Constitutional Political Economy (2023) 34:357–366
https://doi.org/10.1007/s10602-022-09380-y

ORIGINAL PAPER

Selecting a voting method: the case for the Borda count

Donald G. Saari[1]

Accepted: 3 December 2022 / Published online: 16 December 2022

Abstract
Of importance when selecting a voting method is whether, on a regular basis, its outcomes accurately capture the intent of voters. A surprise is that very few procedures do this. Another desired feature is for a decision approach to assist groups in reaching a consensus (Sect. 5). As described, these goals are satisfied only with the Borda count. Addressing these objectives requires understanding what can go wrong, what causes voting difficulties, and how bad they can be. To avoid technicalities, all of this is illustrated with examples accompanied by references for readers wishing a complete analysis. As shown (Sects. 1–3), most problems reflect a loss of vital information. Understanding this feature assists in showing that the typical description of Arrow's Theorem, "with three or more alternatives, no voting method is fair," is not accurate (Sect. 2).

Keywords Borda count · Voting systems · Voting paradoxes

JEL Classification D71 · D72

1 The choice of an election method matters

The choice of a election method matters. To demonstrate, suppose 15 people are to select a group's beverage of choice. Of them ("$\succ$" means "preferred to"),

six prefer Milk $\succ$ Wine $\succ$ Beer,
five prefer Beer $\succ$ Wine $\succ$ Milk,
four prefer Wine $\succ$ Beer $\succ$ Milk.

The plurality outcome is Milk $\succ$ Beer $\succ$ Wine with Milk as the winner. Beer wins a runoff election between Milk and Beer. Wine wins by comparing

Donald G. Saari
dsaari@uci.edu

[1] Department of Mathematics and Department of Economics, University of California Irvine, Irvine, CA 92697-5100, USA

candidates with pairwise majority votes. (Wine $\succ$ Milk by 9:6 and Wine $\succ$ Beer by 10:5.) So,

Milk wins with the standard plurality vote.
Beer wins with the common runoff.
Wine wins with majority vote over pairs.

Each candidate cannot be the group's beverage of choice. Yet each, Milk, Beer, or Wine, is the "winner" with one of these widely accepted methods. The general message: rather than an "election winner" identifying what or who the voters want, the outcome can more accurately reflect the choice of an election approach.

It is much worse. A positional voting method tallies a ballot by assigning a candidate a specified number of points based on the candidate's ballot position. (The numbers satisfy the obvious; e.g., a second ranked candidate cannot receive more points than a top-ranked one.) A three-candidate Borda count (BC) uses (2, 1, 0), meaning that a ballot's top, middle, and bottom positioned candidates receive, respectively, 2, 1, 0 points. The BC outcome of Wine $\succ$ Beer $\succ$ Milk, with the 19:14:12 tally, reverses the plurality beverage ranking. The plurality vote weights are (1, 0, 0).

To identify other voting problems, here is a list of the seven positional-method outcomes for the beverage example that arise by using different vote weights ("$\sim$" means "tied"):

$$\begin{array}{lll} \text{Milk} \succ \text{Beer} \succ \text{Wine}, & \text{Milk} \succ \text{Beer} \sim \text{Wine}, & \text{Milk} \succ \text{Wine} \succ \text{Beer}, \\ \text{Wine} \sim \text{Milk} \succ \text{Beer}, & \text{Wine} \succ \text{Milk} \succ \text{Beer}, & \text{Wine} \succ \text{Milk} \sim \text{Beer}, \\ \text{Wine} \succ \text{Beer} \succ \text{Milk} & & \end{array}$$

They range from the plurality ranking to the BC reversal. Of the 13 possible transitive rankings of three alternatives (including ties), *over half* of them are possible election outcomes from the same profile of preferences. With more candidates, the problem escalates. The 2016 Republican Presidential primaries had 12 candidates; 12-candidate examples exist (Saari, 1992) with 439,084,800 different positional election outcomes where, depending on the election method, each candidate is the winner in some and the loser in others. Remember, voter preferences are fixed; only the election method changes. Which of the hundreds of millions of methods are reliable? Rather than a rare phenomenon, with three candidates and "central limit" assumptions, for 69% of the profiles, each example yields three or more different positional election rankings (Saari & Tataru, 1999)! The choice of a method matters.

More drastic outcomes arise with Approval Voting (AV) (Brams & Fishburn, 2007) where voters vote for all candidates of whom they "approve." For three candidates, then, a voter decides whether to tally the ballot with (1, 0, 0) "vote for one" or (1, 1, 0) "vote for two." Possible outcomes? With the beverage example, any of the 13 ways to rank the three beverages can be the AV election outcome! Rather than reflecting the voters' intent, it is arguable that AV outcomes resemble a lottery. Indeed, a convincing argument (Tabarrok, 2001) shows that had AV been used in the 1992 Presidential election, we could have had President Ross Perot.

An intent of AV is to let voters more accurately reflect their preferences by choosing how to tally their ballot. An admirable goal, but I had to reject such approaches after discovering consequences: their multiple outcomes can distort the voters' views (Brams et al., 1988; Saari & Van Newenhizen, 1988). As an analogy, suppose in a competitive examination, students can determine how to grade their answers. Students may embrace this as "better reflecting what they know," but it compromises the integrity of the final outcome. Similarly, while disappointing, questionable outcomes will arise if voters can determine how to tally their ballots (Saari, 2010).

More bothersome than AV is where a voter can divide a certain number of points, say ten, among the candidates in any desired manner. (Some versions impose constraints, such as "no candidate can receive more than five points.") Voters may like this approach, but society should not: it admits a wild assortment of dubious election outcomes (Saari, 2010).

2 The silo effect

A common source of lost information across disciplines is the "silo effect." An example from the late 1980s to 2008 is when the financier Bernard Madoff carried out a gigantic Ponzi scheme defrauding investors out of billions of dollars. Although investigated by branches of the Securities and Exchange Commission (SEC), nothing conclusive was uncovered. As explained by Prof. Coffee of the Columbia University School of Law in a PBS NewsHour show on March 3, 2009, "… a former chairman of the SEC told me that, in his view, the SEC was like a group of vertical silos. Each silo knew a lot, but they never shared information with the other silos." Once the silos started communicating, missing information was filled in and a successful criminal case was made.

Silo consequences affect voting rules. This is shown by the following six-candidate example, where the preferences of the 30 voters are

Voters	1st	2nd	3rd	4th	5th	6th
10	Ann	Barb	Connie	Deanna	Elaine	Fred
10	Barb	Connie	Deanna	Elaine	Fred	Ann
10	Connie	Deanna	Elaine	Fred	Ann	Barb

Had this information been available, the group would know that Fred is their bottom choice, and, presumably, Connie is their favorite. Yet, by using the "siloed election method" of a common agenda, Fred can be overwhelming elected! The agenda that achieves this follows.

The winner of first vote between Deanna and Elaine is compared with Connie.
The winner of the second election is compared with Barb.
The winner of the third election is compared with Ann.
The winner of the fourth election is compared with Fred.
The winner of this last election is the overall winner.

For outcomes, everyone prefers Deanna to Elaine, so Deanna advances to be compared with Connie. By unanimity, everyone prefers Connie to Deanna, so Connie advances. In a Connie–Barb election, the first 20 voters prefer Barb, so Barb wins with a landslide two-thirds outcome to move to an election with Ann. In the Ann–Barb vote, the first and the last groups of 10 prefer Ann, so Ann wins overwhelmingly and is advanced to be compared with Fred. Here the second and third groups of 10 prefer Fred to Ann.

2.1 Fred is elected with a landslide two-thirds vote!

As these results demonstrate, Fred is the voters' definitive favorite! Except, he is not.

What goes wrong is that a pairwise vote constitutes a separate silo; it ignores information about the full system. With the Barb–Connie silo election, for instance, only {Barb, Connie} data is considered; here Barb beats Connie. But examine the full system; Connie can beat any other candidate with landslide proportions; she loses only to Barb. Similarly, with the Ann–Fred silo, Fred wins this particular election. *However,* Ann is the *only* candidate that Fred can beat; this crucial information is camouflaged by the agenda's silo structure.

The objective is to find the favored of six candidates, not just of a particular pair. Mimicking the SEC example, a way to recapture the system information is to combine results from the different silos. To do so, note that Connie beats Ann by 20:10, Connie loses to Barb by 10:20, Connie beats Deanna, Elaine, and Fred each by 30:0. Adding up all points that would be cast for Connie over all pairs, the $20+10+30+30+30=120$ value is her system total. In comparison, Deanna and Barb earn 90, Elaine and Ann receive 60, and Fred gets a mere 30. While the siloed approach loses this information, this tallying approach (which is equivalent to the BC, which tallies a ballot by assigning 5, 4, 3, 2, 1, 0 points, respectively, to a ballot's first, second,..., last ranked candidate) recaptures system information, leading to the arguably more valid outcome of

$$\text{Connie} \succ [\text{Barb} \sim \text{Deanna}] \succ [\text{Ann} \sim \text{Elaine}] \succ \text{Fred}.$$

The message is clear; to analyze a system, emphasize the system rather than silos. Similar difficulties arise whenever the set of candidates is divided into separate parts; the information lost by silos causes "the sum of the parts need not represent the whole" complexity (Saari, 2019).

This leads to the seminal Arrow (1963) Impossibility Theorem, which is commonly interpreted as "with three or more candidates, no election method is fair." Central to Arrow's result is IIA, "Independence of Irrelevant Alternatives," where if each voter ranks a particular pair the same in two different profiles (i.e., a list of how each voter ranks the candidates), then the pair's outcome is the same for both profiles. It is not difficult to show (Saari, 2019, 2021) that IIA is a filter; it admits only voting rules that independently compare each pair. Thus, if a procedure satisfies IIA, it can be re-expressed as a collection of independent paired comparisons. No positional method satisfies IIA, which only means that they cannot be expressed in this manner. So, rather than describing all possible voting rules, Arrow's result more accurately shows that “with three or more candidates, no voting rule using independent paired comparisons is fair.” The culprit is IIA; it requires using silos of paired comparisons.

3 Positional methods and runoffs

Suppose that when scheduled for surgery, you must select from among five surgeons, which a procedure has ranked by using several criteria. To stress excellence, only the top-ranked surgeon in each criterion receives a point. Sue, who is second ranked over all criteria, receives zero points; Pat is top-ranked only over his office’s appearance and bottom ranked over all other criteria. With his one point, Pat is ranked over Sue. Who would you prefer, Pat or Sue? Clearly, this ranking method cannot be trusted.

An election for leadership in an organization, state, or country can carry greater consequences than minor surgery, yet we use the above procedure—the plurality vote. (Each criterion is a voter.) Check what the plurality vote does; it rewards a voter’s top choice and treats all other candidates, second on down, as bottom ranked losers. With all of this lost information, expect troubling outcomes. In the 1996 New Hampshire Republican Presidential primary, for instance, based on the press, Bob Dole was the overwhelming favorite of the eight Republican candidates. But Dole was ranked second by many (which the plurality vote equates with being a loser) while Pat Buchanan, who appeared to be bottom-ranked by most voters, was top-ranked by enough to win. In just about any election cycle, similar doubtful plurality outcomes can be found in multi-candidate elections.

These problems are illustrated with the following example (Saari, 2018, Chap. 2) designed to demonstrate that the publicized difficulties with the Oscar awards are more subtle than generally recognized. Here, 100 voters rank five actors:

Number	1st	2nd	3rd	4th	5th
40	A	Smith	D	B	C
29	B	Smith	D	C	A
21	C	Smith	D	B	A
10	D	Smith	C	B	A

Everyone has Smith ranked second, which makes it arguable that he is the favorite; he is the “Condorcet winner” (a candidate who beats all others in pairwise majority votes). Also, D appears to be their second choice (dropping Smith, D becomes the Condorcet winner), and it appears that A and C should be at the bottom (A is the Condorcet loser). But the plurality ranking is $A \succ B \succ C \succ D \succ$ Smith: the Condorcet loser wins and the Condorcet winner is bottom ranked. By ignoring distinctions among candidates (the lost information), Smith’s strong support cannot be registered. As this explanation makes clear, such paradoxical outcomes are very likely to occur.

Furthermore, anticipate similar problems with any voting method in which the flawed plurality vote is a component part, such as the Instant Runoff Vote (IRV). In the above, the IRV drops the favorite Smith at the first stage. The second stage outcome is $A \succ B \succ C \succ D$, where, although D now is top or second ranked by all, D is dropped to leave {A, B, C}. Here B is either top or second ranked by all, but B is dropped. The IRV winner is C.

The "lost information" problem extends beyond the plurality vote to scramble outcomes for any "Vote for k" method. For a different "lost information" difficulty, consider what happens when an organization requires voters to vote for a certain number, say 9, of the nominated candidates. Suppose a voter has a high regard for 4 of them. As distinctions cannot be made, the status of the voter's favorites is jeopardized by voting for other candidates. The obvious incentive (which happens) is to waste the 5 extra votes by voting for 5 weaker candidates who probably would not be elected. Should enough voters vote this way, there can be uncomfortable surprises. The message: avoid voting methods where a component tallying part uses only 0's and 1's, such as the plurality vote, instant plurality runoffs, vote-for-k, pairwise voting over several pairs, Approval Voting, etc.

4 Comparing methods

Reasons why procedures can produce questionable election outcomes have been listed, but an uncountable number of voting rules remain. The challenge is to analyze all of them. As all standard methods involve positional and pairwise rules, they are emphasized.

To find criteria over which to compare methods, I was influenced by Fishburn and by Nurmi (2012); both have constructed clever voting paradoxes. Fishburn, for instance, showed that for an $A \succ B \succ C \succ D$ plurality outcome, dropping D could change the outcome to the reversed $C \succ B \succ A$. The "lost information" discussion makes it clear why this can happen, but what else can occur?

"Paradoxes" are important because they identify a method's unexpected, undesired properties. A goal, then, is to determine not just some, but *all possible* ranking paradoxes allowed for any number of candidates and voters. In a series of papers starting with Saari (1989), this has been completed for all possible positional methods.

A troubling result: *anything* can happen with almost all positional methods! To parse this comment with 7 candidates, rank them in any way; maybe A is top ranked and all others are tied. Then, randomly select rankings for each of the 7 6-candidate sets. Similarly, assign a ranking to each of the 21 5-candidate subsets. Do the same for all subsets down to the 21 pairs. So, 120 rankings, which need not have anything to do with one another, have been selected. Now select a positional method for each subset. With almost all choices, which includes all "vote-for-k" rules, a profile exists where for each subset, its positional outcome is the assigned ranking. Thus, almost all methods allow as high of a level of chaotic outcomes as desired to create serous doubt about what the outcomes mean.

A positive property of a method is a particular paradox that can never occur. The method that avoids the largest number of paradoxical behaviors is the BC (tally an n-candidate ballot by assigning $n-j$ points to the jth positioned candidate). For instance, the BC always ranks a Condorcet winner over the Condorcet loser (which was known); this is not true for any other positional method (which is new). To illustrate with the above movie example, should ballots be tallied by 4, 3, 2, 1, 0, the first stage outcome is the more reasonable Smith $\succ D \succ B \succ A \succ C$ by 300:220:187:160:133. Using the BC rather than the plurality vote in the runoff procedure, the former winner C is the first candidate to be dropped, and Smith is the final winner.

If the BC has a paradox, it also plagues all possible positional methods. Conversely, should another method have a positive ranking property (i.e., a paradox that cannot happen), this property is shared with the BC. The BC has the largest number of positive features; e.g., it relates rankings of subsets, which is not true for other positional methods.

The reason the BC enjoys all of these desired features is that it is the natural extension of the majority vote over a pair. To explain, suppose a voter's preference is $A \succ B \succ C \succ D$. In the three paired elections {A, B}, {A, C}, and {A, D}, this voter votes for A. In the two pairs {B, C} and {B, D}, the voter votes for B. The final pair is {C, D}, where the voter votes for C. So, over the six pairs the sum of points cast by the voter are 3, 2, 1, 0, respectively, to A, B, C, D, which agrees with the voter's BC assignment. (This feature holds for all $n \geq 3$.) So, adding how voters vote over all pairs (as in the "Fred" silo example) always yields the BC tally and outcome. As the BC tallies for any subset are associated with all pairwise tallies, the BC enjoys a large number of consistency properties. Indeed, using the above seven candidate illustration, the plurality vote admits over 10^{50} times more paradoxical problems than are allowed by the BC.

5 Core; strategic voting

The striking Gibbard (1973), Satterthwaite (1975)result proves that for three or more candidates, all reasonable voting rules can be gamed. I expected to find that the BC could be gamed more successfully than others. To my surprise, the BC is the least likely of all positional rules to be successfully manipulated (Saari, 1990). The reason is that the symmetry between assigned points causes a smaller boundary between the sincere and manipulated outcomes than for any other positional method; this reduces the likelihood of successful strategic opportunities.

In deliberations, does a decision approach bring participants together or drive them apart? As an example, suppose the bullets in Fig. 1a identify positions of three voters over an issue. A natural choice to position your candidacy in the middle as indicated by the arrow. No. An opponent could beat you by selecting the dashed arrow position in Fig. 1b; the opponent receives the votes of 1 and 2, while you get only 3's vote. Continuing this analysis, it is clear that the only position that cannot be beaten is the arrow in Fig. 1c; right at the median voter's position. This is the

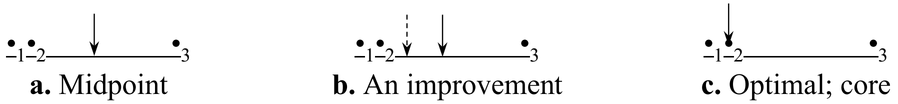

Fig. 1 Selecting a winning position

core; it is a position that cannot be beaten with the specified voting rule. As indicated in Fig. 1, with a core, the dynamic of subsequent positions tends to coalesce toward it.

A seminal, impressive result by McKelvey (1976) and advanced by Schofield (1978) is that with an empty core (which is true in general with two or more issues), the majority vote can drive proposed positions as far from the true wishes of the voters as desired. Rather than attaining group acceptance, an empty core setting can drive a radical separation of views.

So, does any positional method have a non-empty core; is there a method that would attract deliberations toward an acceptable compromise? The surprise is that this is true only for the BC (Saari, 2009). From a technical perspective, this is because the difference between successive weights is the same constant, which eliminates artificial imbalance when evaluating divergent proposals. Conversely, the plurality vote can drive proposals radically far apart.

Does this theoretical conclusion transfer to reality? Based on the experience of Emerson (2020), where he has experimented with various divided groups trying to come together, he finds that only the BC (and obvious modifications) bring groups together.

6 Summary; Borda count

My emphasis betrays that my main interests are theoretical: to identify positive and/or negative consequences of decision rules. Indeed, only in organizations for which decisions affect me do I advocate for any method. As my choice is guided by concrete results, my recommendations are for the BC, or a variant to represent an organization's peculiarities. As a quick review:

The initial challenge was to find a method for which "on a regular basis, its outcomes accurately capture the intent of voters." As shown, the answer is the Borda Count. If a positive feature (an avoided paradox) holds with a standard method, it is shared by the BC. But the BC enjoys many positive features not possible with any other method. It permits distinctions among candidates and reduces the amount and kind of lost information.

Can the BC be gamed? Of course; all methods can. The worst is the plurality vote as voiced in many multi-candidate elections with the strategic "Don't lose your vote, vote for—!" Surprisingly, the BC is the least likely positional method to be successfully gamed.

As for BC variants, when I was a professor at Northwestern, I chaired a committee to hire four faculty members. The department was divided over priorities and, with over 1,000 applicants, it was impossible to use the BC. So, each committee member could select and rank four candidates; the ballots were tallied with a 4, 3, 2, 1 process. The conclusion of the first vote was unanimously embraced.

Similarly, my current advice to an organization that is trying to elect a slate of three, but where each voter is to vote for nine choices, is to replace candidates with "slates;" Here a voter lists three three-candidate slates. Votes are tallied by assigning each candidate in a voter's top, second, and third slates with, respectively, 3, 2, 1 points.

In summary, all available evidence strongly supports using the Borda Count, or variants, in elections.

References

Arrow, K. (1963). *Social choice and individual values* (2nd ed.). Yale University Press.

Brams, S., & Fishburn, P. (2007). *Approval voting* (2nd ed.). Springer.

Brams, S., Fishburn, P., & Merrill, S. (1988). The responsiveness of approval voting: Comments on Saari and Van Newenhizen. *Public Choice, 59*, 121–131.

Emerson, P. (2020). *Majority voting as a catalyst of populism*. Springer.

Gibbard, A. (1973). Manipulation of voting schemes: A general result. *Econometrica, 41*, 587–601.

McKelvey, R. (1976). Intransitivities in multidimensional voting models and some implications for agenda control. *Journal of Economic Theory, 12*, 472–482.

Nurmi, H. (2012). *Voting procedures under uncertainty*. Springer.

Saari, D. G. (1989). A dictionary for voting paradoxes. *Journal of Economic Theory, 48*, 443–475.

Saari, D. G. (1990). Susceptibility to manipulation. *Public Choice, 64*, 21–41.

Saari, D. G. (1992). Millions of election rankings from a single profile. *Social Choice and Welfare, 9*, 277–306.

Saari, D. G. (2009). The core with positional spatial voting. *Homo Oeconomicus, 26*, 501–517.

Saari, D. G. (2010). Systematic analysis of multiple voting rules. *Social Choice and Welfare, 34*, 217–247.

Saari, D. G. (2018). *Mathematics motivated by the social and behavioral sciences*. SIAM.

Saari, D. G. (2019). Arrow and unexpected consequences of his theorem. *Public Choice, 179*, 133–144.

Saari, D. G. (2021). Seeking consistency with paired comparisons: A systems approach. *Theory and Decision, 90*, 1–26.

Saari, D. G., & Tataru, M. (1999). The likelihood of dubious election outcomes. *Economic Theory, 13*, 345–363.

Saari, D. G., & Van Newenhizen, J. (1988). The problem of indeterminacy in approval, multiple, and truncated voting systems. *Public Choice*, *59*, 101–120 and Is approval voting an "unmitigated evil?. *Public Choice*, *59*, 133–147.

Satterthwaite, M. (1975). Strategy-proofness and Arrow's conditions: Existence and correspondence theorems for voting procedures and social welfare functions. *Journal of Economic Theory, 10*, 187–216.

Schofield, N. (1978). Instability of simple dynamic games. *The Review of Economic Studies, 45*, 575–594.

Tabarrok, A. (2001). President Perot, or fundamentals of voting theory illustrated with the 1992 election. *Public Choice, 106*, 275–297.

Publisher's Note Springer Nature remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.