Correction to: Polynomial-based method for determining coast-terminating zero of fuel-optimal time-fixed trajectory

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In the original article, there is an error on page 4 regarding coordinate transformation which is corrected here. This error does not influence the results and contributions of the manuscript. The contents of page 4 except Eq. (15) and the two paragraphs immediately before it are replaced by the following:

- (2) Calculate the projections of the position and velocity vectors onto the new coordinate system; here, the new coordinate system is the perifocal frame, which is termed PF. It will eliminate the inclination i and the sum of the argument of periapsis and right ascension of ascending node, $\omega + \Omega$.

The perifocal frame $O-fgh$ is also shown in Fig. 1. The h axis coincide with the direction of angular momentum so as to eliminate i . The f and g axes are in the orbital plane. The f axis points to the periapsis so as to eliminate $\omega + \Omega$, and is obtained by anticlockwise rotating the ascending node line s about h by the angle ω . The g axis

Fig. 1 Rectangular inertial coordinate system (O-XYZ) and the perifocal frame (O-fgh)

along with h and f complete the right-handed coordinate system.

To transform O-XYZ to O-fgh, one can first rotate the O-XYZ about Z axis by the angle Ω , then rotate the current coordinate system about the current X axis by angle i , and finally rotate the current coordinate system about the new current Z axis by the angle ω . The transformation matrix is given compactly as

$$\begin{aligned} C_{XYZ \rightarrow fgh} &= M_z(\omega) M_x(i) M_z(\Omega) \\ &= \frac{1}{(1+h^2+k^2)\sqrt{f^2+g^2}} C_1 C \end{aligned} \quad (11)$$

where:

$$C_1 = \begin{bmatrix} f & g & 0 \\ -g & f & 0 \\ 0 & 0 & \sqrt{f^2+g^2} \end{bmatrix},$$

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$$\mathbf{C} = \begin{bmatrix} 1+h^2-k^2 & 2hk & -2k \\ 2hk & 1-h^2+k^2 & 2h \\ 2k & -2h & 1-h^2-k^2 \end{bmatrix};$$

$$\mathbf{M}_z(\omega) = \begin{bmatrix} \cos \omega & \sin \omega & 0 \\ -\sin \omega & \cos \omega & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

$$\mathbf{M}_x(i) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos i & \sin i \\ 0 & -\sin i & \cos i \end{bmatrix},$$

$$\mathbf{M}_z(\Omega) = \begin{bmatrix} \cos \Omega & \sin \Omega & 0 \\ -\sin \Omega & \cos \Omega & 0 \\ 0 & 0 & 1 \end{bmatrix};$$

$$\cos \Omega = \frac{h}{\sqrt{h^2+k^2}}, \quad \sin \Omega = \frac{k}{\sqrt{h^2+k^2}},$$

$$\sin(\omega + \Omega) = \frac{g}{\sqrt{f^2+g^2}}, \quad \cos(\omega + \Omega) = \frac{f}{\sqrt{f^2+g^2}},$$

$$\sin i = \frac{2\sqrt{h^2+k^2}}{1+h^2+k^2}, \quad \cos i = \frac{1-h^2-k^2}{1+h^2+k^2},$$

$$\begin{aligned} \sin \omega &= \sin(\omega + \Omega - \Omega) = \sin(\omega + \Omega) \cos \Omega \\ &\quad - \cos(\omega + \Omega) \sin \Omega, \end{aligned}$$

$$\begin{aligned} \cos \omega &= \cos(\omega + \Omega - \Omega) = \cos(\omega + \Omega) \cos \Omega \\ &\quad + \sin(\omega + \Omega) \sin \Omega. \end{aligned}$$

The position and velocity vectors projected on the perifocal frame are:

$$\mathbf{x}_{\text{PF}} = [\mathbf{r}_{\text{PF}}^T, \mathbf{v}_{\text{PF}}^T]^T = \mathbf{C}_{\text{XYZ} \rightarrow \text{fgh}} \mathbf{x}_{\text{rv}} \quad (12)$$

The costate vector with respect to $\mathbf{x}_{\text{PF}}$ is:

$$\boldsymbol{\lambda}_{\text{PF}} = \mathbf{C}_{\text{XYZ} \rightarrow \text{fgh}}^T \boldsymbol{\lambda}_{\text{rv}} \quad (13)$$

where $\boldsymbol{\lambda}_{\text{rv}}$ is the co-state vector with respect to the position and velocity.

(3) Calculate the new modified equinoctial orbital elements $\mathbf{x}_{\text{new}}^0$ and corresponding costates $\boldsymbol{\lambda}_{\text{new}}^0$:

$$\boldsymbol{\lambda}_{\text{new}}^0 = \frac{\partial \mathbf{x}_{\text{PF}}}{\partial \mathbf{x}_{\text{new}}^0} \boldsymbol{\lambda}_{\text{PF}} \quad (14)$$

where the Jacobian matrix $\frac{\partial \mathbf{x}_{\text{PF}}}{\partial \mathbf{x}_{\text{new}}^0}$ is the inverse matrix of $\frac{\partial \mathbf{x}_{\text{new}}^0}{\partial \mathbf{x}_{\text{PF}}}$ that can be obtained by equations given in Appendix of Liu and Tongue (2010).

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