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ORIGINAL PAPER

The case for minimax-TD

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Abstract
In spatial-model computer simulations with artificial voters and candidates, the well-known minimax single-winner voting system far outperformed 10 other systems at picking the best winners. It essentially tied with two others (Schulze and ranked pairs), both of which are far more complex than minimax. Minimax's other advantages include Condorcet consistency, simplicity, monotonicity, and ease of voting because it allows tied and missing ranks. It also makes insincere strategic voting schemes difficult and dangerous for the schemers. The new minimax-TD system modifies minimax in three ways, all of which make it pick better winners, according to simulation studies: (a) TD includes Smith/minimax, (b) TD uses one particular definition of a candidate's "largest loss" in two-way elections, and (c) TD includes a multi-stage tie-breaker which breaks nearly all ties. TD avoids four of the worst anomalies afflicting classic minimax. Four other minimax anomalies can be ignored, leaving TD arguably free of anomalies.

Keywords Minimax · Voting systems · Condorcet · Spatial voting models · Voting anomalies

JEL Classification D71 · D72

1 Introduction

1.1 Summary of the advantages of minimax-TD

A new voting system called minimax-TD has several advantages over other systems. Some of these advantages are shared with other versions of minimax, whereas others are unique to minimax-TD. This section merely lists the major advantages. Some of the evidence supporting these claims appears later in this paper, though much of it is in other papers which are freely available online,

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especially Darlington (2022). Section 2 of that paper also has brief descriptions of most of the voting systems mentioned in this one. Minimax's advantages are:

1. **Simplicity**. Voters can easily remember, and explain to others, how the basic system works. Each voter ranks the candidates. Those ballots are used to run a two-way majority-rules race between each pair of candidates, counting for each candidate the number of voters who ranked them above their opponent in that race. The difference between those two numbers is the **margin of victory** for one and the **margin of loss** for the other. If there is a **Condorcet winner** (someone who wins all their two-way races), they win. If there is a **Condorcet paradox** (no Condorcet winner), we compute the size of each candidate's **largest loss (LL)**, and name as winner the candidate with the smallest LL. There are three main ways to define LL, but for reasons given in Sect. 1.4, minimax-TD defines it as a candidate's largest margin of loss.
2. **Efficiency** – the ability to pick the winners genuinely preferred by the voters. In computer simulations with artificial voters and candidates, in which we know each voter's preference for each candidate, minimax emerges as far more efficient than 10 other voting systems, especially with larger numbers of voters. When each artificial-data trial had 1000 voters, minimax-TD picked a better winner than any of 10 competing systems in at least 60% of the trials in which they picked different winners. (Sect. 1.2 explains what we mean by "better winners.") When TD was compared just to the three most widely-used voting systems—plurality, plurality with runoff between the top two candidates, and Instant Runoff Voting (IRV)—that figure rose to 98% or higher. By our measure of efficiency, TD was essentially tied with two other systems (ranked pairs and Schulze systems), both of which are far more complex than TD.
3. **Ease of voting**. Minimax-TD uses ranked-choice ballots, with tied and missing ranks allowed. That is, voters need not rank candidates with whom they're not familiar, and they may give the same rank to candidates they prefer equally. Both of those facts make voting easier. And unlike plurality (vote for one) or approval ballots (approve as many as you wish, the candidate with most approvals wins), voters are allowed to express all the preferences they have, so voters need not spend time deciding which preferences they will leave unexpressed. If a voter fails to rank a candidate, that candidate receives a rank below all others; that removes any incentive for candidates to remain unknown to voters who they think might rank them poorly. Minimax is not typically used with rating scales, though that could be done, and might make voting even easier.
4. **Resistance to insincere strategic voting**. By some measures, the Condorcet-IRV voting system resists strategic voting schemes better than minimax. But Sects. 3 and 4 argue that under both voting systems, the schemes are difficult and dangerous enough that they seem unlikely to be used. If they are used, it will probably be obvious after the election, so society can decide later whether minimax must be abandoned for future elections.

5. **Condorcet consistency (CC)**. If a voting system picks every Condorcet winner as winner, that system possesses CC. Of the 20-odd best-known voting systems, only about half possess CC. IRV lacks CC, but all forms of minimax possess it.
6. **Monotonicity**. Surprisingly, analysts have discovered artificial situations in which candidate X loses an election because voters moved X *up* (toward the top) in their ranking of candidates, or in which X becomes a loser when newly added voters all rank X at the top. Voting systems subject to this anomaly are sometimes called *nonmonotonic*. Felsenthal and Nurmi (2017) created a more precise definition of monotonicity which is too complex to describe here. Amazingly, they found that of 18 voting systems they analyzed, only four besides minimax enjoyed full monotonicity: plurality, approval, Borda, and range voting, none of which possess CC. Felsenthal and Nurmi (2017) show, in addition, that the Black, Bucklin, Copeland, and Young voting systems are monotonic for a fixed electorate, though not when new voters are added to a previous list of voters. All of these except Bucklin possess CC. However, they all have little if any support for actual use. There are three other voting systems, simplified Dodgson, ranked pairs and Schulze, that Tideman (2006, Chapter 13) shows are monotonic for a fixed electorate and possess CC. The first of these has little if any support for actual use, and the last two, as explained below, almost always produce the same results as Minimax-TD and have other disadvantages described later.
7. **Avoidance of other anomalies**. Anomalies (often called paradoxes) are problems revealed by artificial-data examples designed to demonstrate the disadvantages of particular voting systems. Nonmonotonicity and Condorcet inconsistency are two famous anomalies. Felsenthal (2012) studied the susceptibility of 18 voting systems to 16 anomalies including those two, and concluded that every system suffered at least 6 anomalies. I argue that minimax-TD suffers no serious anomalies, and is matched in this respect only by Schulze and ranked pairs, both of which are far more complex and yet nearly always pick the same winner as minimax-TD.
8. **Resistance to ties**. Minimax-TD has a multi-stage tie-breaker which breaks nearly all ties. Simulations show it chooses substantially better winners than random tie-breakers.

1.2 Artificial-data examples, simulations, and IRV

This section introduces artificial-data examples and simulations to readers new to voting theory, using examples which also demonstrate anomalies in the popular IRV (instant runoff voting) system. IRV is now used in New York City and many other US cities, in Alaska and Maine, and in at least seven countries outside the US. IRV eliminates the candidate with the fewest top ranks, recomputes the ranks, and repeats that process until only one candidate is left.

But suppose a scale of liberalism-conservatism runs from 1 to 12, and there is one voter at each integer along that scale. Suppose voters all prefer the candidates closest to themselves on the scale, and candidates A, B, and C are respectively at 3, 6, and 9. Then the top choices will be A for the 4 voters from 1 to 4, B for the 3 voters from 5 to 7, and C for the 5 voters from 8 to 12. B has the fewest top ranks, so IRV eliminates B first. C then beats A and wins. But we can define the best candidate as the one with the smallest mean distance from voters on the scale. Here that candidate is B, and B would beat either A or C in a two-way race, so IRV has chosen poorly. Similar anomalies appeared in a 2022 US special Congressional election using IRV in Alaska; Graham-Squire and McCune (2022) show that a Condorcet winner lost, and there was also nonmonotonicity since the IRV winner could have lost if *more* people had ranked her first.

I ran 10,000 trials of an artificial simulation, each with 5 candidates and 100 voters, to see how often errors like these might occur. In each trial, the scale scores of both candidates and voters were picked randomly and independently from a univariate (one-variable) bell-shaped normal distribution, since real-world opinions are often distributed roughly normally. Again, each voter was assumed to rank the candidates by their closeness to themselves. By our mean-distance criterion, IRV picked the best of the 5 candidates in only 5856 of the 10,000 trials, so it failed to pick the best winner over 40% of the time. On 66 trials it actually picked the worst of the 5 candidates. Artificial-data examples and simulations like this are used widely in modern voting theory, and they appear throughout this paper.

1.3 Brief history of classic minimax

Minimax was invented independently by Black (1958, p. 175), Simpson (1969), and Kramer (1977), though Black never recommended it. It was little used for years, probably because of some anomalies described in Sect. 2, but its acceptance has increased in recent years. I believe Tideman (2006, p. 238) was the first major electoral theorist to include it in a short list of the apparently best systems. In 2017, computer scientist Andrew Myers adopted it as the default voting system for his Condorcet Internet Voting Service (CIVS). Myers tells me that over 30,000 nongovernmental elections and polls have now used CIVS since he founded it in 2003. Tideman (2019) recommended the Condorcet-IRV, minimax, and IRV systems as apparently the three best systems. But many electoral theorists still prefer other systems.

1.4 Minimax-TD

Minimax-TD stands for minimax-Tideman-Darlington. It differs from other versions of minimax in three ways described in the next three paragraphs.

Nicolaus Tideman suggested to me the use of Smith/minimax, which applies minimax only within the Smith set. The Smith set is the smallest set of candidates in which every member of the set beats all candidates outside the set, in majority-rule two-way races (Smith, 1973). A simulation by Darlington (2022, p. 10), with 4 candidates and 25 voters in each trial, took nearly 46 million trials to find 1000 trials in which Smith/minimax and classic minimax picked different winners. By the mean-distance criterion, Smith/minimax picked a better winner in 752 of those 1000 trials. That improvement might seem too rare to bother with, but Smith/minimax eliminates four anomalies described in Sect. 2. The Smith set can be found quickly even by hand, using only a table showing the winner of each two-way race. First we put into the set the candidate who won the most races. Then we add anyone who beat that candidate, then anyone who beat any of those new entries. We keep doing that until everyone in the set beat everyone outside it. If there is a Condorcet winner (as there usually is), they are the sole member of the Smith set.

Minimax-TD defines candidate X's **largest loss LL** as X's largest *margin* of loss. The *winning votes* version of minimax defines LL as the largest number of voters who voted against X in any two-way race which X lost, and the *pairwise opposition* version defines LL as the largest number of voters who voted against X in any two-way race, regardless of who won that race. These three versions of minimax can pick different winners if there are tied or missing ranks. In simulations by Darlington (2022, p. 23), the margin-based definition of LL picked a better winner than the winning votes version in 76% of the 21,119 trials in which they picked different winners, and a better winner than pairwise opposition in 78% of 21,693 such trials.

Minimax-TD includes a multi-stage tie-breaker which breaks nearly all ties. Computer simulations in Darlington (2016, pp. 16–18) show that it produces better winners than random tie-breakers do. If two candidates are tied by the LL measure defined above, we re-express each margin of loss as a proportion of the number of voters who gave different ratings to the two candidates in that margin, and define each candidate's LL as the largest of those proportions. If there is still a tie, we look at each candidate's second-largest margin of loss, considering first the raw margin and then the proportional margin. If there is still a tie, we go on to the third-largest margins, etc. This procedure admittedly makes TD more complicated than otherwise, but it is easy to understand and it should be needed only rarely.

2 Dismissing eight anomalies attributed to minimax

Smith/minimax eliminates the two most serious anomalies afflicting other forms of minimax: the **Condorcet loser anomaly** (in which a voting system could pick a candidate who loses all their two-way races), and the **absolute loser anomaly** (in which a system could pick a candidate ranked last by over half the voters). Any absolute loser is a Condorcet loser, since anyone ranked last by most voters will lose all their two-way races. These points are illustrated by an example from Brandt et al., (2022, p. 542), which an anonymous reviewer called to my attention. Suppose there are 9 voters and 4 candidates, and the preference profile is

3	ABCD
1	ADBC
3	CDBA
2	DBCA

A is ranked first by 4 voters and last by 5, so A loses to each of the others by one vote. B, C, and D create a Condorcet paradox, since B beats C, D beats B, and C beats D, each 6–3. Thus the LL values are respectively 1, 3, 3, 3, so A wins under classic minimax despite being an absolute loser and Condorcet loser. But A loses under minimax-TD since the Smith set is BCD.

The **preference inversion anomaly** occurs when a voting system picks as winner the same candidate it would pick if each voter's ranking of candidates were inverted. Classic minimax suffers from this anomaly whenever it picks a Condorcet loser or absolute loser as winner, because inverting the ranks turns a Condorcet loser or absolute loser into a Condorcet winner or absolute winner (ranked first by over half the voters). Thus, by being limited to the Smith set, TD eliminates all *those* instances of the preference inversion anomaly. However, Markus Schulze showed me that preference inversion can still occur in TD. It will occur if one candidate wins or loses every race by a small margin, so that all their losses are still small after ranks are inverted, and every other candidate has at least one larger loss, both before and after inversion. But in my simulations, TD sometimes picks the best winner as defined in Sect. 1.2 even when this occurs. The whole point of an anomaly is to show that a voting system must be picking the wrong winner, and minimax picks the best winner in many of these cases.

The **multiple-districts anomaly** occurs if a candidate can win in each of two or more districts, but lose if the districts are merged into one. Example 3.5.11.4 in Felsenthal (2012) illustrates classic minimax's susceptibility to this anomaly. But that anomaly exists in that example only because classic minimax has picked an absolute loser as winner in one of the districts. Minimax-TD never picks absolute losers as winners, so the Felsenthal example doesn't apply to it, and can be ignored. The same is true of an example of the multiple-districts anomaly in Darlington (2016, Table 3, p. 13).

Minimax-TD does suffer the anomaly of violating what Felsenthal (2012) called the **Subset Choice Criterion** (**SCC**). This anomaly exists if removing a loser from the contest can change the winner. Of the 18 voting systems examined by Felsenthal (2012), only two satisfy SCC, and one of those (range or score voting) is widely agreed to be highly susceptible to insincere strategic voting. Thus, acceptance of SCC virtually forces one to adopt the other system satisfying SCC. That's the majority judgment (MJ) voting system of Balinski and Laraki (2010), in which voters rate candidates on a scale (usually with 5 to 10 points), and the winner is the candidate with the highest median rating. MJ includes a tie-breaker which breaks nearly all ties.

But acceptance of SCC forces one to ignore the fact that losing candidates can provide benchmarks that can be used to compare candidates who beat those losers. For instance, suppose candidates A and B both beat C, D, and E in two-way races, and A beats B by MJ, but B beats C, D, and E by much larger margins than A does. SCC says those latter margins are irrelevant, since they wouldn't even be known if C, D, and E hadn't run. But Darlington (2017) invented two voting systems which use *only* information which SCC calls irrelevant, and showed that those voting systems usually outperform MJ, often by substantial margins. Thus SCC must be dismissed. Darlington (2022, pp. 11–13) gives several other reasons for dismissing SCC.

An example in Felsenthal (2012, pp. 62–63) includes three other anomalies which apply to both classic minimax and minimax-TD. They all could theoretically allow voters to benefit by voting insincerely. Space prevents a detailed discussion of this example here, but Sect. 4.2 of Darlington (2022) explains why the example is unrealistic and can be ignored. Section 3 of the current paper gives six reasons why any type of strategic voting is not likely to be a serious problem under minimax; Sect. 4 is also relevant.

For minimax-TD we have just dismissed all eight of the anomalies that Felsenthal (2012) attributes to minimax, so TD is one of the most anomaly-free voting systems known.

3 Strategic voting under minimax

Under minimax, strategic voting schemes contain several problems for the schemers. First, in most schemes designed to work under minimax, a large fraction of the voters must participate. But voters will naturally assume that voting sincerely will be the best way to advance their own interests, so they will be reluctant to try some scheme unless they fully understand it and believe it will work. But it takes some serious thought to even understand most of these schemes. Many voters will assume correctly that their one vote is unlikely to change the winner, so they will be uninterested in studying these anomalies, and they won't participate in the scheme.

Second, many electorates are large enough that no candidate can win without attracting votes from people who don't know them personally. Few candidates want to promote themselves as schemers trying to subvert the will of the majority. Voters may think, "If I vote for them and they win, how sure can I be that they will keep their campaign promises?".

Third, these schemes require unusually accurate information about how everyone will vote, often including the knowledge that there will be a Condorcet paradox or that the schemers will be able to create one with their insincere votes. If they guess wrong about how others will vote, their scheme may backfire and end up electing someone whom they dislike even more than the winner under sincere voting. Any attempt to insincerely defeat one candidate X means you must rank them below some other candidate Y when you actually prefer X to Y. But that means you're giving Y a better rank than they really deserve, which can help them win if other voters like them better than you thought.

Fourth, even if the schemers guess right about how other voters will vote, they must convince the other schemers that they have guessed right. That may be difficult, since any group of people often have widely different views on what other people are thinking. Again, their scheme will not work if they fail to persuade enough people to participate.

Fifth, if the schemers' opponents discover the scheme (as seems likely, since many people must usually be involved), they could easily use the scheme to embarrass the schemers and to encourage opposing voters to actually vote.

Sixth, schemes by two groups can produce results unfortunate for both. For instance, suppose nearly all voters prefer A and B over C, and that fact is widely known. Voters preferring A over B may then strategically rank B below C, and those favoring B over A may rank A below C. Either strategic scheme might work in the absence of the other. But if voters execute both schemes, then C might be elected even though C is nearly everyone's least favorite candidate. Thus, either group of schemers would actually benefit by abandoning its scheme, thereby electing its second-favorite candidate instead of its least favorite.

Theoretical examples often ignore all such problems facing strategic schemers. But for all these reasons, it seems unlikely that strategic voting would be a serious problem under minimax, especially in larger elections. I have argued that schemes like these are likely to be revealed even during the election. But they're even more likely to be revealed when votes are counted, when a very popular candidate receives many ranks even below obscure minor candidates, or when a scheme's participants brag afterwards about their cleverness, or urge others to help them repeat their scheme in the next election. If these schemes become common in the future, we might reevaluate minimax at that time.

4 Is Condorcet-IRV more strategy-resistant than minimax?

Green-Armytage et al. (2016) concluded that minimax and Condorcet-IRV appear to be the best systems at offering good levels of both efficiency and protection against strategic voting. In Condorcet-IRV, any Condorcet winner wins, and IRV is used otherwise. Those authors concluded that minimax was just slightly more efficient than Condorcet-IRV, but noticeably inferior in protection against strategic voting. But in my own simulations, described in Sect. 5, minimax picked a better winner than Condorcet-IRV in at least 76% of the trials in which they picked different winners. See Table 2 in Darlington (2022, p. 17) for more detailed results of that study than appear in this paper. Another study on pages 21–22 of that paper yielded similar results.

Another simulation, in Darlington (2022, pp. 20–21), explored strategic voting under these two systems. It studied trials in which candidate A would be a Condorcet winner under sincere voting, and B would be one if A didn't run. But in the trials, everyone who preferred B to A voted strategically by ranking A at the bottom, to try to make B win. In agreement with Green-Armytage et al. (2016), those simulations found that A was much more likely to remain the winner under Condorcet-IRV than under minimax. And when A did lose, the strategic voters were less likely to be satisfied with the winner under Condorcet-IRV than under minimax. That's also a point in favor of Condorcet-IRV, since it means the schemers would regret scheming. However, under *either* system, the scheme made both A and B lose, and elected one of the less popular candidates, in over half the trials. The schemers' mean satisfaction with the winner actually went down (from their satisfaction with A) in over half the trials, so the schemers would be unlikely to repeat their schemes. Recall also that this analysis assumed that every single voter preferring B to A would vote to bury A. Section 3 shows how unrealistic that assumption is, and therefore how unlikely it is that either minimax or Condorcet-IRV will suffer much from strategic voting.

5 Computer simulations comparing voting systems in efficiency

Electoral theorists use two major types of mathematical model to compare the efficiency of voting systems. **Valence models** assume that voters focus mainly on general-excellence traits like energy, intelligence, honesty, and experience. **Spatial models** ignore general excellence and assume that voters are concerned mainly with policy issues such as taxes and health care. Spatial models represent each voter and each candidate as a dot in multidimensional space, with each axis in that space representing one policy issue. Each voter is presumed to rank the candidates by their Euclidean distance from the voter in that space, with closest candidates ranked best. The best voting systems are presumed to be those that pick candidates with the smallest mean distance from voters, since their policy preferences are most like those of the voters.

There are ways to tell whether a given voting data set conforms better to a valence model or to a spatial model; Darlington (2022, p. 14) describes some of them. Tideman and Plassmann (2012) analyzed the data in dozens of real elections and concluded that spatial models fitted that data far better than valence models did. Thus, I have emphasized spatial models in my own work. I have mostly used two-dimensional models, as when voters and candidates differ from each other on economic policies and separately on social policies.

Most real-life elections have Condorcet winners, and all Condorcet-consistent (CC) voting systems pick those candidates as winners. Study 2 of Darlington (2022) analyzed 100,000 artificial-data trials with Condorcet winners, with 10 candidates and 1000 voters in each trial. Both voters and candidates were drawn randomly and independently from a bivariate normal distribution, to simulate an election with two main political issues. These trials compared CC systems to 9 non-CC systems. For each of these 9 systems, the study looked at the trials in which the non-CC system had picked a different winner than CC had picked, and computed the percentage of those trials which CC had "won" by picking a better winner by the mean-distance measure of Sect. 1.2. For the 9 non-CC systems, those percentages ranged from 87.0 to 98.9%. Thus, the main conclusion from Study 2 was that CC systems are far superior to non-CC systems in trials like these.

Study 4 of Darlington (2022) used similar methods. It compared minimax-TD to 4 CC systems (Schulze, Black, Nanson, and Condorcet-IRV) and 7 non-CC systems (STAR, Coombs, Bucklin, majority judgment, approval, plurality with runoff, and simple plurality). The Dodgson, Young, Kemeny, Copeland, and range voting systems were excluded from these studies for reasons described in Darlington (2022, p. 16). The exact versions of the approval, plurality runoff, and STAR systems used are described in the same source. Study 4 used 10,000 trials, all with Condorcet paradoxes, with 10 candidates and 1000 voters in each trial. Minimax-TD was compared to each of the 11 other systems, based on winning percentages as in Study 2. To avoid biasing the results against systems which yield many ties, any trial was omitted from the entire analysis if it produced a tie in any of the systems studied. That decision favored the Schulze system, which produces many ties, and disfavored minimax-TD, which produces almost none.

Minimax-TD beat all systems except Schulze in at least 60% of the trials in which they picked different winners. Schulze and minimax-TD picked different winners in only 46 of the 10,000 trials, with TD winning 25 of those 46 trials, so the two systems are essentially tied. Since the Schulze and TD systems both possess CC, they also pick all the same winners whenever there is a Condorcet winner. But TD is far simpler than Schulze and suffers from far fewer ties between candidates, so it seems far preferable. Thus, these studies suggest strongly that minimax-TD is the best of all these systems.

The ranked-pairs voting system (Tideman, 1987) was omitted from these studies because of an earlier analysis I had done with Tideman. We compared that system to classic minimax, with 6 candidates and 1001 voters in each of one million trials. The two systems picked the same winner in all but 16 of those trials. Minimax won 13 of those 16 and ranked-pairs won 3. It would be nice to have more trials in which they had picked differently, but ranked-pairs is far more complex than minimax, so the large number of identical picks is another argument for minimax. And minimax did do much better than ranked pairs among the few cases in which they picked different winners.

These studies show minimax-TD to be far more efficient than all other systems studied except Schulze and ranked pairs, both of which nearly always pick the same winner as minimax despite being far more complex. When those points are combined with all the other minimax-TD advantages mentioned in Sect. 1.1, TD clearly seems to be the best voting system known.

6 Have we reached a milestone?

The high level of agreement among the minimax, Schulze, and ranked pairs systems has some interesting implications, which are further bolstered by two other facts. First, when we exclude minimax and Schulze from the Study 4 mentioned just above, on the average any two of the 10 other systems in that study picked different winners on 7133 of the 10,000 trials. The lowest of those $10 \times 9/2$ or 45 values was 3919, between plurality and plurality with runoff. So, the 46 figure for minimax and Schulze is a real outlier. Second, a very complex voting system, named CMO for "constrained multinomial optimization," is described in Sect. 6 of Darlington (2016). In the only study which ever compared it to minimax, the two systems picked the same winners in 99.994% of 9,138,797 trials. Thus, we have four voting systems which nearly always pick the same winners, very unlike all other major systems. Those four systems were suggested by very different lines of thought, they differ greatly in their mechanics, and they seem to be the most efficient systems known. We would not necessarily expect four such systems to nearly always pick the same winners, and it's quite remarkable that they do, especially when other systems disagree so often in their picks. That raises the possibility that these four are the most efficient possible systems, and we'll never find a reasonable voting system that is noticeably more efficient. And minimax is by far the simplest of these.

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Conflict of interest The authors declare no competing interests.

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