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APPLICATION OF THE ROBUST METHODS FOR ESTIMATION OF DISTRIBUTION PARAMETERS WITH A PRIORI CONSTRAINTS ON PARAMETERS IN ECONOMICS AND ENGINEERING*

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Abstract. *The approach to estimating distribution parameters with a priori constraints has been developed. The Lorentz six-sector model has been constructed. Based on this model, the authors have studied the relationships between food, energy, water resources, supply systems, epidemic, and social situations. They also have investigated the impact of the successive transitions of interconnected sectors to the deterministic chaos regime on the operation mode of the whole system. The method of risk assessment for food security and risk management has been elaborated.*

Keywords: *Lorentz model, mathematical modeling, economic development model, optimal control, deterministic chaos, stochastic models.*

INTRODUCTION

A complex global economy network has significantly strengthened the fields of technogenic, climate-related, economic, biomedical, and social risks. The cumulative effects of various stressors emanating from multiple sources become important generators of uncertainty and instability in the present-day society. They produce a set of interlinked crises that multiply each other. Therefore, an integrated policy is needed to minimize their consequences. Thus, this requires elaborating new methods for the integrated modeling and robust risk management. These methods will allow us to investigate the synergistic interactions between risks of various origins, estimate the critical parameters of risk management, expand the horizon of prediction under uncertainty, provide the risk management of irreversible changes in the safety space, and develop a strategy to minimize their devastating consequences.

The study of complex interdependencies between the above dimensions of security of modern society requires using integrated models that combine the various control loops of socio-economic systems. The efficiency of modeling depends on the completeness of information about the parameters of the studied system. The lack of this knowledge due to the incompleteness of statistical data sample leads to insufficient stability of the approaches based on regression analysis. Therefore, there is a need to create new risk analysis methods, given the efficiency of traditional statistical approaches is rapidly declining. The purposes of this work are as follows: 1) to create a method that will improve the quality of estimation of the parameters of distributions when it is possible to create a system of a priori constraints, combining the possibilities of deterministic and probabilistic approaches; 2) to study, using the six-sector Lorenz model, the relationships between food, energy, and water resources, supply systems, epidemic, and social situation; 3) to create a method for assessment and management of risks for food security.

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ESTIMATION OF THE DISTRIBUTION PARAMETERS WITH A PRIORI CONSTRAINTS

In [1], the method of missing data recovery by solving the problem of identification of parameters of the mathematical model used to study the functioning of a complex system is proposed. The underlying concept of the method consists in determining the trajectories of the variables characterizing the system behavior using the mathematical model. The algorithm for missing data recovery is as follows: 1) based on hypotheses about the laws of functioning of the system, the authors construct its mathematical model; 2) using available data about the dynamics of the system, they solve the problem of identification of its parameters; 3) by solving the Cauchy problem, they determine missing data.

Problem statement. Let the available sample be composed of the results of observations of some complex system $(t_i, \vec{x_i})$, $i = \overline{1, n}$, where n is the number of elements in the sample, t_i is the time of the kth observation, $\vec{x_i} = (x_{i1}, \ldots, x_{ij})$ is the vector of variables characterizing the system at time t_i, and j is the number of variables. Denote by S a set of all ordered index pairs $\langle i, j \rangle$, $i = \overline{1, n}$, $j = \overline{1, m}$, which correspond to the values x_{ij}. Determine from the set S the subset $S1$ of index pairs $\langle i, j \rangle$, for which there are no observation results x_{ij}. Let the mathematical model describing the dynamics of the observed system be given as follows:

$$\frac{dx_j}{dt} = F_j(x_1, \ldots, x_j, A_1, \ldots, A_k, t),$$

where $A_k = A_k(t)$ are the model parameters.

The solution to the problem of recovering missing sample data is to find the missing results of observations x_{ij} for index pairs $\langle i, j \rangle \in S1 \subset S$. For this purpose, we solve the following optimization problem: determine the parameters A_k minimizing the functional

$$I_s = \sqrt{\sum_{i=1}^{n} \sum_{j=1}^{m} (x'_{ij} - x_{ij})^2},$$

where x'_{ij} are the values of variables x_j at time t_i obtained as a result of solving the optimization problem, and x_{ij} are the corresponding values x_j from the sample $(t_i, \vec{x_i})$. To get a numerical solution to the problem of identification of the model parameters, we used a modified random search method, i.e., the stochastic gradient method.

Thus, the proposed method makes statistical methods less vulnerable to missing observation data, as it allows for generating additional data quantity.

ORIGIN OF DETERMINISTIC CHAOS IN THE SIX-SECTOR LORENTZ MODEL

In [2], the authors have developed an approach to studying the relationship between food, water, and energy resources using the three-sector Lorenz model. In addition, they have determined conditions of deterministic chaos origin in the minimum economic development model and revealed the possible causes of the increasing vulnerability of the global economy to small changes in the control parameters. They have considered the problem of determining efficient controls to minimize aggregate structural disturbances over the selected time interval. As a result of model experiments, the trajectories of changes in the control parameters, allowing to reduce the number of structural disturbances, have been revealed. Let us consider the problem of developing the six-sector Lorentz model to study the relationship between food, energy, and water resources, supply systems, and medical, epidemical, and social situations.

Let us construct a model that combines in a holistic structure the sectors of the economy described in the same way. Each of them we will consider in terms of the normalized level of productivity (x_i), the normalized number of jobs (y_i), and the normalized level of structural disturbances (z_i) ($i = 1, 2, \ldots, 6$) for food, water, and energy resources, supply systems, medical and epidemical, and social spheres, respectively). Let us take as a basis the basic model presented in [3]. We rely on the following postulates: 1) there is a competition for labor between different sectors of the economy, so the growth of the production system function in one sector slows down the creation of jobs in other sectors; 2) such factors as variability of crop yields, fluctuations in energy prices, weather-induced changes in the water balance, implementation of new technologies that significantly change workplaces productivity, provoke random disturbances $w_{ij}(t)$ which become additional factors in the increase of the level of structural disturbances; 3) processes in the different sectors of the economy can occur at different speeds, with time scaling by sector using parameters ε_i. The mathematical model has the following form:

$$\varepsilon_i \frac{dx_i}{dt} = \sigma_i (y_i - x_i) + \delta_i \dot{w}_{ij},$$
$$\varepsilon_i \frac{dy_i}{dt} = [F_i(x_1, x_2, \ldots, x_6) - z_i] x_i - y_i + \delta_i \dot{w}_{ij}, \qquad (1)$$
$$\varepsilon_i \frac{dz_i}{dt} = x_i y_i - b_i z_i + \delta_i \dot{w}_{ij},$$

where $F_i = r_i \left(1 - \sum a_{ij} x_i\right)$, a_{ij} are the parameters characterizing competition between different sectors of the economy in labor markets $(i \neq j)$, $w_{ij}(t)$ are the independent standard Wiener processes with parameters $E(w_{ij}(t) - w_{ij}(s)) = 0$, and $E(w_{ij}(t) - w_{ij}(s))^2 = |t - s|$, σ_i, r_i, and b_i are the parameters of the Lorentz model, δ_i are the intensities of disturbances.

A study of the canonical Lorentz model shows that, when increasing, the parameter r can take values that are critical for the origin of both periodic and turbulent trajectories [4]. There are several such values of this parameter: $r = 13.926$, $r = 24.06$, and $r = 24.74$. Since in the six-sector model, the linkage between sectors is performed through parameters $F_i(t)$, we actually have Lorentz models with a variable parameter r. Parameters ε_i and a_{ij} were estimated using the data given in [5]. The impact of infectious disease threats is the most significant compared to other threats being of relevance for the next two years. The anti-epidemic measures taken by national governments lead to a supply chain crisis. Therefore, we assume that this crisis is a short-term risk. Socio-economic crises belong to the category of medium-term risks, which will be of importance for 3–5 years. Water, food, and energy crises are the most influential long-term risks, which are essential for 5–10 years. Therefore, for the long-term processes we select values $\varepsilon_i = 1$ $(i = \overline{1, 3})$. For medium-term processes, we select ε_4 and ε_5 equal to 2.5 each, and for short-term ones, we choose $\varepsilon_6 = 5$. The weights a_{ij} are chosen depending on the influence of the corresponding threat. The ranking of these threats based on the data from [5] is presented in Table 1. Different sectors of the economy are characterized by parameters a_{ij} $(i = \overline{1, 6};\ j = \overline{1, 6})$.

According to the results obtained in [2], the parameters σ_i, r_i, and b_i are related to the characteristics determining the functioning of economy sectors as follows:

$$\sigma_i = (\alpha_{1i} \beta_{2i}) / (\alpha_{2i} \gamma_{2i}),\ r_i = (\beta_{1i} \gamma_{1i}) / (\beta_{2i} \gamma_{2i}),\ b_i = (\zeta_i) / (\alpha_{2i} \gamma_{2i}),$$

where α_{1i} and α_{2i} are parameters characterizing the adaptive capabilities; β_{1i} is the demand for activity of ith manufacturing system normalized to the unit of material production system, i.e., to a job in the corresponding branch of manufacturing Y_i; β_{2i} is the level of supply, normalized to the unit of the function of ith manufacturing system X_i; γ_{1i} is the demand for insreasing the number of jobs, normalized to the unit of X_i; γ_{2i} is a share of jobs Y_i involved in the maintenance of X_i; ζ_i is the rate of disturbance elimination.

The transition of parameters r_i to the range of values corresponding to the regime of metastable chaos is achieved by increasing the specific demand for the products of the manufacturing system and the number of jobs employed in it (parameters β_{1i} and γ_{1i}) or by reducing their specific supply (parameters β_{2i} and γ_{2i}).

Based on the modeling results we have investigated how the successive transition of interconnected sectors into the deterministic chaos regime due to changes in parameter r affects the dynamics of system (1) as a whole (Figs. 1 and 2).

When in one or two sectors, the values of r correspond to the deterministic chaos regime, the rest of the sectors suppress it. But when one more sector joins the chaotic sectors, a stable strange attractor appears in the system, and a kind of phase synchronization of auto-oscillations from the six coupled oscillation generators occurs.

Figure 3 shows the dynamics of changes in the single-sector model as the noise intensity δ_j increases. The noise impact leads to the stochastic deformation of deterministic attractors of the Lorentz model. In the case of random perturbations, the trajectories of the stochastic system leave the deterministic attractor and form around it some bundle of trajectories with an appropriate probability distribution. The variance of random states near the deterministic attractor depends on the noise intensity and stability of the local parts of the attractor.

TABLE 1. Ranking of Threats

Threats	Rank of Influence	Parameter Values
Food, Water, and Energy Crises	43.9	$a_{ij} = 0.013$ $(j = 1, 3)$
Supply Risks	38.3	$a_{i4} = 0.012$
Spread of Infectious Diseases	58	$a_{i5} = 0.018$
Socio-economic Risks	43.4	$a_{i6} = 0.013$

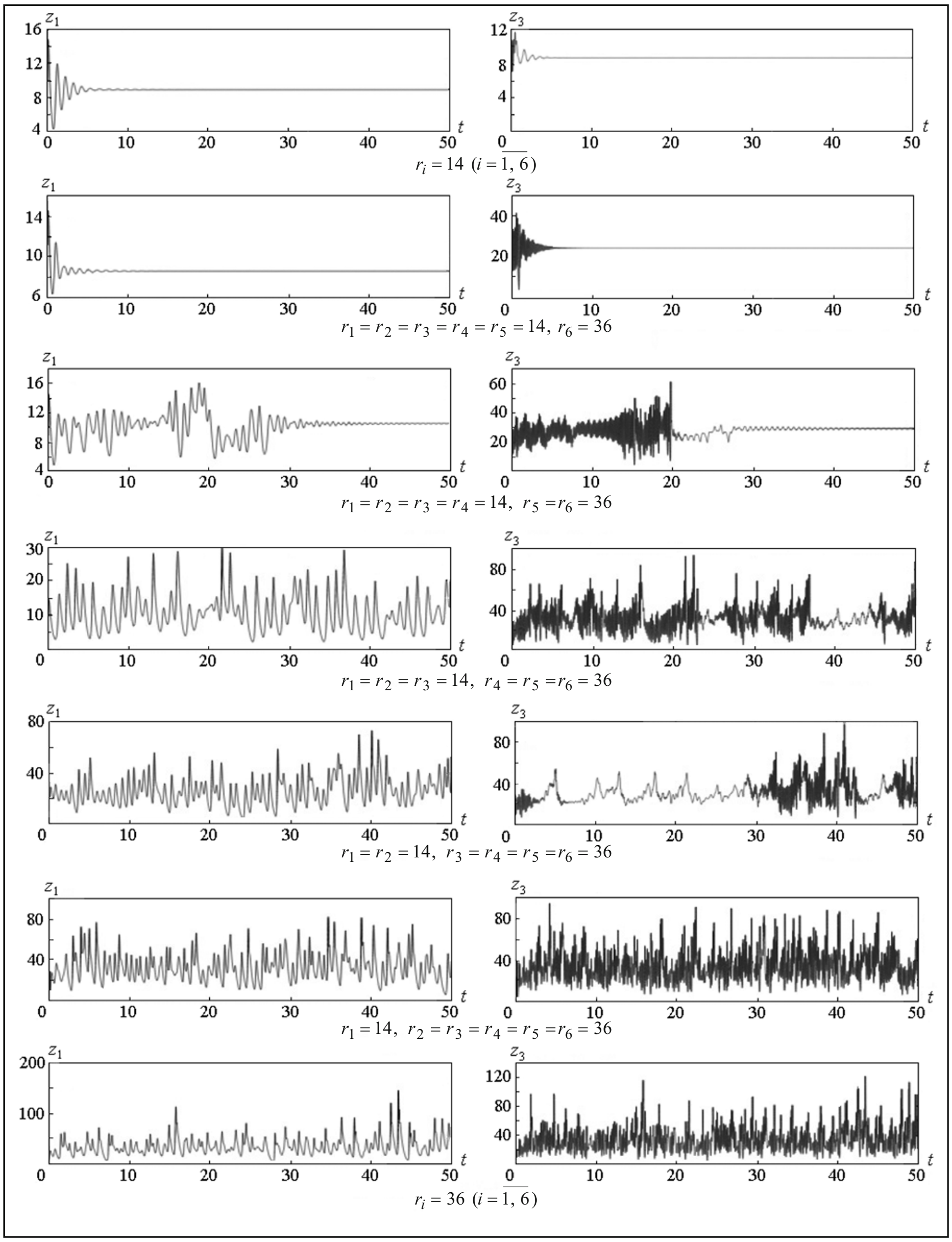

Fig. 1. Dynamics of the level of structural disturbances in the food (z_1) and energy (z_3) sectors under the consecutive crossing of the bifurcation boundaries of the parameter r.

Model (1) can be written in a complex form. As shown in [6], in the complex system of Lorentz equations, the scenario of transition to chaos through a subharmonic cascade of bifurcations of two-dimensional tori is realized. The

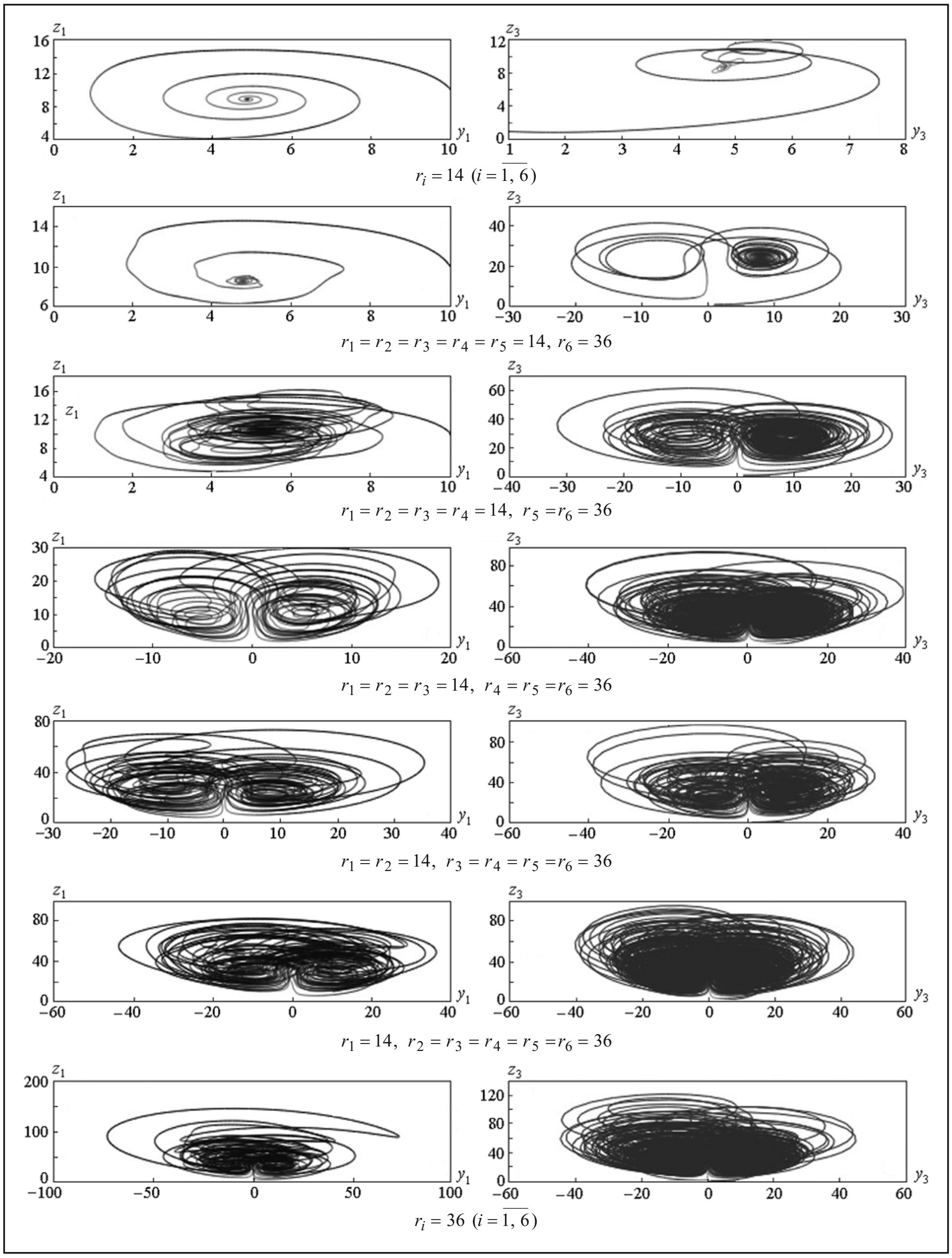

Fig. 2. Projections of the phase space of system (1) on planes $y_1 - z_1$ and $y_3 - z_3$ when the parameter r changes.

study of n economic sectors is reduced to consideration of the behavior of an assembly of n coupled oscillators generating oscillations with frequencies ω_i, respectively. Collective synchronization of these oscillators can be investigated using the Kuramoto model [7], which has the following form:

$$\frac{d\theta_i}{dt} = \omega_i + K / N \sum_{j=1}^{N} \sin(\theta_k - \theta_j), \ i = 1, \dots, N,$$

where $\theta_i \in [0, 2\pi]$ are phase variables, ω_i are natural frequencies, and $K > 0$ is a coupling parameter.

Thus, the problem of managing socio-economic development under emerging chaotic regimes is reduced to the control of the frequency of the field with a non-zero average value, which is generated by the coupled oscillators.

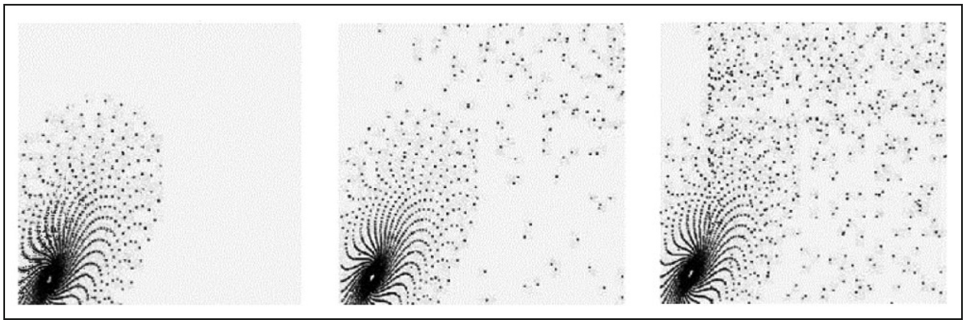

Fig. 3. Effect of random disturbances on the dynamics of model variables.

The study of deterministic chaos resulted in a reinterpretation of its role in the self-organization of complex dynamical systems. It turned out that, due to chaotic modes, it is possible to move the system from one unstable limit cycle to another and to qualitatively change the system behavior through small changes in control parameters. Given this, a transformation of the chaos management process from actions aimed at stopping it to the choice of controls stabilizing some unstable periodic trajectories [6] has occurred.

Resource base depletion resulting from climate change, the economic consequences of pandemics, and other global turbulences, narrows down the opportunities for sustainable development. Resource scarcity allows only a slight change in the parameters of management. Therefore, the search for such development trajectories, which would enable a “jump” from one development trajectory to another by the small changes in control parameters, becomes very important. Strange attractor models can be used to study exactly those controls that enable a phase transition from one limit cycle to another.

As the results of this study show, the above management can be carried out by changing the demand/supply ratio for jobs and products in the different sectors of economy. If the critical values of these ratios are exceeded, the solution of system (1) does not converge to an equilibrium point. It is transformed first into an oscillatory and then chaotic regime. Different values of this ratio correspond to various scenarios of economic development. Based on the analysis of macroeconomic indicators for multiple regions, we can classify them in the space of model parameters and determine the critical parameter values. Upon reaching these values, the saltatory transitions from one economic development trajectory to another occur. We also emphasize that the parameters of the model (1) are determined by more detailed models [8] developed during 2017–2020.

MATHEMATICAL METHOD FOR ASSESSING THE LEVEL OF FOOD SECURITY

Because of global changes, sustainable economic development, food security, and health are significantly interlinked. Therefore, the COVID-19 pandemic posed complex and far-reaching threats not only to the healthcare sector but also to the economic, food, and social sectors. This situation may require significant changes in the daily practices of the global world and affects food security. According to estimates, the impact of the COVID-19 pandemic and measures to address it is fraught with an economic downturn and exacerbate hunger [9]. However, while the pandemic, which has disrupted supply chains, mainly affects access to food, the political destabilization of the world in 2022 may also reduce the level of food resources due to the reduction of crop areas. Thus, the food security state becomes a critical problem for the global economy.

Assume that such parameters as indicators of the sufficiency of consumption (Y_1), differentiation value of food by social groups (the highest and lowest decile groups) (Y_2), differentiation of the daily energy value of the human diet (Y_3), and availability of food (Y_4) characterize the socio-economic and climatic-ecological dimensions of the food system. At the same time, the system is affected by several other factors that are changing much faster. They are associated with the spread of the global pandemic and have disruptive effects on food chains, complicate access to food, alter the balance of supply and demand through reduced economic activity, and lead to a rise in unemployment levels. To characterize these threats, we introduce an index Y_5. We compute it using the following indicators: (1) the level of patients hospitalized, immunized, or dead during the pandemic; (2) the level of unemployment and arrearage of wages due to the imposing of lockdowns and other measures aimed at overcoming the pandemic; (3) the level of degeneration of the health system performance due to the overload during the period of overcoming the pandemic. To the group of rapid factors, we include threats of large-scale military action, which can also have a devastating effect on food security. To characterize the consequences of the impact of military action, we introduce an index Y_6, which considers the reduction of crop areas, the disruption of transport chains, and the shortage of fertilizers.

However, not only the levels of individual indicators of complex hierarchical structures characterize the stability of their functioning. The system's reserve capacities, safety and efficiency depend strongly on the balance of its specific chains. There is a kind of "mobile" of security, i.e., dynamic invariant, where the negative dynamics of some indicators is offset by others. Therefore, the quantitative assessment of the level of food security requires the creation of models that allow formalizing the dependence of critical parameters on environmental, economic, biomedical, and social factors for an integrated robust management of food security.

We determine the level of food security using the mathematical approach to assessing the vulnerability of complex systems presented in [3]. It is based on catastrophe theory formalism and uses the heterogeneity of the phase space of complex dynamic systems. We consider food security management in the space of a limited number of control parameters, each of which is the function of the variables determining the system behavior. Assume that this system satisfies the main properties of potential systems, is described by some potential function $U(x)$ of behavioristic variable x, is continuous, has local extremums, in which the time derivative of variable x is zero. It means that there is a certain number of stationary states. Some of them are stable, and some are unstable. Let us introduce the function $F(x)$ as follows: $U(x) = \int F(x)dx$.

For studying the system behavior near local extrema $U(x)$, the function $F(x)$ is expanded into a series in the neighborhood of stationary points and limited to a few small terms of this expansion. For studying a wide range of systems, the assembly model of catastrophe is the most commonly used, for which functions $F(x)$ and $U(x)$ have the following form:

$$F(x) = x^3 + ax + b,\ U(x) = -x^4/4 - ax^2/2 - bx, \tag{2}$$

where a and b are the control parameters characterizing the reserve capacities of the system and the external load on the system.

According to the Sturm's theorem, a cubic polynomial has three or one real root. The number of roots depends on the discriminant $\Delta = 4a^3 + 27b^2$. If condition $\Delta < 0$ is satisfied, we have three real roots. The system has three steady states, two of which are stable. The first stable steady state characterizes the norm (sufficient level of food security), the second — the food crisis (low level of food security). For $\Delta > 0$, we have one real root and two imaginary ones. The curve $\Delta(a, b) = 0$ is the curve of bifurcation values.

To perform a comprehensive assessment of the food security level, we use model (2). Control parameters a and b can be computed using behavioristic variables. As these variables, we will use previously defined integral indices $Y_i\ (i = \overline{1, 6})$ as follows:

$$a = a_0 + a_1Y_1 + \ldots + a_6Y_6,\ b = b_0 + b_1Y_1 + \ldots + b_6Y_6.$$

Knowing the actual value of a, we obtain its bifurcation value for the parameter b from equation (2). The difference between the bifurcation value and the actual value of parameter b is a measure of risk. The more the current value of parameter b is distant from its bifurcation value, the less the deformation of the safety space. As the level of reserve capacities increases, so does the magnitude of the external load that the system can withstand before the deformation of the food security space occurs. The problem of robust control, which will provide a system transition from a high-risk area to a low-risk one, can be formulated as follows: there is mathematical model (2), and there is a system of constraints that defines the acceptable range of values of model variables and control parameters Y_i. It is necessary to determine control parameters $Y_i(t)$ that minimize the functional $F(Y_i) = |4a^3 + 27b^2|$.

The method proposed makes it possoble to perform the following actions: (1) comprehensive assessment of food security, (2) risk ranking, and (3) identification of efficient ways of adapting agriculture to climate change. Striving to use the same-type approaches for different sectors, we note that this approach can be applied to the environmental, energy, epidemic, technogenic, and social dimensions of security.

CONCLUSIONS

We have proposed the approach to improve the quality of estimation of distribution parameters in cases where it is possible to create a system of a priori constraints. This approach combines the capabilities of the deterministic and probabilistic approaches. Using the six-sector Lorenz model, we have studied the interconnection between food, energy, and water resources, supply systems, and epidemic and social situations and have determined the conditions for the transition of interconnected sectors to a deterministic chaos mode due to changes in unit demand and supply for the products of manufacturing systems and the number of jobs involved. We have created the method for assessing risks for food security and managing them based on the data on sufficiency of consumption, differentiation value of food by social groups, differentiation of the daily energy value of the human diet, economic availability of food, disruption of supply chains, labor force shrinking, rise in unemployment levels, reduction of crop areas, rise in food prices, and shortage of fertilizers.

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