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ORIGINAL PAPER

The Maltese single transferable vote experience: a case study of gerrymandering?

Serhat Hasancebi[1]

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Abstract
Re-drawing the electoral boundaries to provide benefit to one particular political party and thereby damaging the principle of representation in democracy has been a core issue in political science in the recent years. For years social scientists have been advocating the idea of measuring or preventing the potential for damage that may arise from the existence of the above-mentioned redistricting process. Following this discussion, we investigate the possible gerrymandering phenomenon that might have arisen, or whether there are any asymmetries or partisan biases due to boundary delimitation of the electoral constituencies in the case of the Maltese general elections. From the evidence of various statistical tests and simulations, our conclusion is that we find no evidence of gerrymandering in the 2013 and 2017 Maltese general elections.

Keywords Gerrymandering · Redistricting · Single transferable vote · Malta

JEL classification K16

1 Introduction

Redistricting the electoral constituencies in favor of a certain group or party has always been a critical concept for political scientists, who have sought to minimize the opportunities which could help partisan advantages. The process of altering the borders of electoral districts to give a political party or incumbent a significant advantage is called Gerrymandering (Grofman, 1985). Although gerrymandering can weaken any electoral system implemented, it is also detrimental to the concept of democracy's representativeness. In other words, political maneuvering by gerrymandering limits the integrity of electoral processes in which seat-vote casting is not adequately carried out.

Serhat Hasancebi
serhat.hasancebi@upf.edu

1 Department of Economics and Business, Universitat Pompeu Fabra, Barcelona, Spain

Along these lines, in this article, we examine the two recent legislative elections of Malta, the last years for which there are available data, in terms of whether the 2017 and 2013 redistricting processes gave any particular political party any advantage.[1] As a starting point, we hypothetically applied different electoral formulas other than the single transferable vote (STV), which are frequently mentioned in the electoral systems literature, both on the condition that the electoral constituencies remain the same, and as one vast constituency (i.e. nationwide). In other words, we questioned whether the election outcome would have changed if other electoral formulas were implemented in the same electoral districts, and if, alternatively, Malta had had a single nationwide constituency. Second, and as a follow-up to Wang (2016a, 2016b), we applied three gerrymandering statistical tests to measure whether there was any partisan gerrymandering scenario or partisan bias in Malta's 2013 and 2017 general elections. After obtaining the necessary information from the Gerrymandering measurements, we carried out simulations in order to detect possible anomalies that might be caused by the redistricting process. We conclude that the 2013 and 2017 general elections of Malta were fair for political parties in terms of reallocation of constituencies and showed no signs of possible gerrymandering or partisan bias.

More importantly, and as will be discussed further in Sect. 2, the implementation of the "bonus seat" in the Maltese general elections is critical. The bonus seat concept was introduced in the 1987 amendment to assure seat-vote proportionality. That is, a party that obtains a majority of first-preference votes will be granted additional seats to secure the majority in Parliament. This system is intended to ensure that parties are fairly represented in the legislature in proportion to the number of votes they receive. Consequently, we show in this article that implementing such a concept to Maltese general elections increases the proportionality of party representation, thereby reducing the probability of effective gerrymandering.

The rest of the paper is organized as follows. Section 2 describes the single transferable vote system for Malta. Section 3 presents the data and the detail of the statistical tests carried out in the paper. Results of gerrymandering measurements and statistical tests are discussed in Sect. 4. Section 5 concludes. Finally, three formal appendices are included at the end of the paper: Appendix A describes how the single transferable vote system works with a given empirical example. Appendix B compares different electoral formulas for the Maltese general elections. Technical details corresponding to Sect. 4 are discussed in Appendix C.

2 The single transferable vote in Malta

This section is devoted to Malta's electoral system, the single transferable vote, and its place in the world. Why does the Maltese case deserve some attention? First, Malta is good example of *de facto* bipartidism. No parties other than the *Partit*

[1] The results of the 2013 general elections are suppressed for the sake of space saving, however, they are available on request from the corresponding author.

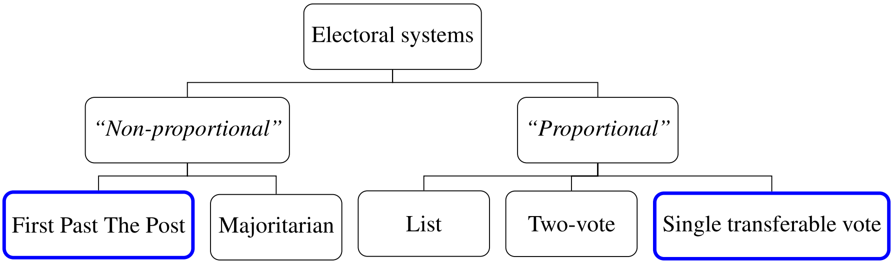

Fig. 1 The five main types of electoral systems

Nazzjonalista/Nationalist Party (PN) and the *Malta Labour Party (MLP)* have obtained representation in parliament since Malta became independent in 1964, so that Malta has proven that it is one of the purest two-party electoral systems in the worldwide (Gallagher, 2010).[2] In this respect, it seems to be a suitable ground for partisan gerrymandering scenarios, and eventually it has proximity to (Wang, 2016a, b) as referenced in this paper. Second, there is a long held concern in the Maltese political class about the likely existence of gerrymandering in the design of the electoral district map for general elections. See Grofman and Lijphart (2003); Bickerstaff (2020); De Miño and Lane (1996) and De Miño and Lane (2010) among others. Along these lines, and as we will discuss below in this section, two constitutional reforms have been implemented with the purpose of achieving a higher degree of proportionality between the vote and seat distributions in the Maltese House of the Representatives.

Farrell (1997) describes 5 main electoral systems as displayed in Fig. 1. We will not examine all electoral systems in this section, but it is useful to make some general definitions. Let us first dwell on the reason for the separation of electoral systems as the concepts of *proportionality* and *non-proportionality*. The most common non-proportional electoral systems consist of First Past The Post (FPTP) and majoritarian systems, while proportional systems include list, two-vote and single transferable vote systems. The difference between the two systems is generally based on different objectives. While non-proportional systems often try to consolidate the stability of governments by obtaining a plurality or overall majority from particular electoral districts, proportional systems, on the contrary, aim to obtain the closest number of seats corresponding to the proportion of votes obtained. Generally speaking, the debates on electoral systems are therefore centered on representativeness and stability of the governments formed (Farrell, 1997). Gerrymandering tests are typically performed for the FPTP system, an example of which is the United States. As we extend these statistical tests to the case of Malta's single transferable vote system, it is also worth describing the electoral systems implemented in both countries.

[2] For convenience, we will refer to the Malta Labour party and the Nationalist Party as MLP and PN, respectively, throughout the article.

The FPTP system is often used in countries such as the United States, Canada, and Britain. By definition it is based on pluralism, meaning that candidates will be considered elected from a constituency with the highest vote share among their competitors (Diamond & Plattner, 2006). For example, in the case of the United States, in a two candidate race in a particular constituency, the candidate with superior vote share will be elected as a representative from that electoral district. The rights of representation of minority groups are one of the most critical arguments in this system. More precisely, all the votes of candidates who do not win generate surplus votes while minimizing the opportunity for smaller parties to be represented, resulting in disproportionate election outcomes (Farrell, 1997). On the other hand, factors such as the single-party government (other than coalition) or the stability of governments are shown as the advantages of the FPTP system.

The single transferable vote (STV) is a proportional representation (PR) system and originates from the 19th century philosophers Thomas Hare and John Stuart Mill. The STV electoral system is generally implemented in countries with small populations such as Ireland and Malta, as well as in the Australian state of Tasmania. It is also used in Cambridge (Massachusetts) to elect school committees in the United States (Tideman, 1995).

The principal reason why the STV is part of proportional electoral systems is that electoral constituencies appoint more than one candidate for a legislative term. In other words, in a given multi-member district (MMD), electors may choose more than one candidate. Thus, the STV differs from other electoral systems such as the FPTP and the majoritarian systems, as it does not feature single-candidate electoral districts. STV's ballot structure and electoral formula are further variances from other electoral systems. STV allows voters to list their preferences on their ballot papers. Among the candidates nominated in each constituency, electors can select their votes as a first, second, or third choice, so ensuring that minority groups are somewhat represented (Bowler & Grofman, 2010). A simple example of the ballot paper is represented in Fig. 2, where each candidate is inserted under the name of the party and ordered alphabetically [Farrell (1997), p. 85].

As indicated previously, a further distinction between the STV and the other electoral systems is that an electoral formula is needed to determine which candidate is to be elected. A quota (called Droop quota) indicates the minimum amount of first-preference votes needed to select a candidate in a certain constituency. More precisely, the Droop quota, DQ, is defined as

$$DQ \equiv \left[\frac{total\ valid\ votes}{total\ number\ of\ seats + 1} \right] + 1. \tag{1}$$

The first candidate from that district to meet the quota is therefore automatically elected when computing the overall number of the first-preference votes (Herron et al., 2018).[3]

[3] See Appendix A for the explanation of how the STV works in a given example.

No. of Members to be elected .... Division ....

Mark order of preference	Badge of Candidate	Names of Candidates
		PARTIT TAL-FJURI
		JONES, (John Jones, of 52 Old Bakery Street, Valletta, Merchant)
		MAGRO, (William David Magro, of 10 Tower Road, Sliema (Painter)
		MIFSUD, (Joseph Mifsud, of 16 Victoria Avenue, Sliema, Labourer)
		MUSCAT, (Francesco Muscat of 1 St. Paul's Str. Zabbar Driver)
		VELLA, (James Vella, of 5 Republic Street, St. Julians Architect
		WILLIAMS, (Francis Williams of 85 Genuis Street, Zurrieq Chemist)
		PARTIT TAL- GHASFUR
		AZZOPARDI, (Spiro Azzopardi, of 13 Marina Street, Zejtun, Printer)
		BORG, (Assuero Borg, of 69 Barbara Street, Mellieha, Clerk)
		CASSAR, (Lela Cassar, of "Dolores", Main Street, Cospiuca, Housewife)
		MIZZI, (Glormu Mizzi, of 70 Two Gates Str. Lija, Lawyer)
		ZARB, (Fortunat Zarb, of 15 Strait Street, Luqa, Clerk)
		PARTIT TAS-SIĠAR
		AZZOPARDI, (Reginald Azzopardi, of 165 St. Domenic Str., Qormi, Clerk)
		ZAMMIT, (Lawrence Zammit of "Josdor", 188 Bwieraq Str. Hamrun, Chemist)
		KANDIDATI INDIPENDENTI
		BUHAGIAR, (Louis Buhagiar, of 55 Republic Street, Zabbar, Merchant)
		GALEA, (Ninu Galea, of 67 B'Kara Lane, Qrendi, Worker)

Fig. 2 Malta single transferable vote ballot paper

Let us consider a basic example below to understand the droop quota better. Suppose that in a constituency with 500 electors, 5 candidates must be elected. Accordingly, the first candidate to receive $(500/(5 + 1)) + 1 = 84.3$ votes will be directly elected from that electoral constituency. The process of counting votes continues until the total number of representatives to be elected from that constituency is completed by transferring the surplus votes of the elected candidate to other candidates. That is, according to the candidate's second, third, etc. preferential votes on the ballots, proportional distribution of the votes for candidates will be spread until the last phase. Thus, essentially, the STV system can also minimize the number of wasted votes which is one of the most discussed about issues with regard to electoral systems (Farrell & Katz, 2014; Bowler & Grofman, 2010). Still, the wasted votes concept can also be an issue in all electoral systems, including the STV system. In the STV case, for instance, the wasted vote problem arises in three situations. The first is the total votes of the candidates who did not reach the quota until the last count, but accumulated votes. Second, representatives elected in the last count produce wasted votes by the amount exceeding the quota. Lastly, the sum of the non-transferable votes accumulated until the last count (Mair & Laver, 1975).

In Malta, the PR-STV system has been implemented since 1921 and elects 65 Members of Parliament (MPs) from the 13 multi-member constituencies, five from each, to the unicameral parliament (known as the House of Representatives).[4] The legislative basis for the electoral process is defined by two bodies of legislation: the Constitution of Malta and the General Elections Act of 1991. An Electoral Commission, whose members are appointed by the Prime Minister, establishes the electoral district borders (De Miño & Lane, 2010, 1996). In these regulation issues, which are frequently mentioned in the electoral systems literature, criteria such as equal population ratios, compactness, interests and representation of other minority groups are taken into consideration.

At this point, another important issue is that of the proportionality of the election results. In other words, the popular vote share of a party in the elections and the number of seats obtained should correspond to each other. There is a general consensus amongst scholars on this issue that the district magnitude should consist of at least five representatives in terms of promising proportional results (Taagepera & Shugart, 1989; Mair, 2003). In this context, the STV system implemented by Malta seems to be a fair practice for political parties from a theoretical point of view.[5] Nevertheless, there were some election processes with disproportionate results. The first one was in the 1981 general elections: while the PN had the majority of votes throughout the country (50.9%), it could not achieve the majority to gain office. And the MLP received 49.1% of the first-preference votes, but it secured the majority of seats in the parliament and formed the government. So much so, that this situation suggested that electoral districts were organized according to gerrymandering

[4] We will use the terms MPs and representatives interchangeably.

[5] As pointed out by a referee, unless there are significant differences in the size of electoral districts when the country's electoral constituencies are redrawn, PR systems are unlikely to exhibit bias (Tan & Grofman, 2018)

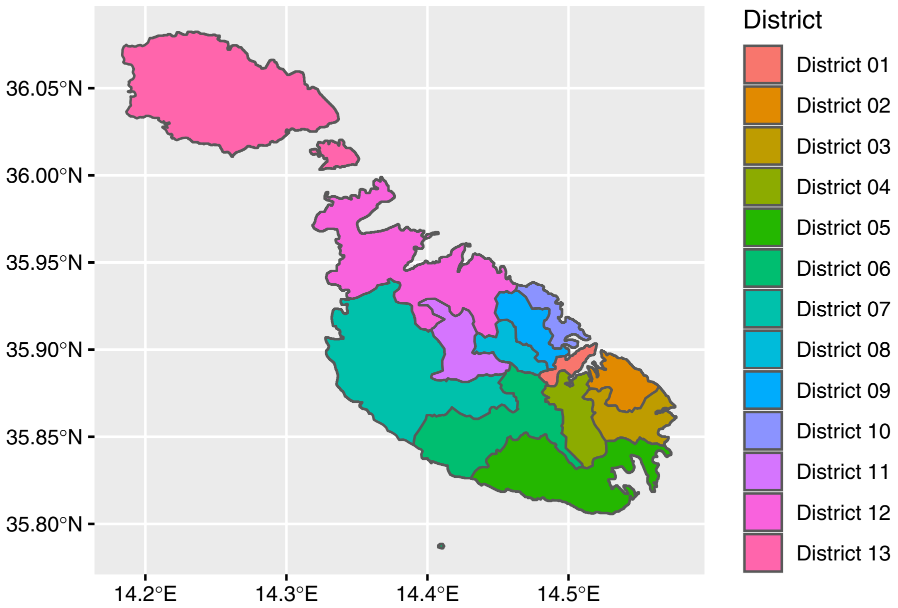

Fig. 3 Malta's electoral constituencies. *Key:* Malta's electoral constituencies as decided for the 2017 legislative elections. There is a total of 13 constituencies, each represented by a different colour

practices during the MLP government (Grofman & Lijphart, 2003). After the boycotts of the PN and the political stalemate, the concept of bonus seats was introduced with the 1987 amendment in order to ensure seat-vote proportionality (see Article 52 of the Constitution). Consequently, a decision was made to give additional seats to a party obtaining a majority of first-preference votes in order to ensure the majority of seats in the House of Representatives (De Miño & Lane, 2010). For instance, 4 bonus seats were given to the PN following the general elections in 1987 and 2008 in order to secure the majority in the Parliament, so that the governments were formed by the PN. Furthermore, 4 bonus seats were granted to the PN in the elections of 2013 and 2 bonus seats in the general elections of 2017, although this time to ensure the proportionality of vote-seat instead of the parliamentary majority.[6]

The 13 constituencies of Malta, determined by the 2017 constitutional amendment, are displayed in Fig. 3.[7] The number of registered voters in the 13 constituencies ranges from 24,884 to 28,680, with a standard deviation of 867.3 and a coefficient of variation of 3.30. The minimal demographic variation in the electoral districts is a prerequisite to first-glance investigation into a gerrymandering situation, thereby preventing potential malapportionment by ensuring that the number of

[6] See data for the bonus seats. https://www.um.edu.mt/electionsdata/elections.

[7] See the details for the 2017 constitutional amendment on redesigning the electoral constituencies. https://electoral.gov.mt/ElectoralDivisions.

Table 1 Descriptive statistics for the 2017 Maltese general elections

Constituency	Registered Voters	Quota	First-preference votes		Seats obtained	
			MLP	PN	MLP	PN
1	26,598	4033	13,844	10,094	3	2
2	26,396	4062	17,353	6761	4	1
3	25,404	3894	16,328	6775	4	1
4	26,095	4033	16,383	7528	4	1
5	25,295	3870	15,259	7720	3	2
6	24,884	3847	13,717	9164	3	2
7	27,106	4147	14,042	10,509	3	2
8	25,982	3968	10,830	12,591	2	3
9	25,636	3853	9712	13,007	2	3
10	26,460	3887	8873	14,058	2	3
11	26,244	3986	10,282	13,207	2	3
12	27,076	3899	11,059	11,982	2	3
13	28,680	4306	13,233	12,361	3	2
Total	341,856	51,785	170,915	135,757	37	28

Key: The results of the 2017 Maltese general elections for each constituency. The number of registered votes in each constituency is shown in the second column, while the third column indicates the quotas that candidates must reach in order to be elected per electoral district. The MLP and the PN votes for the relevant electoral district are represented by columns four and five, and the numbers of parliamentary seats obtained by the parties are displayed in columns six and seven, respectively

electors does not vary considerably between electoral districts (Wang, 2016b; Johnston, 2003).[8]

For example, in the 2017 general elections, the highest quota among the 13 electoral districts was 4,306 votes, and the minimum quota to be reached was 3,847, with a standard deviation of 121.7 and a coefficient of variation of 3.06. While Malta had a total of 8 constituencies in 1921, it was raised to a total of 13 constituencies in 1976 and has continued unchanged until today. For completeness, Table 1 provides some descriptive statistics for Malta's 2017 general elections. The second column indicates the total number of registered voters for each constituency, while the third column shows the quota corresponding to the constituencies. The fourth and fifth columns refer to the first-preference votes and the number of seats the political parties obtained, respectively.

One should note that not every disproportionality necessarily means rigging elections, or any gerrymandering signs, while any disproportional translation of the

[8] Malapportionment can be sometimes described as one of the gerrymandering techniques, but this is not the case for the Maltese parliamentary elections as demographics in all electoral districts do not differ much, and are homogeneous.

votes into seats would not mean malapportionment by nature. That is, reshaping of legislative constituencies by jurisdiction is not always intended to gain an advantage for the political parties, or the advantage might have been gained unintentionally (Chen & Rodden, 2013). In other words, boundary delimitation may yield some advantages for the political parties unintentionally, due to the need to adjust population scales across constituencies, as the population will change in time. Thus, one should bear in mind that investigating the asymmetries in the outcomes of the election should be carried out with care. In the next section, we will describe data and statistical tests that we use to analyze aforementioned disproportionality and asymmetry in the Maltese general election outcomes.

3 Data and statistical tests

For the purpose of investigating Malta's general elections, we use the data set which contains the election results at electoral constituency level for each candidate and political party between 1960 and 2017 for each general election. We obtained the data we used from the archive of the University of Malta and the Electoral Commission of Malta.[9]

First, we pose two questions; *i*) What results would have been achieved by majoritarian and minority political parties if other electoral formulae had been applied to the 2017 Maltese general elections for the respective constituencies? *ii*) If the boundary restriction had not been valid for the general elections, what would the election outcome have been? That is, we consider the possible consequences of invoking one vast constituency scenario. In these two questions, which have been argued over for a long time by political scientists, the answer to the question of which electoral formula ensures fairer representation is sought. As the implemented electoral system and electoral formulae differ, the resulting election outcome will also vary. The remaining question is whether the alternative seat allocation formulas will grant representation to other parties both in the same electoral districts and in a possible single national constituency. In addition, given the STV outcome of the 2017 general elections, we do not have information based on the electors' second, third, etc. preference votes on the ballot paper. Therefore, one of the reasons for applying the two exercises below is also to validate the closest method of replicating the STV outcome of the general elections, as we use this information in one of the gerrymandering tests that we carry out in the following sections. To put it differently, the mentioned test is based on computer simulations and thus requires an electoral formula that gives the best replication of the STV outcome.

Second, and most importantly, we apply three gerrymandering tests introduced in Wang (2016a, 2016b) for the analysis of general elections. We have investigated whether there was any partisan bias or a possible gerrymandering scenario in the 2017 parliamentary election results. More precisely, we have conducted a first test,

[9] Parliamentary election data are available at The University of Malta. https://www.um.edu.mt/electionsdata/elections/parliamentary, and The Electoral Commission of Malta. https://electoral.gov.mt/.

the lopsided-outcomes test, which examines the constituencies by party and analyzes the two parties' average margins of victory to determine whether any difference between those averages could be attributable to chance. A party which always wins its seats with large margins is probably victim of a gerrymandering technique known as packing. With a second gerrymandering test, *the consistent-advantage test*, we examined the differences between the average seat share and the median seat share of both political parties. With this approach, also known as the skewness test, a possible gerrymandering scenario can be monitored if a party's average seat share is considerably higher than its median seat share, as an asymmetry in the distribution of seat share for that party will be observed. The third and last test (which, as the previous one, will require computer simulations), is called *the excess seat test*. The test aims to check whether one party's share of seats won deviates unexpectedly from national norms. As a consequence, it reveals to what extent the predicted result achieved in the election will vary from actual outcomes of the election. The following parts will first discuss the two questions referred to above, and then apply the above-mentioned gerrymandering tests to the political parties.

Concerning the first question, we tried to find any possible difference regarding the electoral results in each of the thirteen electoral divisions under alternative seat allocation methods other than the STV. Needless to say, it is also important to investigate how non-STV systems can affect the representation of other small parties, as different electoral systems will produce different election outcomes (Cox, 1997). Consequently, a total of 14 seat allocation formulas were applied to the given vote shares of the political parties in both Matlab© and R programs.[10][11] A summary of the results follows, the reader being referred to Appendix B for further details.

There are differences in quantitative terms only in the fifth constituency as the d'Hondt electoral formula awards the MLP one additional seat given the first-preference votes in that constituency (see Table 4 in Appendix B). As for the largest remainder formula with the Hare quota, the PN receives one more parliamentary seat compared with the actual STV results in the fourth electoral district. Finally, the Sainte-Lague electoral formula yields the same results as the d'Hondt. In fact, all these findings are consistent with previous research (Balinski & Young, 1984; Lijphart, 2012). Lijphart (2003) discusses the proportionality of the translation of vote shares into seats, concluding that the highest proportionality can be achieved under the STV, the d'Hondt, the largest remainder, and the Sainte-Lague formulae. He additionally concludes that the d'Hondt electoral formula favors the majoritarian

[10] Computations for the seat allocations have been made with R version 4.0.5 and the **electoral** package, see CRAN. As for Matlab©, the **apport** function has been used for the same purpose of allocating vote shares into seat distribution, see apport.

[11] A total of 14 electoral formulae were applied. However, the conclusion to be drawn from them was far removed from what was obtained under the STV system, thus not included. That is, the obtained errors in Eq. (3) were more than those represented in the article. In sum, implemented highest average seat allocation methods are: *Adams, D'Hondt, Danish, Dean, Equal proportions, Hill-Huntington, Imperiali, Modified Sainte-Lague,* and *Sainte-Lague*. Regarding the largest remainders, the performed methods are: *Droop, Hangenbach-Bischoff, Hare, Imperial, and modified Imperial*. For detailed information on the seat allocation methods, see (Herron et al., 2018; Van Eck et al., 2005).

parties in the electoral systems, while the largest remainder gives a representation chance to other smaller parties as well.

The difference in the resulting seat distribution comes from the methods that electoral formulas implement for the given vote shares. That is, as discussed in Sect. 2, STV uses the Droop quota for translating the parties' vote shares into parliamentary seats, whereas the d'Hondt and the Sainte-Lague formulas divide the given vote shares into *1, 2, 3*, etc. divisors (the latter divisors involve the odd integers such as *1, 3, 5*), and allocate parliamentary seats to the political parties with the highest averages. Hence they do not need to specify a quota in order to elect the candidates in a certain district. As for the largest remainder, it uses the so called Hare quota (similar to the STV's Droop quota) for choosing the candidates, so that after electing the candidates in the first counting, the remaining surplus votes are transferred to the other candidates.[12] The main difference between the Droop and Hare quotas is that the latter allows smaller parties to obtain MPs in the initial allocation of seats, while the former makes it harder for smaller parties to obtain seat representation since its formula requires more votes for MPs to be elected. Consequently, the largest remainder formula raises the representation of smaller parties or candidates with smaller vote shares, thereby increasing the proportionality of the electoral system.

Regarding the uniqueness of the electoral constituency, we explore what might happen if Malta had a single national electoral constituency. Instead of dividing the country into numerous constituencies, one might ponder the potential consequences of having one large constituency. A single electoral district scenario, which frequently comes up in electoral system debates, is crucial because in some societies, the representation rights of minority groups are neglected due to electoral district separation. As a result, the single constituency system might enhance the representational chances of at least some minority national groups (Reynolds, 2008). Furthermore, regarding proportionality, a national-level election scenario can increase proportionality enormously, as in the cases of Israel and the Netherlands (Lijphart, 2012; Gallagher & Mitchell, 2006).

Thus, we questioned how the representation of parties in parliament would change if Malta's 2017 general elections were held in one single electoral district. Accordingly, we adopted the electoral formulas mentioned in the previous section in a single electoral constituency scenario.[13] As can be seen in Appendix B, if one vast constituency scenario were applied hypothetically, d'Hondt and Sainte-Lague electoral formulas would appoint only one extra MP to the PN. When evaluated in this respect, d'Hondt and Sainte-Lague are again the electoral formulas that most closely replicate the STV system. On the other hand, under the largest remainder method the right of a new party to be represented in the parliament would be ensured by increasing the chance of representation for small parties. Thus, Alternattiva Demokratika Party (AD) would be able to obtain its representation in the government if its votes

[12] More precisely, the Hare quota is defined as $HQ \equiv \frac{\textit{total valid votes}}{\textit{total number of seats}}$.

[13] As in the previous exercise, the seat allocation methods were also simulated with the R program in the same way. It is worth mentioning that R gives more efficient (i.e. shorter computing time) results than Matlab©.

were not concentrated in a particular electoral district under the largest remainder method (see Table 5 in Appendix B).

In summary, the following conclusions can be drawn from the two exercises applied above. First of all, for the given vote shares (first-preference votes) of the political parties in Malta, different electoral formulas can yield variations in the election outcomes. Applying distinct seat allocation methods, as previously stated out, might give advantage either to the majoritarian or minority parties depending on the methods chosen, so that implementing methods other than the STV may increase the representation of the minority groups, or on the contrary, may favor the majoritarian political parties in the electoral system. Second, the seat distribution across parties is not robust to the electoral district design (one single nationwide district vs the actual 13 district map). A single constituency scenario, in particular, will be crucial to enhancing the rights in terms or representation of minority groups as shown in Table 5, and will also regulate the proportionality of the electoral system. Finally, since there is no information about the second or third preference votes of the electors, different electoral formulae were applied to the distribution of "first-preference votes" (the only available in the data set) amongst political parties in order to find the closest system to the STV in terms of lowest mean absolute error (MAE) [see Eq. (3) in Appendix B]. After applying the d'Hondt formula to first-preference vote distribution, the simulated seat distribution gives the closest number of MPs compared to the observed distribution of seats under the STV. In addition to our findings, there is also consensus that the d'Hondt method most closely represents the STV system for translating the given vote shares into the parliamentary seats (Grofman & Lijphart, 2003; Buhagiar & Lauri, 2009). Consequently, we will perform the simulations needed for the excess-seat test (the third gerrymandering test that we will run in Sect. 4.3), using the d'Hondt formula.

4 Gerrymandering statistical tests

4.1 Lopsided-outcomes test

To begin with, we first run the *lopsided outcomes test,* an analysis of intents for detecting partisan gerrymandering. This test is used to compare the average vote margins in the electoral districts, to check if the difference is due to chance in any sense. In other words, if a political party obtains representatives in certain constituencies by high vote margins, it is likely to be a victim of packing, a gerrymandering strategy (Wang, 2016b, a). Accordingly, the party that designs the electoral districts seeks as many members as possible from constituencies with small margins. Thus, while the opposing party's votes are heavily concentrated in particular constituencies, its chances in the others are diminished. More precisely, we compare the difference between the average vote shares of the MLP and the PN in the electoral constituencies. That is to say, we question the cost in votes per seat for a particular district for the corresponding political party.

Figure 4 depicts how many votes both parties would have received on average for the MPs they will return to the parliament from each electoral district. As noted in the

Sect. 2, as a consequence of the 1987 amendment, the party that wins the majority of the popular vote across the country or for the purpose of adjusting seat-vote proportionality, is awarded bonus seats, which in the current case added two bonus seats to the PN to balance the seat-to-vote ratio, regardless of the majority of votes in the 2017 general elections. Therefore, the first chart in Fig. 4 shows the absence of these bonus seats, whereas the second chart shows the situation after the bonus seats have been added to the PN.

In both graphs, the red dots show the number of votes per seat for the Nationalists, while the blue dots display the number of votes per seat for the MLP in each of the electoral districts. In other words, while the average number of votes per seat for the MLP was 4712, this figure corresponds to an average of 5200 votes per seat for the Nationalists. Regarding the second graph in Fig. 4, that is after two bonus seats were granted to the PN under the Article 52, the average number of votes per seat for the PN decreases to 4752, while for the MLP it remains the same. At first glance, obtaining a particular seat in the constituencies seems to be more costly for the PN than for the MLP. However, the bonus seats concept seems to adjust the average vote per seat for the PN as demonstrated in the second graph in Fig. 4. In order to determine whether the difference in average votes may be attributed to chance, and following Wang (2016a, 2016b), we compute the *t*-statistic (see Appendix C for further details on the t-statistic).

The obtained *t*-statistic as a solution to the Eq. (4) was −1.35 and the *p*-value of the test was 0.19, which is higher than any reasonable significance level. After the 2 bonus seats adjustment in favor of the PN, the *t*-statistic equals −0.13 with the corresponding *p*-value of 0.90. Therefore, we fail to reject the null hypothesis in both cases (before and after granting bonus seats), and conclude that the data do not provide convincing evidence of the difference between the average vote shares of the two political parties. That is to say, given the vote shares of the two political parties, the lopsided outcomes test does not provide any evidence of a possible gerrymandering scenario due to the 2017 boundary changes of the electoral constituencies.

(a) Votes per seat and political party with no bonus seats

(b) Votes per seat and political party with bonus seats

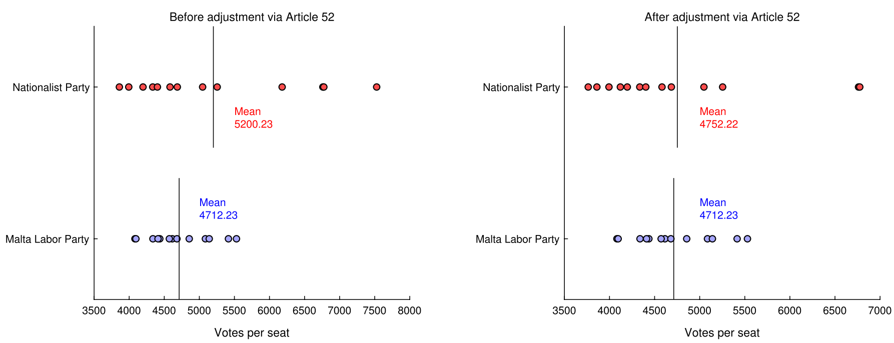

Fig. 4 Analysis of Intents: Lopsided outcomes test. *Key:* Calculated votes per seat for each party in the 2017 general elections. The red points represent the Nationalist Party's effort to obtain a particular seat in each electoral constituency, and the blue points likewise show the same case for the Malta Labour Party. Before [panel (a)] and after the bonus seats were added to the Nationalist Party [panel (b)] respectively

4.2 Consistent-advantage test

As a second test, we examined the difference between average and median seat shares for the two political parties. The mean-median difference is a statistical approach used to test the skewness of the distribution of a random variable, and to explore the asymmetry in the seat share distribution. The case is usually conducted as analyzing the mean-median difference in vote shares for the political parties such as in FPTP with single-member districts. However, the Maltese case differs from this system as it features multi-member constituencies, as previously mentioned in Sect. 2. Therefore, if the average seat share obtained by a given political party across districts is statistically significantly higher than that party's median seat share, the party in question is most likely to succumb to the gerrymandering strategy (Wang, 2016b; Grofman, 2019). The main idea behind the *consistent-advantage test* is that if a party's seat share is intentionally concentrated in a few constituencies with very high margins, the party's average seat share will eventually increase, while the median value will be low because the party's seat share in the *majority* of constituencies would be reduced.[14]

Before proceeding, consider the seat share distribution across the 13 electoral districts in 2017 shown in Table 2.

The median value for the MLP seat share is 56.92%, while the average is 60.0% (lower). The case is the opposite, however, for the PN: the median falls short of the mean, 40.0% and 43.08% respectively. In short, according to these comparisons, it seems that the electoral district distribution favors the MLP against the PN.

Visualization of the data mentioned above is displayed in Fig. 5. The graph on the left shows the distribution of the seat share for the MLP across the 13 electoral districts, the black and the red vertical lines denoting the mean and the median respectively. Thus, the median seat share is slightly higher than the average seat share which implies that no evidence of gerrymandering against the MLP can be found. Likewise, the second graph on the right displays the same calculations for the Nationalists. Unlike the MLP, the average seat share for the Nationalists appears to be slightly higher than the corresponding median. An open question is whether the mean is statistically significantly higher than the median or not. We turn to this next.

In order to statistically analyze the mean-median difference of the parties' seat shares, we applied the skewness test introduced in Wang (2016a, 2016b) which, in turn, follows Lemma 3 in Cabilio and Masaro (1996), p. 351, for testing the symmetry of a distribution function relating to an unknown median (see Appendix C for further details on the skewness test (S_k)).[15]

We next simulate the distribution of the S_k statistic in Eq. (5) in Appendix C under such an assumption. Thus, after running 10 million random samples of size 13 (the total number of electoral districts) out of uniform distributions for the MLP

[14] As Katz et al. (2020) point out, although the mean-median measure may assess partisan asymmetry in several ways, it should not be the only test used to draw conclusions. Similarly, Wang (2016b) highlights the need of performing several statistical tests to avoid false-positive outcomes.

[15] Note that the *true* median of the seat share distribution is known for neither of the two parties.

Table 2 Seat share distribution (%)

MLP	40	40	40	40	60	60	**60**	60	60	80	80	80	80
PN	20	20	20	20	40	40	**40**	40	40	60	60	60	60

Bold values represent each party's average seat share

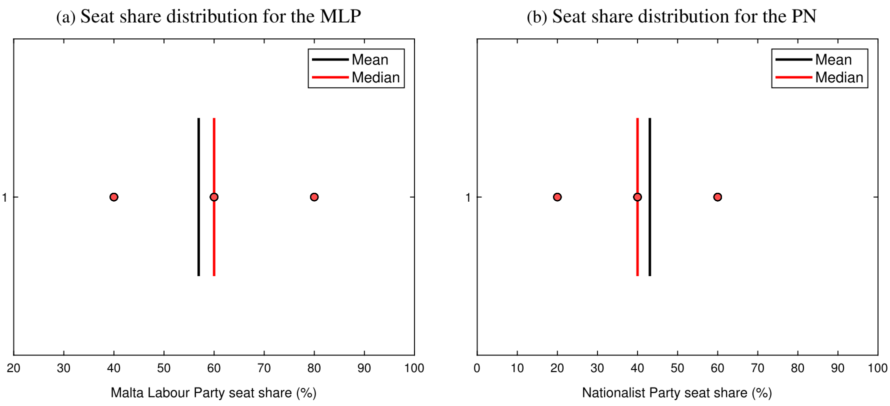

Fig. 5 Analysis of Intents: Mean-median difference in seat shares. *Key:* Seat share distributions across electoral districts. In both figures black (red) bars represent the average (median) seat shares

and the PN seat shares, recall $U(SS^{min}_{MLP}, SS^{max}_{MLP})$ and $U(SS^{min}_{PN}, SS^{max}_{PN})$, we obtain the empirical distributions of S_k for both parties, S^{MLP}_k and S^{PN}_k. The result is shown in Fig. 8 in Appendix C, where the two empirical distributions are represented by the histograms colored blue. For completeness, the figure also shows the corresponding fitted normal distributions in red: the computed mean and variance for S^{MLP}_k are -0.25×10^{-3} and 0.7541, respectively, while those of S^{PN}_k are 0.2×10^{-5} and 0.7542. Finally, the figure also shows the obtained values for the test statistics on the black bar: $S^{MLP}_k = -0.7978$ and $S^{PN}_k = 0.7978$, so that the implied p-values are 0.81 and 0.19 respectively.

We next consider the case with bonus seats granted to the PN. Once again, the mean and median seat shares are represented in Fig. 6 for both political parties. At first glance, the mean seat share is smaller than the median seat share for the MLP, 55.13% and 60.0% respectively. As regards the PN, mean seat share (44.87%) still exceeds the median seat share (40.0%) after the bonus seats were introduced in order to adjust for the balance between the seat and the vote shares at the national level.

For completeness, we also apply the test statistic S_k once more considering the two bonus seats granted to the PN. The results are presented in Fig. 9 in Appendix C where blue histograms represent the two empirical distributions, and red lines show the fitted normal distributions. In this case, the mean and variance for S^{MLP}_k are 0.29×10^{-3} and 0.7539, respectively, whereas for those of S^{PN}_k are 0.87×10^{-4} and 0.7539. Lastly, the black lines depict the test statistics: $S^{MLP}_k = -1.3650$ and $S^{PN}_k = 1.3649$, so that the implied p-values are 0.94 and 0.06 respectively. To conclude: the equality of the mean and the median seat shares for both parties across the 13

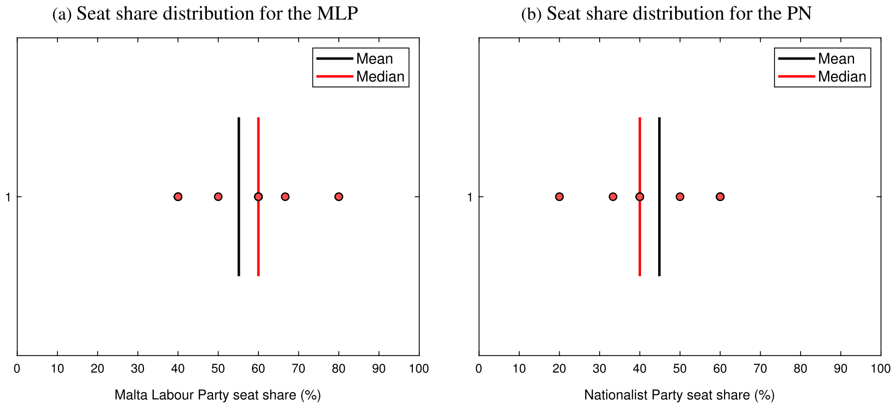

Fig. 6 Analysis of Intents: Mean-median difference in seat shares. *Key:* Seat share distributions across electoral districts. In both figures black (red) bars represent the average (median) seat shares

electoral districts, i.e. the absence of gerrymandering according to the *consistent-advantage test*, can be rejected regardless of whether the bonus seats are considered or not.

4.3 Excess-seats test

Following Wang (2016a, 2016b), we run a third test, the *excess-seats test,* with the purpose of checking whether the vote share obtained by a party given the current 13 electoral district setting unexpectedly deviates from the vote share that this party would have obtained at the national level. More precisely, the experiment that we run is as follows. Imagine that we could build up a set with a large enough number of replicas for the results of the general elections in 2017, so that in each of these we had 13 electoral districts randomly chosen.[16] And, next, among all the possible combinations in this set, we could select only those *fantasy* (or *synthetic*) Maltas in that the difference between *i*) the observed distribution of votes for the six parties participating in the electoral process, and *ii*) the synthetic distribution of votes (at the national level) were equal to zero (or low enough up to some arbitrary point). Finally, we will compare the seats that each party obtained in the 2017 general elections with the average number of seats which that party would have obtained in those fantasy Maltas. The question that we will answer is: what is the probability

[16] A similar analysis was carried out by Wang (2016b) for the case of the state of North Carolina in the United States. In each potential replica, thirteen electoral districts (the number of North Carolina electoral districts) were randomly chosen from the whole (i.e nationwide) set of electoral districts. Among such potential replicas, only those with a vote share distribution "equal" to that of North Carolina were kept: the "synthetic" or fantasy North Carolinas. Finally, a comparison was made between the observed seats obtained by each party in North Carolina, and the averages that those parties would have obtained in the synthetic North Carolinas.

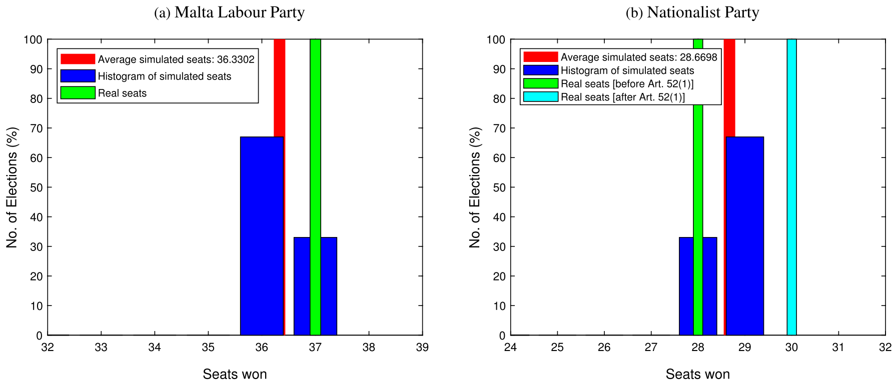

Fig. 7 The excess-seats test: simulated and actual seats. *Key*: Observed and simulated seat numbers. In both figures, the blue bars represent simulated seat numbers, while the green bars show the actual seat. In addition, the turquoise bar in the second figure shows the actual seat number after the bonus seats were granted to the PN under Article 52

that a given party would have obtained a higher number of seats than the ones effectively won if the electoral district distribution had been different?

The first step is to compute all possible combinations of the 13 electoral districts.[17] And from this collection, we preserve only those combinations of electoral districts where the discrepancy between the potential synthetic Malta's vote share distribution and the actual vote share distribution is small enough, i.e. less than the first percentile. As a consequence, we now have 440,159 fantasy Maltas.

As a final step, we next calculate the seats that each party (in practice, only the two majoritarian parties at play) would obtain in each of these fantasy Maltas, thereby obtaining two seat distributions [See Fig. 7]. Consider first the MLP. As the figure shows, the MLP would obtain 36 seats in 66.98% of the elections and 37 seats in the remaining 33.02%, or 36.33 on average, the actual number of seats being higher, 37. In short, the probability for the MLP to obtain a higher number of seats than the one effectively obtained is 0. In other words, this third test does not provide empirical evidence of gerrymandering in favor of the Malta Labour Party.

As the natural counterpart, a similar analysis can be carried out for the Nationalist Party. The PN would have obtained 28 seats and 29 seats in 33.02% and 66.98% of the synthetic Maltas, respectively, implying an average of 28.67 seats. On this occasion, the comparison with the actual number of seats that the PN obtained in 2017 requires some qualification though. Recall that, as a consequence of the constitutional amendment of Article 52, the PN was granted two bonus seats, so that its total number rose from 28 to 30. Thus, before the amendment was implemented, the probability for the PN to obtain a higher number of seats would have been 66.98%. Or, in other words, the electoral district set up would imply the existence of

[17] For a technical explanation of how to calculate all possible combinations of the 13 electoral districts, see Appendix C.

gerrymandering against the Nationalist Party which was corrected by the adjustment for the bonus seats. The bonus seats adjustment, however, would more than offset the initial partisan bias against the PN.

5 Concluding remarks

Whilst the pace of governments collecting data rises, usage of knowledge is also becoming more significant in the rapidly changing technical environment. Many insights can also be collected for democratic campaigns, and various computer systems or algorithms can be used to gain benefits or to undertake various political initiatives. As a result of these factors, in elections the question of fairness is becoming more and more relevant. Political scientists have devised many methods and statistical tests to discourage such politics and, in particular, have sought to reduce the possible anomalies caused by redrawing electoral constituencies.

In the light of the above information, we have examined Malta's two recent general elections in this article. On the one hand, Malta is open to potential gerrymandering possibilities by means of the two-party system; on the other, the single transferable vote system proves itself to be one of the most proportional among electoral systems in terms of converting votes cast into seat distribution. As a first step, we have used statistical tests to detect possible attempts at gerrymandering by analyzing the average vote margins of the MLP and the PN. That is, we have questioned how far political parties go in their attempts to obtain parliamentary seats in the corresponding electoral districts. Consequently, we have posed the hypothesis test to determine whether the average vote shares of each political party in the electoral constituencies were statistically different, and we have measured its significance with the *t*-test. According to the results, we have found that the difference between the average vote shares of the two parties was not statistically significant. As a second analysis of intents, we have measured the mean-median difference in national-level seat shares of both political parties as a skewness test in order to detect partisan asymmetries that might have been caused by political maneuvering. Once again, we have come to the conclusion that the mean-median difference, which we have subjected to significance tests, was not statistically significant. At the last stage, and with the information obtained from the two tests mentioned, we have obtained statistical inference with bootstrap simulations, taking into account the votes of all parties that entered the 2013 and the 2017 general elections. Accordingly, we have attempted to identify possible anomalies or partisan bias by comparing the observed average vote shares of the political parties with the vote shares estimated via the bootstrap simulations. As in the first two tests, we did not find any signs of gerrymandering in the final test, and we can conclude that Malta's two last governmental elections were held in fair election processes.

In general, three more important conclusions can be drawn from this article. First, and frequently discussed in political science literature, the STV favors the majoritarian parties, especially as regards the droop quota, which makes it harder for the smaller parties to return representatives to the government. Thus, the system reinforces governability by reducing the chance of having coalitions in the parliamentary. Note, however, that in our case there are two parties at issue and, as pointed out by a referee, governability is easiest when a single party is in charge. Second, as argued by the political elite in Malta, there would be no change in the overall outcome by having a single constituency for the entire nation. Third and finally, regarding the constitutional amendment of the bonus rule, and contrary to expectations, proportionality of party representation is actually increased.

A Appendix: How the STV works

This appendix illustrates the way the STV works on an empirical base. In Table 3 below, seven candidates from the first electoral division in Malta's 2017 general elections are displayed together with the votes they received. Note that although there are a total of 23 candidates in constituency 1, only 7 are included in the example for simplicity. The second column in the table shows the aforementioned first-preference votes. That is, it corresponds to the total of the first-preferences received by the electors on the ballot paper. At this stage, firstly, the amount of quota candidates have to reach in order to be elected from this constituency will be determined as $(24,196/(5+1))+1=4033$, as stated in Eq. (1). In other words, the listed candidates must have reached a total of 4033 votes in order to be elected. As reflected in the second column, only two of the candidates (Jose Herrera and Mario De Marco) at the first counting of the votes will reach the quota and will be automatically elected from the constituency. Therefore, according to the number of second preference votes on the ballots, the number of votes cast by these candidates as surplus votes after exceeding the quota is transferred to other candidates.

In the Table, the amount of votes to be added to other candidates is indicated in the third column. For example, since Jose Herrera surpassed the quota with 597 votes and Mario De Marco with 688, these surplus votes were distributed among the other candidates in proportion to the electors' second choice preferences on the ballot paper, using the formula

$$\left[\frac{\textit{second preference votes}}{\textit{total votes of the candidate}}\right] \times \textit{surplus votes}. \tag{2}$$

This transfer procedure continues until sufficient candidates (5 in the current example) have met the quota to fill all the seats to be elected. In certain cases, for example, after calculating the number of second preference-votes for candidates, if any candidate fails to reach the determined quota, then the candidate with the fewest votes is excluded and his/her votes are transferred to the voters' second preferences among the candidates. Accordingly, as seen in column three, Silvio Parnis is eliminated at this stage as he has the lowest amount of votes (1506), and his votes are distributed among other candidates to be shared. After the third count votes are calculated, Aaron Farrugia is elected as he reaches the quota. Paula Mifsud will be eliminated at this stage since she has the lowest number of votes (2465). Her votes are therefore transferred to the fourth count. As a consequence, a total of five

Table 3 Election in the first electoral constituency in Malta, 2017

Electorate: 25,598, Total Valid Poll: 24,196, Seats: 5, Quota: 4033

Candidates	First count	Second count		Third count		Fourth count	
Debattista, Deo (MLP)	2378	+194	2572	+357	2929	+1104	*4033*
Farrugia, Aaron (MLP)	3600	+149	3749	+284	*4033*	–	*4033*
Herrera, Jose' (MLP)	4630	−597	*4033*	–	*4033*	–	*4033*
Parnis, Silvio (MLP)	1385	+121	1506	−1506	–	–	–
De Marco, Mario (PN)	4721	−688	*4033*	–	*4033*	–	*4033*
Grech, Claudio (PN)	2606	+215	2821	+738	3559	+474	*4033*
Mifsud, Paula (PN)	2000	+338	2338	+127	2465	−2465	–

Source: The Electoral Comission Malta

Italicized and underlined numbers indicate the amount of droop quotas that each candidate must receive in order to be elected in the constituency

delegates will be appointed as members of the parliament from this district when the fourth count is completed.

B Appendix: Comparison of seat allocation formulas

B.1 Other allocation methods at the district level

As noted in Sect. 3, a total of 14 seat allocation formulas were applied to the given vote shares of the political parties. The gap between the implemented m-th electoral formula and the actual number of the STV seats is given by the mean absolute error. More precisely, the mean absolute error can be defined as

$$MAE_m \equiv \frac{\sum_{d=1}^{13} \sum_{p=1}^{6} |S_{p,d}^{STV} - S_{p,d}^{m}|}{13 \times 6}, \tag{3}$$

where $S_{p,d}^{STV}$ represents the actual number of seats obtained by the p-th political party in the d-th district under the single transferable vote system; and $S_{p,d}^{m}$ stands for the seats obtained by the p-th political party in the d-th district under the m-th electoral method. Therefore, the obtained difference, $S_{p,d}^{STV}$-$S_{p,d}^{m}$, will give the deviations from the actual seats obtained under the STV system. Consequently, this process was applied to all political parties in Malta (in total 6). Regarding the electoral formulas, and for the sake of space saving, only those which attained the lowest MAE in Eq. (3) were considered. Table 4 presents the results of the 2017 general elections in the same constituencies when the other seat allocation methods are applied. The second and third columns display the actual number of seats obtained under the STV, while the fourth and fifth columns indicate the outcome of the d'Hondt method being applied to votes cast in the same districts. Similarly, columns six and seven show the results achieved with the use of the largest remainder electoral formula, and lastly

Table 4 Distinct electoral formulas applied to each electoral constituency

District	STV		D'Hondt		Largest remainder		Sainte-Lague	
	MLP	PN	MLP	PN	MLP	PN	MLP	PN
1	3	2	3	2	3	2	3	2
2	4	1	4	1	4	1	4	1
3	4	1	4	1	4	1	4	1
4	4	1	4	1	***3***	***2***	4	1
5	3	2	***4***	***1***	3	2	***4***	***1***
6	3	2	3	2	3	2	3	2
7	3	2	3	2	3	2	3	2
8	2	3	2	3	2	3	2	3
9	2	3	2	3	2	3	2	3
10	2	3	2	3	2	3	2	3
11	2	3	2	3	2	3	2	3
12	2	3	2	3	2	3	2	3
13	3	2	3	2	3	3	3	3
Total	37	28	38	27	36	29	38	27
MAE[18]	–		0.026		0.026		0.026	

[18] There were other seat allocation methods that happened to attain the same MAE [see Eq. 3)], such as the Imperiali and the Dean. *Key:* In addition to the actual seats obtained under the STV system, implemented electoral formulas d'Hondt, largest remainder, and Sainte-Lague for the given constituency. The last row indicates the mean absolute error as obtained in Eq. (3). Finally, the bold italic numbers show the differences compared to the actual STV in that constituency. Note that results are presented without taking into account possible bonus seats

columns eight and nine show the parliamentary seats that would have been obtained with the implementation of the Sainte-Lague method.

B.2 Uniqueness of the electoral constituency

Table 5 depicts the number of parliamentary seats that would be achieved as a result of adopting the STV, the d'Hondt, the largest remainder, and the Sainte-Lague electoral formulas. Note that the bonus seat concept was not taken into account in the calculations, so that each electoral formula was applied to 65 representatives. The second column in the Table shows the observed seat numbers under the STV system for both parties, while the third, fourth and fifth columns indicate the number of representatives that can be obtained by applying the d'Hondt, largest remainder, and Sainte-Lague electoral formulas, respectively.

Table 5 Electoral formulas under the hypothesis of unique constituency

Parties	STV	D'Hondt	Largest remainder	Sainte-Lague
MLP	37	36	36	36
PN	28	29	28	29
AD	–	–	1	–
Total	65	65	65	65

Key: The hypothetical scenario applied where Maltese general elections of 2017 are held under one single constituency on a nationwide level

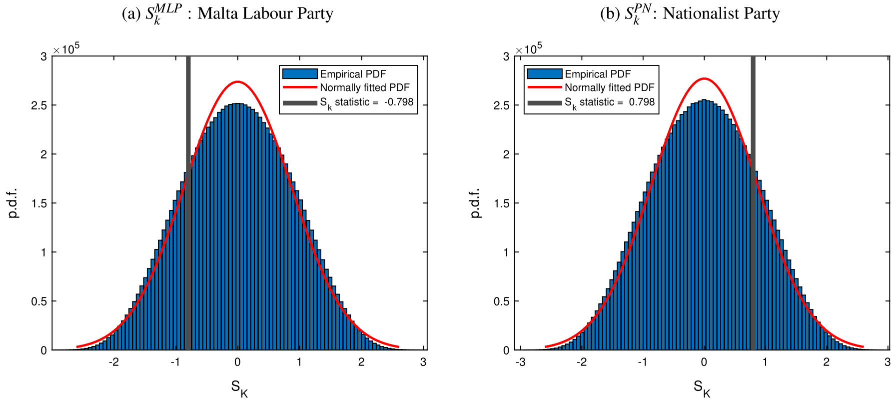

Fig. 8 S_k distributions for the MLP and the PN. *Key:* Skewness test statistic S_k (black) for both parties [See Eq. (5)]. Histograms (blue) show the empirical probability density functions for the MLP [panel (a)] and for the PN [panel (b)]. The red lines represent the normally fitted probability density functions with mean and variance equal to -0.25×10^{-3} and 0.7541, respectively, for the MLP, and 0.2×10^{-5} and 0.7542 for the PN

C Appendix: Statistical tests

C.1 Lopsided-outcomes test

As mentioned in Sect. 4, the corresponding t-statistic is performed to assess whether the difference in average votes may be attributed to chance,

$$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} \tag{4}$$

where

$$s_1^2 \equiv \frac{\sum_{i=1}^{n_1}(x_i - \bar{x}_1)^2}{n_1 - 1}, \qquad s_2^2 \equiv \frac{\sum_{j=1}^{n_2}(x_j - \bar{x}_2)^2}{n_2 - 1}$$

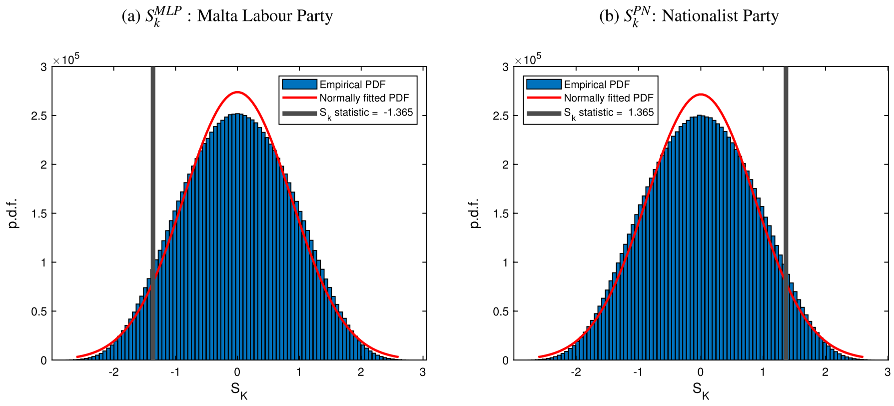

Fig. 9 S_k distributions for the MLP and the PN. *Key:* Skewness test statistic S_k (black) [See Eq. (5)]. Histograms (blue) show the empirical probability density functions for the MLP [panel (a)] and for the PN [panel (b)]. The red lines represent the normally fitted probability density functions with mean and variance equal to 0.29×10^{-3} and 0.7539, respectively, for the MLP, and 0.87×10^{-4} and 0.7539 for the PN

and where $\bar{x}_1$ and $\bar{x}_2$ are the sample means, s_1^2 (s_2^2) is the sample variance of x_1 (x_2), n_1 (n_2) is the sample size of the x_1 (x_2) sample.[18]

More formally, the null hypotheses can be expressed as $H_0 : \mu_1 = \mu_2$, i.e. the data provide convincing evidence that there is no significant difference between the population mean vote shares of the MLP and the population mean vote shares of the PN, the alternative hypothesis being $H_1 : \mu_1 \neq \mu_2$ (in other words, the difference between the two means is statistically significant).

C.2 Consistent-advantage test

As noted in Sect. 4, we used the skewness test to assess the symmetry of a distribution function relating to an unknown median in order to statistically examine the mean-median difference of the parties' seat shares. Thus, assuming a random variable X which follows cumulative distribution function F, it can be shown that if its mean and its median are the same, then (asymptotically)

$$S_k = \frac{\sqrt{n}(\overline{X} - m)}{s} \rightarrow N(0, \sigma_0^2(F)) \tag{5}$$

where $\overline{X}$ and m represent the sample mean and the sample median respectively, s denotes the sample standard deviation, and n stands for the sample size. Regarding

[18] Note that there are two options for the t-test, assuming equal variances and unequal variances. Here only the unequal variances case will be shown since the conclusion of both were identical. This is the so-called Welch-Satterthwaite correction and refers the resulting statistic to the t-distribution instead of the standard normal with a random number of degrees of freedom, see (Lehmann & Romano, 2005; Armitage & Berry, 1994).

the variance of the test statistic, $\sigma_0^2(F)$, and this is a key point, it depends on the *parent* distribution F (normal, uniform,$\cdots$) from which the sample is drawn [see Cabilio and Masaro (1996) for details].[19] We assume that the seat shares of the parties follow uniform distributions, $U(a, b)$, where a and b denote the minimum and maximum seat shares of the party in question: SS_{PN}^{min} and SS_{PN}^{max} for the Nationalist Party, and SS_{MLP}^{min} and SS_{MLP}^{min} for the Malta Labour Party.

C.3 Excess-seats test

As stated in Sect. 4, this subsection of the appendix illustrates how to compute all possible combinations of the 13 electoral districts. In general, we have that the number of combinations with repetition of m elements, taken n at time, $CR_{m,n}$, is given by

$$CR_{m,n} \equiv \binom{m+n-1}{n} = \frac{(m+n-1)!}{(m-1)! \times n!}. \tag{6}$$

In our case, $m = n = 13$, so that we obtain a potential number of 5,200,300 synthetic Maltas.[20] For each i-th combination of these, we compute the (norm of the) difference between the vote share distribution of the i-th potential synthetic Malta, $\boldsymbol{V}_i \in \mathbb{R}^6_{\geq 0}$, and the observed vote share distribution, $\boldsymbol{V}^{obs} \in \mathbb{R}^6_{\geq 0}$, where

$$\boldsymbol{\Delta}_i \equiv \| \boldsymbol{V}^{obs} - \boldsymbol{V}_i \| . \tag{7}$$

And from this set, we keep only those combinations of electoral districts for which the difference between the vote share distribution of the potential synthetic Malta and the observed vote share distribution is small enough, more specifically, less than the first percentile. As a result, we are finally left with 440,159 fantasy Maltas. The result is shown in Fig. 10, where the histogram for $\boldsymbol{\Delta}_i$ is shown in blue color, and a red vertical line represents the 1st percentile of the distribution.

[19] Wang (2016a, 2016b) assume normality for the vote shares of the political parties, and eventually this would allow the vote share to be negative in the simulated (theoretical) distribution function.

[20] This is an instance of the so-called bootstrapping sampling method. Computations were made with the Matlab© combinator function (for more information, see (Efron & Tibshirani, 1994; Dekking et al., 2005)).

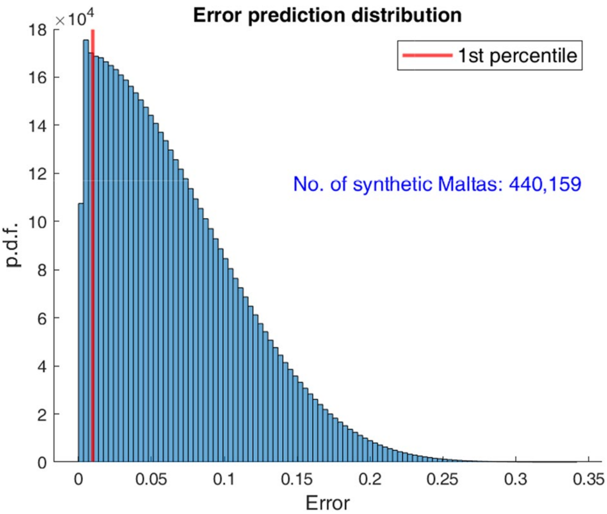

Fig. 10 Error prediction distribution obtained for the simulated general elections of Malta. *Key:* Histogram of Δ_i in blue color [See Eq. (7)]. The vertical red line represents the 1^{st} percentile (0.0023)

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