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CORRECTION

# Correction to: The best Condorcet-compatible election method: Ranked Pairs

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## Correction to: Constitutional Political Economy (2023) 34:434–444 https://doi.org/10.1007/s10602-022-09382-w

An error (with an accompanying erroneous footnote) occurs in the paper cited,[1] as the last clause in the last sentence of the second paragraph of page 8, which paragraph presently reads

"If the order of candidates is to have something to do with merit, then if the candidate who placed first had not run, then the candidate who placed second should have won. Kemeny-Young and Ranked Pairs have that property, but Beatpath fails it in the worst possible way; the candidate who placed second can become ranked *last* of the remaining candidates[11]."

In fact the example referenced by the footnote, while correctly used in its own section, is fallacious in this context. Nor can an alternate example be supplied; it has been proved impossible for the candidate who placed second under Beatpath to become placed last.[2] The necessary argument can nonetheless be completed; replacing the faulty sentence, the revised paragraph reads,

"If the order of candidates is to have something to do with merit, then if the candidate who placed first had not run, then the candidate who placed second should have won. Kemeny-Young and Ranked Pairs have that property, but Beatpath fails in almost the worst possible way. Though it is impossible for the candidate who had placed

[1] Charles T. Munger, Jr., *The best Condorcet-compatible election method: Ranked Pairs* Const Polit Econ (2022). https://doi.org/10.1007/s10602-022-09382-w.

[2] Schulze, M. (2018). The Schulze method of voting. See Section 4.21 on pp. 283–285. Retrieved July 1, 2023 from https://arxiv.org/ftp/arxiv/papers/1804/1804.02973.pdf.

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second to become placed last[1], computer searches,[3] for numbers of candidates from 3 to 18 have shown it is possible for the candidate who placed second to become placed anywhere down to second-to-last. The same searches show examples of the candidate who placed last becoming placed first; indeed, except for the candidate who placed second becoming placed last, and for the candidate who placed third becoming placed first, examples have been found[3] for a candidate who placed anywhere from second down to last in the first election becoming placed anywhere from first down to last in the second."

Victory matrices showing the patterns described follow for numbers of candidates equal to 4 or 5. (Some matrices serve for more than one pattern.)

$N = 4$

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 3 | 6 | −1 | 5 | −2 | 4 | \| | **2** | 3 | 4 |
| 4 | −3 | 6 | 5 | −1 | −2 | \| | 4 | **2** | 3 |
| . | . | . | . | . | . | \| | . | . | ~~**2**~~ |
| . | . | . | . | . | . | \| | ~~**3**~~ | . | . |
| 3 | 6 | −1 | 5 | −2 | 4 | \| | 2 | **3** | 4 |
| 5 | −4 | 3 | 6 | 1 | −2 | \| | 2 | 4 | **3** |
| 4 | −3 | 6 | 5 | −1 | −2 | \| | **4** | 2 | 3 |
| 5 | −4 | 3 | 6 | 1 | −2 | \| | 2 | **4** | 3 |
| 3 | 6 | −1 | 5 | −2 | 4 | \| | 2 | 3 | **4** |

$N = 5$

| | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 6 | −2 | −3 | −5 | 4 | 8 | 9 | −7 | 10 | \| | **2** | 3 | 4 | 5 |
| 7 | −6 | 5 | −3 | 8 | −2 | 9 | −4 | 10 | −1 | \| | 4 | **2** | 3 | 5 |
| 8 | −5 | 9 | 7 | 6 | −2 | −1 | −3 | −4 | 10 | \| | 4 | 5 | **2** | 3 |
| . | . | . | . | . | . | . | . | . | . | \| | . | . | . | ~~**2**~~ |
| . | . | . | . | . | . | . | . | . | . | \| | ~~**3**~~ | . | . | . |
| 1 | 6 | −2 | −3 | −5 | 4 | 8 | 9 | −7 | 10 | \| | 2 | **3** | 4 | 5 |
| 7 | −6 | 5 | −3 | 8 | −2 | 9 | −4 | 10 | −1 | \| | 4 | 2 | **3** | 5 |
| 10 | −6 | −2 | 4 | 8 | 7 | −5 | 3 | −1 | 9 | \| | 2 | 4 | 5 | **3** |
| 7 | −6 | 5 | −3 | 8 | −2 | 9 | −4 | 10 | −1 | \| | **4** | 2 | 3 | 5 |
| 10 | −6 | −2 | 4 | 8 | 7 | −5 | 3 | −1 | 9 | \| | 2 | **4** | 5 | 3 |
| 1 | 6 | −2 | −3 | −5 | 4 | 8 | 9 | −7 | 10 | \| | 2 | 3 | **4** | 5 |
| 7 | 1 | −5 | −3 | 6 | −2 | 10 | 8 | 9 | −4 | \| | 2 | 3 | 5 | **4** |
| −5 | 6 | −4 | 7 | 9 | −8 | −3 | 10 | 1 | −2 | \| | **5** | 2 | 3 | 4 |
| 7 | −3 | −5 | 6 | 8 | −2 | 9 | 10 | −4 | −1 | \| | 2 | **5** | 3 | 4 |
| 7 | 1 | −5 | −3 | 6 | −2 | 10 | 8 | 9 | −4 | \| | 2 | 3 | **5** | 4 |
| 1 | 6 | −2 | −3 | −5 | 4 | 8 | 9 | −7 | 10 | \| | 2 | 3 | 4 | **5** |

Here a victory matrix has under Beatpath the rank order [1234] for $N = 4$, and [12345] for $N = 5$. The above-diagonal elements of the victory matrix are listed with the column index increasing faster than the row index, and the rank order of the election when candidate 1 drops is given. Thus line 7 of the table for $N = 4$ shows that after the winning candidate drops, the victory matrix

$$V = \begin{pmatrix} 0 & 4 & -3 & 6 \\ -4 & 0 & 5 & -1 \\ 3 & -5 & 0 & -2 \\ -6 & 1 & 2 & 0 \end{pmatrix} \text{ produces the rank order } [\mathbf{4}23]$$

[3] Munger, C.T. (2023) Dropping one candidate under Beatpath and Ranked Pairs. https://bettervotingmethods.com/resources-all/technical-aspects-of-election-methods retreived October 10, 2023.

and the boldface directs attention to candidate 4 appearing appearing in first place; that is, the candidate formerly ranked last has become ranked first. Line 13 of the second table is an example of this happening for $N = 5$.

Examining the bolded numbers in the tables shows that, except for the candidate who placed second never becoming the candidate placed last, and the candidate who placed third never being placed first, every candidate can appear anywhere in the new rank order. Matrices showing these same patterns for $N = 6$ and $N = 7$ are available[3].

The victory matrix $V$ has for candidates $j \neq k$ the element $V_{jk}$ equal to the number of ballots on which candidate $j$ is placed ahead of candidate $k$, minus the number on which $k$ is placed ahead of $j$; and the diagonal elements of $V$ are defined to be zero so that $V$ is antisymmetric. In what I call the common case of a victory matrix all the above-diagonal elements have distinct magnitudes, none of which are zero. The number of fundamentally distinct elections in the common case is finite for any $N$, and for $N = 4$ or 5 that number is small enough every case can be examined.

For those two values of $N$, we found that under Beatpath in the common case, every election that yields a single rank order, and when the winning candidate is dropped the new election yields a single rank order, the candidate who had been ranked second must continue to rank ahead of the candidate who had been ranked third. Random samples of $> 10^6$ elections for $N$ from 6 to 18, restricted to elections with an $N$-candidate cycle, found no elections where that property did not continue to hold[3].

We therefore conjecture that property always holds. If so, then under Beatpath in the common case, if when the candidate who had placed second becomes placed second-to-last, the candidate who had placed third will be found to be placed last.

The author acknowledges and thanks Marcus Schulze[2] for pointing out the error.