5120	0.5801	0.1792	10.6860
D-Vine$_{norm}$	6.8079	1.5716	0.0746	25.4647	3.8915	0.8175	0.0523	15.1273	3.8068	0.9451	0.2124	14.8345	2.7219	0.6247	0.1799	11.3054

Table 4 (continued)

Models	Mean	SD	$\mathcal{L}_{VaR}$	Cost	Mean	SD	$\mathcal{L}_{VaR}$	Cost	Mean	SD	$\mathcal{L}_{VaR}$	Cost	Mean	SD	$\mathcal{L}_{VaR}$	Cost
	Minimum risk				Equally weighted				Minimum risk				Equally weighted			
	$\alpha = 1\%$, Full Sample								$\alpha = 5\%$, Full Sample							
D-Vine$_{std}$	9.5496	2.0726	0.0990	34.8607	5.8626	1.1502	0.0681	21.8481	4.8554	1.0368	0.2594	18.3216	3.5988	0.7215	0.2062	14.0327
D-Vine$_{sstd}$	6.4200	1.6153	0.0701	24.1011	3.8950	0.9005	0.0516	15.1584	3.4211	0.8983	**0.1951**	**13.4773**	2.4960	0.5944	0.1798	10.6607
HS	6.9946	0.6144	0.0757	24.7617	4.3304	0.1113	0.0569	15.9967	3.6447	0.2520	0.2080	13.5630	2.6863	0.0955	0.1872	10.8268
DCC	6.3024	1.5094	**0.0694**	**23.7031**	3.5628	0.8138	0.0526	**14.0728**	3.5774	0.8903	0.2022	14.0037	2.5275	0.5977	**0.1785**	10.7402
DCC$_{std}$	6.4611	1.4896	0.0709	24.2091	3.6324	0.8041	0.0524	14.2830	3.7143	0.8724	0.2077	14.4233	2.5971	0.5877	0.1788	10.9164
GO-GARCH	8.0364	5.3532	0.0854	29.7570	4.7278	3.4424	0.0579	17.9620	4.4053	1.7376	0.2400	17.0507	3.1730	1.7683	0.1977	12.8358
$\alpha = 1\%$, 2020-2022									$\alpha = 5\%$, 2020-2022							
R-Vine$_{norm}$	8.1551	3.4364	0.1025	14.6262	4.8699	1.9905	0.1189	9.2214	4.2541	1.8106	0.2809	8.1403	3.2099	1.3046	**0.2978**	7.0453
R-Vine$_{std}$	11.5409	4.8758	0.1186	19.8622	7.5280	3.1959	0.1176	13.3487	5.3987	2.2208	0.3119	9.7073	4.2286	1.7106	0.3007	8.1654
R-Vine$_{sstd}$	7.7348	3.4011	0.1002	13.8615	4.9251	2.0549	0.1147	9.3032	3.7312	1.6587	0.2745	7.4000	2.8602	1.2230	0.3056	**6.7071**
C-Vine$_{norm}$	8.1627	3.4117	**0.0963**	14.6434	4.8603	2.0365	0.1187	9.2807	4.2410	1.8823	0.2846	8.1861	3.1794	1.3625	0.3014	7.0516
C-Vine$_{std}$	11.4240	4.7190	0.1161	19.9388	7.3729	3.0941	**0.1042**	13.0701	5.4177	2.3191	0.3120	9.7545	4.2123	1.7588	0.3016	8.1805
C-Vine$_{sstd}$	7.8163	3.1983	0.1057	14.0175	4.9932	2.0178	0.1198	9.4519	3.7862	1.6645	**0.2714**	7.4399	2.8939	1.1984	0.3045	6.7363
D-Vine$_{norm}$	8.1120	3.4951	0.1046	14.5840	4.8295	1.9583	0.1215	9.2597	4.1978	1.9238	0.2790	8.0619	3.1836	1.3503	0.3005	7.0563
D-Vine$_{std}$	11.4821	4.9635	0.1188	19.9752	7.4919	3.4335	0.1140	13.2865	5.3929	2.2999	0.3114	9.6839	4.2437	1.8252	0.3032	8.2236
D-Vine$_{sstd}$	7.6948	3.3783	0.0965	13.7142	4.8609	2.1017	0.1171	9.2144	3.7206	1.6560	0.2753	**7.3923**	2.8495	1.2067	0.3127	6.7623
HS	7.5784	0.8195	0.1442	13.9892	5.2179	0.4428	0.1798	10.3071	3.2727	0.3664	0.3205	7.3983	2.6330	0.1046	0.3538	6.9271
DCC	7.3193	3.1413	0.1012	**13.1440**	4.2086	1.7643	0.1285	**8.2313**	4.0566	1.7847	0.2753	7.8085	2.9263	1.2453	0.3088	6.8087
DCC$_{std}$	7.5319	3.1203	0.1029	13.5035	4.3026	1.7518	0.1265	8.3586	4.2423	1.7671	0.2805	8.0810	3.0203	1.2348	0.3067	6.8932
GO-GARCH	8.1787	3.0751	0.0992	14.4267	4.8740	1.9633	0.1205	9.1430	4.3155	1.5402	0.2966	8.3253	3.2114	1.1276	0.3098	7.0984

The statistics were applied under the percentage log-returns

Note Values in bold indicate the models with the best result according to realized loss and realized cost

the increase in confirmed COVID-19 cases occurred, an event that also coincided with the period when the World Health Organization (WHO) officially declared the COVID-19 outbreak as a global pandemic. Furthermore, we observe the similarity of returns obtained through the two portfolio construction strategies: minimum risk and equally weighted.

In Tables 3, 4, 5, 6we describe the mean, standard deviation (SD), realized loss ($\mathcal{L}_{VaR}$), and realized cost (Cost) for portfolio risk forecasts with 6 and 12 stocks, respectively, considering $\alpha = 1\%$ and 5%. The results obtained for $\alpha = 2.5\%$ are described in the Appendix. We note, as expected, that the mean value and standard deviation of risk forecasts are higher in portfolios formed with 6 stocks. In a portfolio with 12 stocks, there is a greater reduction in diversifiable (non-systemic) risks than in a portfolio with 6 stocks. We also verified that the significance level equal to 1% presents higher risk estimates when compared to $\alpha = 5\%$. Furthermore, the values of the risk forecasts for the significance level equal to 2.5% are between the forecasts obtained for α 1% and 5%. This result is expected because more extreme levels are associated with higher losses, implying higher risk forecasts.

For the same significance level, we found that, in general, for a smaller rolling estimation window (500), risk predictions tend to be higher. This result does not hold when we evaluate portfolios with 6 stocks and weights obtained from risk minimization. Some studies, such as Kuester et al. (2006); Alexander and Sheedy (2008), and Righi and Ceretta (2015) argue that larger estimation windows tend to improve the accuracy of risk predictions. Smaller sample sizes are usually associated with estimation problems, such as the bias of the estimators of the parameters of the models used to predict risk. To compare the impact of different estimation windows for risk prediction, we recommend Righi and Ceretta (2016). Moreover, we notice that the risk statistics of the minimum risk and equally weighted portfolios differ, which is in line with the descriptive statistics of the portfolio returns. The equally weighted portfolio tends to be less risky when evaluating out-of-sample forecasts for the same rolling window and significance level. However, it should be noted that our intention here is not to compare the results of the portfolios built using both strategies to determine the weights but to evaluate the impact of different weights in selecting the most accurate model to predict risk.

Furthermore, we noticed that the average risk forecasts for the sub-sample from 2020 to April 2022 tend to be higher than the period from 2016 to 2019. This result was expected because the sample from 2020 to April 2022 includes the period of the COVID-19 outbreak. For this period, we also observed an increase in the standard deviation of the risk forecasts. The higher standard deviation in the forecasts for 2020–2022 can be explained by the period of heavy losses around March 2020 and the market recovery after the shock, as can be seen in Fig. 1. Literature exposes some reasons for market recovery, including the low-interest rates and expansive actions taken by central banks (Cantú et al., 2021). Khalfaoui et al. (2021) and Rouatbi et al. (2021) identify that vaccination was also responsible for stabilizing stock markets. The implementation and acceleration of the vaccination program increased market confidence, spurring the stock market recovery (Khalfaoui et al., 2021).

We notice that in general, Vine models have the best result, with the main emphasis on the case in which the marginal distribution is skewed Student′s t. For example, considering the portfolio with 6 stocks and weights obtained from risk minimization with a sample of 500 observations, we observe R-Vine$_{sstd}$ as the model with the lower realized loss for the full sample and the sub-sample 2016-2019, when we use VaR$^{1\%}$ and VaR$^{2.5\%}$. Vine copulas describe the multivariate relationship between stocks considering a cascade of pair-copulas and the marginal densities (Kurowicka & Cooke, 2006). The variety of bivariate copulas that can be considered allows us to construct a rich multivariate distribution class, which models complex and asymmetric dependency structures (Geidosch & Fischer, 2016). Each of the Vine copulas considered provides a way of arranging the pair-copulas in a tree structure (hierarchical), facilitating the analysis of multiple dependencies. Good performance of Vine copulas to forecast risk measures was also observed in the Monte Carlo simulation performed by Müller and Righi (2020), and empirical analyses of Righi and Ceretta (2015), and Trucíos et al. (2020). The superiority of copula models over GARCH approaches, such as GO-GARCH, can be explained by the greater flexibility of the copulas to model the dependence between variables (Kole et al., 2007). Also, it is observed that the results of the realized loss for the Vine copulas tend to be similar for the same marginal distribution. Regarding the marginal distribution, the skewed Student′s *t* tends to do better results because it considers two important stylized facts in the returns: asymmetry and excess kurtosis.

Although we perceive the superiority of copulas, we observed that the performance of the models tends to change as we consider different significance levels, rolling estimation windows, and analysis periods (full sample, 2006-2019, 2020-2022). Regarding different significance levels, we have that lower significance levels are associated with more extreme observations. For this reason, different models may more accurately accommodate the data characteristics for each level. The different performance results for both rolling estimation windows can be explained because the smaller sample size in general, they are affected by the bias of the estimators of the model parameters employed for risk forecasting. A similar result is found by Wong et al. (2012). Furthermore, when considering a rolling window of 1000 observations, we have more information about the dynamics of returns, which can influence the performance of risk prediction models. When analyzing the out-of-sample samples used to present the results, we noticed in many scenarios that the model with the lowest realized loss for the full sample tends to be the same as for the 2016-2019 period. By an illustration, for portfolios with 6 stocks and a rolling estimation window of 1000 observations, considering minimum risk and equally weighted portfolio, and $\alpha = 2.5\%$, the lowest realized loss is presented by D-Vine$_{sstd}$. However, some exceptions are found. For example, for a portfolio of minimum risk, considering a portfolio of 6 stocks, a rolling estimation window of 1000 observations, and $\alpha = 1\%$, the DCC model presents the best result for three out-of-sample periods, that is, the lowest realized loss. We also observe that the model's performance is not generally maintained for the results obtained for the two weighting strategies used to build the portfolios. However, we note that when we consider portfolios with 6 stocks and a rolling estimation window of 1000 observations, the performance of the models tends to coincide for both strategies of determining the weights (this result holds for the full sample and 2016-2019). From a practical point of view, it is clear that when the manager performs the portfolio rebalancing, he does need to worry about choosing a new risk prediction model.

In Fig. 2, we display the evolution of risk estimations considering minimum risk portfolio with 6 stocks and $\alpha = 1\%$ and rolling estimation window of 500 observations. The VaR plotted had its signal converted so that the plot of VaR can be compared to the negative portfolio returns. For brevity, we omit the illustrations from the other risk estimates. However, they are available under request. Visually, we can verify that the risk estimates obtained via the DCC and copula approach, considering the normal and skewed Student's *t* distribution, tend to follow better the behavior of the portfolio returns, which corroborates the good results of both forecasting approaches according to the realized loss. We visually verify that the predictions obtained by the copula approach using Student's *t* distribution as marginal distribution results in predictions that do not follow the evolution of the series and are visually volatile, which is confirmed by checking the standard deviation of the predictions obtained by these models. A similar evolution is displayed by the GO-GARCH model forecasts, mainly between 2016 and 2017. For the full sample and 2016-2019 results, we identified that the GO-GARCH model is among the models with the highest realized loss (worst result). This result does not corroborate the findings of Tiwari et al. (2020), which identify consistent results between GO-GARCH and DCC for risk analysis. Regarding HS, we observe that this approach does not follow the evolution of portfolio returns. Thus, as observed by Müller and Righi (2020), we note that the HS tends to overestimate the risk in calm periods and underestimate the risk in turbulent periods. One of the reasons for this is that HS responds slowly to volatility and price movement changes (Pritsker, 2006). For a discussion of the use of HS in risk forecasting, we recommend Christoffersen and Gonçalves (2005) and Pritsker (2006).

Regarding the results of the cost realized, we verified that the results differ from the loss realized. For a significance level equal to 1%, the DCC model shows the lower realized cost. This model also performs well for $\alpha = 2.5\%$ and an equally weighted portfolio. Corroborating, Weiß (2013) identifies that DCC models are not outperformed by the copula models, in the bivariate sense, for estimating VaR and ES. In the other scenarios, R-Vine$_{sstd}$ and D-Vine$_{sstd}$ present the best results (lower realized cost). Thus, DCC, R-Vine$_{sstd}$ and D-Vine$_{sstd}$ present the best trade-off between the sum of the costs from underestimation and overestimation. The highest values for the realized cost are identified copulas models with Student's *t* distribution. The differences in the results identified by both criteria can be explained by the fact that the VaR loss function penalizes more aggressively those observations for which we observe returns showing risk estimates exceedance and consider only forecasting errors rather than the costs associated with such errors (Righi et al., 2020). However, it is worth noting that the results of the realized cost are conditioned to the costs from risk overestimation and underestimation paid by the manager. A thorough analysis of the costs incurred by incorrect risk forecasting is essential because it reduces profitability (risk overestimation) and increases costs from unexpected and uncovered losses (risk underestimation).

5 Concluding Remarks

This study analyzed the performance of multivariate models to predict VaR. The models considered were HS, GARCH-DCC with multivariate normal and Student's *t* distribution, GO-GARCH, C-Vine, D-Vine, and R-Vine. For copula models, we consider normal, Student's *t* and skewed Student's *t* distribution as marginal distribution. To evaluate the performance of the models, we considered portfolios of minimum ES

Table 6 Descriptive statistics [Mean and standard deviation (SD)], loss ($\mathcal{L}_{VaR}$), and realized cost (Cost) of VaR forecasts, considering portfolios with 6 stocks and 1000 observations to estimate the weights

Models	Mean	SD	$\mathcal{L}_{VaR}$	Cost	Mean	SD	$\mathcal{L}_{VaR}$	Cost	Mean	SD	$\mathcal{L}_{VaR}$	Cost	Mean	SD	$\mathcal{L}_{VaR}$	Cost
	Minimum risk				Equally weighted				Minimum risk				Equally weighted			
$\alpha = 1\%$, Full Sample									$\alpha = 5\%$, Full Sample							
R-Vine$_{norm}$	9.5485	3.8157	0.1140	29.5440	5.1672	2.0152	0.0881	16.4623	5.0098	2.1476	0.3224	16.9808	3.5772	1.4324	0.2623	12.4209
R-Vine$_{std}$	12.8996	4.9515	0.1378	38.8298	7.8611	3.0010	0.0965	23.8141	6.3324	2.4910	0.3647	20.3867	4.6750	1.7895	0.2844	15.1061
R-Vine$_{sstd}$	9.1680	3.7605	0.1159	28.3730	5.3258	2.0978	0.0897	16.7731	4.4719	1.9569	0.3101	15.4372	3.2760	1.2888	0.2623	**11.6838**
C-Vine$_{norm}$	9.6039	3.8453	0.1153	29.7698	5.1169	2.0250	0.0867	16.3322	5.1114	2.1691	0.3225	17.1896	3.6018	1.4371	0.2630	12.4715
C-Vine$_{std}$	12.8786	5.1405	0.1391	38.9108	7.7214	3.0098	0.0980	23.5691	6.3791	2.5415	0.3650	20.4882	4.6732	1.7976	0.2865	15.1517
C-Vine$_{sstd}$	9.1491	3.5703	0.1117	28.2532	5.2893	2.1264	0.0888	16.7630	4.5135	1.8783	0.3113	15.5041	3.3008	1.2830	0.2616	11.7162
D-Vine$_{norm}$	9.6099	3.8157	0.1172	29.9820	5.2065	2.0173	0.0875	16.6314	4.9910	2.1548	0.3167	16.8883	3.5690	1.4200	0.2630	12.4477
D-Vine$_{std}$	13.1076	5.0285	0.1376	39.3832	8.0399	3.1818	0.0936	24.2539	6.3330	2.5528	0.3623	20.3223	4.7074	1.8126	0.2867	15.2032
D-Vine$_{sstd}$	9.2237	3.7758	0.1119	28.5219	5.3743	2.1403	**0.0849**	16.9179	4.4608	1.9039	**0.3028**	**15.3270**	3.2870	1.3021	**0.2602**	11.7006
HS	9.7876	1.1660	0.1482	29.6154	5.6798	0.2844	0.1202	17.6920	4.6189	0.8025	0.3477	15.8007	3.4052	0.3766	0.2954	12.1201
DCC	8.9019	3.6524	**0.1111**	**27.8833**	4.7682	1.8960	0.0911	**15.5815**	4.7851	2.0526	0.3159	16.2334	3.3482	1.3483	0.2612	11.8895
DCC$_{std}$	9.1273	3.6280	0.1128	28.4994	4.8644	1.8861	0.0903	15.8255	5.0075	2.0374	0.3213	16.7788	3.4444	1.3386	0.2615	12.0993
GO-GARCH	11.8147	6.5333	0.1357	37.2063	6.6930	3.9737	0.0937	21.2922	5.9657	2.9684	0.3599	20.1733	4.3181	2.2184	0.2832	14.7505
$\alpha = 1\%$, 2016-2019									$\alpha = 5\%$, 2016-2019							
R-Vine$_{norm}$	9.4468	3.0697	0.0999	36.3079	5.0266	1.5580	0.0645	20.0072	5.2001	1.8908	0.2915	21.0705	3.5788	1.1683	0.2241	15.0377
R-Vine$_{std}$	12.7312	3.7809	0.1304	47.9797	7.6650	2.1300	0.0846	29.2601	6.5090	2.1207	0.3494	25.6046	4.6698	1.3752	0.2627	18.6823
R-Vine$_{sstd}$	9.1097	3.0065	0.0978	35.0850	5.1483	1.5553	0.0662	20.3754	4.6226	1.7198	0.2770	19.1178	3.2902	1.0770	0.2195	**14.1034**
C-Vine$_{norm}$	9.4944	3.1008	0.1010	36.5526	4.9772	1.5600	0.0643	19.8567	5.2823	1.8822	0.2954	21.3074	3.5920	1.1767	0.2246	15.0716
C-Vine$_{std}$	12.6915	3.9121	0.1299	47.9970	7.5073	2.1660	0.0841	28.9002	6.5631	2.1651	0.3503	25.7556	4.6677	1.3876	0.2632	18.7069
C-Vine$_{sstd}$	9.0539	2.9567	0.0973	34.9185	5.1215	1.6217	0.0653	20.3992	4.6731	1.6981	0.2752	19.1607	3.3045	1.0786	0.2185	14.1307
D-Vine$_{norm}$	9.6301	3.1842	0.1014	37.0525	5.1094	1.5750	0.0642	20.2889	5.1968	1.9360	0.2941	21.0886	3.6002	1.1824	0.2258	15.1146

Table 6 (continued)

Models	Mean	SD	$\mathcal{L}_{VaR}$	Cost	Mean	SD	$\mathcal{L}_{VaR}$	Cost	Mean	SD	$\mathcal{L}_{VaR}$	Cost	Mean	SD	$\mathcal{L}_{VaR}$	Cost
	Minimum risk				Equally weighted				Minimum risk				Equally weighted			
$\alpha = 1\%$, Full Sample									$\alpha = 5\%$, Full Sample							
D-Vine$_{std}$	12.9280	3.8097	0.1323	48.7012	7.8280	2.1503	0.0871	29.8938	6.4782	2.1072	0.3475	25.4681	4.6980	1.3773	0.2641	18.7764
D-Vine$_{sstd}$	9.1685	3.0405	0.0977	35.3195	5.2180	1.6401	0.0640	20.6551	4.6182	1.7087	**0.2708**	**19.0191**	3.3019	1.1054	**0.2179**	14.1481
HS	10.2766	0.9213	0.1103	36.9237	5.8653	0.1088	0.0754	21.7023	5.1514	0.4727	0.2998	19.6982	3.6740	0.1520	0.2419	14.7195
DCC	8.9194	3.0505	**0.0966**	**34.6194**	4.7615	1.5665	**0.0639**	**19.1873**	4.8882	1.7853	0.2850	20.0378	3.3680	1.1435	0.2189	14.3749
DCC$_{std}$	9.1461	3.0358	0.0983	35.3746	4.8583	1.5609	0.0640	19.5006	5.1122	1.7757	0.2920	20.7365	3.4648	1.1378	0.2209	14.6608
GO-GARCH	12.9542	6.2947	0.1326	48.4738	7.3129	3.8034	0.0797	27.6129	6.6138	2.8533	0.3586	26.1665	4.7241	2.0988	0.2664	18.8478
$\alpha = 1\%$, 2020-2022									$\alpha = 5\%$, 2020-2022							
R-Vine$_{norm}$	9.7227	4.8304	0.1380	17.9690	5.4078	2.6027	0.1285	10.3959	4.6841	2.4955	0.3754	9.9820	3.5744	1.7976	0.3275	7.9429
R-Vine$_{std}$	13.1878	6.4757	0.1507	23.1716	8.1967	4.0617	0.1167	14.4942	6.0301	2.9991	0.3908	11.4573	4.6838	2.3351	0.3214	8.9861
R-Vine$_{sstd}$	9.2678	4.7841	0.1469	16.8866	5.6296	2.7669	0.1300	10.6086	4.2139	2.2856	0.3667	9.1386	3.2518	1.5877	0.3355	7.5432
C-Vine$_{norm}$	9.7912	4.8593	0.1398	18.1623	5.3561	2.6215	0.1249	10.3007	4.8188	2.5626	0.3688	10.1428	3.6186	1.7983	0.3287	8.0219
C-Vine$_{std}$	13.1988	6.7343	0.1547	23.3615	8.0878	4.0425	0.1217	14.4458	6.0642	3.0573	0.3903	11.4739	4.6827	2.3395	0.3263	9.0676
C-Vine$_{sstd}$	9.3120	4.4257	0.1363	16.8469	5.5765	2.7638	0.1290	10.5403	4.2403	2.1259	0.3732	9.2464	3.2943	1.5735	0.3353	7.5843
D-Vine$_{norm}$	9.5754	4.7067	0.1444	17.8822	5.3726	2.5988	0.1274	10.3723	4.6388	2.4469	**0.3556**	9.7003	3.5155	1.7534	0.3267	7.8838
D-Vine$_{std}$	13.4150	6.6052	0.1467	23.4373	8.4025	4.3991	**0.1047**	14.6023	6.0844	3.1600	0.3876	11.5162	4.7234	2.3808	0.3255	9.0882
D-Vine$_{sstd}$	9.3183	4.7802	0.1363	16.8890	5.6417	2.7773	0.1207	10.5224	4.1914	2.1734	0.3574	**9.0088**	3.2614	1.5838	0.3325	**7.5121**
HS	8.9508	1.0609	0.2132	17.1087	5.3624	0.1985	0.1968	10.8292	3.7076	0.2175	0.4297	9.1307	2.9451	0.0971	0.3870	7.6719
DCC	8.8720	4.5024	**0.1359**	**16.3559**	4.7796	2.3568	0.1378	**9.4109**	4.6089	2.4348	0.3689	9.7228	3.3142	1.6411	0.3336	7.6361
DCC$_{std}$	9.0949	4.4657	0.1376	16.7339	4.8749	2.3417	0.1354	9.5362	4.8285	2.4117	0.3715	10.0059	3.4095	1.6262	0.3309	7.7158
GO-GARCH	9.8648	6.4780	0.1410	17.9242	5.6320	4.0374	0.1176	10.4757	4.8568	2.8324	0.3621	9.9171	3.6234	2.2467	**0.3119**	7.7387

The statistics were applied under the percentage log-returns

Note Values in bold indicate the models with the best result according to realized loss and realized cost

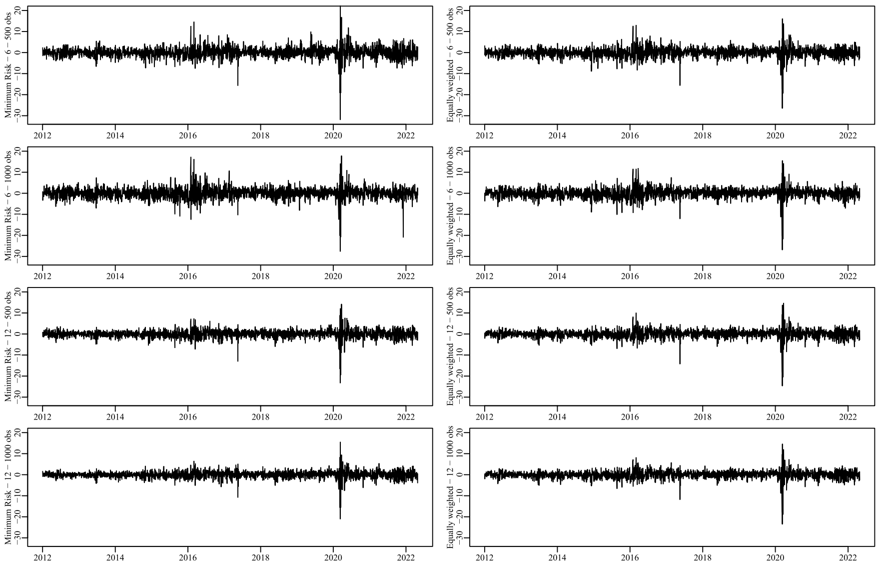

Fig. 1 Returns on portfolios built with 6 and 12 stocks, considering the minimum risk and equally weighted strategies. For portfolios with 6 and 12 stocks, the weights were computed using 500 and 1000 observations to estimate portfolio weights. The sampling period comprises daily data from January 2012 to April 2022. The portfolio returns are multiplied by 100

with cardinality restriction and equally weighted obtained from the stocks belonging to the Ibovespa index considering 500 and 1000 observations to quantify the weights. We form portfolios with 6 and 12 stocks. The risk forecasts are obtained using two rolling estimation windows (500 and 1000) and significance levels equal to 1%, 2.5%, and 5%. To assess VaR forecasting, we consider realized loss and cost. We quantify the VaR forecasts from January 2012 to April 2022.

In general, we observe that risk estimates are sensitive to the choice of significance level, strategy to determine the weights, and estimation window. We observed that, according to the realized loss, the copulas considering skewed Student's *t* distribution tend to present better performance. Another interesting result is that the model indicated by the realized loss, obtained via the score function, does not coincide with the model's best performance according to the realized cost. According to realized cost, DCC, R-Vine$_{sstd}$, and D-Vine$_{sstd}$ present the best performance, i.e., these models have the best trade-off between the sum of the costs of overestimation and underestimation. Knowing the performance of risk estimates and their behavior under different scenarios is important for both regulators and investors since the risk is one of the most important variables, along with return, in financial decision-making. For future research, we recommend further investigating the impact of the weight on stocks that make up the portfolio on the performance of risk forecasting models. In addition, it is suggested that other models be included and portfolios with more stocks investigated than those considered in this study. Finally, we recommend that future work considers data from other markets and different data frequencies, including weekly and intraday data.

Table 5 Descriptive statistics [Mean and standard deviation (SD)], loss ($\mathcal{L}_{VaR}$), and realized cost (Cost) of VaR forecasts, considering portfolios with 12 stocks and 500 observations to estimate the weights

Models	Mean	SD	$\mathcal{L}_{VaR}$	Cost	Mean	SD	$\mathcal{L}_{VaR}$	Cost	Mean	SD	$\mathcal{L}_{VaR}$	Cost	Mean	SD	$\mathcal{L}_{VaR}$	Cost
	Minimum risk				Equally weighted				Minimum risk				Equally weighted			
$\alpha = 1\%$, Full Sample									$\alpha = 5\%$, Full Sample							
R-Vine$_{norm}$	7.7910	2.9980	0.0947	22.7401	4.7999	1.7903	0.0874	14.4176	4.1214	1.6488	0.2583	12.9878	3.2823	1.2251	0.2465	10.8236
R-Vine$_{std}$	10.8361	3.9170	0.1136	31.0808	7.5339	2.6870	0.0950	21.8989	5.2651	1.8921	0.2987	16.0264	4.3534	1.4932	0.2710	13.5348
R-Vine$_{sstd}$	7.2675	2.9310	0.0916	21.3391	4.8190	1.9077	0.0867	14.4814	3.6651	1.4557	**0.2436**	11.7891	2.9481	1.1091	**0.2446**	10.0790
C-Vine$_{norm}$	7.6617	2.9105	0.0906	22.4655	4.7416	1.8090	0.0863	14.2952	4.0394	1.5121	0.2558	12.8277	3.2219	1.1746	0.2456	10.7277
C-Vine$_{std}$	10.6517	3.9758	0.1103	30.4724	7.3042	2.7373	0.0927	21.1338	5.2293	1.9055	0.2931	15.8050	4.3220	1.4900	0.2663	13.3622
C-Vine$_{sstd}$	7.3669	3.0540	0.0909	21.5977	4.9468	2.0194	**0.0856**	14.7287	3.7059	1.4960	0.2465	11.8909	2.9879	1.1588	0.2455	10.1485
D-Vine$_{norm}$	7.6747	2.9789	0.0873	22.4134	4.7725	1.7537	0.0872	14.4142	4.0643	1.5995	0.2563	12.8234	3.2317	1.1994	0.2457	10.7112
D-Vine$_{std}$	10.6596	4.1280	0.1140	30.6151	7.3346	2.6853	0.0983	21.4013	5.2289	1.9180	0.2976	15.8971	4.3185	1.5222	0.2717	13.4541
D-Vine$_{sstd}$	7.1178	2.7881	0.0913	21.0124	4.7218	1.8792	0.0868	14.2158	3.6445	1.4717	0.2451	**11.7688**	2.9126	1.0967	0.2468	**10.0606**
HS	7.8880	1.9865	0.1173	22.4184	5.6790	1.8891	0.1144	15.9950	3.7227	0.6725	0.2724	12.2173	2.9798	0.4371	0.2712	10.4066
DCC	7.0275	2.7015	**0.0873**	**20.7759**	4.2463	1.5883	0.0898	**13.1830**	3.8866	1.5257	0.2515	12.3203	2.9981	1.1186	0.2463	10.2247
DCC$_{std}$	7.2483	2.6973	0.0890	21.3451	4.3495	1.5860	0.0896	13.4358	4.0801	1.5222	0.2576	12.7843	3.1014	1.1161	0.2461	10.4269
GO-GARCH	8.7201	5.7566	0.1024	26.0990	5.3975	3.7196	0.0999	16.6569	4.6666	2.9174	0.2929	14.9261	3.6559	2.2508	0.2740	12.2562
$\alpha = 1\%$, 2016-2019									$\alpha = 5\%$, 2016-2019							
R-Vine$_{norm}$	7.0457	1.7876	0.0804	26.5377	4.3081	0.9271	0.0627	16.7171	3.8242	1.1256	0.2214	15.2393	2.9873	0.7408	0.2007	12.4676
R-Vine$_{std}$	9.9353	2.5118	0.1045	36.7204	6.9095	1.6119	0.0780	25.8922	4.9361	1.3953	0.2677	19.0837	4.0557	0.9811	0.2339	16.0104
R-Vine$_{sstd}$	6.5873	1.7916	0.0735	24.8813	4.2898	1.0332	0.0610	16.6880	3.4100	1.0514	0.2077	13.8060	2.6841	0.7059	**0.1956**	11.5180
C-Vine$_{norm}$	6.9957	1.7540	0.0797	26.3810	4.2458	0.9162	**0.0592**	16.4830	3.8061	1.1144	0.2198	15.1503	2.9704	0.7361	0.1990	12.3839
C-Vine$_{std}$	9.7040	2.4233	0.1006	35.8614	6.6213	1.5275	0.0764	24.8290	4.8911	1.2882	0.2630	18.8192	4.0134	0.9170	0.2306	15.7796
C-Vine$_{sstd}$	6.6330	1.8968	0.0770	25.1085	4.3331	1.0677	0.0622	16.8297	3.4420	1.0697	0.2096	13.9214	2.7010	0.7114	0.1972	11.5714
D-Vine$_{norm}$	6.9786	1.7387	0.0780	26.3204	4.2859	0.9525	0.0606	16.6719	3.7866	1.0858	0.2207	15.0851	2.9550	0.7322	0.2000	12.3542

Table 5 (continued)

Models	Mean	SD	$\mathcal{L}_{VaR}$	Cost	Mean	SD	$\mathcal{L}_{VaR}$	Cost	Mean	SD	$\mathcal{L}_{VaR}$	Cost	Mean	SD	$\mathcal{L}_{VaR}$	Cost
	Minimum risk				Equally weighted				Minimum risk				Equally weighted			
$\alpha = 1\%$, Full Sample									$\alpha = 5\%$, Full Sample							
D-Vine$_{std}$	9.7097	2.4883	0.1031	36.0391	6.6916	1.5985	0.0785	25.2058	4.9018	1.3012	0.2665	18.9290	4.0086	0.9567	0.2320	15.8518
D-Vine$_{sstd}$	6.4610	1.8200	0.0754	24.5902	4.1856	1.0332	0.0597	16.3345	3.3906	1.0509	**0.2064**	**13.7610**	2.6567	0.7201	0.1982	**11.5004**
HS	6.8715	1.1305	0.0760	25.1119	4.4439	0.5442	0.0636	16.9326	3.6065	0.6736	0.2163	14.1496	2.8166	0.3826	0.2043	11.7494
DCC	6.4475	1.7054	**0.0732**	**24.4593**	3.8756	0.9416	0.0593	**15.3376**	3.5996	1.0329	0.2150	14.4161	2.7585	0.6952	0.1971	11.7379
DCC$_{std}$	6.6382	1.6920	0.0747	25.1005	3.9648	0.9344	0.0594	15.6296	3.7675	1.0251	0.2203	14.9454	2.8476	0.6879	0.1978	11.9845
GO-GARCH	7.6146	4.6001	0.0856	30.8902	4.6709	2.7240	0.0702	19.5425	4.1304	2.4847	0.2450	17.5719	3.2191	1.8748	0.2188	14.2587
$\alpha = 1\%$, 2020-2022									$\alpha = 5\%$, 2020-2022							
R-Vine$_{norm}$	9.0642	4.0412	0.1192	16.2525	5.6400	2.4697	0.1296	10.4892	4.6290	2.1899	0.3212	9.1416	3.7862	1.6517	0.3248	8.0151
R-Vine$_{std}$	12.3749	5.2013	0.1292	21.4467	8.6007	3.6506	0.1238	15.0769	5.8271	2.4242	0.3516	10.8036	4.8619	1.9972	0.3345	9.3057
R-Vine$_{sstd}$	8.4294	3.9574	0.1227	15.2879	5.7231	2.5969	0.1306	10.7117	4.1008	1.8853	**0.3050**	**8.3436**	3.3991	1.4699	0.3282	7.6205
C-Vine$_{norm}$	8.7994	3.9563	0.1093	15.7766	5.5886	2.5096	0.1327	10.5578	4.4378	1.9555	0.3171	8.8600	3.6514	1.5879	0.3251	7.8985
C-Vine$_{std}$	12.2707	5.3531	0.1270	21.2661	8.4709	3.7636	**0.1204**	14.8211	5.8070	2.5450	0.3446	10.6558	4.8493	2.0349	0.3272	9.2325
C-Vine$_{sstd}$	8.6207	4.0793	0.1146	15.6001	5.9952	2.7132	0.1256	11.1394	4.1568	1.9464	0.3097	8.4220	3.4780	1.5473	0.3279	7.7178
D-Vine$_{norm}$	8.8638	4.0801	**0.1033**	15.7390	5.6037	2.3855	0.1327	10.5573	4.5387	2.1364	0.3173	8.9596	3.7044	1.6215	**0.3239**	7.9044
D-Vine$_{std}$	12.2823	5.6067	0.1326	21.3492	8.4329	3.6423	0.1321	14.9021	5.7878	2.5658	0.3507	10.7176	4.8478	2.0671	0.3396	9.3580
D-Vine$_{sstd}$	8.2399	3.6626	0.1185	14.9003	5.6377	2.5333	0.1330	10.5964	4.0783	1.9199	0.3112	8.3655	3.3497	1.4392	0.3297	**7.6009**
HS	9.6246	1.9302	0.1878	17.8169	7.7891	1.4489	0.2013	14.3932	3.9211	0.6229	0.3681	8.9164	3.2586	0.3800	0.3854	8.1127
DCC	8.0184	3.6408	0.1113	**14.4834**	4.8794	2.1650	0.1421	**9.5024**	4.3769	2.0263	0.3138	8.7400	3.4076	1.5166	0.3304	7.6398
DCC$_{std}$	8.2905	3.6202	0.1135	14.9298	5.0066	2.1545	0.1411	9.6882	4.6143	2.0081	0.3212	9.0923	3.5348	1.5066	0.3288	7.7658
GO-GARCH	10.6086	6.9287	0.1311	17.9139	6.6389	4.7307	0.1507	11.7274	5.5826	3.3449	0.3749	10.4061	4.4021	2.6158	0.3684	8.8353

The statistics were applied under the percentage log-returns

Note Values in bold indicate the models with the best result according to realized loss and realized cost

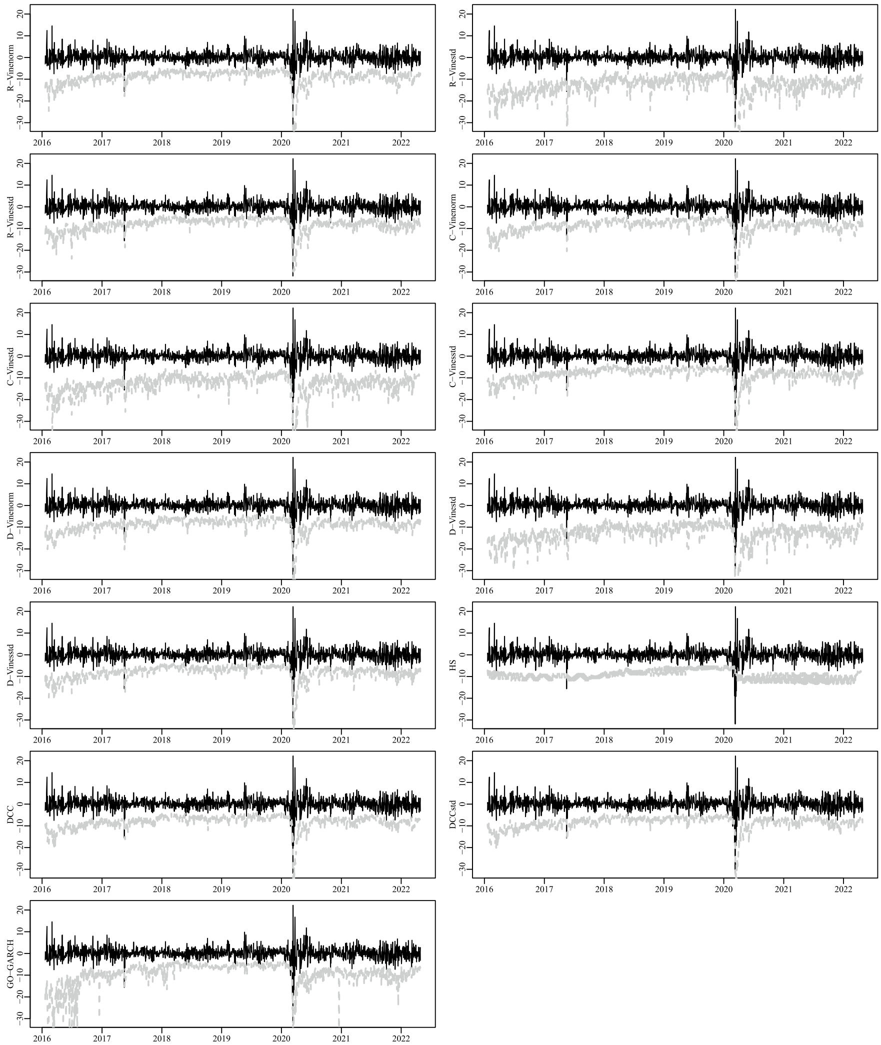

Fig. 2 Returns and risk forecasts on portfolios built with 6, considering the minimum risk portfolio, $\alpha = 1\%$ and a rolling estimation window equal to 500 observations. The out-of-sample period comprises data from January 2016 to April 2022. The risk forecasts are obtained by: R-Vine, C-Vine, and D-Vine, considering normal, Student's t and skewed Student's t distribution for marginal distribution, DCC model using normal and Student's t distribution, HS, and GO-GARCH model. *Note* The solid line refers to the portfolio returns with 6 stocks. We obtain the portfolio weight considering a minimum risk portfolio (see Eq. (2)). The dashed lines refer to the risk forecasts. We illustrate the VaR predictions obtained by the different models we use, considering $\alpha = 1\%$ and a rolling estimation window equal to 500 observations

Appendix

See Tables 7, 8, 9, 10

Table 7 Descriptive statistics [Mean and standard deviation (SD)], loss ($\mathcal{L}_{VaR}$), and realized cost (Cost) of VaR forecasts, considering portfolios with 6 stocks, considering 500 observations to estimate the weights and $\alpha = 2.5\%$

Models	Mean	SD	$\mathcal{L}_{VaR}$	Cost	Mean	SD	$\mathcal{L}_{VaR}$	Cost
	Minimum risk				Equally weighted			
Full sample								
R-Vine$_{norm}$	6.4296	2.4389	0.2037	19.8252	4.6691	1.6370	0.1675	14.5228
R-Vine$_{std}$	8.9005	3.3544	0.2475	26.9255	6.7671	2.4236	0.2021	20.4981
R-Vine$_{sstd}$	5.7510	2.2639	**0.1898**	**18.0063**	4.3150	1.6993	**0.1649**	**13.7126**
C-Vine$_{norm}$	6.3804	2.4835	0.2004	19.7858	4.6058	1.6726	0.1669	14.4331
C-Vine$_{std}$	8.8348	3.2499	0.2475	26.7081	6.6692	2.3774	0.1997	20.2374
C-Vine$_{sstd}$	5.9302	2.4040	0.1949	18.4588	4.4077	1.7071	0.1697	13.9000
D-Vine$_{norm}$	6.4200	2.4758	0.2031	19.8122	4.6654	1.7534	0.1691	14.5801
D-Vine$_{std}$	8.9146	3.4620	0.2517	26.9479	6.7406	2.4236	0.2010	20.4735
D-Vine$_{sstd}$	5.8043	2.3427	0.1984	18.2920	4.3055	1.7001	0.1660	13.7008
HS	6.1341	1.2124	0.2257	18.9902	4.7412	0.8964	0.2024	14.7294
DCC	6.0154	2.3064	0.1945	18.7884	4.2664	1.5834	0.1667	13.7135
DCC$_{std}$	6.2739	2.3028	0.1984	19.4618	4.3974	1.5749	0.1671	14.0314
GO-GARCH	6.7697	3.9022	0.2073	21.9164	4.8720	2.8620	0.1774	16.1685
2016–2019								
R-Vine$_{norm}$	6.1881	1.9412	0.1732	24.0155	4.4022	1.1991	0.1361	17.3148
R-Vine$_{std}$	8.6936	2.7112	0.2305	33.2739	6.5211	1.8204	0.1809	25.0975
R-Vine$_{sstd}$	5.5024	1.9388	**0.1579**	**21.7494**	4.0319	1.2685	0.1346	**16.2965**
C-Vine$_{norm}$	6.1952	2.0259	0.1754	24.1187	4.3829	1.2346	0.1372	17.2872
C-Vine$_{std}$	8.6061	2.6296	0.2267	32.8965	6.4314	1.7950	0.1774	24.7540
C-Vine$_{sstd}$	5.6214	1.9595	0.1613	22.1724	4.0787	1.2360	0.1364	16.3854
D-Vine$_{norm}$	6.2016	1.9512	0.1754	24.0666	4.4217	1.2249	0.1378	17.4151
D-Vine$_{std}$	8.6828	2.7125	0.2294	33.2100	6.5363	1.8450	0.1782	25.1313
D-Vine$_{sstd}$	5.5693	1.9299	0.1661	22.0752	4.0356	1.2644	0.1371	16.3128
HS	5.8928	1.2280	0.1691	22.4225	4.3665	0.7190	0.1417	16.9090
DCC	5.8585	1.9318	0.1670	22.9304	4.1097	1.2404	**0.1345**	16.5006
DCC$_{std}$	6.1147	1.9232	0.1716	23.7825	4.2396	1.2308	0.1355	16.9095
GO-GARCH	6.5385	4.3542	0.1864	27.1957	4.6792	3.1899	0.1539	19.9369
2020–2022								
R-Vine$_{norm}$	6.8421	3.0683	0.2559	12.6668	5.1251	2.1160	0.2210	9.7532
R-Vine$_{std}$	9.2541	4.2123	0.2765	16.0803	7.1874	3.1590	0.2385	12.6406
R-Vine$_{sstd}$	6.1758	2.6801	0.2442	**11.6118**	4.7987	2.1695	0.2168	9.2984
C-Vine$_{norm}$	6.6966	3.0899	0.2433	12.3838	4.9866	2.1793	0.2176	9.5573
C-Vine$_{std}$	9.2255	4.0710	0.2830	16.1363	7.0754	3.0910	0.2378	12.5215
C-Vine$_{sstd}$	6.4578	2.9432	0.2522	12.1146	4.9698	2.1881	0.2267	9.6541
D-Vine$_{norm}$	6.7931	3.1446	0.2506	12.5444	5.0816	2.3442	0.2225	9.7369
D-Vine$_{std}$	9.3107	4.4350	0.2898	16.2502	7.0895	3.1485	0.2400	12.5164
D-Vine$_{sstd}$	6.2056	2.8736	0.2535	11.8290	4.7667	2.1830	**0.2154**	9.2386
HS	6.5463	1.0664	0.3223	13.1266	5.3812	0.8025	0.3062	11.0059
DCC	6.2835	2.8157	**0.2414**	11.7125	4.5342	2.0133	0.2218	**8.9523**
DCC$_{std}$	6.5460	2.8173	0.2441	12.0806	4.6669	2.0051	0.2212	9.1146
GO-GARCH	7.1646	2.9369	0.2430	12.8977	5.2012	2.1543	0.2175	9.7307

The statistics were applied under the percentage log-returns

Note Values in bold indicate the models with the best result according to realized loss and realized cost

Table 8 Descriptive statistics [Mean and standard deviation (SD)], loss ($\mathcal{L}_{VaR}$), and realized cost (Cost) of VaR forecasts, considering portfolios with 12 stocks, considering 500 observations to estimate the weights and $\alpha = 2.5\%$

Models	Mean	SD	$\mathcal{L}_{VaR}$	Cost	Mean	SD	$\mathcal{L}_{VaR}$	Cost
	Minimum risk				Equally weighted			
Full sample								
R-Vine$_{norm}$	5.7820	2.2538	0.1738	17.3533	3.9968	1.4809	0.1544	12.4320
R-Vine$_{std}$	7.6047	2.6981	0.2062	22.3199	5.6513	1.9543	0.1728	16.8857
R-Vine$_{sstd}$	5.1901	2.0531	0.1614	15.7544	3.7474	1.4332	**0.1541**	11.8116
C-Vine$_{norm}$	5.6626	2.1165	0.1698	17.0786	3.9328	1.4609	0.1549	12.3474
C-Vine$_{std}$	7.5265	2.7087	0.2019	22.0444	5.5573	1.9114	0.1718	16.5671
C-Vine$_{sstd}$	5.2784	2.1275	0.1624	15.9520	3.8027	1.4738	0.1543	11.9377
D-Vine$_{norm}$	5.7072	2.1668	0.1676	17.0942	3.9492	1.4627	0.1541	12.3471
D-Vine$_{std}$	7.4946	2.7996	0.2056	22.0055	5.5895	1.9911	0.1775	16.7713
D-Vine$_{sstd}$	5.1302	1.9673	**0.1607**	**15.6078**	3.6897	1.4079	0.1551	11.7381
HS	5.3514	1.0492	0.1912	16.3167	3.8925	0.7600	0.1832	12.2101
DCC	5.3529	2.0860	0.1617	16.1315	3.5831	1.3519	0.1551	**11.5128**
DCC$_{std}$	5.5634	2.0825	0.1663	16.6723	3.6864	1.3490	0.1552	11.7487
GO-GARCH	6.4774	4.1010	0.1906	19.8070	4.4544	2.9230	0.1769	14.2302
2016–2019								
R-Vine$_{norm}$	5.2849	1.4294	0.1483	20.3536	3.6101	0.8282	0.1193	14.3852
R-Vine$_{std}$	7.0236	1.8391	0.1855	26.4820	5.2281	1.1761	0.1474	20.0252
R-Vine$_{sstd}$	4.7738	1.3564	**0.1351**	18.5018	3.3702	0.8098	**0.1151**	13.5649
C-Vine$_{norm}$	5.2372	1.3988	0.1456	20.1549	3.5775	0.8261	0.1187	14.2864
C-Vine$_{std}$	6.9556	1.7394	0.1827	26.1629	5.1038	1.1389	0.1448	19.5394
C-Vine$_{sstd}$	4.8306	1.4015	0.1388	18.7135	3.3913	0.8473	0.1191	13.6893
D-Vine$_{norm}$	5.2370	1.3748	0.1458	20.1213	3.5726	0.8274	0.1186	14.2782
D-Vine$_{std}$	6.9099	1.7413	0.1855	26.0731	5.1392	1.1923	0.1467	19.7663
D-Vine$_{sstd}$	4.7226	1.3400	0.1351	**18.3560**	3.3241	0.8543	0.1185	13.5163
HS	4.9790	0.8650	0.1409	18.8003	3.4485	0.3966	0.1234	13.5824
DCC	4.9228	1.3392	0.1384	18.9824	3.2800	0.8072	0.1163	**13.3354**
DCC$_{std}$	5.1045	1.3284	0.1423	19.5928	3.3692	0.7993	0.1165	13.6086
GO-GARCH	5.6999	3.3696	0.1606	23.4610	3.8856	2.2757	0.1339	16.6319
2020–2022								
R-Vine$_{norm}$	6.6310	3.0223	0.2173	12.2279	4.6573	2.0201	**0.2143**	9.0953
R-Vine$_{std}$	8.5973	3.5201	0.2415	15.2096	6.3743	2.6760	0.2162	11.5223
R-Vine$_{sstd}$	5.9013	2.7350	0.2064	11.0608	4.3919	1.9464	0.2206	8.8163
C-Vine$_{norm}$	6.3894	2.8216	0.2112	11.8233	4.5398	2.0086	0.2168	9.0350
C-Vine$_{std}$	8.5017	3.6345	0.2348	15.0086	6.3322	2.5951	0.2178	11.4894
C-Vine$_{sstd}$	6.0433	2.8257	0.2026	11.2346	4.5055	1.9692	0.2145	8.9453
D-Vine$_{norm}$	6.5104	2.9110	0.2049	11.9230	4.5926	1.9933	0.2149	9.0481
D-Vine$_{std}$	8.4935	3.8056	0.2398	15.0569	6.3588	2.7164	0.2300	11.6548
D-Vine$_{sstd}$	5.8267	2.5794	0.2044	**10.9128**	4.3144	1.8726	0.2174	8.7005

Table 8 (continued)

Models	Mean	SD	$\mathcal{L}_{VaR}$	Cost	Mean	SD	$\mathcal{L}_{VaR}$	Cost
	Minimum risk				Equally weighted			
HS	5.9876	1.0309	0.2772	12.0740	4.6510	0.6192	0.2854	9.8657
DCC	6.0877	2.8061	**0.2016**	11.2612	4.1010	1.8481	0.2216	**8.3991**
DCC_{std}	6.3473	2.7868	0.2074	11.6829	4.2282	1.8373	0.2213	8.5714
GO-GARCH	7.8056	4.8359	0.2420	13.5647	5.4261	3.5795	0.2503	10.1272

The statistics were applied under the percentage log-returns

Note Values in bold indicate the models with the best result according to realized loss and realized cost

Table 9 Descriptive statistics [Mean and standard deviation (SD)], loss ($\mathcal{L}_{VaR}$), and realized cost (Cost) of VaR forecasts, considering portfolios with 6 stocks, considering 1000 observations to estimate the weights and $\alpha = 2.5\%$

Models	Mean	SD	$\mathcal{L}_{VaR}$	Cost	Mean	SD	$\mathcal{L}_{VaR}$	Cost
	Minimum risk				Equally weighted			
Full sample								
R-Vine_{norm}	7.1717	2.9862	0.2152	22.9395	4.3188	1.7142	0.1634	14.2485
R-Vine_{std}	9.1915	3.5916	0.2518	28.3726	5.9798	2.2785	0.1788	18.6221
R-Vine_{sstd}	6.5331	2.7164	0.2057	**20.9714**	4.1407	1.6346	0.1631	13.7185
C-Vine_{norm}	7.2699	3.0040	0.2185	23.1889	4.3187	1.7103	0.1627	14.2482
C-Vine_{std}	9.2195	3.6329	0.2531	28.5284	5.9424	2.2897	0.1798	18.5694
C-Vine_{sstd}	6.5674	2.6244	0.2058	21.0486	4.1510	1.6118	0.1634	13.7741
D-Vine_{norm}	7.1512	2.8759	0.2147	22.9489	4.3228	1.6926	0.1634	14.3229
D-Vine_{std}	9.2107	3.5903	0.2499	28.4313	6.0633	2.3264	0.1796	18.8413
D-Vine_{sstd}	6.5338	2.6931	**0.2050**	21.0358	4.1624	1.6396	**0.1609**	13.7773
HS	6.9380	0.8026	0.2454	21.7799	4.4696	0.4273	0.1995	14.7650
DCC	6.7653	2.8081	0.2106	21.7462	4.0143	1.6109	0.1634	**13.5544**
DCC_{std}	6.9898	2.7867	0.2144	22.3449	4.1105	1.6013	0.1633	13.7892
GO-GARCH	8.5903	4.3661	0.2469	27.7773	5.3666	2.9782	0.1759	17.5909
2016–2019								
R-Vine_{norm}	7.2512	2.5010	0.1954	28.4447	4.2584	1.3525	0.1318	17.3247
R-Vine_{std}	9.2470	2.8700	0.2425	35.4801	5.9029	1.6819	0.1624	23.0226
R-Vine_{sstd}	6.6092	2.2957	0.1824	**26.0800**	4.0889	1.2896	0.1310	16.7199
C-Vine_{norm}	7.3251	2.4829	0.1974	28.6649	4.2487	1.3511	0.1321	17.3056
C-Vine_{std}	9.2768	2.9282	0.2422	35.6242	5.8509	1.6827	0.1618	22.8939
C-Vine_{sstd}	6.6462	2.2891	0.1822	26.1463	4.0883	1.3038	0.1306	16.7626
D-Vine_{norm}	7.2891	2.5219	0.1968	28.5866	4.3050	1.3574	0.1329	17.4859
D-Vine_{std}	9.2583	2.8588	0.2412	35.5123	5.9742	1.6808	0.1650	23.2960
D-Vine_{sstd}	6.6384	2.3152	**0.1810**	26.2019	4.1165	1.3143	**0.1287**	16.8166
HS	7.4548	0.4826	0.2050	27.2997	4.7611	0.2166	0.1499	18.1798
DCC	6.8389	2.3881	0.1866	26.9850	4.0219	1.3459	0.1295	**16.5904**
DCC_{std}	7.0651	2.3754	0.1914	27.7355	4.1186	1.3400	0.1303	16.8946
GO-GARCH	9.4314	4.2260	0.2451	36.1454	5.8712	2.7912	0.1614	22.7211

Table 9 (continued)

Models	Mean	SD	$\mathcal{L}_{VaR}$	Cost	Mean	SD	$\mathcal{L}_{VaR}$	Cost
	Minimum risk				Equally weighted			
2020–2022								
R-Vine$_{norm}$	7.0356	3.6692	0.2491	13.5185	4.4220	2.1968	0.2175	8.9841
R-Vine$_{std}$	9.0965	4.5709	0.2677	16.2094	6.1113	3.0363	0.2067	11.0917
R-Vine$_{sstd}$	6.4030	3.3131	0.2456	12.2291	4.2294	2.0957	0.2179	8.5821
C-Vine$_{norm}$	7.1755	3.7311	0.2545	13.8179	4.4385	2.1886	0.2149	9.0160
C-Vine$_{std}$	9.1214	4.5960	0.2718	16.3853	6.0989	3.0565	0.2107	11.1690
C-Vine$_{sstd}$	6.4326	3.1133	0.2462	12.3249	4.2584	2.0302	0.2196	8.6598
D-Vine$_{norm}$	6.9152	3.3866	**0.2454**	13.3012	4.3532	2.1492	0.2156	8.9100
D-Vine$_{std}$	9.1294	4.5805	0.2647	16.3135	6.2157	3.1330	0.2046	11.2179
D-Vine$_{sstd}$	6.3548	3.2337	0.2460	**12.1950**	4.2410	2.0805	0.2162	8.5762
HS	6.0536	0.3287	0.3145	12.3339	3.9709	0.1428	0.2843	8.9213
DCC	6.6393	3.4075	0.2516	12.7810	4.0013	1.9853	0.2215	**8.3590**
DCC$_{std}$	6.8608	3.3748	0.2538	13.1201	4.0966	1.9708	0.2197	8.4749
GO-GARCH	7.1510	4.2272	0.2498	13.4570	4.5031	3.0902	**0.2008**	8.8115

The statistics were applied under the percentage log-returns

Note Values in bold indicate the models with the best result according to realized loss and realized cost

Table 10 Descriptive statistics [Mean and standard deviation (SD)], loss ($\mathcal{L}_{VaR}$), and realized cost (Cost) of VaR forecasts, considering portfolios with 12 stocks, considering 1000 observations to estimate the weights and $\alpha = 2.5\%$

Models	Mean	SD	$\mathcal{L}_{VaR}$	Cost	Mean	SD	$\mathcal{L}_{VaR}$	Cost
	Minimum risk				Equally weighted			
Full sample								
R-Vine$_{norm}$	5.4771	1.8164	0.1586	16.4626	3.5239	1.1708	0.1388	11.1267
R-Vine$_{std}$	7.2409	2.3003	0.1923	21.0665	4.9132	1.6099	0.1534	14.5774
R-Vine$_{sstd}$	4.9654	1.7448	**0.1484**	15.0429	3.3267	1.1458	0.1388	10.6607
C-Vine$_{norm}$	5.4095	1.8793	0.1573	16.2294	3.4981	1.2205	**0.1377**	11.0752
C-Vine$_{std}$	7.1847	2.3938	0.1924	20.9749	4.8518	1.6297	0.1524	14.4667
C-Vine$_{sstd}$	4.9878	1.7109	0.1504	15.0796	3.3469	1.1031	0.1387	10.6927
D-Vine$_{norm}$	5.4328	1.8623	0.1572	16.3168	3.5136	1.1809	0.1403	11.1524
D-Vine$_{std}$	7.1985	2.4035	0.1939	21.0106	4.9232	1.6989	0.1551	14.6707
D-Vine$_{sstd}$	4.9489	1.7527	0.1487	**14.9915**	3.3147	1.1356	0.1416	10.6779
HS	5.0107	0.4226	0.1727	15.1394	3.4047	0.1956	0.1661	10.9870
DCC	5.0629	1.7331	0.1498	15.2823	3.1952	1.0887	0.1418	**10.4118**
DCC$_{std}$	5.2285	1.7278	0.1532	15.6954	3.2738	1.0823	0.1414	10.5813
GO-GARCH	5.9837	2.6602	0.1717	18.2467	3.9069	2.2825	0.1504	12.4720
2016–2019								
R-Vine$_{norm}$	5.1752	1.2317	0.1402	19.6715	3.2649	0.7084	0.1049	12.9878
R-Vine$_{std}$	6.8612	1.4347	0.1800	25.3506	4.5327	0.8346	0.1283	17.1850

Table 10 (continued)

Models	Mean	SD	$\mathcal{L}_{VaR}$	Cost	Mean	SD	$\mathcal{L}_{VaR}$	Cost
	Minimum risk				Equally weighted			
R-Vine$_{sstd}$	4.7128	1.1816	0.1313	18.0425	3.0974	0.7034	0.1037	12.4464
C-Vine$_{norm}$	5.0929	1.1938	0.1388	19.3519	3.2358	0.7168	0.1043	12.9123
C-Vine$_{std}$	6.7902	1.4522	0.1790	25.1695	4.4735	0.8763	0.1269	17.0296
C-Vine$_{sstd}$	4.7217	1.1493	0.1318	18.0551	3.1109	0.6916	**0.1027**	12.4649
D-Vine$_{norm}$	5.1412	1.2129	0.1397	19.5281	3.2655	0.7157	0.1051	13.0121
D-Vine$_{std}$	6.8005	1.4217	0.1791	25.1841	4.5472	0.8861	0.1289	17.2860
D-Vine$_{sstd}$	4.6986	1.2078	**0.1289**	**17.9729**	3.0941	0.7230	0.1036	12.4544
HS	5.0587	0.3685	0.1383	18.2401	3.3440	0.1379	0.1133	12.7908
DCC	4.7987	1.1700	0.1317	18.3189	3.0041	0.6910	0.1037	**12.1848**
DCC$_{std}$	4.9450	1.1494	0.1349	18.7807	3.0738	0.6813	0.1039	12.3846
GO-GARCH	5.9805	2.9266	0.1591	22.5622	3.8872	2.6617	0.1181	15.1371
2020–2022								
R-Vine$_{norm}$	5.9936	2.4360	0.1902	10.9712	3.9670	1.5967	0.1968	7.9420
R-Vine$_{std}$	7.8908	3.1885	0.2133	13.7350	5.5645	2.2735	0.1966	10.1150
R-Vine$_{sstd}$	5.3977	2.3612	**0.1777**	9.9096	3.7190	1.5721	0.1988	7.6049
C-Vine$_{norm}$	5.9513	2.5845	0.1891	10.8858	3.9470	1.6864	**0.1949**	7.9314
C-Vine$_{std}$	7.8598	3.3495	0.2153	13.7969	5.4992	2.2867	0.1961	10.0808
C-Vine$_{sstd}$	5.4431	2.3139	0.1824	9.9878	3.7507	1.4916	0.2003	7.6599
D-Vine$_{norm}$	5.9319	2.5493	0.1873	10.8214	3.9382	1.6193	0.2006	7.9700
D-Vine$_{std}$	7.8795	3.3886	0.2193	13.8684	5.5667	2.4150	0.1998	10.1951
D-Vine$_{sstd}$	5.3772	2.3554	0.1827	9.8894	3.6922	1.5423	0.2068	7.6376
HS	4.9285	0.4914	0.2317	**9.8333**	3.5086	0.2327	0.2564	7.9002
DCC	5.5152	2.3418	0.1807	10.0858	3.5222	1.4932	0.2072	**7.3776**
DCC$_{std}$	5.7137	2.3381	0.1845	10.4155	3.6161	1.4827	0.2057	7.4951
GO-GARCH	5.9891	2.1307	0.1933	10.8615	3.9405	1.4167	0.2056	7.9112

The statistics were applied under the percentage log-returns

Note Values in bold indicate the models with the best result according to realized loss and realized cost

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Declarations

Conflict of interest Author Fernanda Maria Müller declares that she has no conflict of interest. Author Marcelo Brutti Righi declares that he has no conflict of interest

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