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ORIGINAL RESEARCH

Intermediate factors and precedential constraint

Trevor Bench-Capon[1]

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Abstract

This paper explores the extension of formal accounts of precedential constraint to make use of a factor hierarchy with intermediate factors. A problem arises, however, because constraints expressed in terms of intermediate factors may give different outcomes from those expressed only using base level factors. We argue that constraints that use only base level factors yield the correct outcomes, but that intermediate factors play an important role in the justification and explanation of those outcomes. The discussion is illustrated with a running example.

Keywords Precedential constraint · Legal reasoning · Factors · Factor hierarchy · Explanation

1 Introduction

Extending the theory of precedential constraint proposed in Horty and Bench-Capon (2012) to hierarchies with intermediate factors between the issues and base level factors has been the subject of a number of recent papers: (Bench-Capon 2023; Canavotto and Horty 2023b; Van Woerkom et al. 2023b; Canavotto and Horty 2023a). Introducing intermediate factors gives rise to two notions of constraint: the original notion in Horty and Bench-Capon (2012), in which only the base level factors appear in constraints (*flat* or F-constraint), and one in which the constraints can also contain intermediate factors (*hierarchical* or H-constraint).

That these notions of constraint are different was established in Canavotto and Horty (2023b), where it was shown both that F-constrained cases may be not H-constrained (called *Type B* cases in Bench-Capon 2023), and that H-constrained cases may not be F-constrained (called *Type C* cases in Bench-Capon 2023). It was argued in Bench-Capon (2023) that in both Type B and Type C cases F-constraint gives the correct outcome.

✉ Trevor Bench-Capon
tbc@liverpool.ac.uk

[1] Department of Computer Science, University of Liverpool, Liverpool, UK

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In Type C cases (Bench-Capon 2023) claimed that this was because the children of an intermediate factor may have different strengths, and so the intermediate factor has more or less force in different situations. For H-constraint to work all the children of the intermediate factor must establish it with the same strength. But whether this is so must be decided in precedent cases: thus if a type C case arises, the decision *may* effectively follow the H-constraint, showing that the different children are indeed of the same strength. But the H-constraint does not force that decision: the judge may equally well decide for the other party, establishing that the factors are of different strengths, and that the H-constraint is not applicable.

For Type B cases (Bench-Capon 2023) claimed that the flattening mechanism proposed in Canavotto and Horty (2023b) did not produce the correct F-constraint. Applying the reason model of Horty and Bench-Capon (2012), Bench-Capon (2023) argued, gives rise to a different F-constraint, which does not constrain the problem case. This was responded to in Canavotto and Horty (2023a), using the idea of *enablers*, factors which, while not forming part of the reason governing the case, are required for that reason to have effect. Enablers will be discussed in detail in Sect. 4.

In this paper we will explore the notion of precedential constraint using intermediate factors. Section 2 will give a brief outline of formal approaches to precedential constraint. In Sect. 3 we will introduce our running example, and go through a series of example cases to show how the notions of constraint diverge and why. Section 4 will introduce enablers as proposed in Canavotto and Horty (2023a). Section 5 will present and discuss an implementation of the constraints in the running example and Sect. 6 will offer some concluding remarks.

2 Formal precedential constraint

Precedential constraint is based on the idea that precedents constrain judges to decide future cases so as to be consistent with them. In the formal accounts of precedential constraint discussed in this paper, cases are represented as sets of *factors*. Factors were introduced[1] in CATO (Aleven and Ashley 1995), where factors are stereotypical patterns of facts favouring one side or the other. CATO organised factors into hierarchies of the sort found in Fig. 1, with one or more layers of *intermediate factors* between the root and the *base level factors*.[2]

Since factors favour a particular side, a case can be seen as the union of the set of pro-plaintiff factors (P) and the set of pro-defendant factors (D). Thus

– $Case \equiv P \cup D$

[1] Although factors were mentioned in Ashley's book about the HYPO system (Ashley 1990), they did not appear in the original presentations of HYPO (e.g. Rissland and Ashley 1987) which used dimensions rather than factors. Dimensions continued to be the primary technique in Ashley (1990), but factors replaced dimensions entirely in CATO.

[2] In CATO the roots were issues, but formal accounts of precedential constraint typically follow (Horty 2011) and have the case outcome at the root, with base level factors as its children, as in the rules R1-R3 below.

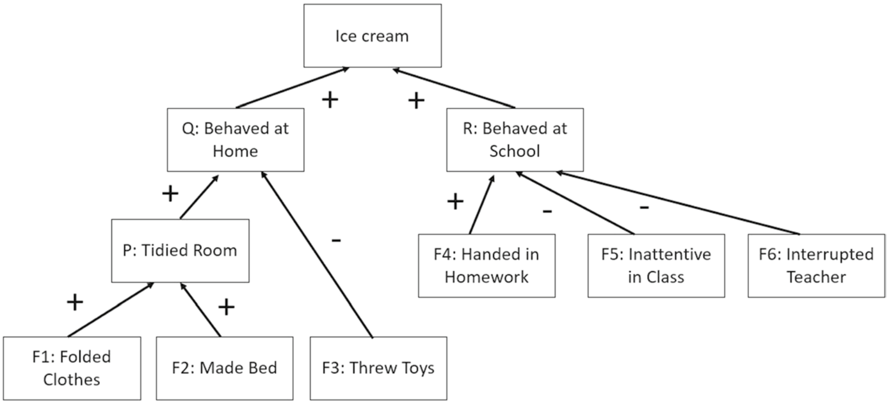

Fig. 1 Hierarchy for treat domain adapted from (Canavotto and Horty 2023a)

If we have a precedent in which the plaintiff won, this shows that P was preferred to D. This was exploited in Prakken and Sartor (1998), to represent a precedent as three rules:

- R1: $P \rightarrow \pi$
- R2: $D \rightarrow \delta$
- R3: $R1 \succ R2$ if the plaintiff won, and $R2 \succ R1$ if the defendant won

These rules were then deployed in a dialogue game. This idea, that the precedent shows the strongest reason for the winner is preferred to the strongest reason for the loser is what Horty in (2011) terms the *result* model (Alexander 1989). The result model was also used in Bench-Capon (1999), which proposed a method for using precedents to develop an ordering over all possible sets of factors.

In Horty (2011) and Horty and Bench-Capon (2012), Horty proposed the *reason* model, for which he cited (Lamond 2005). The idea here is that the winner might need only a subset of the available factors to defeat all the loser's factors. The subset would be the reason for deciding for the winner. So now, given $S \subset P$ in a case won by the plaintiff rule R1 would become

- R4 $S \rightarrow \pi$

Whereas the result model allows only *a fortiori* reasoning, the reason model allows for more cases to be constrained. A formal account of constraint in terms of consistency with an existing case base of precedents is given in Horty (2011) and Horty and Bench-Capon (2012): if either decision is consistent with the existing case base, the judge is free to choose, but otherwise the judge is obliged to decide so as to maintain consistency. The account was further developed in Rigoni (2015).

Horty extended the reasons to include factors with magnitude and dimensions in Horty (2019). This was critiqued by Rigoni in Rigoni (2018) and later modified by Horty in Horty (2021) to avoid the collapse of the result and reason models in the original account in Horty (2019). The various accounts are summarised and compared in Prakken (2021).

All these accounts (other than Prakken and Sartor 1998) give a single step argument from base level factors to outcome.[3] In recent papers both Canavotto and Horty (2023b) and Van Woerkom and his colleagues (Van Woerkom et al. 2023b), attempted to produce multi-step arguments, as recommended in Prakken and Sartor (1998), by restoring the intermediate factors of CATO's original hierarchies and providing an account of constraint in terms of these intermediate factors, which we will term hierarchical (or H-) constraint.

As explained in the introduction this raised some problems in that H-constraint diverged from constraint expressed in terms of base level factors (flat, or F-constraint). It is these problems that we will now explore.

3 Example cases

We will base our example on one taken from (Canavotto and Horty 2023a), concerning whether or not children should be given or denied a treat, depending on their behaviour[4] which we shall call the *treat* domain. Figure 1 gives the factor hierarchy adapted from that used in Canavotto and Horty (2023a).[5] Jack and Jo are the parents of two children, Emma and Max, and Jack and Jo, motivated by a desire for consistency, use rules to determine whether requests for treats should be granted. The basic rule is that treats will only be awarded for good behaviour, considering both behaviour at school and behaviour at home. Essentially the treat is for good behaviour at home, subject to misbehaviour at school.

On the basis of this we can look at some case law. The base level factors present in the cases discussed are shown in Table 1.

[3] Moving immediately from base level factors to outcomes, was seen in Prakken and Sartor (1998) as "limiting cases of a richer framework" and "Without those higher level arguments, which substantiate a rationale for decisions on controversial points, judicial reasoning would in some cases appear impoverished and arbitrary." Single step arguments can be seen as examples of Loui and Norman's *compression rationales* (Loui and Norman 1995), in which intermediate steps providing the rationale of the rule are omitted. Issues were restored as a step between base level factors and outcome in Bench-Capon and Atkinson (2021) to give two step arguments, and the papers discussed in this paper attempt to reintroduce intermediate factors to allow the full multi-step arguments to be produced.

[4] Like (Canavotto and Horty 2023a), we use a non-legal example, in the tradition of Twining and Miers (1999), where examples of parental rules were used to discuss legal interpretation.

[5] We present our hierarchies in the familiar style of CATO (Aleven and Ashley 1995). In Canavotto and Horty (2023a), all intermediate factors also have an explicit contrary. Such contraries somewhat change the nature of factors as understood from CATO, where they are either present or absent, and such contraries, if relevant, are represented by distinct factors (as with *SecurityMeasures* and *NoSecurityMeasures* in CATO).

Table 1 Base level factors in the cases discussed. *MaxWed*, *EmmaWed* and *MaxThursday* will be discussed in Sect. 5

Case	F1	F2	F3	F4	F5	F6
MaxMonday	Yes				Yes	
EmmaMonday		Yes				Yes
EmmaTuesday	Yes			Yes		Yes
MaxTuesday	Yes					Yes
MaxWed	Yes	Yes				Yes
EmmaWed	Yes	Yes	Yes	Yes		
MaxThursday				Yes		

3.1 Case of *MaxMonday*

The first case is that of *MaxMonday*. Max asks Jo for ice cream. The facts are that he folded his clothes (F1), but was inattentive in class (F5). Jo has to balance the good behaviour at home with the bad behaviour at school. In this case she decides that Max can have his ice cream, on the grounds that the good behaviour outweighed the bad. This will set a precedent, but questions arises as to the *ratio* of the case.

There are (at least) two possible interpretations.[6] We could see the case as giving rise to the H-constraint that behaviour at home (Q) is more important than behaviour at school (R). This is quite a sweeping judgement.

Or we could argue for the more limited finding, the F-constraint, that folding clothes (F1) is more important than inattention in class (F5), remaining silent as to the relative importance of other forms of behaviour.

So we have two interpretations:

- MMH $Q \succ R$
- MMF $F1 \succ F5$

Jo may give clues as to her interpretation. If she says *ok, you tidied your room, which meant you behaved at home, so you can have ice cream even though you misbehaved at school*, she is clearly thinking in hierarchical terms. If, on the other hand she says *ok, you folded your clothes so you can have ice cream even though you were inattentive in class,* she is clearly thinking in terms of MMF. But she might say *ok, your behaved at home by folding your clothes, so you can have ice cream even though you misbehaved at school by being inattentive*, in which case it is less clear which constraint is intended. The *ratio* of cases are often not explicitly stated: they are identified by the subsequent judges when citing them as precedents.

[6] The interpretation of decisions in precedent cases is the subject of Rigoni (2024). He discusses the need for multiple interpretations and argues that allowing multiple reasonable interpretations of cases and modelling precedential constraint as a function of what all reasonable interpretations compel may be advantageous.

3.2 Case of *EmmaMonday*

The next case is *EmmaMonday*, in which Emma asks Jack for ice cream. Like Max, Emma had also shown mixed behaviour. She had made her bed (F2), but had interrupted her teacher (F6). Emma will offer the interpretation MMH to argue that she should get ice cream because, like Max, she had misbehaved at school but behaved at home. In contrast MMF suggests that *MaxMonday* sets no precedent because there are no base level factors in common. This is a Type C case, H-constrained, but not F-constrained.

Jack must first consider the relative strengths of F1, the good home behaviour in *MaxMonday* and F2, the good behaviour in *EmmaMonday*. Suppose he decides they are of similar strength, and establish the intermediate factor *TidiedRoom* with the same force. He must then consider the relative strengths of F5, the factor representing misbehaviour at school in *MaxMonday*, and F6, the misbehaviour at school in *EmmaMonday*. If he also considers these to be of similar strength he should follow *MaxMonday* and assert the new F-constraint:

- $\mathrm{EMF}_{\pi}\ F2 \succ F6$

This also yields the H-constraint MMH, but does not yet establish its validity. For that we would need further decisions confirming that $F1 \succ F6$ and $F2 \succ F5$. Even with MMF and EMF_{π}, a future judge is free to decide that F1 is weaker than F2, and F6 stronger than F5 so that $F6 \succ F1$ holds, which means MMH will give the wrong answer. Only when all possibilities where we have to balance good behaviour at home with bad behaviour at school have been tested can we say that the H-constraint is valid.[7] But then all the possibilities will be covered by F-constraints, so that F- and H-constraint will coincide and no Type C cases will arise for that H-constraint.

Jack, however, is not constrained to find for Emma. He may well consider that the general disruption resulting from interrupting the teacher is more serious that inattention, which affects only Emma, and therefore outweighs the bed making. He therefore denies ice cream, adopting the more restrictive interpretation MMF of *MaxMonday* and setting the new constraint,

- $\mathrm{EMF}_{\delta}\ F6 \succ F2$

The decision in this case can be simply *no, even though you made your bed, you interrupted your teacher*. In order to remove any suspicion that he is overruling Jo's decision in *MaxMonday*, however, Jack may clarify by saying that interrupting the teacher is serious misbehaviour (R+) whereas inattention is only simple misbehaviour (R) and thus can also offer a justification in terms of H-constraint:

[7] Of course, it is open to the court to make a more sweeping ruling such as "good behaviour at home always trumps bad behaviour at school". But this will be tested by attempts to distinguish new possibilities as the cases arise.

– EMH $R+ \succ Q$

This time the decision will be *no, even though you behaved at home by tidying your room, interrupting the teacher is serious misbehaviour at school, worse than just being inattentive in class.* Note, however, that this move involves accepting that intermediate factors can have different strengths, depending on which base level factors established them.[8]

3.3 Case of *EmmaTuesday*

On Tuesday Emma folds her clothes (F1), hands in her homework (F4), but interrupts her teacher (F6). This is like the case c3 in Canavotto and Horty (2023a). Emma pleads MMH as a precedent, distinguishing *EmmaMonday* by having handed in her homework. Here the question before Jack is whether handing in the homework is sufficient to mitigate the interruption. Jack may decide that the homework reduces the seriousness of the behaviour so that R rather than $R+$ applies, and then he can simply follow MMH. Or he may think that homework is so important that it negates entirely the interruption, in which case there is no reason at all to deny the ice cream, so that R does not apply at all. In discussing their case 3 in Canavotto and Horty (2023a), Canavotto and Horty, seem to adopt the latter position, giving the decision:

> "Emma tidied up her room because she folded her clothes and she behaved at school because she turned in her homework. Since she tidied up her room, Emma behaved at home. Because of this, my decision is that Emma can have ice cream" (p19).

In this case the H-constraint is simply

– ETH $Q \succ []$

What of the F-constraint in this case? Canavotto and Horty proposed a method for flattening cases in Canavotto and Horty (2023b), which produces the F-constraint:

– ETF_{ch} $F1 \succ F6$

In Bench-Capon (2023) Bench-Capon reasons differently. He argues that the plaintiff reason from *EmmaTuesday* cannot be simply F1, giving the preference ETF_{ch}, because F1 was never tested against F6 in *EmmaTuesday*, since F6 had been neutralised by F4. There is no certainty that Emma would have got her ice cream had F4 been absent: indeed *EmmaMonday* strongly suggests otherwise. Therefore,

[8] The use of dimensions in hierarchical constraint was explored in Van Woerkom et al. (2023a). We prefer to use, as in CATO, only factors in our constraints, but allow factors of different strengths to be derived from a given dimension when the facts are stated in terms of dimensions. The relationship between dimensions and factors is discussed in Bench-Capon and Atkinson (2021).

since F1 could be defeated by F6 unless F4 is present to neutralise F6, Bench-Capon argues that the F-constraint from *EmmaTuesday* should have *both* pro-ice cream factors:

– $\mathrm{ETF}_{bc}\ F1 \wedge F4 \succ F6$

Let us suppose Jack allows Emma her ice cream, leaving open for the moment whether he is thinking in terms of ETF_{ch} or ETF_{bc}.

3.4 Case of *MaxTuesday*

We now have the case of *MaxTuesday*. Max had folded his clothes (F1) but interrupted his teacher (F6). But unlike Emma he did not hand in his homework.

EmmaTuesday does not H-constrain the case in favour of Max, since F6 was cancelled by F4, so that *R* was not present in that case. But given the flattening of Canavotto and Horty (2023b), yielding the F-constraint ETF_{ch}, Jo is F-constrained to find for Max, simply because he folded his clothes. This is consistent with EMF_{δ}, although it does suggest that clothes folding in more important than bed making. This is the example of a Type B case in Canavotto and Horty (2023b), F-constrained but not H-constrained.

If, however, we adopt Bench-Capon's interpretation of the reason for *EmmaTuesday*, ETF_{bc}, in the absence of F4 Jo is not F-constrained and can find against Max, adding the constraint:

– $\mathrm{MTF}_{\delta}\ F6 \succ F1$

Note that Jo is not constrained to decide this way. She is entitled to find for Max, establishing the constraint

– $\mathrm{MTF}_{\pi}\ F1 \succ F6$

The possibility of adopting MTF_{π} would mean that, since F2 is not (from EMF_{δ}), but F1 might be, preferred to F6, we have to recognise that tidied room (*P*) cannot have uniform strength since, given the possibility of MTF_{π}, it may, or may not, defeat F6. The strength of *P* would then depend on which of the base level factors established it, which makes it unreliable for use in constraints: it needs to be replaced by the appropriate base level factor. Moreover, even if Jo does decide against Max, asserting MTF_{δ}, it would still be possible for the *combination* of F1 and F2 to defeat F6, so we still may need the idea that *P* may be established with different strengths.

Most plausible is that Jo will follow MTF_{δ}, although we may conjecture that in a future case (*MaxWed*, see Sect. 5) it may be established that F1 and F2 can, in combination, defeat F6, giving the constraint:

– MWF $F1 \wedge F2 \succ F6$

showing that there is potentially a stronger version of P, $P+$, and this in turn will mean that Q also can have different strengths. In hierarchical terms the constraint from *MaxWed* is:,

– MWH $Q+ \succ R+$

Whether MWF and MWH hold or not will be resolved in a case presented in Sect. 5. If we did want to represent intermediate factors with different strengths in the hierarchy in such a way as to allow H-constraint, we could add a number of additional nodes to the hierarchy in Fig. 1 to provide a different intermediate factor for each strength. This is shown (accepting MWF) in Fig. 2. Note that every intermediate factor can be rewritten as a specific combination of base level factors, so that every H-constraint has an equivalent F-constraint. In the hierarchy shown in Fig. 2 we can, if we wish, use H-constraints, and it may help our explanations if we do. But we can use the H-constraint only because we are confident that H-constraint and F-constraint will coincide because in this hierarchy precedents have shown that all the children of intermediate factors establish them with equal strength. And, as discussed in Sect. 3.2, we need to have precedents showing us that this condition holds *before* we can use the H-constraint.

4 Enablers

In Canavotto and Horty (2023a) Canavotto and Horty defended their approach against the criticisms of their proposed flattening in Bench-Capon (2023) by introducing the concept of *enablers*, an idea derived from (Dancy 2004). In Canavotto and Horty (2023a) they say that rather than a reason, an enabler:

> "is an external consideration that allows the reason that a child folded their clothes to support a decision in favour of ice cream, despite not being itself a reason for ice cream. Hence, our notion of flattening - which returns reasons alone, rather than enablers as well - seems to yield the correct result in this case" (p21).

They maintain that the flattening to ETF_{ch} is correct because handing in homework is to be expected and does not in itself provide a reason for a treat: it can defeat con ice cream factors, but cannot itself be a reason for ice cream. As a solution to the problem raised in Bench-Capon (2023) they suggest that, rather than simply including the enabler in the reason, as in ETF_{bc}, we should

> "refine the standard reason model by introducing a distinction between reasons and enablers that would prevent cases like the one presented by Max from being F-constrained" (p21).

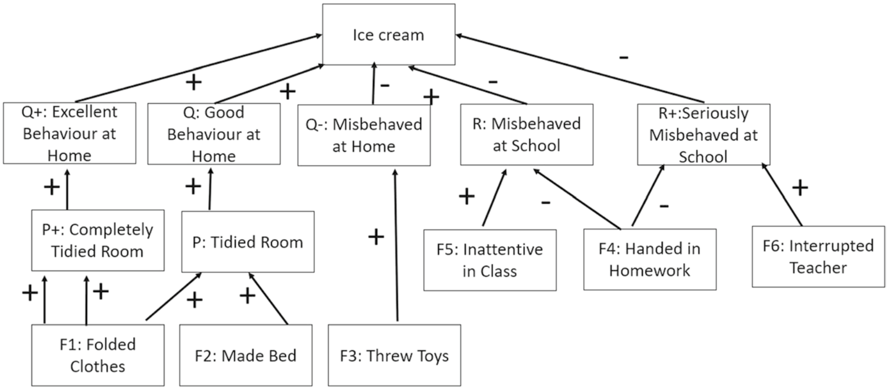

Fig. 2 Hierarchy for treat domain with intermediate factors of different strengths

A key motivation for hierarchical constraint for Canavotto and Horty is the explanatory role of intermediate factors. In Canavotto and Horty (2023a) they say:

> "The intermediate factor tidying up one's room, for example, is not just an empty cipher indicating that the base level factors folding clothes and making one's bed can be substituted for each other in certain arguments. Instead, the intermediate factor plays an explanatory role" (p20).

And, motivating the need for enablers,

> "The problem is that, since the standard reason model does not contain a distinction between reasons and enablers, it cannot capture the information that, absent its enabler, folding clothes may fail to support ice cream" (p21).

That intermediate factors represent important information and are needed for satisfying explanations is also affirmed by Bench-Capon. In Bench-Capon (2024) he argues that intermediate factors do indeed capture important information which is needed to give satisfying explanations. He argues that sometimes the point at dispute should be seen as a conflict between intermediate factors, and using only base level factors would fail to explain why and how they matter. He says that, in some cases,

> "the judgement between intermediate factors is an important part of the reason for the decision, and so should be reflected in the explanation" (p26). And this requires the intermediate factors.

He insists, however, that *constraint* should be done solely in terms of the base level factors: that including hierarchical factors in the reasons would give rise to incorrect decisions when the intermediate factors support an intermediate factor with different strengths. This problem is discussed in detail in Bench-Capon and Gordon (2022). He concludes (Bench-Capon 2024) with:

> "the intermediate factors play an important cognitive role in aiding understanding of the domain, but play no role in the logic of precedential constraint" (p31).

As discussed in footnote 3, F-constraints can be seen as examples of *compression rationales* (Loui and Norman 1995). The intermediate factors are needed to unpack them to discover the rationale for the rule, and so represent the richness of the original argument, but the logic remains the same. We will discuss this more fully in the next section.

4.1 Discussion of enablers

If, in the case of *EmmaTuesday* we ask why Emma could have ice cream, a good answer would indeed be *because she behaved at home* or *because she folded her clothes*, that is either the intermediate factor, or the flattening suggested in Canavotto and Horty (2023b), ETF_{ch}. But if the question is why was she not denied ice cream after having interrupted her teacher, the answer is rather *because she handed in her homework*. This seems to support the contention of Canavotto and Horty that the correct reason is ETF_{ch}, but that the enabler F4 also needs to be considered.

However, from the point of view of logic, leaving all considerations of cognitive aspects such as satisfying explanation aside, enablers achieve little.

Enablers suggest a logical statement like

– $Enabler \rightarrow (Factor \rightarrow Outcome)$

But this is logically equivalent to

– $(Enabler \land Factor) \rightarrow Outcome$

the reason ETF_{bc} identified in Bench-Capon (2023).

This being so, while enablers do undoubtedly play a cognitive role in enabling us to unpack the compression rationale represented by the F-constraint, the distinction plays no real role in the *logic* of precedential constraint.

There remains, however, a distinction between enablers and other factors, albeit not a logical one. This is that enablers cannot be used, on their own, as the reason for a decision. As Canavotto and Horty say in Canavotto and Horty (2023a), "parents often expect their children to turn in their homework and they would not give the children a treat simply because they did what they were expected to do." So in a case with only F4 and F6, we should find against ice cream. But this has nothing to do with the relative merits of F4 and F6. This is because no ice cream is the default: the purpose of the rules is to ensure that a positive reason for the treat is required, and, as conceived in Canavotto and Horty (2023a), handing in homework is not such a positive reason, even though it can neutralise the adverse effects of F6. Indeed even if F4 were the only factor present, the finding would still be against ice cream because no behaviour worthy of reward would have been performed. Of course, our view that homework is expected rather than worthy of reward has not yet been tested in the courts. In a future case with just F4 present Jack would be free to decide for ice cream showing that F4 had been a *bona fide* reason rather than an enabler all along.[9]

4.2 Cognitive role of intermediate factors

The *ratio* of the case is fundamentally part of the justification and explanation of the decision. This suggests that the *ratio* should incorporate the cognitive aspects, such as intermediate factors and enablers. But does this mean that we need to modify the reason model? Instead we can see the situation as similar to the difference between an argument providing a justification and a proof.

In providing an argument to justify a claim we often omit information that we believe to be known by our audience or too trivial to need stating. Suppose I say "Rover is old" and am asked to justify this. I will typically say "He is 20", knowing that my questioner knows that dogs become old around the age of 12. Such an explanation will normally satisfy.

But it would not do as a proof. 20 is not old for a person, let alone a house or a city, so we would need to explicitly state that Rover is a dog. Moreover, for a proof we would need to explicitly state that 20 is greater than 12. And if there were a breed of dog, say the Himalayan Yetihound found in Shangri-La, which typically lives to the age of 50, we would have to add that Rover is not an Himalayan Yetihound.

Once we accept this distinction between what is required for justification and explanation and what is required for logical constraint, we can see that the *ratio* of the case need not coincide with the reason required by the reason model, the subset of winner's factors preferred to the loser's set of factors used to constrain future cases. It seems that, in Canavotto and Horty (2023a), an enabler is not part of the reason but an external consideration that allows the reason to support a decision. The *ratio*, however, would have to include enablers since they are not assumed, but discussed in course of reaching the decision. It may be that the *ratio* can use the abstractions offered by intermediate factors while the reason model can continue to operate using only F-constraints.

Restricting intermediate factors to an explanatory role means that we need not ensure that all children of an intermediate factor have the same strength. Because their role is only to group factors cognitively rather than to apply constraints, we can accept, for example, that both inattention in class and interrupting the teacher are bad behaviour at school, even if only one of them can defeat making the bed. This enables us to keep the simpler hierarchy of Fig. 1 rather that the complicated hierarchy of Fig. 2, but prevents us from using hierarchical constraint to determine the outcome.

[9] Such a decision would require a modification to the hierarchy, introducing a factor for good behaviour at school in addition to the existing misbehaviour factor.

5 Using the constraints

We can now consider how the constraints identified can be represented to produce a working system. We will use the Angelic II methodology (Atkinson and Bench-Capon 2023). In the Angelic II methodology there is a factor hierarchy of the sort shown in Fig. 1 but all nodes are additionally associated with a set of acceptance conditions, prioritised rules for determining their status, and each acceptance condition is associated with its source. Each node has a default acceptance condition.

We make one addition to the method described in Atkinson and Bench-Capon (2023). In Atkinson and Bench-Capon (2023) all the intermediate factors were Boolean, so that the acceptance conditions were rules with their conclusion either “ACCEPT" or “REJECT". To accommodate different strengths we allow the heads to be any of the statuses defined for that node, and add a status property to enumerate the statuses available for each node. The sources for the acceptance condition can be found either in statute or case law or, where case law does not yet constrain all cases, expert commentary. The four cases discussed so far do not settle all the points that could arise.

So let us add a few more cases. In the first, *MaxWed*, Max has both folded his clothes (F1) and made his bed (F2), but interrupted his teacher (F6). This tests the conjecture MWF in Sect. 3.4. Suppose MWF is confirmed and Max has his ice cream. This additionally confirms the conjecture MWH, if we want to think in terms of H-constraint, but note that MWF does rely on there being a strong version of Q, $Q+$, so that home behaviour has to be excellent rather than merely good in order to counteract the serious misbehaviour at school represented by interrupting of the teacher.

In *EmmaWed*, however, while Emma has both folded her clothes (F1) and made her bed (F2) and handed in her homework (F4), she has also had a tantrum and thrown her toys (F3). Jack takes a very serious view of this violent and antisocial behaviour and rules that it cancels out all her good behaviour and refuses ice cream. Thus we can see that F3 is decisive in finding against ice cream:

- EWF $F3 \succ F1 \wedge F2 \wedge F4$

5.1 Angelic Design Model (ADM)

With these additional cases we can produce the Angelic model of intermediate factors shown in Table 2. Acceptance conditions without sources (mainly the defaults) are suggested by the expert. We have followed the expert opinion of Canavotto and Horty (2023a) and modelled F4 as an enabler, so that it never provides a reason for ice cream. Note also that we can say F4 neutralises F5 or F6 because *EmmaTuesday* shows that it neutralises F6, and we have reason to believe that F5 is weaker than F6.

Table 2 Angelic representation of intermediate factors

ID	Status	Node	Children	Acceptance conditions and source
Root	Yes	Ice Cream	Q	No if Q = Bad (EmmaWed)
	No		R	Yes if Q= Excellent (MaxWed)
				No if R= Serious (EmmaMonday)
				Yes if Q= Good (MaxMonday)
				No
Q	Excellent	Home	P	Bad if F3 (EmmaWed)
	Good		F3	Excellent if P=Excellent (MaxWed)
	Neutral			Good if P= Good (MaxMonday)
	Bad			Neutral
R	Serious	School	F4	Neutral if F4 and (F5 or F6) (EmmaTuesday)
	Bad		F5	Serious if F6 (EmmaMonday)
	Neutral		F6	Bad if F5 (MaxMonday)
	Good			Neutral
P	Excellent	Tidied Room	F1	Excellent if F1 and F2 (MaxWed)
	Good		F2	Good if F1 (MaxMonday)
	Reject			Good if F2 (EmmaMonday)
				Reject

The last acceptance condition in each node is the default

5.2 Realisation

The ADM readily supports implementation as a logic program (e.g. Al-Abdulkarim et al. 2016b), more procedural Prolog (e.g. Bench-Capon and Atkinson 2018), an imperative program (e.g. as a Java implementation Atkinson et al. 2021), a web-based application (e.g. using Javascript Collenette et al. 2023), a set of argumentation schemes (e.g. using Carneades Bench-Capon and Gordon 2022), or a mobile phone application (using Logiak Atkinson et al. 2019). A Prolog implementation of the model in Table 2 is given in the Appendix.

5.3 Execution

We will now illustrate the operation of the program. We will use the four cases from Sect. 3 so that the output can be related to the discussion in that Section. The program takes as input cases described using the factors in Table 1.

First, *MaxMonday*:

```
[room,was,tidied,because,folded,clothes]
    [cite,maxMonday]
[behaviour,at,home,was,good,because,tidied,room]
    [cite,maxMonday]
[yes,behaviour,at,home,was,good]
     [cite,maxMonday]
```

As this was the first case in the domain, it is the citation for all the points established. Here the decision makes no reference to the behaviour at school, since the home behaviour is what merited the ice cream. Next, *EmmaMonday*:

```
[behaviour,at,school,was,seriously,bad,because,
    interrupted,teacher]
    [cite,emmaMonday]
[no,behaviour,at,school,was,seriously,bad]
    [cite,emmaMonday]
```

Again the decision cites itself as the case for the points it established. Also note that the decision this time contains no reference to the behaviour at home, since the school behaviour was the reason for the decision.

Next *EmmaTuesday*:

```
[room,was,tidied,because,folded,clothes]
   [cite,maxMonday]
[behaviour,at,home,was,good,because,tidied,room,
   cite,maxMonday]
[yes,behaviour,at,home,was,good,
   cite,maxMonday]
```

This decision simply follows *MaxMonday*. Although Emma had interrupted her teacher (F6), handing in her homework neutralised this, and made the school behaviour irrelevant to the decision.

Next, *MaxTuesday*:

```
[behaviour,at,school,was,seriously,bad]
  [because,interrupted,teacher]
  [cite,emmaMonday]
[no,behaviour,at,school,was,seriously,bad]
  [cite,emmaMonday]
```

This case is governed by *EmmaMonday*, Since he did not hand in his homework, Max's seriously bad behaviour at school was decisive.

Finally we have modelled F4 as an enabler. Thus if we have a case with only F4, say MaxThursday, we get simply:

```
[no,done,nothing,to,earn,it]
```

No case is cited for this: it is part of the conception of the scheme that treats must be earned by good behaviour at home, and that handing in homework is expected rather than meritorious.

5.4 Discussion of realisation

The explanations of the decisions produced in this way have a number of features

- They are grounded in base level factors. The input in only base level factors, and so the program imposes F-constraint.
- They make use of hierarchical factors to explain exactly how and why the base level factors matter.
- They give only the reason for the winning side, albeit expressed at various degrees of abstraction. No mention of any strengths for the losing side is made.

Is the last point a strength or a weakness? It does explain why the winner won, and it can be inferred that whatever the reasons for the other side, the winner's reasons were preferred to them, so it has the advantage of economy. It corresponds to the *how?* explanations of early logic programs such as (Sergot et al. 1986). On the other hand, the loser may feel dissatisfied that there is no explanation of why the case was lost. The explanation presented is the one that falls out naturally from the Angelic design. It would, however, be possible to write an explanatory program that processes the output to give an explanation in terms of the Issue-Rule-Application-Conclusion (IRAC) methodology as described in Bench-Capon (2020) and Bench-Capon (2024) if a more expansive explanation were required.[10] An IRAC explanation of *EmmaMonday* would look like:

> The issue in *EmmaMonday* is whether the seriously bad behaviour at school is sufficient to outweigh the good behaviour at home. The rule is that seriously bad behaviour at school is sufficient to deny ice cream, even if home behaviour is good. Interrupting her teacher was an example of seriously bad behaviour and so ice cream is denied.

and in *MaxWed*, which established the strength of excellent behaviour:

> The issue in *MaxWed* is whether the seriously bad behaviour at school is sufficient to outweigh the excellent behaviour at home. The rule is that excellent behaviour at home is sufficient to allow ice cream, even when school behaviour is seriously bad. Max tidied his room completely, both folding his clothes and making his bed, so his behaviour at home was excellent. Therefore ice cream is allowed.

Note that the issues concerns intermediate factors, even though the base level factors determine the decision. Perhaps the most interesting case is *EmmaTuesday*, which established the role of homework:

[10] Alternatively the design could be realised using Carneades as in Bench-Capon and Gordon (2022), so that the defendant reasons and why they were defeated can be shown in an argument graph, giving a visual explanation of the competing arguments and their resolution.

> The issue is whether Emma handing in homework was sufficient to outweigh the interruption of the teacher. The rule is that behaviour at school is neutral if homework was handed in, even if the teacher was interrupted. Emma did hand in her homework. Therefore, given also that behaviour at home was good, ice cream is allowed.

Here the issue is the conflict concerning F4, the enabler in Canavotto and Horty (2023a), and F6. There are no issues relating to behaviour at home, and so the reasons why behaviour at home were good are not stated. However, since behaviour at school was only neutral, the good behaviour at home is needed to justify the granting of ice cream. This very closely corresponds to the distinction between enablers and reasons in Canavotto and Horty (2023a).

Some cases remain to be tested. Suppose a child tided the room and handed in the homework, but was both inattentive and interrupted the teacher. As the domain is currently modelled, this would deny ice cream, unless the room had been completely tidied (i.e $F5 \wedge F6 \succ F1 \wedge F4$ but $F1 \wedge F2 \wedge F4 \succ F5 \wedge F6$). If this proves to be incorrect when such a case arises, the acceptance conditions will need to be modified to reflect the actual decision (Al-Abdulkarim et al. 2016a).

6 Concluding remarks

In this paper we have argued that hierarchical constraint can give rise to incorrect outcomes. It can conflict with constraint in terms of base level factors and in such cases the flat constraints give the correct outcome. In Canavotto and Horty (2023a), Canavotto and Horty introduced the notion of *enablers* in order to defend hierarchical constraints: these, however, we have shown to have cognitive rather than logical significance.

This points to the conclusion that hierarchical constraints are important cognitively, although not logically. They are essential for giving satisfying explanations and justifications: justifications in terms of base level factors alone often do not supply the information as to why and how the base level factors matter. Moreover sometimes the point at dispute can concern the strength with which an intermediate factor applies (as with seriously bad and merely bad behaviour at school in the discussions above). Thus intermediate factors do play an important role, but only in explanation, not in constraint.

We have illustrated the above with a detailed consideration of a particular domain discussed in Canavotto and Horty (2023a), and shown how the developing case law can be modelled and realised, and the kind of explanations and justifications that result.

Appendix: Prolog Code

```
go(Case):-case(Case,Factors), iceCream(Factors).

iceCream(Factors):-
iceCream(Factors):-home(bad,Factors),
   write([no,behaviour,at,home,was,bad),
   write([cite,emmaWed]),nl.
home(excellent,Factors),
  write([yes,behaviour,at,home,was,excellent,
  write([cite,maxWed]),nl.
iceCream(Factors):-
   school(serious,Factors),
   write([no,behaviour,at,school]),
   write([was,seriously,bad,cite,emmaMonday]),nl.
iceCream(Factors):-home(good,Factors),
   write([yes,behaviour,at,home,was,good,cite,maxMonday]),nl.
iceCream(Factors):-write([no,done,nothing,to,earn,it]),nl.

home(bad,[_,_,y,_,_,_]):-
   write([behaviour,at,home,was,bad,because]),
   write([threw,toys,cite,emmaWed]),nl.
home(excellent,[y,y,n,_,_,_]):-
   write([behaviour,at,home,was,excellent]),
   write({because,room,completely,tidied,room,cite,maxWed]),nl.
home(good,Factors):-
   tidied(good,Factors), write([behaviour,at,home,was,good]),
   write([because,tidied,room,cite,maxMonday]),nl.
home(neutral,F).

school(neutral,[_,_,__,y,n,y]):-
   write([behaviour,at,school,was,ok,because]),
   write([although,interrupted,teacher,handed,in,homework]).
   write([cite,emmaTuesday]),nl.
school(neutral,[_,_,__,y,y,n]):-
   write([behaviour,at,school,was,ok,because],
   write([although,inattentive,handed,in,homework]),nl.
school(serious,[_,_,__,n,_,y]):-
   write([behaviour,at,school,was,seriously,bad,because]),
   write{[interrupted,teacher,cite,emmaMonday]),nl.
school(bad,[_,_,__,_,y,_]):-
   write([behaviour,at,school,was,bad,because,inattentive]),
   write([cite,maxMonday]).nl.
school(neutral,F).
tidied(excellent,[y,y,_,_,_,_]):-
   write([room,was,completely,tidied,because]),
   write([both,folded,clothes,and,made,bed,cite,maxWed]),nl.
tidied(good,[y,_,_,_,_,_]):-
   write([room,was,tidied,because]),
   write([folded,clothes,cite,maxMonday]),nl.
tidied(good,[_,y,_,_,_,_]):-
   write([room,was,tidied,because]),
   write([made,bed,cite,emmaMonday]),nl.
tidied(no,F).

case(emmaMon,[n,y,n,n,y,n]).
case(maxMonday,[y,n,n,n,n,y]).
case(maxTuesday,[y,n,n,y,n,y]).
case(emmaTuesday,[y,n,n,n,n,y]).
case(maxThursday,[n,n,n,y,n,n]).
```

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