Effects of Two-Dimensional Steep Hills on the Performance of Wind Turbines and Wind Farms

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Abstract

We use large-eddy simulations with an immersed boundary method to study the performance of wind turbines and wind farms in hilly terrain. First, we analyze the performance of wind turbines in the vicinity of a two-dimensional hill. For turbines that are significantly taller than the hill, the performance improves as the flow speeds up over the hill. For turbines that have approximately the same or a smaller height than the hill, the impact of the hill on the turbine performance depends on the positioning of the turbine in relation to the hill. For these turbines, the performance is better at the hilltop. However, the power production of these turbines is reduced due to blockage effects when they are placed at the base of the hill. The performance of turbines placed on the windward side of the hill is well predicted by superimposing the wind-turbine wake profile for the flat terrain on the hilly-terrain flow field. In contrast, we show that this approach is invalid when the turbine is placed on the leeward side of the hill where flow separation occurs. Subsequently, we consider wind farms with a hill in the middle. The hill wake is very pronounced due to which the performance of turbines located behind and close to the hill is mainly determined by the flow dynamics induced by the hill instead of the wind-turbine wakes. Finally, we study a wind farm located between two hills. We find that, for this particular configuration, there is a unique turbine spacing that maximizes the wind-farm power production in the valley.

Keywords Atmospheric boundary layer · Power production · Steep hill · Wind farm · Wind turbine

1 Introduction

Energy provision is one of the greatest challenges facing our society today. Wind energy will likely provide a significant contribution to the growing need for clean and renewable energy (van Kuik et al. 2016). There is no doubt that more and larger onshore and offshore

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wind farms will be commissioned, with most onshore sites located in complex terrain due to the lack of alternatives (Alfredsson and Segalini 2017). Because complex terrain strongly influences flow dynamics and wake development, it is crucial to understand the effects of complex terrain on wind-farm performance (Stevens and Meneveau 2017; Porté-Agel et al. 2020).

Taylor and Smith (1991) presented a wind-tunnel investigation on the performance of turbines situated on a two-dimensional flat-topped hill. They found that the wakes generated by wind turbines on the hilltop could delay the flow separation on the leeward side of the hill. Tian et al. (2013) performed measurements on an array of five turbines located on a two-dimensional shallow hill, and compared the power production and fatigue loads for turbines in flat and hilly terrains. They found that the wind speed was much higher and the turbulence intensity was relatively low on the top and the windward side of the hill. Thus, they recommended placing turbines in these locations. Howard et al. (2015, 2016) experimentally studied the performance of a wind turbine located downstream of a three-dimensional steep hill or another wind turbine. They observed that the performance of the downstream turbine was reduced when it was in the wake of the upstream hill or turbine, where the thermal stability conditions also played a significant role. Hyvärinen and Segalini (2017a), Hyvärinen and Segalini (2017b) and Hyvärinen et al. (2018) measured the thrust and power coefficients for turbines located on top of a series of sinusoidal hills. They showed that the turbine wakes recovered more rapidly in hilly terrain than on flat terrain, and concluded that the undulating hills could have a favourable effect on the measured thrust and power coefficients of turbines, especially further downstream in the wind farm.

Morfiadakis et al. (1996) measured the turbulence characteristics in a wind farm on the island of Andros, Greece. The measured spectra of the three velocity components were analyzed by applying the von Kármán formulation. The analysis revealed that the von Kármán spectrum was suitable for the structure of the turbulence measured at some locations when the wind turbines were not operational. However, the pronounced topography and turbine wake effects were not adequately modelled by this formulation. Subramanian et al. (2016) measured the wake evolution downstream of multi-MW wind turbines of the Mont Crosin wind farm in complex terrain and the Altenbruch II wind farm on flat terrain. Results showed that the near-wake region in complex terrain extended up to two rotor diameters and was about 35% shorter than that over flat terrain. However, the further downstream wake evolution in flat and complex terrains revealed similar wake characteristics. Hansen et al. (2016) performed measurements on a wind farm in Shaanxi, China, and showed that in hilly terrain wind-turbine wakes were deflected upwards or downwards depending on the thermal stability conditions. During the daytime, the wakes were deflected upwards, while at night-time wakes were deflected downwards and followed the terrain topography. A similar dependence of the wake propagation was observed in the field measurements conducted in Perdigoão, Portugal (Menke et al. 2018; Barthelmie and Pryor 2019).

Apart from wind-tunnel and field measurements, various numerical simulations have also been used to investigate the effect of complex terrain on wind-turbine and wind-farm performance. Generally, the simulation results agree well with measurements (see, for example, Shamsoddin and Porté-Agel 2017; Berg et al. 2017; Sessarego et al. 2018; Yang et al. 2018). In particular, Shamsoddin and Porté-Agel (2017) performed large-eddy simulations (LES) of turbulent flows over five wind turbines sited on a two-dimensional shallow hill under neutral stratification conditions. In that study the streamwise velocity and turbulence intensity profiles from the simulations agree well with the wind-tunnel measurements of Tian et al. (2013), and wind-turbine wakes were observed to follow the terrain topography. In contrast, Berg et al. (2017) presented simulations of the Perdigoão site in Portugal where the flow sep-

arated behind the ridges. They found that the wind-turbine wakes did not follow the terrain topography, but instead were deflected slightly upwards.

The review of Porté-Agel et al. (2020) explained that topography affects the development of wakes in three ways, namely, due to (1) non-zero pressure gradients, (2) variable elevation of the wake-centre trajectory, and (3) flow separation. Hyvärinen and Segalini (2017b) showed that the Jensen wake model cannot accurately capture the wake modulations induced by the hills. However, reasonable results were still obtained by merely superposing the turbine wake for the flat terrain case and the flow over the hilly terrain without turbines. Feng and Shen (2014) proposed an adapted Jensen wake model by assuming that the wake centreline follows the terrain rather than staying at a constant elevation above sea level. This model was later used to optimize the wind-farm layout in complex terrain. In their study, the flow field for the terrain under consideration without turbines was obtained by numerical simulations. Shamsoddin and Porté-Agel (2018a) developed an analytical model of turbulent axisymmetric planar wakes under pressure gradient conditions, with the assumption that the mean velocity-deficit profiles are self-similar and have a Gaussian shape function. The model was validated by comparison with an LES dataset. Later, Shamsoddin and Porté-Agel (2018b) developed an analytical modelling framework to model wake flows over two-dimensional shallow hills, with the effect of the hill-induced pressure gradient accounted for by the analytical model discussed above (Shamsoddin and Porté-Agel 2018a) and the effect of the hill-induced streamline distortion by a linearized perturbation approach (Hunt et al. 1988).

Based on these and other results, Porté-Agel et al. (2020) concluded that modelling approaches can be successfully employed to predict the effect of shallow hills on the performance of wind turbines and wind farms. However, for steep hills, the effects are much more difficult to model due to the flow separation that occurs. Few works have systematically investigated the performance of wind turbines and wind farms on such complex topography. The objective of the present study is to gain insight into the effect steep hills may have on the performance of nearby turbines. For simplicity, we only consider truly neutral atmospheric boundary-layer (ABL) flow over a two-dimensional steep hill. The mean slope of the hill is assumed to be greater than 0.3, such that significant flow separation occurs (Mason and King 1985). This means that no analytical modelling approach is available to consider this situation (e.g., Shamsoddin and Porté-Agel 2018b; Porté-Agel et al. 2020). While we find that the performance of turbines placed on the windward side of the hill is well predicted by superimposing the wind-turbine wake profile for the flat terrain on the hilly-terrain flow field (Hyvärinen and Segalini 2017b), this approach does not work for turbines on the leeward side of the hill.

The remainder of the paper is structured as follows: in Sect. 2 we discuss the adopted numerical method and provide a validation of this method against wind-tunnel measurements by Cao and Tamura (2006). We note that there are also other measurements for flow over steep two-dimensional hills, such as Ross et al. (2004) and Loureiro et al. (2007, 2009). We selected the measurements by Cao and Tamura (2006) because their study is well documented and thus often used for validations. In Sect. 3, we analyze the effect of a two-dimensional hill on the performance of a nearby wind turbine and its influence on the wake recovery. In Sect. 4 we study the performance of a wind farm with a hill in the middle and a wind farm located between two hills. We conclude with a summary of the main findings in Sect. 5.

2 Numerical Method and Validation

We use LES in combination with an immersed boundary method to simulate the turbulent flow over two-dimensional hills. We consider a truly neutral ABL such that the flow can be simulated by solving the filtered incompressible Navier–Stokes equations,

$$\nabla \cdot \tilde{\mathbf{u}} = 0, \quad (1)$$

$$\partial_t \tilde{\mathbf{u}} + \tilde{\boldsymbol{\omega}} \times \tilde{\mathbf{u}} = \mathbf{f} - \nabla \tilde{p} - \nabla \cdot \boldsymbol{\tau}. \quad (2)$$

Here, $\tilde{\mathbf{u}}$ is the velocity, $\tilde{\boldsymbol{\omega}} = \nabla \times \tilde{\mathbf{u}}$ is the vorticity, $\mathbf{f}$ is the external force (which includes the constant pressure gradient, the force exerted by turbines, and the force caused by the immersed boundary), $\tilde{p}$ is the modified pressure, and $\boldsymbol{\tau}$ is the subgrid-scale stress tensor, which is modelled using the Smagorinsky model (Smagorinsky 1963; Mason and Thomson 1992). We use a pseudo-spectral discretization, and thus periodic boundary conditions in the horizontal directions. In the vertical direction, we use a staggered second-order finite difference method. At the top boundary, we enforce a zero vertical velocity and a zero shear stress boundary condition, and we use a classic wall-layer model at the bottom surface and the immersed boundary to determine the wall stress at grid points closest to the solid boundaries (Moeng 1984; Bou-Zeid et al. 2005; Chester et al. 2007). Time integration is performed using the second-order Adams–Bashforth method (Canuto et al. 1988). The projection method is used to ensure the divergence-free condition of the velocity field (Chorin 1968), and we use the concurrent precursor method to simulate finite-size wind farms and to generate turbulent inflow conditions that match atmospheric turbulence (Stevens et al. 2014).

The wind turbines are modelled using a filtered actuator disk model (Shapiro et al. 2019). In short, when the freestream velocity U_∞ is used to calculate the turbine force F_t , it is given by

$$F_t = -\frac{1}{2} \rho C_T U_\infty^2 A, \quad (3)$$

where ρ is the density of fluid, C_T is the thrust coefficient based on the free-stream velocity U_∞ , and A is the area of the disk. However, if a turbine is behind another turbine or in complex terrain, the freestream velocity is no longer easily available. Meyers and Meneveau (2010) pointed out that in such a case the disk-averaged velocity u_d is a better candidate and then actuator disk theory gives that the force can be written as

$$F_t = -\frac{1}{2} \rho C'_T u_d^2 A, \quad (4)$$

and that the power output is given by (Stevens and Meneveau 2014)

$$P = -F_t u_d = \frac{1}{2} \rho C'_T u_d^3 A, \quad (5)$$

where C'_T is the thrust coefficient based on u_d . In our study, we retain a constant thrust coefficient $C_T = 0.75$, which is equivalent to $C'_T = 4/3$.

In all simulations, we consider the following two-dimensional steep hill, which was used in the experimental study by Cao and Tamura (2006):

$$z_w(x) = h \cos^2\left(\frac{\pi x}{2l}\right), \quad -l \leq x \leq l, \quad (6)$$

where h and l are the height and half-width of the hill, respectively. The maximum slope is $\pi/5$ such that a significant separation region exists on the leeward side of the hill. The

Fig. 1 Time- and spanwise-averaged, **a** streamwise velocity component and **b** its variance as function of height for the ABL flow over a two-dimensional steep hill. Solid lines: simulation results; Open circles: experimental data by Cao and Tamura (2006)

experiments were conducted in an open circuit wind tunnel. The height and half-width of the hill are $h = 40$ mm and $l = 100$ mm, respectively, the roughness length is $z_0 = 0.004$ mm, and the boundary-layer height is $\delta = 250$ mm. We use a domain of $5\delta \times \delta \times \delta$, in the streamwise, spanwise, and wall-normal direction, respectively. The used grid resolution is $192 \times 64 \times 97$.

Figure 1 shows that the vertical profiles of the streamwise velocity component and its variance obtained by the simulations agree well with the measurements at all downstream locations. To further illustrate the characteristics of the hill wake, we show the time-averaged streamwise velocity component and its variance in the streamwise–vertical plane in Fig. 2. A significant feature of the flow is the recirculation zone, which is established on the leeward side of the hill. As a result, the velocity downstream of the hill is significantly reduced, while the velocity fluctuations increase. The edge of the recirculation zone, which is identified by $\langle u \rangle = 0$, where $\langle \cdot \rangle$ denotes the time average, intersects with the ground at $x/h \approx 5.4$, which is in good agreement with the result obtained from the wind-tunnel measurements (Cao and Tamura 2006).

Hereafter, we use the same hill and numerical resolution as used in the validation case discussed above, to ensure that the essential flow features are captured accurately. As mentioned above, this hill geometry is widely used in the literature, and allows us to study the effects that steep hills have on the performance of turbines. We do not study the effect of the hill shape, which will be investigated in future.

3 A Wind Turbine Near a Hill

To investigate the performance of a wind turbine located close to a steep hill, behind which significant flow separation takes place, we use the hill geometry given by Eq. 6. The roughness length z_0 is set to $z_0/D = 10^{-4}$, where D is the diameter of turbines equal to the hill height

Fig. 2 Time- and spanwise-averaged, **a** streamwise velocity component, and **b** its variance for the ABL flow over a two-dimensional steep hill. The white lines indicate the profile at various downstream locations and the black dashed line indicates the edge of the recirculation zone

h . To keep the discretization of the hill identical that one used in Sect. 2 we use a domain size of $30D \times 6D \times 6D$ and a grid resolution of $192 \times 64 \times 97$ in the streamwise, spanwise, and vertical directions, respectively. We perform simulations with the turbine located at various streamwise locations, i.e., $x_t/D = -2.5, -1.25, 0, 1.25, 2.5, 7.5$, where $x_t = 0$ indicates the location of the hilltop. This means that we test the performance of the turbine at the base, top, and the end of the hill. Besides, we test the performance for turbines placed halfway on the windward and leeward sides, respectively so the tested turbine locations are distributed uniformly over the extent of the hill. We also test the turbine performance further downstream at $7.5D$ behind the hill to obtain more insight into the long-range effect of the hill wake.

To study the effect of the ratio of the turbine hub height to hill height we vary the turbine height between $0.75D$ and $3D$. The lower limit ensures that the turbine blades do not hit the ground. The maximum hub height allows us to explore the limit for which the turbine is much taller than the hill. The selected hill heights are distributed equidistantly over this interval. We note that the maximum hub height pushes the limit of the ratio of hub height versus diameter that is seen in practical applications, but we consider it here since it allows us to see what happens when the turbines are much taller than the hill. For each hub height, we perform a reference simulation in flat terrain to normalize the results. This means that 70 simulations are performed, i.e., ten different hub heights for six different streamwise locations and the corresponding reference simulation in flat terrain. This comprehensive set of simulations allows us to investigate the effect of the relative location and height of the turbine compared to the hill.

3.1 Effect of a Steep Hill on the Power Production of Wind Turbines

Figure 3 compares the power production of the turbine located at the different streamwise locations and for various turbine heights with the production of the corresponding reference turbine located in flat terrain. The symbols and solid lines indicate the measured power production by the turbines. The hill has a large effect on the performance of nearby wind turbines. For turbines that are significantly taller than the hill, i.e., $h_{\text{hub}}/D \geq 1.75$, the power production of a turbine that is placed close to the hill is higher than that on flat terrain, and is

Fig. 3 The power production for turbines located at the, **a** windward, and **b** leeward side of the hill compared to the power production for a turbine on flat terrain $P_{\text{hill}}/P_{\text{flat}}$. The symbols and solid lines indicate the measured power production by the turbines. The top panel indicates a sketch of the used turbine locations. The vertical dotted–dashed line denotes the unit ratio of power. The upper dotted–dashed line denotes the ratio of height above which the ratio of power is greater than unity. The lower dotted–dashed line denotes the unit ratio of height. As we assume that the thrust coefficient is constant, a prediction for the expected power production can be obtained by determining $(U_{\text{hill}}(z)/U_{\text{flat}}(z))^3$, where $U_{\text{hill}}(z)$ and $U_{\text{flat}}(z)$ are the streamwise velocity profiles, for the hill and the flat terrain cases, respectively. The coloured dashed lines give this prediction

due to the speed-up of flow over the hill, which benefits turbines placed on the hilltop most. The situation is more complicated for shorter wind turbines for which the hub-height is the same as the hill height, i.e., $h_{\text{hub}}/D = 1$. Due to the blockage effect of the hill, a turbine located at $x_t/D = -2.5$ produces less power than on flat terrain. The production of a turbine on the hilltop can be about 2.5 times higher than on flat terrain. Interestingly, turbines on the leeward side ($x_t/D = 1.25$) produce more power than turbines on the windward side ($x_t/D = -1.25$). Due to the hill wake, the performance of turbines located downstream of the hill is severely affected. The worst position is at the end of the hill at $x_t/D = 2.5$, where the turbine produces almost no power when $h_{\text{hub}}/D = 0.75$. However, further downstream the performance loss is still notable (at $x_t/D = 7.5$ the production loss is about 50% compared to the flat terrain case). Figure 3 reveals that the effect of the hill on the turbine performance increases when the turbine height is reduced compared to the hill height. This effect is most pronounced at the leeward side ($x_t/D = 1.25$) as at this location a drastic drop in the power production of the turbine is observed for $h_{\text{hub}}/D < 1$, because the turbine is then in the recirculation zone behind the hill.

As we assume that the thrust coefficient of the turbines is constant, we can compare the measured power production of the turbines with a prediction that is obtained from a comparison of the flow profile obtained from the simulations of the flat terrain and the hill case, respectively. Under the given assumptions the prediction for the effect of the hill can be determined from $(U_{\text{hill}}(z)/U_{\text{flat}}(z))^3$, where $U_{\text{hill}}(z)$ and $U_{\text{flat}}(z)$ indicate the streamwise velocity profile for the hill and the flat terrain case with z the distance from the ground. The prediction for the different turbine locations is obtained by determining this ratio for the respective turbine positions. Figure 3a shows that the measured power production agrees very well with these predicted values for turbines placed on the windward side of the hill. However, Fig. 3b reveals that on the leeward side, the production of several of the shorter turbines is

Fig. 4 Normalized velocity deficit $\langle u_{\text{def}} \rangle / u_d$ in the mid-plane of the turbine for, **a** the flat terrain case, and **b–e** for flow over a steep hill at $x/D = 0$ with the turbine located at **b** $x_t/D = -2.5$, **c** $x_t/D = 0$, **d** $x_t/D = 1.25$, and **e** $x_t/D = 7.5$. The hill height is $h/D = 1$. The hub-height is (**a–c**, **e**) $h_{\text{hub}}/D = 1$ and **d** $h_{\text{hub}}/D = 0.75$. The white dashed lines denote the location of the wake-centre trajectory

significantly lower than predicted. For turbines located at $x_t/D = 1.25$, this happens when the turbine hub-height is smaller than the hill height $h = D$. For $x_t/D = 2.5$ and $x_t/D = 7.5$ this effect is significant for hub-heights smaller than approximately $1.5D$ and $2D$, respectively. This is because both the hill and the turbine have a significant effect on the flow. Their combined effect is different than what would be predicted by a simple superposition of their effects. To illustrate this, we compare the flow field from the simulations with the turbine with the flow field when the turbine is not present. Figure 3b shows that when the turbine located at $x_t/D = 1.25$ and the hub-height $h_{\text{hub}}/D = 0.75$ the turbine power production is much lower than predicted. The positive values in front of the turbine in Fig. 4d, which will be explained in more detail below, reveal that the velocity there is much lower when the turbine is present than when the turbine is not present. The reason is that the flow is forced around the turbine, and in this particular case, there is a pronounced effect due to geometry. This confirms that the effect of steep hills on the performance of nearby wind turbines cannot be estimated by using a simple superposition approach. For this hill, we find that the power production of a turbine on the leeward side of the hill can be significantly lower than the production estimate that is obtained from the flow field over the hill without the turbine.

3.2 Effect of a Steep Hill on Wake Recovery

Figure 4 shows the normalized velocity deficit in the vertical mid-plane of the domain for a turbine on flat terrain and four different turbine positions. Similar to Shamsoddin and

Fig. 5 Normalized velocity deficit $\langle u_{\text{def}} \rangle / u_d$ along the wake-centre trajectory for a wind turbine on flat terrain (solid line) and for flow over a steep hill at $x/D = 0$ with the turbine located at the base of the hill ($x_t/D = -2.5$: dashed-dotted line); on the hilltop ($x_t/D = 0$: dashed line); or behind the hill ($x_t/D = 7.5$: dotted line)

Porté-Agel (2018a), the velocity deficit is defined as

$$\langle u_{\text{def}} \rangle = \langle u_{\text{nw}} \rangle - \langle u_{\text{w}} \rangle, \quad (7)$$

where $\langle \cdot \rangle_t$ denotes the time average, and u_{w} and u_{nw} are the streamwise velocity components over the same terrain with and without a wind turbine, respectively. We normalize the velocity deficit $\langle u_{\text{def}} \rangle$ by the disk-averaged velocity u_d , which is related to the power output P of a wind turbine as follows $P \propto u_d^3$ when the turbine thrust coefficient is assumed to be constant (Calaf et al. 2010). The benefit of selecting this characteristic velocity is that it allows us to compare the wake recovery for the different cases in which the turbine power production is not the same. Figure 4 also shows the location of the wake centre as a function of the downstream location. We determine the location of the wake centre in the symmetry plane behind the turbine by determining the location of the maximum velocity deficit (Shamsoddin and Porté-Agel 2018a),

$$z_c(x) = \arg \max_z \langle u_{\text{def}}(x, y_t, z) \rangle, \quad (8)$$

where z_c is the z -coordinate of the wake centre and y_t is the spanwise location of the turbine. Evidently, the hill can significantly affect the wake development when the turbine is placed in the vicinity of the hill. We find that the wake tends to follow the terrain. To be specific, the wake moves upwards when generated upstream of the hill and moves down a bit afterwards. Interestingly, the wake also is deflected downwards when the turbine is located downstream of the hill, which does not happen for the flat-terrain case. These findings are qualitatively similar to the theoretical analysis and numerical observations of Shamsoddin and Porté-Agel (2018b) for the development of wakes over two-dimensional shallow hills.

Figure 5 shows the normalized velocity deficit $\langle u_{\text{def}} \rangle / u_d \propto \langle u_{\text{def}} \rangle / P^{1/3}$, again under the assumption that C_T is constant, along the wake-centre trajectory. We see that when the wind turbine is located upstream or downstream of the hill, the turbine wake recovers more rapidly than for the flat terrain reference case. However, when the turbine is located on the hilltop the wake recovery is much slower. These wake recovery characteristics are a result of two competing effects. First of all, the hill significantly increases the turbulence level downstream of the hill (see Fig. 6), which enhances momentum diffusion and therefore promotes wake recovery (Yang et al. 2015). However, the wake recovery is also affected by the pressure gradients created by the hill. The hill generates a favourable pressure gradient on its windward side and an adverse pressure gradient on its leeward side. These pressure effects enhance the wake recovery for turbines in front of the hill and reduce the wake recovery

Fig. 6 The normalized streamwise velocity variance σ_u/U_h in the mid-plane of the turbine for, **a** the flat-terrain case, and **(b–e)** for flow over a steep hill at $x/D = 0$ with the turbine located at **b** $x_t/D = -2.5$, **c** $x_t/D = 0$, **d** $x_t/D = 1.25$, and **e** $x_t/D = 7.5$. The hill height is $h/D = 1$. The hub-height is (a, b, c, e) $h_{\text{hub}}/D = 1$ and **d** $h_{\text{hub}}/D = 0.75$

for turbines located on the hilltop (e.g., Politis et al. 2012; Yang et al. 2015; Shamsoddin and Porté-Agel 2017, 2018a,b). In this case, the pressure effects are so large that the wake recovery is slower for a turbine located on the hilltop than for a turbine on flat terrain, even though the velocity fluctuations are significantly higher downstream of the hill (see Fig. 6).

Figure 6 shows the normalized streamwise velocity variance σ_u/U_h , where σ_u is the standard deviation of the streamwise velocity component and U_h is the flow speed at hub-height over a flat terrain without any turbines (Shamsoddin and Porté-Agel 2018b). One can observe that the velocity fluctuations introduced by the hill are high compared to the velocity fluctuations in the wake behind the turbine on flat terrain. Figure 6b, c shows that the velocity fluctuations behind the hill are reduced due to the existence of an upstream turbine (Shamsoddin and Porté-Agel 2017). Figure 6e shows that the region with higher velocity fluctuations that is formed behind the hill can extend more than $7D$ downstream. This means that even turbines that are placed far downstream of a steep hill may be subjected to higher velocity fluctuations, which increases the unsteady turbulence loading experienced by the turbine (Stevens and Meneveau 2017; Porté-Agel et al. 2020).

4 Wind Farm in Hilly Terrain

To investigate the performance of wind farms located close to a steep hill, we analyze the performance of a wind farm with a steep hill in the middle in Sect. 4.1 and a wind farm

Fig. 7 Sketch of wind-turbine arrangement in different cases for the aligned wind farm. **a** Flat terrain (case 1), **b** wind farm located around a steep hill (case 2), **c** the two turbine rows upstream of the hill removed (case 3), and **d** the two turbine rows downstream of the hill removed (case 4). The colours of the turbines are the same as the colours of the corresponding results presented in Fig. 8. The hill height and hub-height are equal to the turbine diameter, $h = h_{\text{hub}} = D$

between two parallel steep hills in Sect. 4.2. For simplicity, the rotor diameter, the hub-height, and the hill height are the same, i.e., $D = h_{\text{hub}} = h$. We use the same hill geometry as before, which is given by Eq. 6 with $l/h = 2.5$, and $z_0/D = 10^{-4}$. To keep the discretization of the hill the same, we use a domain size of $100D \times 25D \times 8D$ and a grid resolution of $768 \times 192 \times 193$ in the streamwise, spanwise, and vertical directions, respectively.

4.1 Wind Farm with a Hill in the Middle

To evaluate the influence of a hill on the performance of wind farms, we performed simulations of aligned and staggered wind farms, which consist of 13 rows of five turbines. Both the streamwise and spanwise spacings between the turbines are set to $5D$. We consider a reference wind farm on flat terrain (case 1) and a case in which row number 7 is located on the hilltop (case 2). We also consider a wind farm in which the two rows in front of the hill are removed (case 3), and one in which the two turbine rows behind the hill are removed (case 4). Sketches for these different wind farm cases are provided in Fig. 7.

Figure 8 shows the normalized power production P/P_1 for the different cases as a function of the downstream position x for the aligned and staggered wind farms. Here P_1 is the power output of the first row of turbines, which is located at the origin such that the centre of the hill is at $x/D = 30$. Far upstream of the hill ($x/D \leq 20$), the performance of wind farms of case 1 and case 2 is nearly identical, which indicates that the hill has no significant effect on the flow in this region. However, for the aligned wind farm of case 2, the power production of row 6 is about 14% lower than the corresponding row in the reference wind farm (case 1). For the staggered wind farms, this difference is about 18%. This production loss is caused by the flow blockage induced by the hill and is similar to the effect we observed in Sect. 3. The production of the turbine on the hilltop, i.e., row 7, is obviously much higher than in the reference wind farm. This is in agreement with the results obtained in Sect. 3.

Another interesting result is that the power production for rows 8 and 9, i.e., the rows just downstream of the hill, is almost the same for the aligned and staggered wind farms of case 2. The reason is that the hill wake dominates the flow in this region. This conjecture is confirmed by the results of cases 3 and 4. In particular, removing turbines upstream of the hill (case 3) does not significantly affect the performance of turbines downstream of the hill,

Fig. 8 The normalized power P/P_1 as a function of the downstream position x/D , **a** aligned, and **b** staggered wind farms. Case 1: flat terrain; case 2: wind farm located around a steep hill; case 3: the two turbine rows upstream of the hill removed; case 4: the two turbine rows downstream of the hill removed. The hill height and hub-height are equal to the turbine diameter, $h = h_{\text{hub}} = D$. See Fig. 7 for a sketch of the different cases

even though it increases the power production of the turbines on the hilltop. At the same time, removing turbines just downstream of the hill (case 4) does not affect the performance of rows even further downstream. This indicates that the hill wake is the dominating flow feature for a very significant region behind the hill. Interestingly, the power production in the last turbine row, which is located $30D$ behind the hilltop, is still lower than the flat-terrain case. This indicates that the flow requires a very long distance to recover fully.

To analyze the impact of the hill on the flow further, we show the normalized mean streamwise velocity $\langle u \rangle / U_h$ at a fixed distance D above the ground for the aligned wind farms in Fig. 9. The velocity profiles upstream of the hill are almost the same as for the reference wind farm. This finding is in agreement with the power production data shown in Fig. 8, which is nearly identical for all cases for the first four rows. The figure shows the flow acceleration over the hill due to which the power production of the turbines in row 7 is much higher. Removing the two turbine rows upstream of the hill (case 3) significantly increases the power output of the turbines on the hilltop compared to case 2. However, the wake effect of the upstream rows is still visible as the normalized power production of the turbines on the hilltop is still about 30% lower than for an isolated turbine located on top of the hill (see Fig. 3). Downstream of the hill, the hill wake overshadows the effect of the wakes created by turbines upstream of the hill. Due to the dominant effect of the hill wake removing the two turbine rows downstream of the hill (case 4) does not significantly affect the velocity downstream of the hill. In agreement with the power production results, we find that at $30D$ behind the hill the effect of the hill wake is still visible.

Figure 10 shows the corresponding normalized streamwise velocity variance σ_u / U_h at a fixed distance D above the ground. Upstream of the hill, the velocity fluctuations are nearly the same as that in flat terrain. Just behind the hill, the velocity fluctuations increase significantly due to the flow separation. Further downstream of the hill ($x/D > 50$), the velocity fluctuation contours in complex terrain are almost the same again, which is very similar to the streamwise velocity case (see Fig. 9). This indicates that the wind-farm performance downstream of the steep hill is also independent of the turbines upstream of the hill. Removing the two turbine rows upstream (case 3) or downstream (case 4) of the hill does not significantly affect the velocity fluctuations downstream of the hill. In agreement with the single turbine case (see

Fig. 9 Normalized mean streamwise velocity component $\langle u \rangle / U_h$ at a fixed distance D above the ground for the aligned wind farm, **a** flat terrain (case 1), **b** wind farm located around a steep hill (case 2), **c** the two turbine rows upstream of the hill removed (case 3), and **d** the two turbine rows downstream of the hill removed (case 4). The hill height and hub-height are equal to the turbine diameter, $h = h_{\text{hub}} = D$. The hilltop is located at $x = 30D$. Small solid lines indicate the turbine positions. See Fig. 7 for a sketch of the different cases

Fig. 6), we find the velocity fluctuations can also be significantly reduced via the interaction with mainly the hilltop turbine.

Figure 11 shows the normalized vertical kinetic energy flux, $-\langle u \rangle \langle u'w' \rangle / U_h^3$ at a fixed distance $2D$ above the ground, where u' and w' indicate the streamwise and vertical velocity fluctuations. For the flat terrain case, the increase of the vertical kinetic energy flux with downstream location is determined by the increased turbulence induced by the wake and the growth of the internal boundary layer that forms at the start of the wind farm. The figure also reveals that the vertical kinetic energy flux more than doubles downstream of the hill. This increased vertical flux ensures that the flow behind the hill recovers and thus compensates for the flow disruption caused by the hill. We notice that although removing the two turbine

Fig. 10 Normalized streamwise velocity variance σ_u/U_h at a fixed distance D above the ground for the aligned wind farm, **a** flat terrain (case 1), **b** wind farm located around a steep hill (case 2), **c** the two turbine rows upstream of the hill removed (case 3), and **d** the two turbine rows downstream of the hill removed (case 4). The hill height and hub-height are equal to the turbine diameter, $h = h_{\text{hub}} = D$. The hilltop is located at $x = 30D$. See Fig. 7 for a sketch of the different cases

rows downstream of the hill (case 4) does not significantly affect the streamwise velocity and the velocity fluctuations further downstream, it does decrease the vertical kinetic energy flux when compared to cases 2 and 3. Nevertheless, the vertical kinetic energy fluxes downstream of the hill are significantly larger than the flat terrain case, which indicates the strong effect of the hill.

Fig. 11 Normalized mean vertical kinetic energy flux $-10^3 \langle u \rangle \langle u'w' \rangle / U_h^3$ at a fixed distance $2D$ above the ground for the aligned wind farm, **a** flat terrain (case 1), **b** wind farm located around a steep hill (case 2), **c** the two turbine rows upstream of the hill removed (case 3), and **d** the two turbine rows downstream of the hill removed (case 4). The hill height and hub-height are equal to the turbine diameter, i.e. $h = h_{\text{hub}} = D$. The hilltop is located at $x = 30D$. See Fig. 7 for a sketch of the different cases

4.2 Wind Farm Between Two Parallel Hills

In this section, we consider an aligned wind farm located between two parallel hills, separated by $60D$, with seven rows of five turbines. As the optimum turbine performance is obtained at the hilltop, the first and last turbine rows are located on hilltops. The spanwise spacing between the turbines is $5D$. We vary the streamwise spacing s_x between the turbines in the valley, as indicated in the sketch provided in the top panel of Fig. 12. We place the turbines closer to the second hill to prevent turbines being placed in the wake of the upstream hill as much as possible. We do this as we have seen that upstream hills have an enormous effect on

Fig. 12 **a** The normalized total power P_{total}/P_1 as a function of the streamwise spacing s_x/D and **b** the normalized power P/P_1 as a function of the downstream position x/D . The circles in **a** corresponds to the curves in **b**. The top panel sketches the wind turbine arrangement for the wind farm between two steep hills. The hill height and hub-height are equal to the turbine diameter, $h = h_{\text{hub}} = D$

the performance of downstream turbines. Thus the streamwise distance between all turbine rows is the same, except for between the first two turbine rows. The results below confirm the view that in a case like this — in which space is limited — it is beneficial for the wind-farm power output that the turbines are placed in the wake of an upstream hill as little as possible.

Figure 12a shows that the normalized total wind farm power production P_{total}/P_1 reaches a maximum value when $s_x = 7D$. Here, P_1 denotes the time-averaged power production of the first row and P_{total} the output of the entire farm. Figure 12b shows the power production as a function of the downstream position x for four typical cases indicated in Fig. 12a. The figure shows that with decreasing s_x the power production of the first turbine in the valley increases, but the production of all other turbines in the valley and the turbine on the top of the second hill significantly decreases due to the inter-turbine wake effects. For $s_x \geq 7D$ the power production of the first turbine in the valley decreases rapidly with increasing s_x , while the turbines in the valley only marginally benefit from the increased inter-turbine distance. Thus, the maximum wind-farm power production for $s_x = 7D$ is a result of two competing effects: When s_x is too small, the effect of the wind-turbine wakes affects the power production of the turbines too much. However, when s_x is too large, the second turbine row is located too closely behind the first hill, and this severely limits its power production.

In this case, the existence of a unique spacing for which the total power production of the wind farm reaches a maximum value is a result of the space limitation and the flow separation downstream of the hill. Although this is an interesting observation, it is hard to generalize as it depends on many parameters and considerations. Nevertheless, we confirm that the steep hill has a significant effect on the wind-farm performance.

5 Conclusions

We used LES to study the effect of two-dimensional steep hills on the performance of wind turbines and wind farms. Throughout the entire study, we assume that the turbine thrust coefficient $C_T = 3/4$. We find that steep hills have a significant impact on the power production

of turbines, and turbines that are significantly taller than the hill benefit from the speed-up of the flow over the hill. For shorter turbines, the power production strongly depends on the turbine location with respect to the hill but reaches its maximum value on the hilltop while locations just downstream of the hill are worst. While previous studies have shown that it is possible to obtain reasonable predictions for the effect of a shallow hill on the performance of nearby turbines, it is much more challenging to model the effect of a steep hill (Porté-Agel et al. 2020). Here we find that the performance of turbines placed on the windward side of the hill is well predicted by superimposing the wind-turbine wake profile for the flat terrain on the hilly-terrain flow field (Hyvärinen and Segalini 2017b). However, we show that such a prediction is not accurate for turbines placed on the leeward side of the hill.

The hill wake effect is very pronounced when the hill is located in the middle of the wind farm. In particular, removing turbines upstream of the hill has no significant effect on the power production of turbines downstream of the hill. Even removing turbines just downstream of the hill only leads to a minimal benefit for turbines located further downstream. This indicates that the recirculation zone of the hill is the dominant flow feature, and the wind turbines have only a limited effect on the development of the hill wake. The effect of the hill wake is observed up to at least $30D$ behind the hill, implying that steep hills influence the performance of turbines in a significant region.

Furthermore, we find that there is a unique turbine spacing for the wind farms located between two parallel hills such that the power production of the wind farm reaches its maximum value. The existence of such a unique spacing is the result of two competing effects created by the existence of a steep upstream hill and a limited available downstream space.

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