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ORIGINAL PAPER

On the Origin of Venn Diagrams

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Abstract
In this paper we argue that there were several currents, ideas and problems in 19th-century logic that motivated John Venn to develop his famous logic diagrams. To this end, we first examine the problem of uncertainty or over-specification in syllogistic that became obvious in Euler diagrams. In the 19th century, numerous logicians tried to solve this problem. The most famous was the attempt to introduce dashed circles into Euler diagrams. The solution that John Venn developed for this problem, however, came from a completely different area of logic: instead of orienting to syllogistic like Euler diagrams, Venn applied Boolean algebra to improve visual reasoning. Venn's contribution to solving the problem of elimination also played an important role. The result of this development is still known today as the 'Venn Diagram'.

Keywords Logic diagrams · Visual reasoning · Visual representation · Boolean algebra · Euler diagrams · Venn diagrams · Uncertainty · Overspecification · Syllogistics

1 Introduction

Venn diagrams are nowadays used not only in logic, but in almost all areas of science, for example in biology, translation science or artificial intelligence (Edwards 2004; Meulen 1990; Nakatsu 2009). They are used to represent relations between entities, concepts, classes, or more generally to present information. Together with Euler diagrams, tree diagrams, squares of opposition etc. Venn diagrams belong to

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the group of logic diagrams in the field of visual representation and reasoning (Moktefi and Shin 2012). Like many other types of diagrams, Venn diagrams not only have the property of presenting information clearly, but in the best case they can also generate new information than was not intended when the diagram was created (Shimojima 1996).

Venn diagrams and Euler diagrams are often mentioned in the same breath: Since both diagram systems are mostly formed by using circles, appear intuitively understandable at first glance, and in some cases even look similar, this confusion is not surprising. However, research since the 1990s has shown that both diagram systems have their own syntax and semantics (Shin 1994). In fact, Euler diagrams are much older than Venn diagrams, and there are also numerous Euler diagrams that cannot be Venn diagrams. Therefore, the question of how Venn diagrams originated in the first place has been around for a long time.

It is well known that John Venn first published his celebrated logic diagrams in 1880 (Venn 1880c). This method of representation consists in a two-step procedure. First, one draws a primary diagram that represents the combinations between the terms involved in a proposition or an argument. Then, one adds syntactic signs on this general framework to indicate the state of the compartments. For instance, shading a compartment expresses its emptiness. Let us consider the proposition 'No x is y'. It involves two terms: x and y. Hence, we need a two-term diagram where the four combinations xy, x non-y, non-xy and non-x non-y are represented, as shown in Fig. 1a. The proposition asserts that the class x non-y is empty. Hence, we simply shade the compartment x non-y to obtain the diagram of the proposition, shown in Fig. 1b.

It is unclear when Venn invented these diagrams. There is indeed no mention of them in Venn's earlier writings or correspondence prior to their publication. In his autobiographical sketch, written in 1887 (and subsequently updated), Venn reported that he "first hit upon" Eulerian diagrams in 1862 but that his own diagrams "did not occur to [him] till much later" (Venn 2022, p. 70). Now, Venn also indicated that his diagrams were suggested to him by the study of George Boole's logic (Venn 1880c, pp. 4–5). Although Venn knew of Boole's work in the late 1850s already (Venn 2022, p. xi), he wrote that it was only in the late 1870s that he eventually managed to "rationalize" Boole's processes (Venn 2022, p. 57). It is likely that Venn invented his diagrams during this period.

Venn says very little on the journey that led him to his diagrams. All we are told is that he

> tried at first, as others have done, to represent the complicated propositions [of Boole] by the old [Eulerian] plan; but the representations failed altogether

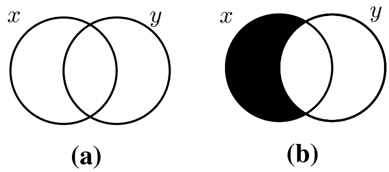

Fig. 1 Venn diagrams taken from Venn (1880c)

> to answer the desired purpose; and after some consideration [he] hit upon the plan here described (Venn 1880c, p. 4).

Although this narrative suggests a sudden or lucky in-sight at work, it also reveals the two ideas that Venn was investigating when he hit upon his diagrams, namely Eulerian diagrams and Boolean logic. The aim of our paper is to determine what Venn diagrams owe to these two traditions. We thus take the above quotation seriously and ask specifically: What inspired Venn from the Eulerian tradition? What does Venn adopt from the Boolean tradition?

Venn and his contemporaries give several hints on what the Eulerian tradition and the Boolean tradition are all about. These hints can therefore be helpful in determining and answering the two questions more precisely. On the one hand, Venn's diagrams appear to be an amended version of Euler's (Bennett 2015). As early as 1881, William Stanley Jevons described Venn's scheme as "a complete and consistent system of diagrammatic reasoning, which carries the Eulerian idea to perfection" (Jevons 1881, p. 233). Yet, Venn regarded his diagrams as a "special, and almost entirely new device" (Venn 2022, p. 70). On the other hand, Venn acknowledged that his method was "founded" on Boole's system, but insisted that it was not "in any way directly derived from" Boole himself. Indeed, the latter "does not make employment of diagrams himself, nor does he give any suggestions of their introduction" (Venn 1881, p. 104). However, many logicians at the time were working on the problem of elimination that originated with Boole.[1] Also Venn was looking for a method to find out how to get from the premises to the conclusion and explained in his own words that he was searching for a

> new scheme of diagrammatic representation which shall be competent to indicate imperfect knowledge on our part; for this will at once enable us to appeal to it step by step in the process of working out our conclusion. (Venn 1880c, p. 4)

Venn's diagrams thus appear to his contemporaries to be improved versions of Euler's diagrams and, moreover, Venn himself says that the problem of 'imperfect knowledge' or 'uncertainty' played an important role in connection with Boolean logic. We can thus further specify the two questions above: What did Venn improve on Euler's diagrams? What does the problem of elimination in Boolean logic have to do with the development of Venn diagrams?

In the following sections, we identify several ideas that opened the way to Venn's invention, even if the latter often failed to acknowledge their merits. However, we are not concerned with the question of who was the first to have which idea before Venn, but that certain problems and proposed solutions existed at the time when Venn also invented his method. We first explore post-Euler conventions that expressed uncertainty regarding the knowledge conveyed by traditional syllogistic propositions. Then, we discuss how Boolean logicians developed a step-by-step

[1] For a characterisation of the intellectual atmosphere during Venn's lifetime, cf. (Dunning 2021; Verburgt 2022)

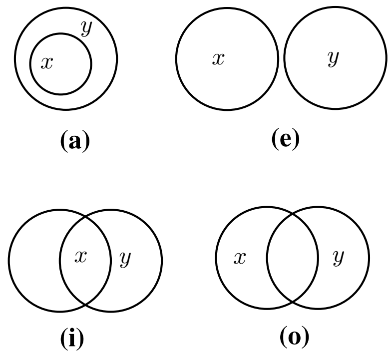

Fig. 2 Euler diagrams taken from (Euler 1803, I, p. 398)

method for the representation of propositions and sets of propositions. Then, we argue that Venn's merit was precisely to combine these two traditions to achieve the desired scheme.

2 The Eulerian idea

Euler first published his diagrams in 1867, in the second volume of his *Letters to a German Princess* (Euler 1768). After explaining the canonical propositions of syllogistic, he exposed their representation by means of circles and their relations:

> These four species of propositions may likewise be represented by figures, so as to exhibit their nature to the eye. This must be a great assistance towards comprehending more distinctly wherein the accuracy of a chain of reasoning consists [...]. As a general notion contains an infinite number of individual objects, we may consider it as a space in which they are all contained. Thus, for the notion of man we form a space [...] in which we conceive all men to be comprehended. (Euler 1833, p. 339)

Since spaces stand for terms, the topological relations between those spaces readily represent the logical relations between the corresponding terms. For instance, a proposition 'Every *A* is *B*' is simply represented with a circle *A* inside a circle *B* (cf. Fig. 2). Similarly, a proposition 'No *A* is *B*' is depicted with two disjoint circles *A* and *B* (cf. Fig. 2).

Euler's method was widespread in the 19th century. Yet, these efforts suffered from some defects that subsequent logicians attempted to overcome. Venn pointed out that the "weak point about [Euler's circles] consists in the fact that they only illustrate in strictness the actual relations of classes to one another. Accordingly they will not fit in with the propositions of common logic" (Venn 1880a, p. 49). Indeed, such propositions generally express, purposely or not, a so-called 'imperfect knowledge' about the relations between terms. Because of this imperfect knowledge, logicians at that time also often spoke of 'uncertainty'. For instance, when we are told that 'All *x* are *y*', it is unclear whether (1) *x* is strictly part of *y* or (2) *x* fully coincides with *y*. The problem had long been known in syllogistics and initially had nothing to do with logic diagrams. Through Euler's diagrams, however, the problem became obvious, since a separate diagram had to be drawn for each uncertain proposition.

Venn identified in his writings several resulting flaws in Euler's diagrams that make their use "very questionable" (Venn 1881, p. 15). For instance, he observed that "every fresh proposition demands a diagram new from the beginning" (Venn 1880a, p. 52). He also argued that "none of the moods in the syllogism can in strict propriety be represented by [Eulerian] diagrams" (Venn 1881, p. 16). Euler diagrams are also criticised for their inability to "break up a complicated problem into successive steps which can be taken independently" (Venn 1880c, p. 3). These shortcomings, directly or not, spring from the inability of Euler diagrams to represent uncertain knowledge in an effective way. Venn precisely worked on a scheme that would overcome this key weakness.

It is important for our purpose to consider how pre-Venn logicians who appealed to Euler diagrams tackled this difficulty. It is true that many authors and textbook writers made use of Euler's circles without pointing out that problem or, when they did, without attempting to overcome them. However, several logicians offered solutions that may be regarded as steps towards what Venn was about to do decades later.[2]

Venn noted that Euler's treatment of particular propositions shows that he must have been aware of this problem (Venn 1880a, p. 50). Indeed, the mere relation of the circles does not suffice to distinguish the proposition 'Some *x* are *y*' from 'Some *x* are not *y*'. Hence, Euler had to introduce a convention related to the position of the label *x*, which is inserted in the region whose existence is asserted. For 'Some *x* are *y*', letter *x* is found in the region common to circles *x* and *y*, as shown in Fig. 2 on the left below. For 'Some *x* are not *y*', letter *x* is rather inserted in the region of circle *y* which is outside circle *y*, as shown in Fig. 2 on the right below. In a sense, the two intersecting circles may be regarded as a primary diagram on which the position of the label indicates the proposition that is represented. In some diagrams, Euler even occasionally replaced this labelling convention with a star ∗ inserted in the compartment whose existence is asserted. It is unclear however if this symbol indicates

[2] It has recently been suggested that Aaron Schuyler may be regarded as "a good candidate for the missing link between Euler and Venn diagrams" (Aznar et al. 2021, p. 196). This claim principally rested upon a diagram where a circle stands for *S* while the outer space stands for *P*. Hence, the absence of a region for non-*S* non-*P* indicates that 'No non-*S* is non-*P*'. It is said that "Schuyler anticipated Venn's idea of representing propositions by removing regions from Euler diagrams" (Aznar et al. 2021, p. 201). From our perspective, such a diagram rather belongs fully to the Eulerian family. Indeed, Schuyler represents directly the relation between terms *S* and *P*. It is precisely the essence of Euler, and not Venn, diagrams to remove the regions of the classes that are known not to exist. In Venn's scheme, such regions are shaded but still are present, while they are absent in Euler's. The fact that class *P* is not represented by a circle does not play any logical role here since the shape of figures has no logical meaning (as was stated by Euler himself). This being said, we agree with the statement that Schuyler's diagram "deserve[s] to be considered an advance." (Aznar et al. 2021, p. 201) However, this advance is not made in the direction of Venn diagrams, but rather towards the determination of the relations between terms and their opposite, a path that was previously explored by Joseph Gergonne and Arthur Schopenhauer and that will eventually lead to what is sometimes known as Keynes' relations (Moktefi 2020).

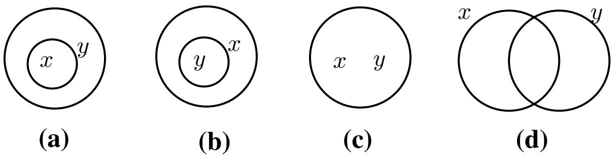

Fig. 3 Eulerian diagrams taken from Ueberweg (1871, p. 217)

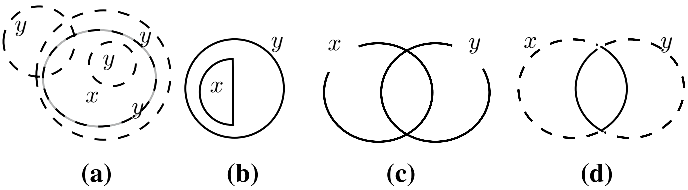

Fig. 4 Conventions for the proposition 'Some *x* are *y*'

existence itself or merely points out to the compartment whose existence is asserted within the text that accompanies the diagram (Euler 1833, pp. 351ff.).

Venn argued that there was only one "legitimate course" for the user of the Eulerian diagrams who wishes to represent uncertainty. It consists in offering "an alternative choice of diagrams, admitting frankly that, though one of these must be appropriate to the case in question, we cannot tell which it is" (Venn 1880c, p. 2–3). For instance, if one wishes to represent the proposition 'Some *x* are *y*', one needs to represent all the possible relations between two spaces that are permitted by that propositions. We obtain the four diagrams shown in Fig. 3. This method was used by some pre-Venn logicians (Ueberweg 1871, pp. 216-7), but is evidently unsuitable when one wishes to combine several propositions involved in an argument.

Several other solutions have been proposed by pre-Venn logicians to represent uncertainty. Most of them were merely sketched and did not lead to a systematic or consistent usage by the logicians who proposed them. In the following, we merely illustrate how such conventions would apply on the case of the proposition 'Some *x* are *y*', if we were to adopt them. We also indicate logicians who used these conventions but we do not imply that they were their inventors or sole users, nor that they did actually treat the specific proposition 'Some *x* are *y*' which we discuss here:

(a) One convention simply consists in visualizing within one diagram all the alternative occurrences of one term (*x*) depending on what relation it may have with the other term (*y*). These duplicated circles are drawn with discontinuous line in order to express our uncertainty as to their existence, as shown (Fig. 4a). This diagram combines the four alternatives shown in (Fig. 3) within a single figure. This convention is, for instance, found in (Bergmann 1879, p. 372).

(b) A second convention to express our uncertainty regarding the proposition 'Some *x* are *y*' consists in drawing half a circle *x* within a full circle *y*, as shown in (Fig. 4b). This convention is found in (Thomson 1842, p. 113). The incompleteness of circle x indicates our uncertainty regarding the location of the second half of x, since it may be inside y, outside y, coincide with the part of y that is not within the first half of x, or contain that part of y that is not within the first half of x. These four instances precisely correspond to the four possibilities depicted in (Fig. 3)

(c) A third convention consists in drawing two intersecting incomplete circles x and y, as shown in (Fig. 4c). This openness indicate our uncertain knowledge regarding the existence of that boundary. This technique is found in Wilson (1856, p. 68). The four possible combinations between the openness of circle x and the openness of circle y produce precisely the four alternative diagrams in (Fig. 3).

(d) A fourth convention consists in using dotted lines for the boundaries whose existence is doubtful. This line may subsequently be made continuous or be erased, depending on whether further information confirms or denies its existence. This technique is older than the previous conventions since it was used in linear diagrams prior to Euler himself. It also enjoyed some popularity since many logicians have made use of it. For instance, it is found in (Ueberweg 1871, p. 217–8). It may be said that the first and third techniques are variations of this fourth convention. If one wishes to use it to represent adequately the proposition 'Some x are y', it suffices to draw the uncertain boundaries with discontinuous lines, as shown in (Fig. 4d). Here again, the various combinations of the discontinuous lines produce the four alternative diagrams in (Fig. 3)

These various techniques show that pre-Venn logicians using Eulerian diagrams knew about this uncertainty problem and attempted to solve it. Their approach resemble what Venn was about to do later. They provided a primary diagram that embraces the uncertainty and is easily amended if further information occurs. Venn knew of some of these techniques, especially the dotted lines method, but argued that all "modifications of this sort seem [...] wholly mis-aimed and ineffectual" (Venn 1880a, p. 50). It is true that such schemes prove unpractical when the number of propositions increases. However, it is unclear why Venn viewed them as misaimed since they precisely address the main weakness that he objected to Euler's scheme. One may even argue that such techniques embody Venn's idea and truly open the way to his scheme. Venn's dissatisfaction with Euler's scheme and subsequent modifications of it encouraged him to search for a new diagrammatic method. His invention of what became known as Venn diagrams was eventually suggested by the burgeoning Boolean logic.

3 The Boolean idea

Venn stated in several places that his scheme was both motivated and suggested by the new logic that was developing in his time in the footsteps of George Boole's work (Venn 1880c, pp. 4–5, Venn 1881, p. 104). Boole famously published a mathematical theory of logic in which propositions are expressed in the form of equations, and hence, logical problems are reduced to a system of equations (Boole 1854). An interesting aspect of this development is that Boole and his followers regarded traditional syllogisms as simple instances of a more general problem, known as the problem of elimination. It consists in determining what conclusion follows from any set of propositions offered as premises, involving any number of terms (Wilson and Moktefi 2019). Boolean logicians designed symbolic, diagrammatic and even mechanical solutions to this problem. They also often engaged in a friendly, and sometimes not so friendly, contest by comparing their notations and methods (Moktefi 2019)[3]. This 'research program' required new types of diagrams since the older schemes, notably Euler diagrams, were not suitable for tackling such complex problems.

Venn championed Boole's logic and was part of this network of logicians who worked on the so-called 'elimination problem' (Venn 2022). The point is to find a procedure in which one eliminates in an inference all those elements of the premises that are not to appear in the valid conclusion. For example, in the traditional syllogism, the middle term must be eliminated, thereby obtaining in the conclusion the relationship between the two terms, each of which is given individually in one of the two premises. Venn first addressed such complex problems with the old (Eulerian) methods. However, he soon observed their failure to tackle problems for which they simply were not designed (Venn 1880c, pp. 13–14). Hence, the need for a "reformed scheme of diagrammatic notation" (Venn 1881, p. 103). It is this scheme that he eventually invented and that opened the way to a family of 'Boolean' diagrams intended to tackle the complex problems of the so-called 'new logic' (Moktefi and Edwards 2011; Moktefi 2019)[4]. Venn described the difference between his scheme and the old plan as follows:

> Whereas the Eulerian plan endeavoured at once and directly to represent propositions, or relations of class terms to one another, we shall find it best to begin by representing only classes, and then proceed to modify these in some way so as to make them indicate what our propositions have to say (Venn 1880c, p. 5).

To understand the source of this two-step method we need to consider further the influence of Boolean logic. Venn argued that his diagrammatic scheme "may be said to be underlie Boole's method, and to be the appropriate diagrammatic representation of it" (Venn 1880a, pp. 4–5). This method consists in listing all the combinations between the terms involved in the argument before excluding those that are not permitted by the premises. Venn elegantly summarized it as follows: "Our method,

[3] Venn and his diagrams were engaged in this contest, and had to face the rivalry of symbolic, mechanic and, of course, other diagrammatic methods. For instance, Hugh MacColl acknowledged the ingenuity of Venn diagrams but objected to their superiority and promoted his own symbolic methods (MacColl 1897). Venn (and Jevons) strongly rejected MacColl's methods (Verburgt 2020). This opposition eventually discouraged MacColl who abandoned the study of logic for over thirteen years (Abeles and Moktefi 2011).

[4] Early reactions to the publication of Venn diagrams included the design of rival scheme by Allan Marquand in 1881 and Alexander Macfarlane in 1885 (Lemanski 2019). Both aimed at overcoming some difficulties that Venn faced in the design of diagrams for a high number of terms, notably regarding the continuity and regularity of the terms (Moktefi et al. 2013). Subsequently, Lewis Carroll also offered alternative diagrams but his prior familiarity with Venn's is unclear (Moktefi 2017).

in fact, is to start with all the possibilities, and then ascertain which of them are actualities" (Venn 1880b, p. 256).

To determine all the possibilities offered by a given number of terms, Venn simply proceeds by successive (dichotomy) divisions of the logical universe. Venn considered this process to be "[a]t the basis of [his] Symbolic Logic, however represented, whether by words by letters or by diagrams" (Venn 1881, p. 101). This process was known to Boole as expansion or development, and was usually followed in the solution of logical problems by the processes of elimination and reduction. For Boole, a given expression involving a number of terms can be expanded into a sum of constituents, each multiplied by its corresponding coefficient (Boole 1854, pp. 75–76). For instance, the expression '$1 - x$' can be expanded into '$0xy + 0x(1 - y) + 1(1 - x)y + 1(1 - x)(1 - y)$'. The constituents, which correspond to what Venn calls compartments, "represent the several exclusive divisions of the universe of discourse, formed by the predication and denial in every possible way of the qualities" involved in the expression (Boole 1854, p. 81). Boole noted that these constituents "are in form independent of the form of the function to be expanded " (Boole 1854, p. 75).

Jevons, one of Boole's first followers, exposed at length this method in several works from the 1860s onward. In his *Principles of Science* (1879), he described the method as follows:

> When we want to effect at all a thorough solution of a logical problem it is best to form, in the first place, a complete series of all the combinations of terms involved in it [...]. Now if we have any premise, [...] we must ascertain which of these combinations will be rendered self-contradictory by substitution [...]. I propose to call any such series of combinations the Logical Alphabet. It holds in logical science a position the importance of which cannot be exaggerated (Jevons 1879, pp. 91–93).

To implement this method, Jevons published columns of such alphabets for up to six terms (Jevons 1879, pp. 94), (Jevons 1880, p. 181). He also designed mechanical devices (slate, abacus, and machine) to make the manipulation of the alphabet "evident to the eye and easy to the mind and hand" (Jevons 1879, p. viii)[5]. However, one still needs to mark the appropriate combinations to express propositions. In his writings, Jevons often simply split his lists of combinations into groups of those that are consistent with the premises and those that are not. His mechanical devices made it possible to visualize the exclusion of some combinations by removing material components (for instance, pieces of paper or wood) that stand for those combinations [Jevons 1879, pp. 104–114].

[5] In the first edition of his *Principles of Science* (1874), Jevons called his method the 'Abecedarium', but renamed it as the 'Logical Alphabet' in the second edition (1877). Jevons acknowledge that Johann Christian Lange's logical square may be regarded as an ancestor of his method "but Lange had not arrived at a logical system enabling him to use his invention for logical inference in a the manner of the Logical Abacus." (Jevons 1879, pp. xxii–xxiv). On Lange's matrix, see (Lemanski 2021, sect. 3.2.3).

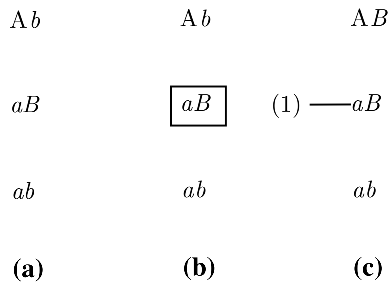

Fig. 5 Diagram taken from Jevons (1880, p. 212)

Interestingly, Jevons later proposed "logical diagrams which almost explain themselves", based on his logical alphabet, to represent propositions and solve logical problems (Jevons 1880, p. 215). In this scheme, combinations are listed in columns, then syntactic devices are added to mark the combinations which one wishes to exclude. To illustrate them, let us consider the proposition 'All *A* are *B*'. To represent this proposition, we need a logical alphabet for two terms, that is a column of four combinations: *AB*, *A* b, *aB* and *ab* (where *a* stands for non-*A*, and *b* for non-*B*), as shown in (Fig. 5). To express our proposition, we need to exclude the combination *Ab*. Jevons made use at least two conventions for such purposes. One technique, although seldom used, consists in putting a box around the combination(s) that one wishes to exclude, as shown in (Fig. 5) (Jevons 1880, p. 212). Another convention, more commonly used, consists in numbering premises, then pointing out the combination(s) with a numbered segment (or a bracket when more than one combination is targeted) to express its inconsistency with the corresponding premise, as shown in (Fig. 5) (Jevons 1880, pp. 215–217).

Jevons' diagrams may be said to be analogous in their construction to Venn's. The logical alphabet operates in a manner similar to Venn's primary diagram. Then, a syntactic device is added to express the erasure of the combination(s) that are forbidden by the premises. Martin Gardner already acknowledged that Venn's method did not differ from Jevons' and that both methods spring from Boole's work (Gardner 1958, p. 102). As he did not consider Jevons' method to be diagrammatic (Gardner 1958, p. 53), Gardner argued that one may regard "Venn circles as a diagrammatic form of Jevons's alphabet" (Gardner 1958, p. 102).

Although Jevons' Alphabet is much older, his diagrams, found in his *Studies in Deductive Logic* (1880), actually appeared few months after Venn's. Jevons reported that the text of his book was completed and sent to the printer before the publication of Venn's diagrams (Jevons 1880, p. xviii) (see also (Venn 2022, pp. 173–174)). If so, one may think of Jevons and Venn diagrams as independent simultaneous innovations that offered a diagrammatic analogue for Boole's method. Our interest here is not in settling a priority dispute but rather in ascertaining that Venn's method of representation was known to his contemporaries. Venn's merit, over Jevons, was to combine this Boolean idea with the older tradition of circular diagrams. Yet, it must be noted that Venn was not the first to do that, neither.

Indeed, in his *Der Operationskreis des Logikkalkuls* (1977), Ernst Schröder made use of circular diagrams to illustrate logical operations. For the purpose, "circular

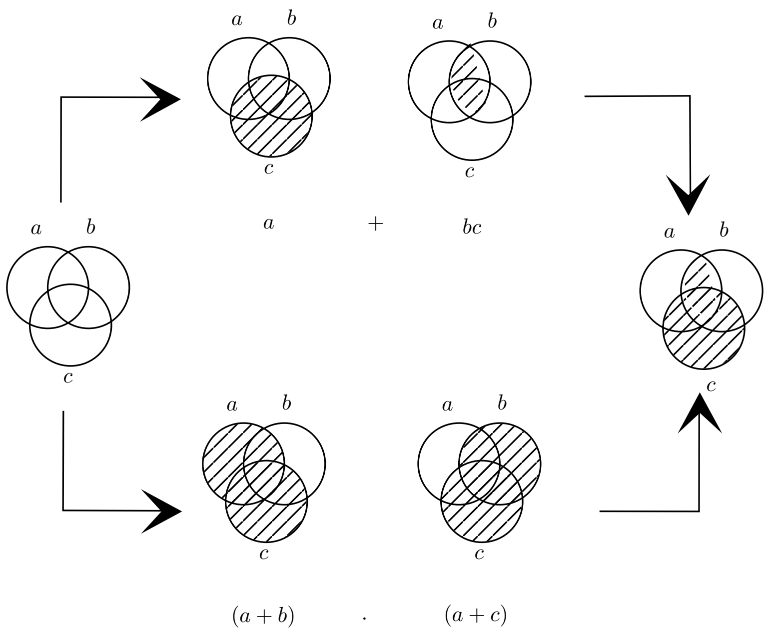

Fig. 6 Schröder Diagrams

areas" are said to be "geometrical illustrations" of classes (Schröder 1877, p. 6). Then, primary diagrams, similar to those of Venn, are produced, and subsequently, shaded to mark specific classes. This device is effectively used to demonstrate some logical laws. It suffices to illustrate each side of an equation, then observe that their shaded parts coincide, and hence, attest "the coinciding value of both sides of the equation" (Schröder 1877, p. 10). Consider Schröder's distribution law: $a + bc = (a + b)(a + c)$. Here, concatenation stands for intersection, '+' for union, and '=' for equality. The left side '$a + bc$' is illustrated by saving all the shaded areas that are designated by either a or bc. The right side $(a + b)(a + c)$ is illustrated by saving only the shaded areas that are designated by both $(a + b)$ and $(a + c)$. Figure 6 shows that the resulting diagrams coincide, which establishes the "the coinciding value" of the expressions that form the two sides of the equation.

Venn acknowledged that Schröder's scheme made use of a primary diagram similar to his, notably the three-circle diagram, but contended that it did "not adopt the subsequent step of using them as a basis for representing propositions" (Venn 1881, p. 472). It is true that Schröder marked his compartments with syntactical devices, here shadings, as Venn was about to do in his diagrams. However, Venn argued that Schröder's shadings are used "simply to direct attention to the compartments under consideration, and not, as here, with the view of expressing propositions" (Venn 1881, p. 112).

Schröder did not contradict Venn in this respect, however, one already finds the introductory remark in the *Operationskreis* that Schröder does not want to go into more detail here on the "derivation of the syllogisms of the old logic, so that the present publication does not become too extensive" (Schröder 1877, p. v). Schröder thus extends Euler's diagrams by using a primary diagram and shading. He also deals with Boole's problem of elimination (Schröder 1877, pp. 22ff.), but without using visualisations. This may be due, among other things, to the fact that Schröder's primary aims were not the visualisations, since one only finds a few at the beginning of the book, or that he did not know how to visualise the propositions.

Propositions with Venn-like diagrams were published by Scheffler (1880), Venn even admitted this himself (Venn 1894, pp. 123, 485). Scheffler also extends Euler's diagrams and relates them to Boole's approach, however-as Venn himself pointed out-Scheffler's diagrams were published in the same year as Venn's diagrams.

4 Conclusion

In this paper we have followed the indications of Venn and his contemporaries and have taken a closer look at two traditions. These two traditions brought Venn to his diagrams, which then became increasingly popular in the 20th century: Firstly, the Eulerian tradition of circular diagrams, and secondly, Boolean algebra.

Both traditions are connected with two problems that Venn wanted to solve with the help of his diagrams: First, the problem of uncertainty, and second, the problem of elimination. Euler diagrams already revealed the problem of uncertainty inherent in syllogistics. After all, one had to draw several diagrams to find out whether a syllogism was valid or not. The problem of elimination was initiated by Boole and treated by several authors in the late 19th century, who also already used similar forms of diagrams as Venn, albeit with different objectives or properties.

We can thus say that Venn's idea, which brought him to the now famous diagrams, was already in the air at that time. Perhaps Venn would not have come up with his idea had it not been of the many already similar ideas that inspired Venn. And perhaps Jevon's, Schröder's or Scheffler's diagrams would have become better known at some point if John Venn had not published his idea. However, the fact is that the idea for the diagrams did not fall from the sky. It was a product of numerous approaches to solving well-known problems that and his contemporaries worked on.

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