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ORIGINAL PAPER

A Dodgson-Hare synthesis

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Abstract
In 1876, Charles Dodgson (better known as Lewis Carroll) proposed a committee election procedure that chooses the Condorcet winner when one exists and otherwise eliminates candidates outside the Smith set, then allows for re-votes until a Condorcet winner emerges. The present paper discusses Dodgson's work in the context of strategic election behavior and suggests a "Dodgson-Hare" method: a variation on Dodgson's procedure, for use in public elections. This method allows for candidate withdrawal and employs Hare's plurality-loser-elimination method to resolve persistent cycles. Given plausible assumptions about how candidates decide whether to withdraw when there is a cycle, Dodgson-Hare outperforms Hare, Condorcet-Hare, and 12 other voting rules in a series of spatial-model simulations that count how often each rule is vulnerable to coalitional manipulation. In the case of a one-dimensional spatial model, *all* coalitional voting strategies that are possible under Condorcet-Hare can be undone in Dodgson-Hare, by the withdrawal of candidates who have incentive to withdraw.

Keywords Voting rules · Dodgson · Hare · Condorcet · Borda · Plurality · Approval

JEL Classification D71 · D72

1 Introduction

Two years after he published *Through the Looking Glass*, Charles Dodgson entered in his journal, "Began writing a paper (which occurred to me last night) on 'Methods of Election', in view of our election of a Lee's Reader in Physics and a Senior Student next Wednesday." Though he seemingly was not aware of either Borda's or Condorcet's ideas on voting, he at first proposed a system much like Borda's; and then, after reconsidering at some point during the next six months, he proposed a system much like Condorcet's. However, unlike most modern voting rules based

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on Condorcet's pairwise comparison idea, the method Dodgson arrived at provides that in the event of a majority rule cycle, voters may *discuss* the issue further before potentially amending their votes.[1]

Although Dodgson was developing voting procedures for use by small committees of well-acquainted colleagues[2] in which strategic behavior was a limited concern, one may apply essential features of his final proposal to large, anonymous, high-stakes elections. To this end, this paper defines a "Dodgson-Hare" procedure[3] for conducting public elections that hybridizes Dodgson's method with Thomas Hare's idea of iteratively eliminating plurality losers,[4] and argues that the resulting voting rule may prove unusually resistant to strategic manipulation.

2 Dodgson's *A method of taking votes on more than two issues* (1876)

As both an academic mathematician and a voting member of the Governing Body of Christ Church, Oxford, Charles Dodgson became interested in the mathematical theory of committee elections in December of 1873; and between this time and February of 1876, he wrote three pamphlets on the subject for distribution among his colleagues. In *A method of taking votes on more than two issues*—which is both the last and most sophisticated of the three[5]—he proposes the following voting procedure.

First, each voter indicates one candidate, and the majority winner is elected if one exists. If there is no majority winner, voters cast ranked ballots, and the Condorcet winner is elected if one exists.

Of the next step, Dodgson wrote, "If an issue be found which has a majority over every other taken separately, it shall be formally moved as in Rule II: but if none be found, the majorities being 'cyclical', opportunity shall be given for further debate." At this point, Dodgson suggested eliminating all candidates outside what is now called the Smith set.[6]

After that, he recommended, "When the issues to be further debated consist of, or have been reduced to, a single cycle, the Chairman shall inform the meeting how many alterations of votes each issue requires to give it a majority over every other separately." Thus, Dodgson here defined what is now sometimes[7] called a "Dodgson score"—which is zero for a Condorcet winner, and positive for any other candidate.

[1] For both the text of Dodgson's pamphlets and their historical context, I rely on Black (1958).

[2] These committees decided things like academic appointments, and at one time the design of a new belfry.

[3] Green-Armytage (2015) described a similar idea (referring to it as a "Dodgsonesque procedure"), but only in brief.

[4] Hoag and Hallett (1926, 162–95) indicate that Hare began advocating this provision (as a refinement of the single transferable vote) in 1865.

[5] For more detail about the other two pamphlets, see Section A2 of the online Appendix to this paper.

[6] Dodgson wrote, "If the issues cannot be all arranged in one cycle, but form a cycle and a set of issues each of which is separately beaten by each of the cycle, it shall be formally moved that this cycle be retained and all other issues struck out, and, if none object, this shall be done." This definition aligns closely with Smith (1973).

[7] For example, by Nurmi (1983), and by Bartholdi et al. (1989).

However, Dodgson did *not* propose for this score to automatically determine the outcome in the case of a cycle. Instead, he concluded his proposal as follows: "If, when the majorities are found to be cyclical, any elector wish to alter his paper, he may do so: and if the cyclical majorities be thereby done away with, the voting shall proceed by former Rules: but if, when none will make any further alteration, the majorities continue cyclical, there shall be no election." Thus, he did not want to resolve cycles by algorithmic force based on the initial set of ranked ballots; rather, he preferred to proceed by way of *conversation*.

After completing his proposal, Dodgson went on to critique two possible algorithmic methods of resolving cycles: The first is analogous to Condorcet-plurality, while the second is analogous to Condorcet-Hare. Here, he argued by constructing alphanumeric examples (and analyzing them with the aid of pairwise matrices) in which each method fails to select the candidate that has the lowest Dodgson score. However, since he still did not suggest that cycles should be resolved by electing the candidate with the lowest Dodgson score, the modern convention of referring to such a rule as "Dodgson's method" is misleading.[8]

3 The problem of strategy in single-winner elections

Gibbard (1973) and Satterthwaite (1975) showed that when there are three or more candidates, any election rule that satisfies universal domain and non-dictatorship must give incentives for strategic voting in some cases.[9] This poses a profoundly serious problem for constitutional design: When selecting a single-winner election rule, it is necessary to consider not only whether the final outcome represents the aggregate of the voters *expressed* preferences in a satisfying way, but also whether the voters' expressed preferences are likely to accurately reflect their *actual* preferences. If not, the collective choice process may devolve to a case of "garbage in, garbage out."

This section proceeds by example (as Dodgson did), illustrating some of the possible consequences of strategic behavior in eight commonly discussed election rules. To complement this anecdotal approach, Sect. 6 reports the frequency with which each of fifteen rules including Dodgson-Hare is vulnerable to strategic voting, in a series of issue-space models.

8 This metamorphosis of a Dodgson *score* into determinative Dodgson *election rule* occurs in Fishburn (1973), who wrote under the header "Dodgson's Inversion Method": "Our final function is based on C. L. Dodgson's idea of taking inversions in the orders in *D*, and will therefore be referred to as *Dodgson's function*." Nurmi (1983) defined "Dodgson's method" equivalently, and Bartholdi et al. (1989) referred to the same as "the Dodgson voting scheme.".

9 Their proofs established that *individual* voters could sometimes get a preferred outcome by reporting false preferences; this implies in turn that *groups* of voters can also sometimes get a mutually preferred outcome by reporting false preferences. In large elections, it is unlikely that any individual will have an opportunity to change the result by voting strategically; hence, in selecting a rule for deciding an election with many voters, vulnerability to coalitional strategy (rather than individual strategy) is the more pressing concern.

3.1 Plurality[10]

Example 1: 45 ABC; 20 BAC; 20 BCA; 15 CBA.

Let this notation indicate that the candidates are A, B, and C, that 45 voters sincerely prefer A ≻ B ≻ C, and so on. If everyone orders the candidates sincerely, the plurality rule chooses A. However, if the 15 CBA voters place B first instead, B wins with 55 points.

This voting behavior may be called *compromising*: a type of strategy that entails voters raising the ranking (or rating) of a candidate in order to cause that candidate to win.[11] In the short run, the use of a compromising strategy may at times *improve* the election outcome, in that it causes the election of the sincere Condorcet winner. However, in the long run, such strategies may lead to systemic dysfunction. That is, the compromising strategy is more effective when employed by a larger group of voters who agree on *which candidate* to compromise on; thus, the custom of conducting party primaries results from each group's need to answer this question in order to compete. Further, once one such party has formed that includes close to half the electorate, any other coalition that includes much less than half the electorate has little chance of winning. Therefore, except in periods of disequilibrium, plurality voting is unlikely to produce more than two serious competitors in each race. Thus, the plurality rule tends to produce *two-party* political systems: a result known as Duverger's law.[12] Such systems can be dysfunctional to the extent that they limit voter choice, create a zero-sum game between the two parties (which disincentivizes cooperation), and reduce political competition (along with the anti-corruption effects it might otherwise provide).[13]

3.2 Borda, approval, and range[14]

Example 2: 40 ABC; 60 BAC.

Applying the Borda rule to Example 2, if votes are sincere, Borda awards 140 points to A, 160 points to B, and 0 points to C, so that B wins. However, if the ABC

[10] Here I assume that voters may submit complete rankings, but plurality uses only their first preferences.

[11] Green-Armytage (2014) defined "burying" and "compromising" strategies while attributing both terms to Blake Cretney, as they appeared in essays on his (no longer active) web site Condorcet.org during the early 2000s.

[12] See Duverger (1964), and note that the two dominant parties may vary from district to district within the same nation.

[13] Green-Armytage (2014) provides evidence that plurality is particularly susceptible to both the compromising strategy and to strategic exit: defined as a candidate exiting a race in order to change the outcome to one they prefer. Both of these results suggest that plurality has an especially strong tendency to reduce the number of parties to two.

[14] Here I define Borda, approval, and range voting as the following rules which share the feature that the candidate with the most points wins. In Borda, a first-choice vote is worth $\boldsymbol{N} - 1$ points (where $\boldsymbol{N}$ is the number of candidates), a second-choice vote is worth $\boldsymbol{N} - 2$ points, a third-choice vote is worth $\boldsymbol{N} - 3$ points, and so on. In approval voting, each voter may assign each candidate either one point or zero points. In range voting, each voter may assign each candidate any score within a predetermined closed interval.

voters change their votes to ACB, A wins instead—a strategic incursion that would not be possible in plurality, runoff, Hare, or any Condorcet-consistent system.

This type of behavior may be called *burying*: a strategy in which voters worsen the ranking (or rating) of one candidate in order to cause a more-preferred candidate to win. Since the most commonly used single-winner systems (viz., plurality, runoff, and Hare) are never vulnerable to burying, its long-term effects are substantially less well-understood than those of the compromising strategy.

Example 3: 40 A|BC; 35 BC|A; 25 CB|A.

This notation indicates that 40 voters sincerely prefer A $\succ$ B $\succ$ C and sincerely approve only of A, and so on. Applying the approval rule to Example 3 under sincere voting, there is a *tie* between B and C, who each receive 60 points to A's 40. However, if one BC|A or one CB|A voter alters their ballot to approve only their favorite candidate, that candidate wins. As Nagel (2007) shows, this type of example creates a strategic dynamic between the BC|A and CB|A voters analogous to a game of chicken, in which approving both candidates is analogous to swerving, approving only one candidate is analogous to driving straight ahead, and the election of A is analogous to the car crash. Such a dynamic lacks an obvious practical resolution; and since it does not require an exact tie in the sincere outcome, it may not be uncommon.

Under range voting, if voters maximize the differential they add between the two perceived front-runners, they will cast ballots that are functionally equivalent to approval ballots. Hence, approval and range voting may produce similar strategic results.[15]

3.3 Runoff and Hare[16]

Example 4: 45 ABC; 10 BAC; 10 BCA; 35 CBA.

Given Example 4 and sincere voting, both runoff and Hare eliminate B first, and elect A. However, if the CBA voters compromise in favor of B, they can ensure that B wins. Thus, as in Example 1 with plurality, strategic voting can restore the Condorcet winner, but at the cost of suppressing information about voters' sincere preference for candidate C—and thus (in some pivotal cases) limiting the ability

[15] Green-Armytage (2014) provides evidence that Borda, approval, and range voting are vulnerable to strategic voting with unusually high frequency, and also that Borda has an unusual vulnerability to strategic *entry*, defined as a candidate entering a race to change the outcome to one they prefer, without winning. Hence, whereas strategic nomination in plurality tends to reduce the number of candidates, strategic nomination in Borda would most likely *increase* the number of candidates, leading to unforeseen consequences.

[16] Define runoff as the rule by which voters rank the candidates, the two candidates with the most first-choice votes proceed to a runoff election, and the candidate with the most votes in the runoff election wins. Define Hare as the rule by which voters rank the candidates, and the candidate with the fewest first-choice votes is eliminated (and deleted from the rankings) in each of a series of rounds, until one candidate remains.

of a new party to effectively challenge an existing major party within a two-party system.[17]

3.4 Minimax[18]

In three-candidate elections, minimax is equivalent to various other Condorcet-consistent rules, including ranked pairs and beatpath[19]; hence, these rules should have similar rates of strategic vulnerability. For Example 1 (45 ABC; 20 BAC; 20 BCA; 15 CBA) any of these rules elects the Condorcet winner B if votes are sincere; but if the ABC voters bury B by voting ACB, they can elect A. This strategy works by creating a *false cycle*, where although B still defeats A pairwise, A also defeats C, *who defeats B in turn*.

Since Condorcet rules are not yet used for public elections, we lack practical experience on which to build an understanding of how often such a false-cycle strategy might occur in practice. However, if and when it *did* occur, and was used to change the result, it may justifiably be regarded as a serious breach of the election's integrity.

3.5 Condorcet-Hare

Whereas Green-Armytage (2014) shows that Hare frequently resists strategic voting and Condorcet-consistent rules frequently resist strategic nomination,[20] Green-Armytage (2011) shows that *Condorcet-Hare* rules[21] resist both strategic voting *and* strategic nomination more frequently than most other rules. Further, Green-Armytage et al. (2016) and Durand et al. (2016) both prove that for most single-winner voting rules including Hare, adding a provision to elect the Condorcet winner when one exists can never make the rule vulnerable to strategy in cases where it was not vulnerable already. Hence, Condorcet-Hare resists strategy at least as often as

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[17] Green-Armytage (2014) provides evidence that Hare is rarely vulnerable to strategic voting, but both Hare and runoff are quite vulnerable to strategic *exit*. The latter result fits with the intuition from Example 4: That is, even if the CBA voters do not compromise in order to elect the Condorcet winner, candidate C (who is losing anyway) could achieve the same result by *withdrawing* before the race, so that B is elected. Since all of C's supporters prefer B to A, it is likely that C will as well, in which case C has both the means and the motive to exit the race.

[18] Defining $\boldsymbol{P}$ as the pairwise matrix and thus $\boldsymbol{P}_{XY}$ as the number of voters who rank candidate X above Y, define minimax as the rule by which the winner is the candidate Y with the smallest maximum value of $\boldsymbol{P}_{XY}$.

[19] See Tideman (1987) and Schulze (2003) for the definitions of ranked pairs and beatpath, respectively.

[20] That is, both strategic exit and entry. Intuitively, whenever the voting rule is Condorcet-consistent and there is a sincere Condorcet winner, no candidate will have an incentive to exit or enter strategically.

[21] That is, voting rules that elect the Condorcet winner when one exists, and otherwise use Hare's method of successively eliminating the plurality loser. Green-Armytage (2011) defines and compares four such rules, which are equivalent in the three-candidate case.

Hare, which the literature often finds to be more resistant to strategy than most other single-winner rules.[22]

However, election strategy is still a legitimate concern in Condorcet-Hare. To understand how, consider again Example 4 (45 ABC; 10 BAC; 10 BCA; 35 CBA). Whereas Hare elects A when votes are sincere, Condorcet-Hare chooses B to begin with; yet, the fact that the Condorcet winner and the Hare winner are different still provides an opportunity to manipulate Condorcet-Hare—in this case, in the opposite direction. That is, if the ABC voters cast their ballots as ACB, this creates a false cycle in which C pairwise-beats B; and Hare resolves the cycle in favor of A. Again, if strategists succeed in taking an office from the sincere Condorcet winner in this way, this will constitute a serious subversion of the election process.

4 A proposed Dodgson-Hare synthesis

4.1 Dodgson-Hare

For the purposes of this paper, define "Dodgson-Hare" as the voting procedure described by the following four steps.

Step 1: Cast ranked ballots, and elect the Condorcet winner if one exists.

Step 2: If there is a cycle, eliminate candidates outside the Smith set, as well as any candidate who voluntarily withdraws. Elect the Condorcet winner if one exists among remaining candidates.

Step 3: If there is still a cycle, cast a new set of ballots ranking the remaining candidates. Eliminate candidates outside the Smith set, as well as any candidates who voluntarily withdraw. If the cycle persists, eliminate the candidate with the fewest first-choice votes. Elect the Condorcet winner if one exists among remaining candidates.

Step 4: Repeat Step 3 until it produces a Condorcet winner.

4.2 Limited-round Dodgson-Hare

Dodgson-Hare only requires one round of voting in cases with either a Condorcet winner or a cycle that resolves via voluntary candidate withdrawal; and in the case of a top cycle with N candidates, it requires at most $N - 1$ rounds. Nonetheless, some societies may prefer an alternate version with a lower upper limit on rounds.

For example, a society that is comfortable with one extra vote at most (as in a runoff) could adopt the following modified procedure. First, cast ranked ballots, and elect the Condorcet winner when one exists. Second (if there is a cycle), eliminate the non-Smith candidates and allow candidate withdrawal. Third (if there is still a

[22] See for example Chamberlin (1985), Lepelley and Valognes (2003), Favardin and Lepelley (2006), Tideman (2006), Green-Armytage (2014), and Green-Armytage et al. (2016).

cycle), have a second vote, then allow candidate withdrawal again before applying a Smith-Hare[23] tally to the new set of ballots.

Limited-round Dodgson-Hare procedures (which can be constructed with any round limit, including one) may reduce the discursive benefits of Dodgson-Hare while also moving it further from the spirit of Dodgson's proposal; but in terms of the formal strategic resistance analysis that follows in the next two sections, Dodgson-Hare and limited-round Dodgson-Hare would reach equivalent results.

5 Theoretical properties of Dodgson-Hare

5.1 General properties

The Dodgson-Hare procedure derives philosophically from Dodgson's recommendation in that it selects the Condorcet winner when one exists, and otherwise allows for discussion and a possible alteration of votes. It differs from Dodgson's proposal by including a separate provision for the voluntary withdrawal of candidates, and employing a modified version of Hare's elimination method in some cases. These added features are intended to adapt Dodgson's idea to the problem of public elections. For example, whereas Dodgson was content to declare "no election" in the case of a persistent cycle (presumably because a committee can always return to the issue in its next meeting), the Hare elimination feature hastens a final decision.

If there is a sincere Condorcet winner and no voter strategy, Dodgson-Hare elects the sincere Condorcet winner. If there is a sincere cycle and no strategy, the winner will come from the sincere Smith set.[24]

5.2 General strategic properties

Proposition 1: If there is a sincere Condorcet winner X, and voters who prefer candidate Y change their votes to create a Smith set that includes Y, that Smith set must also still include X.[25]

In cases where a sincere Condorcet winner exists, this result may enable a well-informed public to exert powerful resistance against any strategic incursions. To illustrate this, consider again Example 4 (45 ABC; 10 BAC; 10 BCA; 35 CBA), where the ABC voters are able to make A the Condorcet-Hare winner by voting ACB. However, if they attempted this in *Dodgson*-Hare, there would first be an

[23] Define Smith-Hare as a rule that eliminates all candidates outside the Smith set, then conducts a Hare tally.

[24] In the latter case, if candidates' decisions to withdraw depend on whether doing so produces a winner who is ideologically closer to them, more "central" candidates (relative to the other candidates) are more likely to win. For example, in a spatial model with a three-candidate cycle among A, B, and C, if C is more distant from both A and B than they are from each other, then candidate C will not win Dodgson-Hare.

[25] For proofs of all formal propositions in this paper, see Section A1 of the online Appendix.

opportunity for the public to *scrutinize and reconsider* this result before it became final.

Then, the critical question is this: Given what they know about the candidates and their supporters, will the public be able to *discern* that many or all of the ACB votes are insincere? (In this example, it might be quite easy, since the sincere preferences are consistent with a one-dimensional spatial model, with A and C at opposite ends of the spectrum.) If a sufficient number of voters and/or candidates come to believe this, they can infer that B is the sincere Condorcet winner, and restore the victory to B in one of two ways: First, candidate C can voluntarily withdraw, thus electing B without the need for a second vote. Failing this, the B ≻ A voters (who comprise a majority) can alter their votes on the second ballot by ranking B first, so as to deprive the A ≻ B voters of any further opportunities for strategy.

As this anecdote suggests, Dodgson-Hare's potential to resist strategic incursions lies not only in the *tally algorithm* rejecting candidates whose victories rely on strategic votes, but also in the ability of the *public* to identify such candidates. Given a sufficiently sophisticated public discussion of a false first-round cycle, it may well be possible for defenders of the sincere Condorcet winners to identify and nullify nearly all strategic incursions against the same. This could make Dodgson-Hare a promising alternative for single-winner elections in strategy-prone electorates. However, since the ability of voters to discern insincerity in Dodgson-Hare is difficult to *quantify*, the formal analyses in this section and the next make no attempt to do so.

Proposition 2: If X is the Smith-Hare winner with sincere votes, no group of voters who prefer Y to X can make Y the Smith-Hare winner without including at least one member of the sincere Smith set in the observed Smith set.

Proposition 2 generalizes Proposition 1 to accommodate cases where sincere voting produces a majority rule cycle. That is, when the sincere Smith set contains only one candidate, the two propositions are equivalent; but for cases with a sincere cycle, Proposition 2 clarifies that even if a group of voters alters their ballots to change the result, at least one member of the sincere Smith set will reach Step 2.

5.3 Strategic properties in a spatial model

To provide some additional structure to the analysis of strategic behavior, and in particular to motivate an endogenous decision rule for candidate withdrawal, define a spatial model as one in which all voters and candidates can be located in an *S*-dimensional issue space, such that both voters and other candidates prefer candidates who are closer to them in this space.

Proposition 3: In a one-dimensional spatial model, if voters who prefer Y to the sincere Condorcet winner X make Y the Condorcet-Hare winner by voting insincerely, there must exist some candidate or set of candidates who have the incentive to drop out in Step 2 of Dodgson-Hare, so that X still wins.

Green-Armytage et al. (2016) found that a one-dimensional spatial model produced more instances of strategic vulnerability in Condorcet-Hare than most other data-generating processes; intuitively, this occurs whenever the central Condorcet winner is "squeezed" too closely on both sides by two other candidates, so that the

Condorcet winner does not have sufficient first-choice support to avoid elimination in a Hare count. Thus, it may be valuable to consider whether Dodgson-Hare can offer a remedy in such an environment.

Proposition 3 shows that, in the special case of a one-dimensional space, *every* incursion that succeeds in Condorcet-Hare can be thwarted by a candidate or group of candidates who are inclined to withdraw—if not because they know that their defeat of the sincere winner was arranged insincerely, then at least because they prefer the sincere winner to the insurgent candidate. Intuitively, if B is the sincere Condorcet winner, supporters of a candidate A to the *west* of B can only make A the Condorcet-Hare winner by way of a false cycle in which some other candidate C to the *east* of B pairwise-beats B. But since C is closer in the issue space to B than to A, C has an incentive to withdraw, which restores the victory to B and renders the strategy ineffective.

In a spatial model with *more than one* dimension, it *is* possible for a voting strategy that succeeds in Condorcet-Hare to also succeed in Dodgson-Hare, even if candidates withdraw when it brings the outcome closer to their position.[26] Hence, Sect. 6 below attempts to quantify the frequency of such a situation.

Further, although candidate withdrawal in Dodgson-Hare can favorably resolve many of the instances in which Condorcet-Hare is vulnerable, it does introduce the possibility of another type of strategy, with which a particularly devious group of partisans *might* be able to succeed if the rest of the electorate is especially oblivious. For instance, turning again to Example 4 (45 ABC; 10 BAC; 10 BCA; 35 CBA) and supposing that candidate B prefers A to C, if 32 of the 45 ABC voters cast ACB ballots while the remaining 13 vote BAC, candidate *C* wins in Smith-Hare. This initially makes the strategic voters *worse off*, *but* if candidate B (the sincere Condorcet winner) is utterly naïve about the situation, they *may* agree to withdraw from the race in order to change the winner from C to A. Thus operates what I refer to as the "cooperative victim strategy," because it relies on the willing cooperation of the candidate being targeted.

If a candidate thus targeted becomes aware of the deception and refuses to drop out, the cooperative victim strategy is not effective (as it makes the strategizing voters worse off). Thus, the cooperative victim strategy may be implausible in most circumstances. However, for the sake of completeness, Sect. 6 below reports two strategic resistance scores for Dodgson-Hare in each model: one that assumes that the cooperative victim strategy never works, and another that assumes that the cooperative victim strategy works whenever it is *mathematically* feasible (hence, without regard for whether it is *socially* feasible).

[26] For example (supposing for simplicity that candidates are not also voters), place candidate A and 45 voters at (0,0) in a two-dimensional space, B and 10 voters at (0,1), C at (2,0), 10 voters at (1.125,1), and 35 voters at (2,1).

Table 1 Strategic resistance of selected voting rules in six spatial-model simulations

Dimensions	1	1	2	2	4	4
Voters	9	999	9	999	9	999
Condorcet Winners	1.0000	1.0000	0.9884	0.9998	0.9834	0.9997
Dodgson-Hare without CVS	1.0000	1.0000	0.9841	0.9991	0.9775	0.9987
Condorcet-Hare	0.8918	0.8149	0.9348	0.9361	0.9503	0.9724
Hare	0.8918	0.8149	0.9348	0.9361	0.9503	0.9724
Condorcet-anti-plurality	0.9351	0.9197	0.8786	0.8025	0.8632	0.7697
Minimax	0.8954	0.8403	0.8721	0.8038	0.8658	0.7973
Baldwin	0.8590	0.7642	0.8705	0.7838	0.8771	0.7921
Condorcet-Plurality	0.8701	0.7720	0.8547	0.7750	0.8541	0.7871
Black	0.8868	0.8185	0.8306	0.7197	0.8130	0.6927
Condorcet-Coombs	0.8802	0.8164	0.8387	0.7119	0.8246	0.6714
Plurality	0.8086	0.7075	0.8269	0.7647	0.8401	0.7862
Dodgson-Hare with CVS	0.8120	0.6999	0.8148	0.6619	0.8316	0.6659
Coombs	0.8645	0.8100	0.7600	0.6533	0.7226	0.5934
Borda	0.5644	0.4830	0.6099	0.5305	0.6271	0.5540
Approval	0.4744	0.4423	0.5424	0.5005	0.5767	0.5318
Anti-plurality	0.5104	0.5445	0.5161	0.4725	0.5210	0.4452
Range	0.1357	0.0189	0.1583	0.0293	0.1819	0.0384

6 Resistance to strategy in spatial-model simulations

Table 1 shows the results of six simulations, each of which measures the strategic resistance of a voting rule as the share of trials in which, beginning with sincere voting, it is *impossible* for any coalition of voters to secure a preferred outcome by voting insincerely. All simulations assume a spatial model with three candidates, and employ 100,000 trials; but they differ in the number of issue dimensions (ranging from one to four) and voters (ranging from nine to 999).[27] The top panel reports these parameters as well as the share of trials in which a Condorcet winner exists, while the table's main body reports strategic resistance scores for 15 voting rules (counting Dodgson-Hare only once). Rules are arranged in descending order of their average strategic resistance score.

In all six simulations, Dodgson-Hare performs strictly better than all other voting rules considered here, provided the assumption that the cooperative victim strategy (denoted as CVS) is not feasible. On the other hand, if it *is* feasible, Dodgson-Hare receives low-to-middling marks for strategic resistance. Hence, one may say that Dodgson-Hare displays extraordinarily frequent strategic resistance in this model if and only if one agrees that the cooperative victim strategy is unrealistic.

[27] Section A3 of the online Appendix provides descriptions of the algorithms used to determine whether coalitional manipulation is possible.

7 Conclusions

In the normative theory of democracy and constitutional design, a voting rule that resists the most disturbing effects of strategic manipulation is a critical desideratum. Above, I briefly argue that all of the single-winner voting systems most widely discussed in the literature may indeed lead to disturbing strategic consequences in plausible scenarios; and thus, that this desideratum has yet to be obtained.

In the ensuing pages, I consider the possibility that Charles Dodgson's idea of allowing for *deliberation* in the event of a cycle may provide a valuable check on strategic voting behavior *that other voters (and/or candidates) can correctly identify as strategic*. I am currently unable to quantify the likelihood with which result-altering strategy may be so discovered in the event of a cycle, but I do suggest that in some cases (such as a political environment in which candidates' positions may be approximately mapped to a one-dimensional issue space), identifying false cycles may be relatively straightforward.

Finally, as a first (imperfect) attempt at measuring the strategic resistance of a proposed Dodgson-Hare hybrid, I employ a series of spatial-model simulations. These provide preliminary evidence that Dodgson-Hare may be non-manipulable with a higher frequency than most other commonly considered voting rules.

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Declarations

Competing interests The authors declare no competing interests.

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