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Interactive group decision making method based on probabilistic hesitant Pythagorean fuzzy information representation

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Abstract
Interactive group evaluation is a decision-making method to obtain group consensus by constantly modifying the initial weight of experts. Probabilistic hesitant Pythagorean fuzzy set (PrHPFS) is to be added the corresponding probability values for each membership degree and non-membership degree on the hesitant Pythagorean fuzzy set (HPFS). It is not only a generalization of HPFS and the Pythagorean fuzzy set (PFS), but also a more comprehensive and accurate reflection of the initial decision information given by experts. Especially, it can deal with the decision-making problem of multi-attribute fuzzy information in a wider area. In this paper, some basic definitions and related operations of the probabilistic hesitant Pythagorean fuzzy numbers (PrHPFNs) are first reviewed, and propose score function and accuracy function in PrHPFNs environment. Secondly, the concepts of Hamming distance measure, weighted distance measure and degree of similarity are put forward in PrHPFNs space, and the degree of similarity of two probabilistic hesitant Pythagorean fuzzy matrices (PrHPFMs) is suggested through the aggregation operator formula of PFNs. Finally, an interactive group decision-making method is designed based on the PrHPFM and the degree of similarity under the PrHPFNs environment, the effectiveness of the method is verified by an example, so as to overcome the hesitant psychological state of experts and achieve the consistent consensus evaluation of group preference.

Keywords Pythagorean fuzzy number (PFN) · Probabilistic hesitant Pythagorean fuzzy number (PrHPFN) · Hamming distance measure · Degree of similarity · Interactive group decision making

1 Introduction

Group decision-making method is to aggregate the multi-attribute index information of multiple experts into a comprehensive index information according to certain rules. It is not only an important part of decision-making theory, but also can effectively gather different experts and their wisdom, so as to improve the scientific effectiveness of decision-making. However, in the face of practical problems, it is difficult for decision-makers to obtain satisfactory decision-making solutions only through one-time information aggregation. In the process of evaluation, they often need to be constantly modified and adjusted the weight vector of experts, and even need to aggregate the information for many times to get the final satisfactory decision-making scheme. Usually, due to various uncertainties in data information, traditional fuzzy sets only describe fuzzy phenomena by membership degree. If non-membership degree and hesitation degree are abandoned, then some useful information may be lost, and even lead to the wrong conclusion.

In 1986, Professor Atanassov [1] first suggested the concept of intuitionistic fuzzy set when considering both degree of membership and degree of non-membership, and applied it to medical diagnosis, information fusion and data mining. At present, the traditional fuzzy sets have been extended to many forms, such as the Pythagorean fuzzy set [2–4] and hesitant fuzzy set [5], and they are widely applied in the theoretical and practical problems of decision science. In 2013, Yager [2] first proposed the concept of Pythagorean fuzzy number (PFN) on the basis of intuitionistic fuzzy set, thus, the scope of application is extended to the sum of the squares of membership and

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non-membership degrees less than or equal to 1. Nowadays, the Pythagorean fuzzy set has become an effective tool to deal with multi criteria information decision-making problems. In fact, the research on Pythagorean fuzzy multi-attribute decision-making mainly focuses on three problems: Firstly, aggregation operator problem; Secondly, distance measure and similarity problems; The third is to explore new decision-making methods. In 2016, Ren and Xu et al. [6] gave the extended some arithmetic operations and the method of aggregation operator in Pythagorean fuzzy environment, and designed TODIM method for multi-attribute decision-making problems. Meanwhile, Zhang [7] put forward the Pythagorean fuzzy hierarchy QUALIFLEX method and fuzzy projection method. See [8, 9]. In 2018, Wei and Mao [10] proposed the concept of Pythagorean fuzzy weighted Maclaurin symmetric mean operator. In 2020, Garg [11] suggested the Pythagorean fuzzy aggregation operator model in view of the idea of neutral operation, so that it can deal with the relationship between membership and non-membership in a neutral way. Wang and Li [12] give the weighted Pythagorean fuzzy power Bonferroni mean operator by applying the power mean operator. See [13, 14]. These results include some operation formulas, ranking criteria, distance measures, aggregation operators and closeness of Pythagorean fuzzy sets from different aspects, and gave a new decision-making method.

In order to describe people's hesitation in the evaluation of objective things, Professor Torra put forward the concept of hesitant fuzzy set (HFS) in 2010, that is, the existence of multiple possible values can be allowed in the hesitant fuzzy set. In 2012, Zhu and Xu et al. [15] proposed dual hesitant fuzzy set (DHFS) by intuitionistic fuzzy sets and hesitant fuzzy sets. See [16]. In 2017, Xu and Zhuo [17] introduced the concept of probabilistic hesitant fuzzy set (PHFS) by adding probability values to each membership degree in HFS. In fact, PHFS not only increases the number of membership values from the perspective of hesitancy, but also consider the probability of membership degree. See [18]. Later, Gao and Xu et al. [19] proposed a decision-making method by adding probability to membership degree, non-membership degree and hesitant degree. See [20]. In 2017, Liang and Xu [21] proposed the concept of hesitant Pythagorean fuzzy set (HPFS) by combining PFS and HFS. In 2018, Garg [22] improved the aggregation operator of hesitant PFNs and applies it to multi-attribute decision-making problems. These good results have developed the hesitant fuzzy set theory.

In recent years, some scholars widely applied HPFS to many different research fields. See [23–26]. In 2018, Hao and Xu et al. [27] combined the hesitant fuzzy set with HPFSs to propose the probabilistic dual hesitant fuzzy set (PDHFS). See [28, 29]. In 2019, Luo and Liu [30] put forward the probabilistic interval-value hesitation PFNs and applied it to the selecting processes of project private partner. In 2020, Gao and Liu et al. [31] proposed the decision-making algorithm for sharing accmmodation by using PrHFSs and bipartite network projection, a bipartite graph connecting users and alternatives is established. See [32]. In 2021, Batool and Abdullah et al. [33] introduced the probability hesitation Pythagorean fuzzy numbers and arithmetic operations by adding probability to the membership and non-membership, and proposed six aggregation operators. In 2022, Liu and Wu et al. [34] studied the probabilistic hesitant fuzzy taxonomy method based on the analysis of indifference threshold-based attribute ratio. See [35]. These extended hesitant fuzzy sets are added decision-making methods to deal with multi-attribute fuzzy information from different angles.

The interactive group decision-making is realized by experts repeatedly modifying the initial weight. In 2012, Xu [36] introduced the intuitionistic fuzzy set into the interactive evaluation method and proposed the interactive intuitionistic fuzzy decision-making method. In 2013, Zeng and Su [37] determined the expert weight based on the similarity between the individual and the group of experts, a new interactive group decision-making method is suggested by using the similarity of intuitionistic fuzzy number (IFN). In 2014, Liao and Xu [38] first proposed an interactive decision-making method based on satisfaction in an incomplete weight under hesitant fuzzy environment. See [39]. In 2017, Ding and Xu et al. [40] defined the proximity coefficient of alternatives by introducing the distance measure of PrHFNs. In 2018, Wang and Duan [41] combined polygonal fuzzy number and IFN to describe multi-attribute information, proposed a new TOPSIS method. See [42–44]. In 2022, Sun and Li et al. [45] pointed out the confusion of the current ranking of PFNs and IFNs by counterexamples, and unified IFNs into the Pythagorean fuzzy environment by the centroid coordinate transformation and proposed a new ranking method. Later, Sun and Wang et al. [46] improved the ranking method and applied it to study the decision-making problem of multi attribute information. These methods not only expand the basic operations and ranking criteria of traditional IFNs, but also widely apply them to multi-attribute information decision-making problems.

The main motivation of this paper is to establish a new interactive group decision-making method through the representation of probabilistic hesitant Pythagorean fuzzy number (PrHPFN) in multi-attribute information environment. Because the PrHPFNs can describe fuzzy information by adding corresponding probability values to each membership degree and non-membership degree on hesitant Pythagorean fuzzy sets, and an interactive group decision-making is realized by introducing negotiation mechanism and constantly modifying evaluation information to achieve consensus of group preference. The main innovation of this paper is to propose the distance measure (Formula (3), Lemmas 1–2 and Theorem 4.1) through the standardized method (Definition 4.1) in PrHPFNs environment, and then put forward the weighted distance measure, PFNs family similarity, PFNs fuzzy matrix similarity and expert weight formula according to the new distance measure. In addition, a new score function and accuracy function (Definition 3.4) are introduced, and an interactive group decision-making method is designed in PrHPFNs environment.

The main contents of this paper are as follows. In Section 2, some related concepts and operations of HPFS are reviewed. In Section 3, the concept of PrHPFS is introduced, and the basic operations, aggregation operator and ranking method of PrHPFNs are studied. In Section 4, the concepts of PrHPFNs Hamming distance measure, generalized weighted distance measure and degree of similarity are proposed in PrHPFNs space, and their rationality is proved. In Section 5, the probabilistic PrHPFM is put forward based on the initial evaluation information given by experts, and the expert's weight is determined according to the degree of similarity of PrHPFM. Then a new interactive group decision method is established. In Section 6, the effectiveness of the method in dealing with multi-attribute indexes information is illustrated by a practical example.

2 HPFNs and its operations

As is known to all, HPFS is based on PFS by increasing the overall number of membership and non-membership values, that is, HPFS membership and non-membership are a finite set. This section first reviews some basic concepts and related mathematical expressions of HPFS.

Definition 2.1 [5] Let X be a fixed set, then a hesitant fuzzy set (HFS) on X is a function from X to a subset of [0, 1]. To understand the hesitant fuzzy set we write this mathematically as $H = \{<x, g_H(x)>| \ x \in X\}$, where $g_H(x) \subseteq [0, 1]$ denotes the set of some values belonging to [0, 1], that is the possible membership degree of the element x to the set H.

Definition 2.2 [2] Let X be a fixed set, then a Pythagorean fuzzy set (PFS) P on X can be defined as $P = \{<x, \mu_P(x), \upsilon_P(x)>| \ x \in X\}$, where $\mu_P : X \to [0, 1]$ denotes the membership function and $\upsilon_P : X \to [0, 1]$ denotes the non-membership function of the element x to the set P, with the condition $\mu_P^2(x) + \upsilon_P^2(x) \leq 1$, for all $x \in X$.

Let $\pi_P(x) = \sqrt{1-\mu_P^2(x)-\upsilon_P^2(x)}$, then it is called the degree of hesitation of Pythagorean fuzzy index of element x to the set P, and $0 \leq \pi_P(x) \leq 1$, for all $x \in X$.

Definition 2.3 [21] Let X be a fixed set. A hesitant Pythagorean fuzzy set abbreviated as HPFS on X is an object with the following form

$$P_H = \{< x, F_{P_H}(x), G_{P_H}(x) > | x \in X\},$$

where $F_{P_H}(x)$ and $G_{P_H}(x)$ are two sets of some values in [0, 1], they denote the possible degree of membership and degree of non-membership of element $x \in X$ on P_H, respectively, and for each element $x \in X$, for arbitrary $\gamma_{P_H}(x) \in F_{P_H}(x)$, there is a $\gamma'_{P_H}(x) \in G_{P_H}(x)$ such that $0 \leq \gamma^2_{P_H}(x) + \gamma'^2_{P_H}(x) \leq 1$ and for arbitrary $\delta'_{P_H}(x) \in G_{P_H}(x)$, there is a $\delta_{P_H}(x) \in F_{P_H}(x)$ such that $0 \leq \gamma^2_{P_H}(x) + \gamma'^2_{P_H}(x) \leq 1$.

Let $\Pi_{P_H}(x) = \bigcup_{\gamma_{P_H}(x) \in F_{P_H}(x), \gamma'_{P_H}(x) \in G_{P_H}(x)} \left\{ \sqrt{1-\gamma^2_{P_H}(x)-\gamma'^2_{P_H}(x)} \right\}$, then $\Pi_{P_H}(x)$ is said to be the degree of hesitation of x to P_H.

Note 1 If we take a $x_0 \in X$, and the set $\{< x_0, F_{P_H}(x_0), G_{P_H}(x_0) >\}$ contains only one element (a single point set), then it said to be a hesitant Pythagorean fuzzy number (HPFN) and is denoted by $\widehat{\gamma} =< F_{\widehat{\gamma}}, G_{\widehat{\gamma}} >$, where $F_{\widehat{\gamma}} = F_{P_H}(x_0)$ and $G_{\widehat{\gamma}} = G_{P_H}(x_0)$ are finite sets, such as $F_{\widehat{\gamma}} = \left\{ \mu^{(1)}_{\widehat{\gamma}}, \mu^{(2)}_{\widehat{\gamma}}, \cdots, \mu^{(t)}_{\widehat{\gamma}} \right\}$ and $G_{\widehat{\gamma}} = \left\{ \upsilon^{(1)}_{\widehat{\gamma}}, \upsilon^{(2)}_{\widehat{\gamma}}, \cdots, \upsilon^{(p)}_{\widehat{\gamma}} \right\}$, where $\mu^{(i)}_{\widehat{\gamma}}, \upsilon^{(j)}_{\widehat{\gamma}} \in [0.1]$, $i = 1, 2, \cdots, t; j = 1, 2, \cdots, p$. For arbitrary $x \in X$, if $F_{P_H}(x)$ and $G_{P_H}(x)$ are single point sets, then the HPFN become a PFN; if the non-membership degree set $G_{P_H}(x) = \{0\}$, then the HPFN become a HFN. Therefore, HPFN is not only an extension of PFN, but also a generalization of HFN.

For simplicity, we use the symbols HPFN(X) to abstractly represent all hesitant Pythagorean fuzzy numbers (HPFNs) on X, that is

$$\mathrm{HPFN}(X) = \left\{ \widehat{\gamma} =< F_{\widehat{\gamma}}, G_{\widehat{\gamma}} >, F_{\widehat{\gamma}} = \left\{ \mu^{(1)}_{\widehat{\gamma}}, \mu^{(2)}_{\widehat{\gamma}}, \cdots, \mu^{(t)}_{\widehat{\gamma}} \right\} , G_{\widehat{\gamma}} = \left\{ \upsilon^{(1)}_{\widehat{\gamma}}, \upsilon^{(2)}_{\widehat{\gamma}}, \cdots, \upsilon^{(p)}_{\widehat{\gamma}} \right\}, \left(\mu^{(i)}_{\widehat{\gamma}} \right)^2 + \left(\upsilon^{(j)}_{\widehat{\gamma}} \right)^2 \leq 1, . \right\}$$

Definition 2.4 [21] Let $\widehat{\gamma}_1 = < F_{\widehat{\gamma}_1}, G_{\widehat{\gamma}_1} >, \widehat{\gamma}_2 = < F_{\widehat{\gamma}_2}, G_{\widehat{\gamma}_2} > \in \mathrm{HPFN}(X)$, some of their arithmetic operations can be expressed as

$$\widehat{\gamma}_1 \oplus \widehat{\gamma}_2 = < \bigcup_{\mu_{\widehat{\gamma}_1} \in F_{\widehat{\gamma}_1}, \mu_{\widehat{\gamma}_2} \in F_{\widehat{\gamma}_2}} \left\{ \sqrt{\mu^2_{\widehat{\gamma}_1} + \mu^2_{\widehat{\gamma}_2} - \mu^2_{\widehat{\gamma}_1} \mu^2_{\widehat{\gamma}_2}} \right\}, \bigcup_{\upsilon_{\widehat{\gamma}_1} \in G_{\widehat{\gamma}_1}, \upsilon_{\widehat{\gamma}_2} \in G_{\widehat{\gamma}_2}} \left\{ \upsilon_{\widehat{\gamma}_1} \upsilon_{\widehat{\gamma}_2} \right\} >;$$

$$\widehat{\gamma}_1 \otimes \widehat{\gamma}_2 = < \bigcup_{\mu_{\widehat{\gamma}_1} \in F_{\widehat{\gamma}_1}, \mu_{\widehat{\gamma}_2} \in F_{\widehat{\gamma}_2}} \left\{ \mu_{\widehat{\gamma}_1} \mu_{\widehat{\gamma}_2} \right\}, \bigcup_{\upsilon_{\widehat{\gamma}_1} \in G_{\widehat{\gamma}_1}, \upsilon_{\widehat{\gamma}_2} \in G_{\widehat{\gamma}_2}} \left\{ \sqrt{\upsilon^2_{\widehat{\gamma}_1} + \upsilon^2_{\widehat{\gamma}_2} - \upsilon^2_{\widehat{\gamma}_1} \upsilon^2_{\widehat{\gamma}_2}} \right\} >;$$

$$\lambda \widehat{\gamma}_1 = < \bigcup_{\mu_{\widehat{\gamma}_1} \in F_{\widehat{\gamma}_1}} \left\{ \sqrt{1 - \left(1 - \mu^2_{\widehat{\gamma}_1}\right)^{\lambda}} \right\}, \bigcup_{\upsilon_{\widehat{\gamma}_1} \in G_{\widehat{\gamma}_1}} \left\{ \upsilon^{\lambda}_{\widehat{\gamma}_1} \right\} >;$$

$$\widehat{\gamma}_1^{\ \lambda} = < \bigcup_{\mu_{\widehat{\gamma}_1} \in F_{\widehat{\gamma}_1}} \left\{ \mu^{\lambda}_{\widehat{\gamma}_1} \right\}, \bigcup_{\upsilon_{\widehat{\gamma}_1} \in G_{\widehat{\gamma}_1}} \left\{ \sqrt{1 - \left(1 - \upsilon^2_{\widehat{\gamma}_1}\right)^{\lambda}} \right\} > .$$

Definition 2.5 [21] Let a group HPFNs as $\widehat{\gamma}_i = < F_{\widehat{\gamma}_i}, G_{\widehat{\gamma}_i} >, i = 1, 2, \cdots, n$, and $w = (w_1, w_2, \cdots, w_n)$ be the weight vector corresponding to HPFNs $\widehat{\gamma}_i$ with $w_i \in [0, 1]$ $(i = 1, 2, \cdots, n)$ and $\sum_{i=1}^n w_i = 1$. If a mapping HPFWA : $\mathrm{HPFN}(X)^n \rightarrow \mathrm{HPFN}(X)$ satisfies

$$\mathrm{HPFWA}\left(\widehat{\gamma}_1, \widehat{\gamma}_2, \cdots, \widehat{\gamma}_n\right) \triangleq \sum_{i=1}^{n} \omega_i \widehat{\gamma}_i = < \bigcup_{\substack{\mu_{\widehat{\gamma}_1} \in F_{\widehat{\gamma}_1} \\ \mu_{\widehat{\gamma}_2} \in F_{\widehat{\gamma}_2} \ \dots \\ \mu_{\widehat{\gamma}_n} \in F_{\widehat{\gamma}_n}}} \left\{ \sqrt{1 - \prod_{i=1}^{n} \left(1 - \mu^2_{\widehat{\gamma}_i}\right)^{\omega_i}} \right\}, \bigcup_{\substack{\upsilon_{\widehat{\gamma}_1} \in G_{\widehat{\gamma}_1} \\ \upsilon_{\widehat{\gamma}_2} \in G_{\widehat{\gamma}_2} \ \dots \\ \upsilon_{\widehat{\gamma}_n} \in G_{\widehat{\gamma}_n}}} \left\{ \prod_{i=1}^{n} \upsilon^{\omega_i}_{\widehat{\gamma}_i} \right\} >,$$

then the HPFWA is called to be a hesitant Pythagorean fuzzy weighted average operator.

In fact, the HPFNs can not only describe the fuzzy phenomenon that the sum of membership degree and non-membership degree exceeds 1 (the sum of squares does not exceed 1), but also express the hesitant degree of decision-makers to the evaluation information of objective things. Due to the decision maker may have many different evaluation values or preferences in the hesitant Pythagorean fuzzy environment, adding the corresponding probability value for each membership degree and non-membership degree on the hesitant Pythagorean fuzzy number can describe the fuzzy phenomenon more comprehensively and carefully. In the next section, we will combine hesitant fuzzy sets and Pythagorean fuzzy sets by adding probability to membership and non membership, propose the probabilistic hesitant Pythagorean fuzzy sets (PrHPFSs), and apply them to the interactive group decision making with multi-attribute index information.

3 Probabilistic hesitant Pythagorean fuzzy number

In general, experts often have a hesitant psychological state when evaluating attribute indicators, and this hesitant phenomenon can be characterized by probability. Therefore, we can more comprehensively describe fuzzy information by increasing the probability values of membership and non membership in the environment of hesitant Pythagorean fuzzy set (HPFS). It not only generalizes the hesitant Pythagorean fuzzy set, but also reflects the initial decision information given by experts more comprehensively and accurately. Inspired by the probabilistic dual hesitant fuzzy set (PDHFS) proposed by Hao and Xu et al. [27], we extend the hesitant Pythagorean fuzzy set (HPFS) to the probabilistic hesitant Pythagorean fuzzy set (PrHPFS) by adding probability values to the membership and non membership of HPFS, so as to deal with the decision-making problem of multi-attribute information in a wider region.

Definition 3.1 Let the universe $X = \{x_1, x_2, \cdots, x_n\}$, the $F_{P_H}(x)$ and $G_{P_H}(x)$ are the same as Definition 2.3, and $F^+_{P_H}(X) = \underset{x\in X}{\cup}\{\max F_{P_H}(x)\}$, $G^+_{P_H}(X) = \underset{x\in X}{\cup}\{\max G_{P_H}(x)\}$. A probabilistic hesitant Pythagorean fuzzy set abbreviated as PrHPFS on X is an object with the following form:

$$\Re=\{\langle x, F_{P_H}(x)/p(x), G_{P_H}(x)/q(x)\rangle \mid x\in X\},$$

where the components $F_{P_H}(x)/p(x)$ and $G_{P_H}(x)/q(x)$ are two sets of some possible elements in [0, 1] with regard to $x \in X$, $p(x)$ and $q(x)$ represent the probability values corresponding to the membership and non-membership, respectively, and for any $\gamma^+_x\in F^+_{P_H}(X)$, $\eta^+_x\in G^+_{P_H}(X)$, the $(\gamma^+_x)^2 + (\eta^+_x)^2\le 1$ is satisfied. Besides, they need to satisfy $\sum_{i=1} p(x_i) = 1$ and $\sum_{i=1} q(x_i) = 1$, where $p(x_i), q(x_i) \in [0, 1]$.

It is not difficult to see from Definition 3.1 that if $x \in X$ is taken, then $\mu\le\gamma^+_x$ and $\upsilon\le\eta^+_x$ are obviously true, for any $\mu\in F_{P_H}(x)$ and $\upsilon\in G_{P_H}(x)$, so there is always $\mu^2 + \upsilon^2\le(\gamma^+_x)^2 + (\eta^+_x)^2\le 1$.

For simplicity and intuition, we can express PrHPFS as $F_{P_H}(x)/p(x) = \{\mu_1/p_1, \mu_2/p_2, \cdots, \mu_s/p_s\}$, $G_{P_H}(x)/q(x) = \{\upsilon_1/q_1, \upsilon_2/q_2, \cdots, \upsilon_t/q_t\}$, where $\mu_i^2 + \upsilon_j^2 \le 1$, $\mu_i, \upsilon_j \in [0, 1]$, $i = 1, 2, \cdots, s$; $j = 1, 2, \cdots, t$, the set of probability values is $p(x)= \{p_1, p_2, \cdots, p_s\} \subseteq [0, 1]$, $q(x) = \{q_1, q_2, \cdots, q_t\} \subseteq [0, 1]$, and satisfy $\sum_{i=1}^{s} p_i = 1$ and $\sum_{j=1}^{t} q_j = 1$,where s represents the number of elements in the membership set $F_{P_H}(x)$ and t represents the number of elements in the non membership set $G_{P_H}(x)$.

For example, if $X = \{x_1, x_2, x_3\}$, the membership sets of all additional probability values are $F_{P_H}(x_1) = \{0.1/1\}$, $F_{P_H}(x_2) = \{0.2/0.5, 0.3/0.5\}$ and $F_{P_H}(x_3) = \{0.2/0.3, 0.6/0.7\}$; the non-membership sets of all additional probability values are $G_{P_H}(x_1) = \{0.2/0.7, 0.4/0.3\}$, $G_{P_H}(x_2) = \{0.5/1\}$ and $G_{P_H}(x_3) = \{0.6/1\}$. Then the probabilistic hesitant Pythagorean fuzzy set can be expressed as

$$\Re=\{\langle x_1, \{0.1/1\}, \{0.2/0.7, 0.4/0.3\}\rangle, \langle x_2, \{0.2/0.5, 0.3/0.5\}, \{0.5/1\}\rangle, \langle x_3, \{0.2/0.3, 0.6/0.7\}, \{0.6/1\}\rangle\}$$

In addition, for arbitrary $x \in X$, let

$$\Pi_P(x) = \bigcup_{\mu_i(x)\in F_{P_H}(x),\upsilon_j(x)\in G_{P_H}(x)} \left\{\sqrt{1-\mu_i^2(x)p_i-\upsilon_j^2(x)q_j}/p_iq_j\right\},$$

then the set $\Pi_P(x)$ is called a hesitation degree of the probabilistic hesitant Pythagorean fuzzy set $\Re$ with regard to $x \in X$.

Particularly, if the probability values in $p(x)$ and $q(x)$ satisfy $p_1 = p_2 = \cdots = p_s$ and $q_1 = q_2 = \cdots = q_t$, respectively, then the PrHPFS degenerates to a HPFS. Similarly, if $G_{P_H}(x) = \varnothing$ (there is also $q(x) = \varnothing$) and the probabilistic values in $p(x)$ satisfy $p_1 = p_2 = \cdots = p_s$, then the PrHPFS degenerates to a HFS. Hence, PrHPFS is a generalization of HPFS or HFS.

Note 2 Let $x = x_0 \in X$, the pair $< x_0, F_{P_H}(x_0)/p(x_0), G_{P_H}(x_0)/q(x_0) >$ is regarded as a special probabilistic hesitant Pythagorean fuzzy set, we call it a probabilistic hesitant Pythagorean fuzzy number (PrHPFN), it can be abstractly expressed as $<g/p, h/q>$, where the $g = F_{P_H}(x_0)$, $h = G_{P_H}(x_0)$, $p = p(x_0)$ and $q = q(x_0)$ are finite sets, and the number of elements of the sets g and p is the same, the number of elements of the sets h and q is the same, and the elements in the sets are arranged in order. That is to say, all PrHPFNs are expressed in order of the memberships (or non-memberships) from small to large in the sets g and h. If they are the same, then they are ranked in order of probability values from small to large.

PrHPFS(X) is used to represent the set composed of all probability hesitant Pythagorean fuzzy sets on domain X, PrHPFN(X) is used to represent the set composed of all probability hesitant Pythagorean fuzzy numbers on domain X.

For example, let $\gamma = \ < \{0.3/0.2, 0.5/0.3, 0.6/0.5, \}, \{0.4/0.7, 0.8/0.3\} > \ \in$ PrHPFN(X), then the membership set $g = \{0.3,0.5,0.6\}$ by Note 2, its corresponding probability value set is $p = \{0.2,0.3,0.5\}$; the non membership set is $h = \{0.4,0.8\}$, its corresponding probability set is $q = \{0.7,0.3\}$, that is, PrHPFN γ indicates that the membership degree may be 0.3 (probability is 0.2) or 0.5 (probability 0.3) or 0.6 (probability is 0.5); the non membership degree may be 0.4 (probability is 0.7) or 0.8 (probability is 0.3).

Next, we give the arithmetic operations on PrHPFN(X) space by imitating Ref. [21].

Definition 3.2 [29] Let $\gamma_1 = \ < g_1/p_1, h_1/q_1 >$, $\gamma_2 = \ < g_2/p_2, h_2/q_2 > \ \in$ PrHPFN(X), where the sets p_i and q_i are paired values with the values of sets g_i and h_i, respectively, i=1,2, then their arithmetic operations can be defined as follows:

1) $\gamma_1 \oplus \gamma_2 = < \bigcup_{\mu_1 \in g_1, \ \mu_2 \in g_2} \left\{ \sqrt{\mu_1^2 + \mu_2^2 - \mu_1^2 \mu_2^2} / p_{\mu_1} p_{\mu_2} \right\}, \bigcup_{\upsilon_1 \in h_1, \upsilon_2 \in h_2} \left\{ \upsilon_1 \upsilon_2 / q_{\upsilon_1} q_{\upsilon_2} \right\} >;$

2) $\gamma_1 \otimes \gamma_2 = < \bigcup_{\mu_1 \in g_1, \mu_2 \in g_2} \left\{ \mu_1 \mu_2 / p_{\mu_1} p_{\mu_2} \right\}, \bigcup_{\upsilon_1 \in h_1, \upsilon_2 \in h_2} \left\{ \sqrt{\upsilon_1^2 + \upsilon_2^2 - \upsilon_1^2 \upsilon_2^2} / q_{\upsilon_1} q_{\upsilon_2} \right\} >;$

3) $\lambda \gamma_1 = < \bigcup_{\mu_1 \in g_1} \left\{ \sqrt{1 - (1 - \mu_1^2)^{\lambda}} / p_{\mu_1} \right\}, \bigcup_{\upsilon_1 \in h_1} \left\{ \upsilon_1^{\lambda} / q_{\upsilon_1} \right\} >, \lambda > 0;$

4) $\gamma_1{}^{\lambda} = < \bigcup_{\mu_1 \in g_1} \left\{ \mu_1^{\lambda} / p_{\mu_1} \right\}, \bigcup_{\upsilon_1 \in h_1} \left\{ \sqrt{1 - (1 - \upsilon_1^2)^{\lambda}} / q_{\upsilon_1} \right\} >, \lambda > 0$

For example, let $\gamma_1 = <\{0.3/0.2,\ 0.5/0.3,\ 0.6/0.5,\}, \{0.4/0.7,\ 0.8/0.3\}>, \gamma_2 = <\{0.2/0.8,\ 0.7/0.2,\ \}, \{0.3/0.4,\ 0.5/0.4,\ 0.7/0.2\}> \in$ PrHPFN(X), then it is not difficult to obtain that the membership (non membership) set and the corresponding probability value set are

$$\begin{cases} g_1 = \{0.3, 0.5, 0.6\}, p_1 = \{0.2, 0.3, 0.5\}; h_1 = \{0.4, 0.8\}, q_1 = \{0.7, 0.3\} \\ g_2 = \{0.2, 0.7\}, p_2 = \{0.8, 0.2\}; h_2 = \{0.3, 0.5, 0.7\}, q_2 = \{0.4, 0.4, 0.2\} \end{cases}.$$

According to Definition 3.2–1), we immediately know that

$$\gamma_1 \oplus \gamma_2 = \left\langle \begin{array}{l} \{0.35/0.16, 0.73/0.04, 0.53/0.24, 0.79/0.06, 0.62/0.40, 0.82/0.10\}, \\ \{0.12/0.28, 0.20/0.28, 0.28/0.14, 0.24/0.12, 0.40/0.12, 0.56/0.06\} \end{array} \right\rangle$$

Theorem 3.1 [29] Let $\gamma_1 = < g_1/p_1, h_1/q_1 >, \gamma_2 = < g_2/p_2, h_2/q_2 >, \gamma_3 = < g_3/p_3, h_3/q_3 > \in$PrHPFN($X$), then the following properties 1)-8) are true.

1. $\gamma_1 \oplus \gamma_2 = \gamma_2 \oplus \gamma_1$;
2. $(\gamma_1 \oplus \gamma_2) \oplus \gamma_3 = \gamma_1 \oplus (\gamma_2 \oplus \gamma_3)$;
3. $\lambda(\gamma_1 \oplus \gamma_2) = \lambda\gamma_1 \oplus \lambda\gamma_2, \lambda > 0$;
4. $\gamma_1 \otimes \gamma_2 = \gamma_2 \otimes \gamma_1$;
5. $(\gamma_1 \otimes \gamma_2) \otimes \gamma_3 = \gamma_1 \otimes (\gamma_2 \otimes \gamma_3)$;
6. $\lambda(\gamma_1 \otimes \gamma_2) = \lambda\gamma_1 \otimes \lambda\gamma_2, \lambda > 0$;
7. $(\gamma_1 \otimes \gamma_2)^{\lambda} = \gamma_1{}^{\lambda} \otimes \gamma_2{}^{\lambda}, \lambda > 0$;
8. $\gamma_1{}^{\lambda_1 + \lambda_2} = \gamma_1{}^{\lambda_1} \otimes \gamma_1{}^{\lambda_2}, \lambda_1, \lambda_2 > 0.$

Definition 3.3 [29] Let $\gamma_i = < \alpha_i/p_i, \beta_i/q_i > \in$ PrHPFN(X), $i = 1, 2, \cdots, n$, the mapping PrHPFWA : PrHPFN$(X)^n \to$ PrHPFN(X). If

$$\mathrm{PrHPFWA}(\gamma_1, \gamma_2, \cdots, \gamma_n) \triangleq \omega_1\gamma_1 \oplus \omega_2\gamma_2 \oplus \cdots \oplus \omega_n\gamma_n = < \bigcup_{\mu_i \in g_i} \left\{ \sqrt{1 - \prod_{i=1}^{n} (1 - \mu_i^2)^{\omega_i}} / \prod_{i=1}^{n} p_{\mu_i} \right\}, \bigcup_{\upsilon_i \in h_i} \left\{ \prod_{i=1}^{n} \upsilon_i^{\omega_i} / \prod_{i=1}^{n} q_{\upsilon_i} \right\} > \quad (1)$$

then the mapping is called a probability hesitant Pythagorean fuzzy weighted average operator, and it is also abbreviated as PrHPFWA operator, where $\omega = (\omega_1, \omega_2, \cdots, \omega_n)$ is the weight vector corresponding to PrHPFNs γ_i, and satisfies $\omega_i \in [0, 1]$ and $\sum_{i=1}^{n} \omega_i = 1$.

Obviously, the evaluation values of multiple experts can be aggregated into a comprehensive PrHPFN by the linear operation of Definitions 3.2 and 3.3. In 2017, Liang and Xu [21] proposed the ranking method of hesitant Pythagorean fuzzy numbers, and applied them to multi-attribute group decision-making problems. In this paper, a new ranking method of PrHPFNs is proposed by adding probability value on the basis of the proposed method in [21].

Definition 3.4 If $\gamma = < g/p, h/q > \in$ PrHPFN(X), where the membership set $\{g/p\} = \{\mu_1/p_1,\ \mu_2/p_2,\ \cdots,\ \mu_s/p_s\}$, the non-membership set $\{h/q\} = \{\upsilon_1/q_1,\ \upsilon_2/q_2,\ \cdots,\ \upsilon_t/q_t\}$, the s and t are the total number of elements in the membership set and non-membership set, respectively. Let

$$\rho(\gamma) = \frac{1}{s} \sum_{i=1}^{s} \mu_i{}^2 p_i - \frac{1}{t} \sum_{j=1}^{t} \upsilon_j{}^2 q_j,$$

$$\delta(\gamma) = \frac{1}{s} \sum_{i=1}^{s} \mu_i{}^2 p_i + \frac{1}{t} \sum_{j=1}^{t} \upsilon_j{}^2 q_j,$$

then $\rho(\gamma)$ is called the score function of γ, and $\delta(\gamma)$ is called the accuracy function of γ.

Definition 3.5 Let $\gamma_1 = < g_1/p_1, h_1/q_1 >, \gamma_2 = < g_2/p_2, h_2/q_2 > \in$ PrHPFN(X), then

1) If $\rho(\gamma_1) > \rho(\gamma_2)$, then γ_1 is bigger than γ_2, it is written as $\gamma_1 \succ \gamma_2$;
2) If $\rho(\gamma_1) = \rho(\gamma_2)$, then

(a) If $\delta(\gamma_1) > \delta(\gamma_2)$, then γ_1 is bigger than γ_2, it is written as $\gamma_1 \succ \gamma_2$;
(b) If $\delta(\gamma_1) = \delta(\gamma_2)$, then γ_1 and γ_2 are said to be equivalent, it is written as $\gamma_1 \sim \gamma_2$.

Further, let the ordered sets $P = \{\gamma_1, \gamma_2, \cdots, \gamma_n\}$, $Q = \{\beta_1, \beta_2, \cdots, \beta_n\} \subseteq$ PrHPFN(X), and satisfies $\gamma_i \sim \beta_i$, $i = 1, 2, \cdots, n$, then P and Q are said to be equivalent, it is written as $P \sim Q$.

In fact, the structure of PrHPFNs is to be added the probability information corresponding to membership and non-membership on the basis of HPFN. It not only extends the probability dual hesitation fuzzy set proposed in [27], but also extends the scope of application to the Pythagorean fuzzy set space where the sum of the squares of membership and non-membership is less than or equal to 1. At this time, decision makers can better describe the fuzzy information through PrHPFN, it also depicts contingency and cognitive uncertainty at the same time, so as to reduce the loss of useful information in the decision-making process.

4 Distance and similarity of PrHPFNs

In 2010, Prof. Torre first proposed the concept of hesitant fuzzy set in [5], especially the concepts of distance and similarity in hesitant fuzzy number space are very important. In fact, distance and similarity describe the degree of difference between two things through the value in [0, 1], it plays an important role in multi-attribute group decision-making; the greater the value of similarity, the smaller the degree of difference. Generally, in order to calculate the distance between two PrHPFNs, standardization must be carried out first to make the two PrHPFNs have the same number of elements.

Next, the standardization method of elements in the membership and non-membership sets of PrHPFNs is introduced according to Refs. [19, 21], as follows.

Definition 4.1 Let $\gamma_1 = < g_1/p_1, h_1/q_1 >$, $\gamma_2 = < g_2/p_2, h_2/q_2 > \in$ PrHPFN(X), $|g_i|$ and $|h_i|$ represent the total number of elements contained in membership set g_i and non-membership set h_i, respectively, and $|g_1| \neq |g_2|$ or $|h_1| \neq |h_2|$. Let $\mu_i^+ = \max g_i$, $\mu_i^- = \min g_i$, $\upsilon_i^+ = \max h_i \upsilon_i^- = \min h_i$, $i = 1, 2$. For a given $\theta \in [0, 1]$, if the following 1) - 2) are satisfied:

1) If $|g_1|>|g_2|$, adding $|g_1|-|g_2|$ elements of the form $\left(\theta\mu_2^+ + (1-\theta)\mu_2^-\right)/0$ to the set g_2; If $|g_1|<|g_2|$, adding $|g_2|-|g_1|$ elements of the form $\left(\theta\mu_1^+ + (1-\theta)\mu_1^-\right)/0$ to the set g_1, so that $|g_1|=|g_2|$.
2) If $|h_1|>|h_2|$, adding $|h_1|-|h_2|$ elements of the form $\left(\theta\upsilon_2^+ + (1-\theta)\upsilon_2^-\right)/0$ to the set h_2; If $|h_1|<|h_2|$, adding $|g_2|-|g_1|$ elements of the form $\left(\theta\upsilon_1^+ + (1-\theta)\upsilon_1^-\right)/0$ to the set h_1, so that $|h_1|=|h_2|$.

Then the process of properly adding elements is called a standardization of probability hesitation Pythagorean fuzzy numbers.

For example, if $\gamma_1 = < \{0.3/0.3, 0.8/0.7\}, \{0.3/1\} >$, $\gamma_2 = < \{0.5/1\}, \{0.6/0.4, 0.7/0.6\} > \in$ PrHPFN(X), let $\theta = 0$, after standardization according to Definition 4.1, they can be expressed as $\overline{\gamma}_1 =< \{0.3/0.3, 0.8/0.7\}, \{0.3/1, 0.3/0\} >$, $\overline{\gamma}_2 =< \{0.5/1, 0.5/0\}, \{0.6/0.4, 0.7/0.6\} >$.

Thus, it is obtained that two PrHPFNs have the same number of elements, and after adding the degree of hesitation (DH) information through Eq. (1), they are expressed as

$$\begin{cases} \overline{\gamma}_1(DH) =< \{0.3/0.3, 0.8/0.7\}, \{0.3/1, 0.3/0\} > \cup \mathrm{DH}\{0.6797/0.7, 0.9397/0.3\} \\ \overline{\gamma}_2(DH) =< \{0.5/1, 0.5/0\}, \{0.6/0.4, 0.7/0.6\} > \cup \mathrm{DH}\{0.6753/0.6, 0.7785/0.4\} \end{cases} \tag{2}$$

Note 3 Let $\gamma_1 = < g_1/p_1, h_1/q_1 >$ and $\gamma_2 = < g_2/p_2, h_2/q_2 >$ be PrHPFNs standardized by Definition 4.1 under considering the degree of hesitation, as shown in Formula (2), and $|g_1|=|g_2|= s$ and $|h_1|=|h_2|= t$. If the membership set and non-membership set of PrHPFNs γ_1 and γ_2 after adding probability value are $\{g_i/p_i\} = \{\mu_{i1}/p_{i1}, \mu_{i2}/p_{i2}, \cdots, \mu_{is}/p_{is}\}$ and $\{h_i/q_i\} = \{\upsilon_{i1}/q_{i1}, \upsilon_{i2}/q_{i2}, \cdots, \upsilon_{it}/q_{it}\}$, $i = 1, 2$, respectively. Here, the membership sets $g_1 = \{\mu_{11}, \mu_{12}, \cdots, \mu_{1s}\}$ and $g_2 = \{\mu_{21}, \mu_{22}, \cdots, \mu_{2s}\}$; the non-membership sets $h_1 = \{\upsilon_{11}, \upsilon_{12}, \cdots, \upsilon_{1t}\}$ and $h_2 = \{\upsilon_{21}, \upsilon_{22}, \cdots, \upsilon_{2t}\}$; the membership probability sets $p_1 = \{p_{11}, p_{12}, \cdots, p_{1s}\}$ and $p_2 = \{p_{21}, p_{22}, \cdots, p_{2s}\}$; the non-membership probability sets $q_1 = \{q_{11}, q_{12}, \cdots, q_{1t}\}$ and $q_2 = \{q_{21}, q_{22}, \cdots, q_{2t}\}$. In addition, all elements in these ordered sets are valued on [0, 1], and they are ranked from small to large according to the membership or non-membership. Once the membership or non membership is equal, they are ranked from small to large according to the corresponding probability values.

So far, by Note 3 we can define the Hamming distance of PrHPFNs as follows. Let

$$D(\gamma_1,\gamma_2)=\frac{1}{2}\left(\frac{1}{s}\sum_{k=1}^{s}|\mu_{1k}^2p_{1k}-\mu_{2k}^2p_{2k}|+\frac{1}{t}\sum_{l=1}^{t}|\upsilon_{1l}^2q_{1l}-\upsilon_{2l}^2q_{2l}|+\frac{1}{st}\sum_{j=1}^{st}|\pi_{1j}^2-\pi_{2j}^2|\right) \tag{3}$$

However, whether D (γ_1, γ_2) constitutes a distance or not still needs to be proven theoretically. For this reason, we first give two Lemmas as follows.

Lemma 1 For any real numbers $a, b \in [-1, 1]$, and $|a+b| \le 1$ is satisfied, then there must be $|a| + |b| + |a+b| \le 2$.

Proof According to the assumption $|a+b| \le 1$ if and only if $-1 \le a+b \le 1$. Now, we will discuss it step by step.

Step 1. If $a+b \ge 0$, that is, $a \ge -b$. At this time, if $b \le 0$, then $a \ge -b \ge 0$, and then, $|a|+|b|+|a+b| = a-b+a+b = 2a \le 2$.

$$\text{If } b>0,\text{ then}\begin{cases}\text{when } a\le 0,\text{ then}|a|+|b|+|a+b|=-a+b+(a+b)=2b\le 2\\ \text{when } a>0,\text{ then}|a|+|b|+|a+b|=a+b+(a+b)=2(a+b)\le 2\end{cases}.$$

Step 2. If $a+b<0$, that is, $a<-b$. At this time, if $b \ge 0$, then $a < -b \le 0$, we have $|a|+|b|+|a+b| = -a+b-(a+b) = -2a \le 2$.

$$\text{If } b<0,\text{ then}\begin{cases}\text{when } a\ge 0,\text{ then}|a|+|b|+|a+b|=a-b-(a+b)=-2b\le 2\\ \text{when } a<0,\text{ then}|a|+|b|+|a+b|=-a-b-(a+b)=-2(a+b)\le 2\end{cases}.$$

To sum up, we immediately get $|a| + |b| + |a+b| \le 2$.

Lemma 2 Let $\gamma_1 = < g_1/p_1, h_1/q_1 >, \gamma_2 = < g_2/p_2, h_2/q_2 > \in$ PrHPFN(X), the membership set and non membership set of γ_i as $\{g_i/p_i\} = \{\mu_{i1}/p_{i1}, \mu_{i2}/p_{i2}, \cdots, \mu_{is}/p_{is}\}$ and $\{h_i/q_i\} = \{\upsilon_{i1}/q_{i1}, \upsilon_{i2}/q_{i2}, \cdots, \upsilon_{it}/q_{it}\}$, $i = 1, 2$, respectively, other symbols are shown in Note 3, then there must be

$$|\mu_{1i}^2p_{1i}-\mu_{2i}^2p_{2i}|+|v_{1i}^2q_{1i}-v_{2i}^2q_{2i}|+|\mu_{1i}^2p_{1i}-v_{i1}^2q_{i1}-(\mu_{2i}^2p_{2i}+v_{2i}^2q_{2i})|\le 2, i=1,2.$$

Proof Let $a=\mu_{1i}^2p_{1i}-\mu_{2i}^2p_{2i}$,$b=\upsilon_{1i}^2q_{1i}-\upsilon_{2i}^2q_{2i}$, where $\mu_{1i}, \mu_{2i}, p_{1i}, p_{2i}, \upsilon_{1i}, \upsilon_{2i}, q_{1i}, q_{2i} \in [0, 1]$, $i = 1, 2$. Clearly, $a, b \in [-1, 1]$, and it can be deduced that

$$a+b=\left(\mu_{1i}^2p_{1i}-\mu_{2i}^2p_{2i}\right)+\left(\upsilon_{1i}^2q_{1i}-\upsilon_{2i}^2q_{2i}\right)$$
$$=\left(\mu_{1i}^2p_{1i}+\upsilon_{1i}^2q_{1i}\right)-\left(\mu_{2i}^2p_{2i}+\upsilon_{2i}^2q_{2i}\right)$$
$$\le\mu_{1i}^2p_{1i}+\upsilon_{1i}^2q_{1i}\le\mu_{1i}^2+\upsilon_{1i}^2\le 1.$$

On the other hand, according to the restricted condition $\mu_{2i}^2+\upsilon_{2i}^2\le 1$ and probability values $p_{2i}, q_{2i} \in [0, 1]$ of PFNs, there must be

$$a+b=\left(\mu_{1i}^2p_{1i}+\upsilon_{1i}^2q_{1i}\right)-\left(\mu_{2i}^2p_{2i}+\upsilon_{2i}^2q_{2i}\right)\ge-\left(\mu_{2i}^2p_{2i}+\upsilon_{2i}^2q_{2i}\right)$$
$$\ge-\left(\mu_{2i}^2+\upsilon_{2i}^2\right)\ge-1.$$

Therefore, the real numbers a and b satisfy $|a+b| \le 1$. By Lemma 1, for any $i = 1, 2$, we immediately obtain that

$$|a|+|b|+|a+b|=$$
$$|\mu_{1i}^2p_{1i}-\mu_{2i}^2p_{2i}|+|\upsilon_{1i}^2q_{1i}-\upsilon_{2i}^2q_{2i}|+|\mu_{1i}^2p_{1i}+\upsilon_{i1}^2q_{i1}-(\mu_{2i}^2p_{2i}+\upsilon_{2i}^2q_{2i})|\le 2$$

Theorem 4.1 Let γ_1 = <$g_1/p_1, h_1/q_1$ > , γ_2 = < $g_2/p_2, h_2/q_2$>, γ_3 = < $g_3/p_3, h_3/q_3$ > $\in$ PrHPFN(X), and they have been standardized by Definition 4.1, then D (γ_1, γ_2) in Formula (3) satisfies the following properties 1) - 4).

1) $0 \le D(\gamma_1, \gamma_2) \le 1$;
2) If $D(\gamma_1, \gamma_2) = 0$, then $\gamma_1 \sim \gamma_2$;
3) $D(\gamma_1, \gamma_2) = D(\gamma_2, \gamma_1)$;
4) $D(\gamma_1, \gamma_2) \le D(\gamma_1, \gamma_3) + D(\gamma_3, \gamma_2)$.

Proof All component sets of PrHPFNs γ_1,γ_2 and γ_3 are denoted according to Note 3, and the specific representation process is omitted.

1) From Formula (3), it is obvious that $D(\gamma_1, \gamma_2) \ge 0$. Next, we prove that $D(\gamma_1, \gamma_2) \le 1$.

Since γ_1,γ_2 and γ_3 are standardized, let $|g_1| = |g_2| = |g_3| = s$ and $|h_1| = |h_2| = |h_3| = t$. By the representation in Note 3, let the component sets of PrHPFNs γ_1,γ_2 and γ_3 be

$$\{g_i/p_i\} = \{\mu_{i1}/p_{i1}, \mu_{i2}/p_{i2}, \cdots, \mu_{is}/p_{is}\}, \{h_i/q_i\} = \{\upsilon_{i1}/q_{i1}, \upsilon_{i2}/q_{i2}, \cdots, \upsilon_{it}/q_{it}\},$$
$$DH\Pi_i = \{\pi_{i1}/p_{i1}q_{i1}, \pi_{i2}/p_{i2}q_{i2}, \cdots, \pi_{i,st}/p_{i,st}q_{i,st}\}, i = 1, 2, 3,$$

where $\pi_{ij} = \sqrt{1-\mu_{ij}^2 p_{ij} - \upsilon_{ij}^2 q_{ij}}$, $j = 1, 2, 3, \ldots, st$. Since $s \ne 0$ and $t \ne 0$, according to Formula (2), it is obvious that

$$\begin{aligned} D(\gamma_1, \gamma_2) &= \frac{1}{2}\left(\frac{t}{st}\sum_{k=1}^{s}|\mu_{1k}^2 p_{1k} - \mu_{2k}^2 p_{2k}| + \frac{s}{st}\sum_{l=1}^{t}|\upsilon_{1l}^2 q_{1l} - \upsilon_{2l}^2 q_{2l}| + \frac{1}{st}\sum_{j=1}^{st}|\pi_{1j}^2 - \pi_{2j}^2|\right), \\ &= \frac{1}{2}\left(\begin{array}{l}\frac{1}{st}\sum_{l=1}^{t}\sum_{k=1}^{s}|\mu_{1k}^2 p_{1k} - \mu_{2k}^2 p_{2k}| + \frac{1}{st}\sum_{l=1}^{t}\sum_{k=1}^{s}|\upsilon_{1l}^2 q_{1l} - \upsilon_{2l}^2 q_{2l}| \\ +\frac{1}{st}\sum_{l=1}^{t}\sum_{k=1}^{s}|\mu_{1k}^2 p_{1k} + \upsilon_{1l}^2 q_{1l} - (\mu_{2k}^2 p_{2k} + \upsilon_{2l}^2 q_{2l})|\end{array}\right) \\ &= \frac{1}{2st}\left(\begin{array}{l}\sum_{l=1}^{t}\sum_{k=1}^{s}\left(|\mu_{1k}^2 p_{1k} - \mu_{2k}^2 p_{2k}| + |\upsilon_{1l}^2 q_{1l} - \upsilon_{2l}^2 q_{2l}|\right. \\ \left. +|\mu_{1k}^2 p_{1k} + \upsilon_{1l}^2 q_{1l} - (\mu_{2k}^2 p_{2k} + \upsilon_{2l}^2 q_{2l})|\right)\end{array}\right)\end{aligned}$$

This moment, from Lemma 2, we can immediately get that

$$D(\gamma_1, \gamma_2) \le \frac{1}{2st}\sum_{l=1}^{t}\sum_{k=1}^{s} 2 = \frac{1}{2st}2st = 1.$$

2) When $D(\gamma_1, \gamma_2) = 0$, there must be $\mu_{1k}^2 p_{1k} = \mu_{2k}^2 p_{2k}$ and $\upsilon_{1l}^2 q_{1l} = \upsilon_{2l}^2 q_{2l}$ through Formula (3), $k = 1, 2, \ldots, s$;$l = 1, 2, \ldots, t$. According to Definition 3.4, their score functions $\rho(\gamma_1)$ and $\rho(\gamma_2)$ are equal, that is,

$$\begin{aligned} \rho(\gamma_1) &= \frac{1}{s}\sum_{k=1}^{s}\mu_{1k}^2 p_{1k} - \frac{1}{t}\sum_{l=1}^{t}\upsilon_{1l}^2 q_{1l} \\ &= \frac{1}{s}\sum_{k=1}^{s}\mu_{2k}^2 p_{2k} - \frac{1}{t}\sum_{l=1}^{t}\upsilon_{2l}^2 q_{2l} = \rho(\gamma_2). \end{aligned}$$

Similarly, it is easy to verify that their accuracy functions are also equal, that is, $\delta(\gamma_1) = \delta(\gamma_2)$. According to Definitions 3.4–3.5, there must be $\gamma_1 \sim \gamma_2$.

3) Exchange the positions of γ_1 and γ_2 according to Formula (3), it is not difficult to see that $D(\gamma_1, \gamma_2) = D(\gamma_2, \gamma_1)$.
4) For PrHPFN γ_3, its membership set $\{g_3/p_3\}$, non membership set $\{h_3/q_3\}$ and the hesitation set $DH\Pi_i$ can be expressed in the following form, respectively

$$\{g_3/p_3\} = \{\mu_{31}/p_{31}, \mu_{32}/p_{32}, \cdots, \mu_{3s}/p_{3s}\},$$
$$\{h_3/q_3\} = \{\upsilon_{31}/q_{31}, \upsilon_{32}/q_{32}, \cdots, \upsilon_{3t}/q_{3t}\},$$
$$DH\Pi_i = \{\pi_{31}/p_{31}q_{31}, \pi_{32}/p_{32}q_{32}, \cdots, \pi_{3,st}/p_{3,st}q_{3,st}\}.$$

According to Formula (3) and the properties of absolute value inequality, it is not difficult to get

$$\begin{aligned}
D(\gamma_1,\gamma_2) &= \frac{1}{2s}\sum_{k=1}^{s}\left|\mu_{1k}^2 p_{1k}-\mu_{2k}^2 p_{2k}\right| + \frac{1}{2t}\sum_{l=1}^{t}\left|\upsilon_{1l}^2 q_{1l}-\upsilon_{2l}^2 q_{2l}\right| + \frac{1}{2st}\sum_{j=1}^{st}\left|\pi_{1j}^2-\pi_{2j}^2\right| \\
&\le \frac{1}{2s}\sum_{k=1}^{s}\left(\left|\mu_{1k}^2 p_{1k}-\mu_{3k}^2 p_{3k}\right| + \left|\mu_{3k}^2 p_{3k}-\mu_{2k}^2 p_{2k}\right|\right) + \\
&\quad \frac{1}{2t}\sum_{l=1}^{t}\left(\left|\upsilon_{1l}^2 q_{1l}-\upsilon_{3l}^2 q_{3l}\right| + \left|\upsilon_{3l}^2 q_{3l}-\upsilon_{2l}^2 q_{2l}\right|\right) + \frac{1}{2st}\sum_{j=1}^{st}\left(\left|\pi_{1j}^2-\pi_{3j}^2\right| + \left|\pi_{3j}^2-\pi_{2j}^2\right|\right) \\
&= \frac{1}{2}\left(\frac{1}{s}\sum_{k=1}^{s}\left|\mu_{1k}^2 p_{1k}-\mu_{3k}^2 p_{3k}\right| + \frac{1}{t}\sum_{l=1}^{t}\left|\upsilon_{1l}^2 q_{1l}-\upsilon_{3l}^2 q_{3l}\right| + \frac{1}{st}\sum_{j=1}^{st}\left|\pi_{1j}^2-\pi_{3j}^2\right|\right) + \\
&\quad \frac{1}{2}\left(\frac{1}{s}\sum_{k=1}^{s}\left|\mu_{3k}^2 p_{3k}-\mu_{2k}^2 p_{2k}\right| + \frac{1}{t}\sum_{l=1}^{t}\left|\upsilon_{3l}^2 q_{3l}-\upsilon_{2l}^2 q_{2l}\right| + \frac{1}{st}\sum_{j=1}^{st}\left|\pi_{3j}^2-\pi_{2j}^2\right|\right) \\
&= D(\gamma_1,\gamma_3) + D(\gamma_3,\gamma_2)\cdot
\end{aligned}$$

So far, Theorem 4.1 has been proved.

In fact, with the conclusions of Theorem 4.1, we can confirm that $D\ (\gamma_1,\ \gamma_2)$ does constitute a distance, which is also called a Hamming distance measure.

Next, we continue to define the weighted distance measure and similarity through Hamming distance D in PrHPFNs space.

Definition 4.2 Let $P_1 = \{\alpha_1,\ \alpha_2,\ \cdots,\ \alpha_n\}$ and $P_2 = \{\beta_1,\ \beta_2,\ \cdots,\ \beta_n\}$ are two ordered set families with the same number of elements, where $\alpha_j,\ \beta_j \in$ PrHPFN(X),$j = 1, 2, \ldots, n, \omega = (\omega_1,\ \omega_2,\ \cdots,\ \omega_n\ )$ is a weight vector, and satisfy $0 \le \omega_j \le 1$ and $\sum_{j=1}^{n}\omega_j = 1$. For any parameter $\eta > 0$, let

$$H_{\text{PrHPFWA}}(P_1,P_2) = \left(\sum_{j=1}^{n}\omega_j D\left(\alpha_j,\beta_j\right)^{\eta}\right)^{\frac{1}{\eta}}, \tag{4}$$

then $H_{\text{PrHPFWA}}(P_1,\ P_2)$ is called a weighted distance measure of set families P_1 and P_2, where $D(\alpha_j,\ \beta_j)$ is Hamming distance of PrHPFNs aj and β_j in the form of Formula (3).

In particular, with different values of the parameter η, we can obtain different forms of weighted distance measures H_{PrHPFWA}. For example, when $\eta = 1$, the $H_{\text{PrHPFWA}}(P_1,\ P_2)$ degenerates into a probability hesitant Pythagorean fuzzy Hamming weighted distance measure, that is,

$$H_{\text{PrHPFWA}}(P_1,P_2) = \sum_{j=1}^{n}\omega_j D\left(\alpha_j,\beta_j\right).$$

When $\eta = 2$, the $H_{\text{PrHPFWA}}(P_1,\ P_2)$ degenerates into a probability hesitant Pythagorean fuzzy Euclidean weighted distance measure, that is,

$$H_{\text{PrHPFWA}}(P_1,P_2) = \sqrt{\sum_{j=1}^{n}\omega_j D\left(\alpha_j,\beta_j\right)^2}.$$

Theorem 4.2 Given two set families $P_1 = \{\alpha_1,\ \alpha_2,\ \cdots,\ \alpha_n\}$ and $P_2 = \{\beta_1,\ \beta_2,\ \cdots,\ \beta_n\}$, where $\alpha_j,\ \beta_j \in$ PrHPFN(X), the $\omega = (\omega_1,\ \omega_2,\ \cdots,\ \omega_n\ )$ is a weight vector, and satisfy $0 \le \omega_j \le 1$ and $\sum_{j=1}^{n}\omega_j = 1$. For an arbitrary parameter $\eta > 0$, then the weighted distance measure H_{PrHPFWA} satisfies the following properties (1) - 3):

1) $0 \le s\,(P_1,P_2) \le 1$;
2) $s\,(P_1,P_2) = s\,(P_2,P_1)$;
3) If $s(P_1,P_2) = 1$, then $P_1 \sim P_2$.

Proof 1) By Formulas (3)–(4), it is obvious that $D\ (\alpha_j,\ \beta_j) \ge 0$, and $H_{\text{PrHPFWA}}(P_1,\ P_2) \ge 0$. Because $0 \le D(\alpha_j,\ \beta_j) \le 1$, there is still $0 \le D(\alpha_j,\ \beta_j)^{\eta} \le 1$, for any $\eta > 0$, then,

$$\begin{aligned}
&0 \le \mathrm{D}_{\text{PrHPFWA}}(P_1,P_2) \\
&= \left(\sum_{j=1}^{n}\omega_j D\left(\alpha_j,\beta_j\right)^{\eta}\right)^{\frac{1}{\eta}} \le \left(\sum_{j=1}^{n}\omega_j\right)^{\frac{1}{\eta}} = 1^{1/\eta} \\
&= 1.
\end{aligned}$$

2) By Formula (4), it is obvious that $H_{\text{PrHPFWA}}(P_1,\ P_2) = H_{\text{PrHPFWA}}(P_2,\ P_1)$.
3) If $H_{\text{PrHPFWA}}(P_1,\ P_2) = 0$, then $D(\alpha_j,\ \beta_j) = 0$, and $\alpha_j \sim \beta_j$ can be known by Theorem 4.1, and then $P_1 \sim P_2$ must be obtained by Definition 3.5.

Generally speaking, the similarity of two PrHPFNs can be defined by distance measure, and the smaller the similarity, the greater the difference between the two PrHPFNs; the greater the similarity, the smaller the difference. Next, we will give the definition of the similarity of two groups of PrHPFNs through Formula (4).

Definition 4.3 Given two set families $P_1 = \{\alpha_1, \alpha_2, \cdots, \alpha_n\}$ and $P_2 = \{\beta_1, \beta_2, \cdots, \beta_n\}$, where $\alpha_j, \beta_j \in$ PrHPFN(X), the $\omega = (\omega_1, \omega_2, \cdots, \omega_n)$ is a weight vector, and satisfy $0 \le \omega_j \le 1$ and $\sum_{j=1}^{n} \omega_j = 1$. For any parameter $\eta > 0$, let

$$s(P_1, P_2) = \frac{1 - H_{\mathrm{PrHPFWA}}(P_1, P_2)}{1 + H_{\mathrm{PrHPFWA}}(P_1, P_2)}, \tag{5}$$

then $s(P_1, P_2)$ is called the similarity between P_1 and P_2, where $H_{\mathrm{PrHPFWA}}(P_1, P_2)$ is a weighted distance measure as shown in Eq. (4).

Theorem 4.3 Suppose that all conditions are identical to Definition 4.2, then the similarity $s(P_1, P_2)$ satisfies the following properties 1) - 3).

1) $0 \le s(P_1, P_2) \le 1$;
2) $s(P_1, P_2) = s(P_2, P_1)$;
3) If $s(P_1, P_2) = 1$, then $P_1 \sim P_2$.

Proof By Theorem 4.2, it is obvious that $0 \le s(P_1, P_2) \le 1$ and $s(P_1, P_2) = s(P_2, P_1)$ hold.

3) If $s(P_1, P_2) = 1$, then $H_{\mathrm{PrHPFWA}}(P_1, P_2) = 0$. By Theorem 4.2–3), there must be $P_1 \sim P_2$.

In fact, the distance measure and similarity are important tools to measure two information quantities in Pythagorean fuzzy environment. With the necessary preparation of Definitions 4.1–4.3 and Theorems 4.1–4.3, we can introduce the similarity of probabilistic hesitant Pythagorean fuzzy matrix (PrHPFM), and then propose a new interactive group evaluation method.

5 Interactive group decision making method

Interactive group decision-making is to add interactive feedback opinions among experts on the basis of group evaluation, it can achieve group consensus by repeatedly modifying and adjusting the weight of experts. However, the probability hesitation Pythagorean fuzzy numbers can not only describe some multi-attribute information from the aspects of membership and non-membership, but also reflect the evaluation of experts more comprehensively through the corresponding probability values. Therefore, a new interactive group decision-making method can be proposed in the probability hesitant Pythagorean fuzzy environment.

Suppose that the alternative scheme set is $\{Y_1, Y_2, \cdots, Y_m\}$, the expert set is $\{l_1, l_2, \cdots, l_q\}$, the attribute index set is $\{c_1, c_2, \cdots, c_n\}$, the expert weight vector is $\lambda = (\lambda_1, \lambda_2, \cdots, \lambda_q)$, and satisfies $\sum_{k=1}^{q} \lambda_k = 1$. In addition, the weight vector of attribute indexes is $\omega = (\omega_1, \omega_2, \cdots, \omega_n)$, and satisfies $\sum_{j=1}^{n} \omega_j = 1$, where $0 \le \omega_j \le 1$. Generally, experts may hesitate to evaluate the scheme and attribute index due to their limited understanding of complex objective things. We assume that the evaluation values of all attributes c_j ($j = 1, 2, \cdots, n$) to each scheme Y_i by each expert l_k are PrHPFNs, as follows:

$$\gamma_{ij}^{(k)} = < g_{\gamma_{ij}}^{k} / p_{\gamma_{ij}}^{k}, h_{\gamma_{ij}}^{k} / q_{\gamma_{ij}}^{k} >$$
$$(k = 1, 2, \cdots, q; i = 1, 2, \cdots, m; j = 1, 2, \cdots, n).$$

Then, the evaluation value of expert l_k on all attributes c_i ($j = 1, 2, \cdots, n$) of the scheme Y_i is expressed as the set family of PrHPFNs $P_i^{(k)}$ as follows:

$$P_i^{(k)} = \left\{ < g_{\gamma_{i1}}^{k} / p_{\gamma_{i1}}^{k}, h_{\gamma_{i1}}^{k} / q_{\gamma_{i1}}^{k} >, < g_{\gamma_{i2}}^{k} / p_{\gamma_{i2}}^{k}, h_{\gamma_{i2}}^{k} / q_{\gamma_{i2}}^{k} >, \cdots, < g_{\gamma_{in}}^{k} / p_{\gamma_{in}}^{k}, h_{\gamma_{in}}^{k} / q_{\gamma_{in}}^{k} > \right\}.$$

Without losing generality, let the probability hesitation Pythagorean fuzzy matrix (PrHPFM) $R^{(k)}$given by expert l_k be

$$R^{(k)} = \left(\gamma_{ij}^{(k)}\right)_{m \times n} = \begin{pmatrix} < g_{\gamma_{11}}^{k} / p_{\gamma_{11}}^{k}, h_{\gamma_{11}}^{k} / q_{\gamma_{11}}^{k} > & < g_{\gamma_{12}}^{k} / p_{\gamma_{12}}^{k}, h_{\gamma_{12}}^{k} / q_{\gamma_{12}}^{k} > & \cdots & < g_{\gamma_{1n}}^{k} / p_{\gamma_{1n}}^{k}, h_{\gamma_{1n}}^{k} / q_{\gamma_{1n}}^{k} > \\ < g_{\gamma_{21}}^{k} / p_{\gamma_{21}}^{k}, h_{\gamma_{21}}^{k} / q_{\gamma_{21}}^{k} > & < g_{\gamma_{22}}^{k} / p_{\gamma_{22}}^{k}, h_{\gamma_{22}}^{k} / q_{\gamma_{22}}^{k} > & \cdots & < g_{\gamma_{2n}}^{k} / p_{\gamma_{2n}}^{k}, h_{\gamma_{2n}}^{k} / q_{\gamma_{2n}}^{k} > \\ \vdots & \vdots & \vdots & \vdots \\ < g_{\gamma_{m1}}^{k} / p_{\gamma_{m1}}^{k}, h_{\gamma_{m1}}^{k} / q_{\gamma_{m1}}^{k} > & < g_{\gamma_{m2}}^{k} / p_{\gamma_{m2}}^{k}, h_{\gamma_{m2}}^{k} / q_{\gamma_{m2}}^{k} > & \cdots & < g_{\gamma_{mn}}^{k} / p_{\gamma_{mn}}^{k}, h_{\gamma_{mn}}^{k} / q_{\gamma_{mn}}^{k} > \end{pmatrix} \triangleq \begin{pmatrix} P_1^{(k)} \\ P_2^{(k)} \\ \cdots \\ P_m^{(k)} \end{pmatrix},$$

where $P_i^{(k)}$ ($i = 1, 2, \cdots, m$) represents the set of all elements in row i of the matrix $R^{(k)}$, and each representative element PrHPFN is

$$< g^k_{\gamma_{ij}}/p^k_{\gamma_{ij}}, h^k_{\gamma_{ij}}/q^k_{\gamma_{ij}} >=< \left\{\mu^{k(1)}_{\gamma_{ij}}/p^{k(1)}_{\gamma_{ij}}, \mu^{k(2)}_{\gamma_{ij}}/p^{k(2)}_{\gamma_{ij}}, \cdots, \mu^{k(s)}_{\gamma_{ij}}/p^{k(s)}_{\gamma_{ij}}\right\}, \left\{\upsilon^{k(1)}_{\gamma_{ij}}/q^{k(1)}_{\gamma_{ij}}, \upsilon^{k(2)}_{\gamma_{ij}}/q^{k(2)}_{\gamma_{ij}}, \cdots, \upsilon^{k(t)}_{\gamma_{ij}}/q^{k(t)}_{\gamma_{ij}}\right\} >,$$

and all relevant parameters of them meet $\mu^k_{\gamma_{ij}}, \upsilon^k_{\gamma_{ij}}, p^k_{\gamma_{ij}}, q^k_{\gamma_{ij}} \in [0, 1], k = 1, 2, \cdots, q; i = 1, 2, \cdots, m; j = 1, 2, \cdots, n.$

According to PrHPFWA aggregation operation in Definitions 3.2-3.3, let the group evaluation matrix integrated by q experts be

$$R = \left(\gamma_{ij}\right)_{m\times n} = \begin{pmatrix} < g_{\gamma_{11}}/p_{\gamma_{11}}, h_{\gamma_{11}}/q_{\gamma_{11}} > & < g_{\gamma_{12}}/p_{\gamma_{12}}, h_{\gamma_{12}}/q_{\gamma_{12}} > & \cdots & < g_{\gamma_{1n}}/p_{\gamma_{1n}}, h_{\gamma_{1n}}/q_{\gamma_{1n}} > \\ < g_{\gamma_{21}}/p_{\gamma_{21}}, h_{\gamma_{21}}/q_{\gamma_{21}} > & < g_{\gamma_{22}}/p_{\gamma_{22}}, h_{\gamma_{22}}/q_{\gamma_{22}} > & \cdots & < g_{\gamma_{2n}}/p_{\gamma_{2n}}, h_{\gamma_{2n}}/q_{\gamma_{2n}} > \\ \vdots & \vdots & \vdots & \vdots \\ < g_{\gamma_{m1}}/p_{\gamma_{m1}}, h_{\gamma_{m1}}/q_{\gamma_{m1}} > & < g_{\gamma_{m2}}/p_{\gamma_{m2}}, h_{\gamma_{m2}}/q_{\gamma_{m2}} > & \cdots & < g_{\gamma_{mn}}/p_{\gamma_{mn}}, h_{\gamma_{mn}}/q_{\gamma_{mn}} > \end{pmatrix} \triangleq \begin{pmatrix} P_1 \\ P_2 \\ \cdots \\ P_m \end{pmatrix},$$

where the representative element PrHPFN γ_{ij} is obtained according to the aggregation operator of Formula (1), that is,

$$\gamma_{ij} = \lambda_1\gamma_{ij}^{(1)} \oplus \lambda_2\gamma_{ij}^{(2)} \oplus \cdots \oplus \lambda_q\gamma_{ij}^{(q)}, \quad i = 1, 2, \cdots, m; j = 1, 2, \cdots, n.$$

Let P_i be the set of comprehensive evaluation values of the scheme Y_i for each attribute index c_j, and its specific form can be expressed as

$$P_i = \left\{< g_{\gamma_{i1}}/p_{\gamma_{i1}}, h_{\gamma_{i1}}/q_{\gamma_{i1}} >, < g_{\gamma_{i2}}/p_{\gamma_{i2}}, h_{\gamma_{i2}}/q_{\gamma_{i2}} >, \cdots, < g_{\gamma_{in}}/p_{\gamma_{in}}, h_{\gamma_{in}}/q_{\gamma_{in}} >\right\}.$$

At this time, the similarity $s\left(P_i^k, P_i\right)$ ($i = 1, 2, \cdots, m$) between $P_i^{(k)}$ and P_i are calculated by Formulas (3)–(5), and then calculate the similarity $S(R, R^{(k)})$ between the group evaluation matrix R and each expert evaluation matrix $R^{(k)}$ through the following formula, that is,

$$S\left(R, R^{(k)}\right) = \frac{1}{m}\sum_{i=1}^{m} s\left(P_i^k, P_i\right) \tag{6}$$

then $S(R, R^{(k)})$ is also called the similarity of probability hesitation Pythagorean fuzzy matrix (PrHPFM) of R and $R^{(k)}$, where $s\left(P_i^k, P_i\right)$ is the similarity between the PrHPFNs P_i^k and P_i. Similarly, it is not difficult to obtain the similarity properties of PrHPFM as follows.

Theorem 5.1 Let $R^{(k)}$ be the evaluation matrix given by the k-th expert, $k = 1, 2, \cdots, q$, and R be the group evaluation matrix aggregated according to Formula (1), then the PrHPFM similarity $S(R, R^{(k)})$ satisfies the following properties 1) - 3):

1) $0 \le S(R, R^{(k)}) \le 1$;
2) $S(R, R^{(k)}) = S(R^{(k)}, R)$;
3) If $S(R, R^{(k)}) = 1$, then $R = R^{(k)}, k = 1, 2, \cdots, q$.

Proof By Theorem 4.3 and Formula (6), the conclusions 1) and 2) are obviously true.

3) According to Theorem 4.3–1), clearly $0 \le s\ (P_1, P_2) \le 1$, especially when the similarities $S(R, R^{(k)}) = 1$, we have

$$1 = S\left(R, R^{(k)}\right) = \frac{1}{m}\sum_{i=1}^{m} s\left(P_i^k, P_i\right) \le \frac{1}{m}\sum_{i=1}^{m} 1 = 1.$$

Hence, the equal sign holds only when the similarities $s\left(P_i^k, P_i\right)$ reaches the maximum 1, that is, $s\left(P_i^k, P_i\right) = 1$. From Theorem 4.3–4), we know that there must be $P_i^k = P_i, k = 1, 2, \cdots, q; i = 1, 2, \cdots, m$. Therefore, $R = R^{(k)}$.

If PrHPFM similarities $S(R^{(k)}, R)$ do not meet the given threshold for all k, $k = 1, 2, \cdots, q$, we need to adjust the expert weights accordingly. The adjustment method is that for the minimal PrHPFM similarity, the corresponding expert weight is increased by a certain value; at the same time, for the maximal similarity, the corresponding expert weight is reduced by the same value.

In fact, the PrHPFM similarities $S(R, R^{(k)})$ reflects the degree of difference between expert evaluation matrix $R^{(k)}$ and group comprehensive evaluation matrix R. Because experts have different preferences for objective things, they may give too high or too low evaluation value to the object. In order to eliminate the impact of too high or too low evaluation information on decision-making results, we calculate the similarities $S(R, R^{(k)})$ to give experts higher or lower weight value, so as to effectively ensure the rationality of decision results.

To sum up, we propose a new group interactive decision-making method in the Pythagorean fuzzy information environment as follows:

Step 1. Constructing the probability hesitation Pythagorean fuzzy matrix $R^{(k)}$according to the evaluation of each expert l_k, $k = 1, 2, \cdots, q$, and set the similarity threshold $\beta \in (0, 1)$ and the initial expert weight vector as $\lambda = (1/q, 1/q, \cdots, 1/q)$.

Step 2. All PrHPFMs $R^{(k)} = \left(\gamma_{ij}^{(k)}\right)$ are aggregated into a group evaluation matrix $R = (\gamma_{ij})_{m \times n}$ by PrHPFWA aggregation operator Formula (1) in Definition 3.3.

Step 3. Calculating the similarities $S(R^{(k)}, R)$ between the aggregated group evaluation matrix R and each expert evaluation matrix $R^{(k)}$, $k = 1, 2, \cdots, q$.

Step 4. If $S(R^{(k)}, R) > \beta$, go to Step 5. Otherwise, for the minimum similarity $S(R^{(k)}, R)$, the weight $\lambda_k \to \lambda_k + \theta$; for the maximum similarity $S(R^{(k)}, R)$, the weight $\lambda_k \to \lambda_k - \theta$ simultaneously, then return to Step 3, where θ is the step size of iteration, $k = 1, 2, \cdots, q$.

Step 5. Aggregating the attribute index elements of each row in the group evaluation matrix $R = (\gamma_{ij})_{m \times n}$ according to Formula (1), the obtained PrHPFNs $\widetilde{\gamma}_i$ is the amount of information of the corresponding alternative schemes Y_i, $i = 1, 2, \cdots, m$.

Step 6. Calculating the score function $\rho(\widetilde{\gamma}_i)$ or accuracy function $\delta(\widetilde{\gamma}_i)$ of each PrHPFN $\widetilde{\gamma}_i$ according to Definition 3.4, and give a comprehensive ranking through Definition 3.5, in which the first ranking is the corresponding optimal scheme.

In fact, in the PrHPFNs information environment, the new group interactive decision-making method can not only deal with the multi-attribute decision-making problem from the three aspects of membership, non-membership and probability, but also reflect the hesitation of experts and the interactive feedback among some groups, and modify the expert weight and integration operator according to the set threshold to obtain a reasonable group decision-making scheme. The main flow chart of the proposed method is given in Fig1:

In addition, in the PrHPFNs environment, the technical route flow chart of the group interactive decision-making method proposed in this paper can be expressed in Fig2.

6 Example analysis

At present, although the infection and incidence rate of New Coronavirus (COVID-19) has been well controlled worldwide, some countries and regions are still in an increasingly serious state. For this reason, World Health Organization (WHO) and major epidemic prevention agencies are actively studying the new measures and new crown vaccines to fight for early recovery of normal economic life, social life and spiritual life. Under the leadership of government, it is very important to unify people's thinking and actively do a good job in disease prevention and emergency rescue.

Example 1 The center for Disease Control and Prevention of a province in China invited three experts to evaluate the disease prevention emergency management capacity of five cities under its jurisdiction. These five cities are successively recorded as a alternatives set $\{Y_1, Y_2, Y_3, Y_4, Y_5\}$, four attribute indicators are set: detection and early warning capacity c_1, material support measures c_2, emergency system and plan c_3, transportation and incoming communication technology c_4. Now, three experts l_1, l_2 and l_3 are invited to make a comprehensive evaluation to the emergency management capacity of the cities $\{Y_1, Y_2, Y_3, Y_4, Y_5\}$ according to the attribute indexes $\{c_1, c_2, c_3, c_4\}$, and the evaluation value is expressed by PrHPFNs. It is assumed that the initial evaluation results given by three experts are shown in Tables 1, 2 and 3:

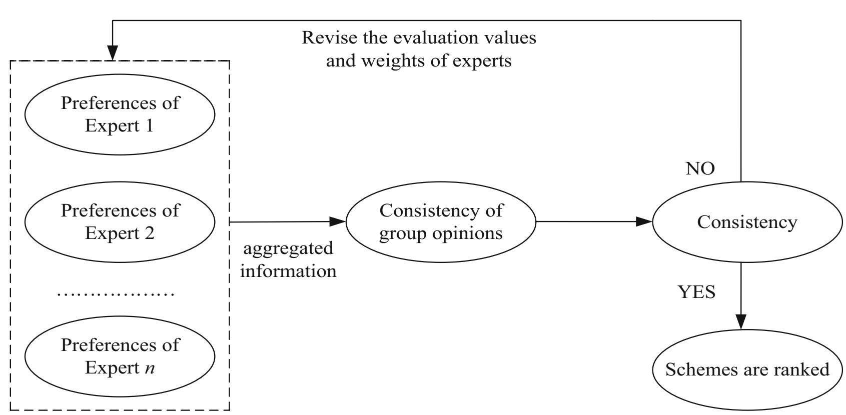

Fig. 1 The flow chart of the proposed group interactive decision-making method

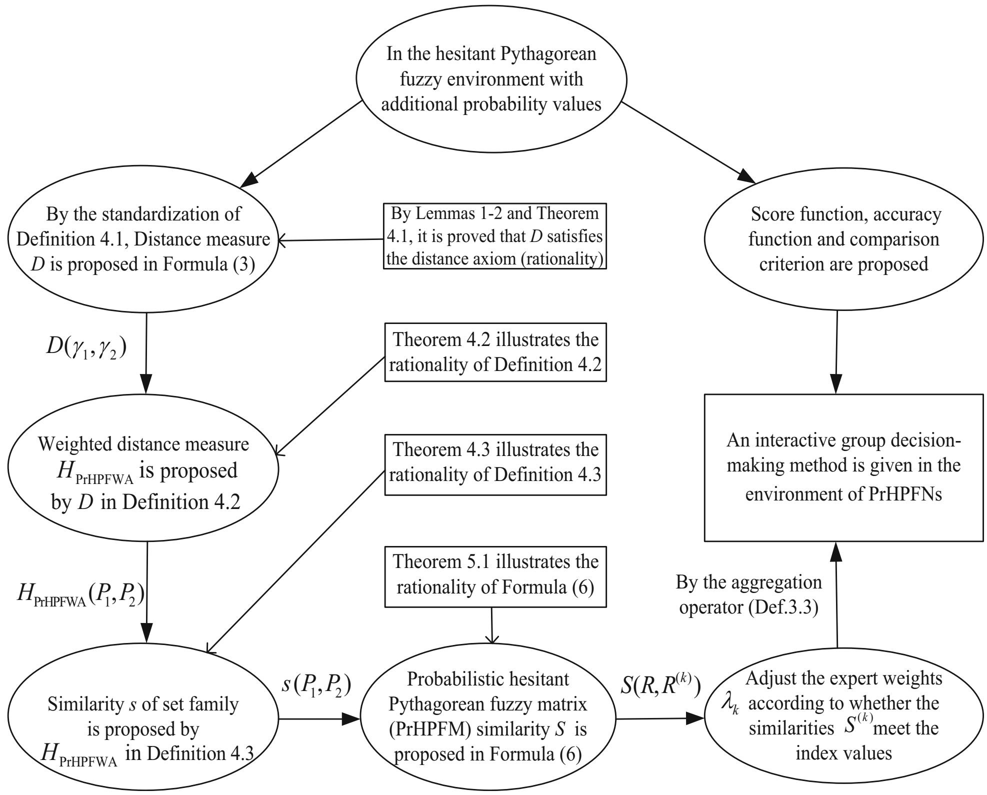

Fig. 2 The flow chart of the technical route of the proposed method

According to the proposed decision-making method **Steps 1–6**, we can summarize the whole decision-making process into an intuitive flow chart (see Fig. 3):

The following results are calculated by MATLAB software programming.

Assuming that the initial value of expert weight vector $\lambda = (1/3,\ 1/3,\ 1/3)$, the comprehensive evaluation matrix $R = (\gamma_{i\ j})_{5\times 4}$ can be aggregated according to Formula (1). See Table 4.

Let the attribute weight vector be $\omega = (0.1,0.4,0.4,0.1)$, $\eta = 1$, follow the steps in flowchart (Fig. 3), all distance measures $D\left(\gamma_{ij}^{(k)},\gamma_{ij}\right)$ and the weighted distance measures $H_{\mathrm{PrHPFWA}}\left(P_i^{(k)},P_i\right)$ can be calculated by Formulas (3)–(4). For example,

$$D\left(\gamma_{11}^{(1)},\gamma_{11}\right)=\frac{1}{2}\left(\begin{array}{c}\frac{1}{1}\left|0.6^2\times 1{-}0.538^2\times 1\right|+\frac{1}{2}\left(\left|0.4^2\times 1{-}0.493^2\times 0.4\right|+\left|0.4^2\times 1{-}0.504^2\times 0.6\right|\right)\\ +\frac{1}{2}\left(\left|0.7698^2{-}0.7831^2\right|+\left|0.7673^2{-}0.7512^2\right|\right)\end{array}\right)=0.1002.$$

Similarly, we can obtain other distance measures and weighted distance measures are

$$D\left(\gamma_{12}^{(1)},\gamma_{12}\right)=0.0057, D\left(\gamma_{13}^{(1)},\gamma_{13}\right)=0.0148, D\left(\gamma_{14}^{(1)},\gamma_{14}\right)=0.0280,$$

$$H_{\mathrm{PrHPFWA}}\left(P_1^{(1)},P_1\right)=\sum_{j=1}^{4}\omega_j D\left(\gamma_{1j}^{(1)},\gamma_{1j}\right)=0.1002\times 0.1+0.0057\times 0.4+0.0148\times 0.4+0.0280\times 0.4=0.0210,$$

$$\begin{cases}H_{\mathrm{PrHPFWA}}\left(P_2^{(1)},P_2\right)=0.0234, H_{\mathrm{PrHPFWA}}\left(P_3^{(1)},P_3\right)=0.0112,\\ H_{\mathrm{PrHPFWA}}\left(P_4^{(1)},P_4\right)=0.0258, H_{\mathrm{PrHPFWA}}\left(P_5^{(1)},P_5\right)=0.0209.\end{cases}$$

Table 1 Evaluation matrix $R^{(1)}$ given by the first expert l_1

Schemes	Attribute indices			
	c_1	c_2	c_3	c_4
Y_1	<{0.6/1},{0.4/1}>	<{0.18/1},{0.6/1}>	<{0.6/0.4,0.7/0.6},{0.7/1}>	<{0.1/0.3,0.2/0.7},{0.45/1}>
Y_2	<{0.57/1},{0.6/0.4,0.7/0.6}>	<{0.45/1},{0.1/1}>	<{0.4/1},{0.49/1}>	<{0.31/1},{0.75/1}>
Y_3	<{0.16/1},{0.5/1}>	<{0.2/0.4,0.36/0.6},{0.59/1}>	<{0.3/1},{0.9/1}>	<{0.4/1},{0.75/1}>
Y_4	<{0.7/0.4,0.8/0.6},{0.43/1}>	<{0.6/1},{0.7/1}>	<{0.8/1},{0.23/1}>	<{0.3/1},{0.4/1}>
Y_5	<{0.2/0.4,0.5/0.6},{0.7/1}>	<{0.8/1},{0.1/1}>	<{0.2/1},{0.71/1}>	<{0.1/1},{0.6/1}>

Table 2 Evaluation matrix $R^{(2)}$ given by the second expert l_2

Schemes	Attribute indices			
	c_1	c_2	c_3	c_4
Y_1	<{0.43/1},{0.75/0.4,0.8/0.6}>	<{0.32/0.4,0.5/0.6},{0.53/1}>	<{0.3/1},{0.25/1}>	<{0.32/1},{0.55/1}>
Y_2	<{0.3/1},{0.8/1}>	<{0.1/0.3,0.3/0.7},{0.3/1}>	<{0.6/1},{0.4/1}>	<{0.3/0.4,0.5/0.6},{0.42/1}>
Y_3	<{0.2/0.4,0.3/0.6},{0.7/1}>	<{0.29/1},{0.8/1}>	<{0.2/1},{0.4/1}>	<{0.3/1},{0.73/0.4,0.8/0.6}>
Y_4	<{0.1/1},{0.23/1}>	<{0.36/1},{0.47/1}>	<{0.67/1},{0.45/0.3,0.6/0.7}>	<{0.3/1},{0.45/1}>
Y_5	<{0.5/1},{0.2/0.3,0.3/0.7}>	<{0.6/0.3,0.7/0.7},{0.35/1}>	<{0.79/1},{0.3/1}>	<{0.7/1},{0.3/0.2,0.5/0.8}>

Table 3 Evaluation matrix $R^{(3)}$ given by the third expert l_3

Schemes	Attribute indices			
	c_1	c_2	c_3	c_4
Y_1	<{0.56/1},{0.4/1}>	<{0.1/1},{0.5/0.3,0.7/0.7}>	<{0.8/1},{0.2/1}>	<{0.2/1},{0.4/1}>
Y_2	<{0.25/0.4,0.5/0.6},{0.4/1}>	<{0.52/1},{0.2/1}>	<{0.4/1},{0.2/0.4,0.5/0.6}>	<{0.1/1},{0.5/1}>
Y_3	<{0.6/1},{0.45/1}>	<{0.2/1},{0.3/1}>	<{0.2/0.4,0.4/0.6},{0.3/1}>	<{0.2/1},{0.6/1}>
Y_4	<{0.7/1},{0.3/1}>	<{0.2/0.4,0.4/0.6},{0.5/1}>	<{0.6/1},{0.5/1}>	<{0.1/0.3,0.5/0.7},{0.4/1}>
Y_5	<{0.6/1},{0.8/1}>	<{0.5/1},{0.7/1}>	<{0.2/0.1,0.4/0.9},{0.35/1}>	<{0.6/1},{0.7/1}>

Table 4 Group evaluation matrix R of three experts after aggregation

Schemes	Attribute indices			
	c_1	c_2	c_3	c_4
Y_1	<{0.538/1}, {0.493/0.4,0.504/0.6}>	<{0.222/0.4,0.323/0.6}, {0.542/0.3,0.606/0.7}>	<{0.637/0.4,0.670/0.6}, {0.327/1}>	<{0.227/0.3,0.248/0.7}, {0.463/1}>
Y_2	<{0.410/0.4,0.477/0.6}, {0.577/0.4,0.607/0.6}>	<{0.410/0.3,0.437/0.7}, {0.182/1}>	<{0.483/1}, {0.340/0.4,0.461/0.6}>	<{0.257/0.4,0.353/0.6}, {0.540/1}>
Y_3	<{0.397/0.4,0.415/0.6}, {0.540/1}>	<{0.234/0.4,0.292/0.6}, {0.521/1}>	<{0.239/0.4,0.313/0.6}, {0.476/1}>	<{0.313/1}, {0.690/0.4,0.711/0.6}>
Y_4	<{0.603/0.4,0.658/0.6}, {0.310/1}>	<{0.433/0.4,0.473/0.6}, {0.548/1}>	<{0.705/1}, {0.373/0.3,0.386/0.7}>	<{0.253/0.3,0.383/0.7}, {0.416/1}>
Y_5	<{0.477/0.4,0.537/0.6}, {0.482/0.3,0.552/0.7}>	<{0.666/0.3,0.695/0.7}, {0.290/1}>	<{0.546/0.1,0.573/0.9}, {0.421/1}>	<{0.560/1}, {0.501/0.2,0.594/0.8}>

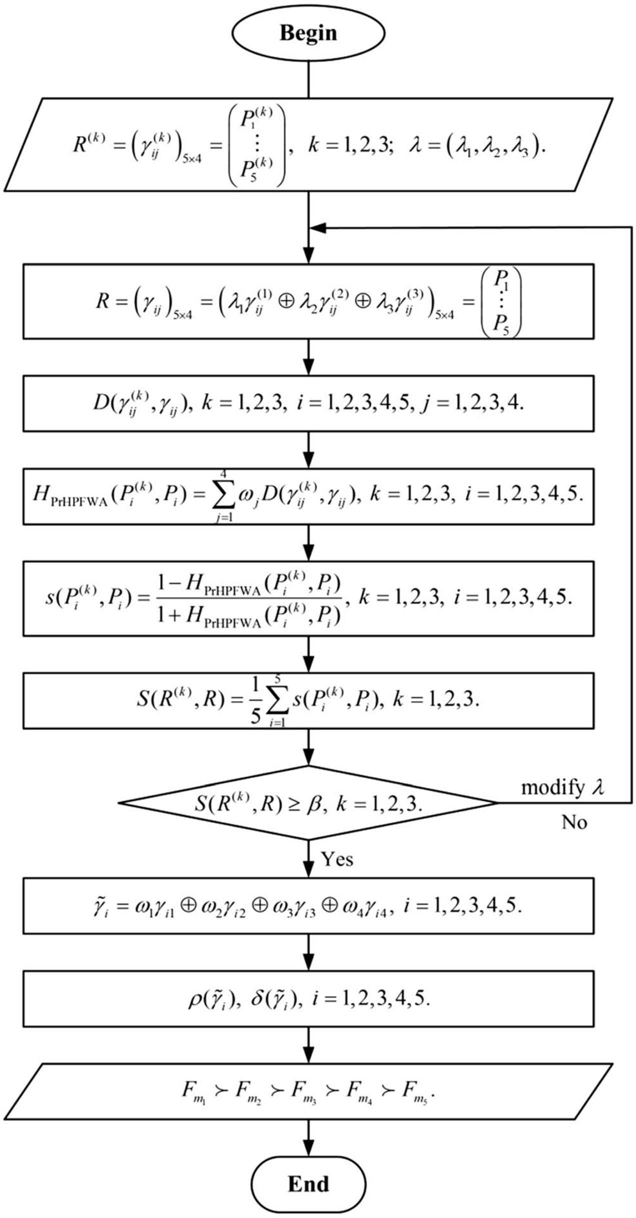

Fig. 3 An intuitive flow chart of the whole decision-making process

According to Formulas (5)–(6), all similarities $s\ (P_i^{(k)},\ P_i)$ and the comprehensive similarity $S\ (R^{(k)},\ R)$ can be calculated. For example,

$$s\left(P_1^{(1)}, P_1\right) = \frac{1 - H_{\text{PrHPFWA}}\left(P_1{}^{(1)}, P_1\right)}{1 + H_{\text{PrHPFWA}}\left(P_1{}^{(1)}, P_1\right)} = \frac{1 - 0.0210}{1 + 0.0210} = 0.9588,$$

Similarly, we can also get that the similarities of other sets $P_i^{(1)}$ and P_i are $s\left(P_2^{(1)}, P_2\right) = 0.9543, s\left(P_3^{(1)}, P_3\right) = 0.9778,$ $s\left(P_4^{(1)}, P_4\right) = 0.9497, s\left(P_5^{(1)}, P_5\right) = 0.9662.$

Therefore, the similarity between the matrices $R^{(1)}$ and R can be obtained as

$$S\left(R^{(1)}, R\right) = \frac{1}{5}\sum_{i=1}^{5} s\left(P_i{}^{(k)}, P_i\right) = \frac{0.9588 + 0.9543 + 0.9778 + 0.9497 + 0.9662}{5} = 0.9613.$$

In a similar way, we can also get other similarities as $S(R^{(2)}, R) = 0.9517$, $S(R^{(3)}, R) = 0.9486$.

At this time, if the similarity threshold value β is set to $\beta = 0.95$. Since $S(R^{(3)}, R) < 0.95$, it is necessary to modify the value of expert weight vector. Let the adjustment step size $\theta = 0.04$, after iterative adjustment, the corrected expert weight vector can be obtained as $\lambda = (0.09, 0.34, 0.57)$, and then by using PrHPFWA aggregation Formula (1) and the corrected weight vector λ, a new group evaluation matrix R' can be got as shown in Table 5.

According to the data of Table 5 and Formulas (5)–(6), the similarities $S(R^{(k)}, R')$ between $R^{(k)}$ and R' can be calculated as

$$S\left(R^{(1)}, R'\right) = 0.9572, S\left(R^{(2)}, R'\right) = 0.9527, S\left(R^{(3)}, R'\right) = 0.9500.$$

Obviously, these similarities satisfy $S(R^{(k)}, R') \geq 0.95$, $k = 1, 2, 3$. At this time, through the given attribute index weight vector $\omega = (0.1, 0.4, 0.4, 0.1)$ and PrHPFWA aggregation operator, we change the attribute index evaluation information in row i of the modified group evaluation matrix R' into a comprehensive evaluation values $\widetilde{\gamma}_i$, that is,

$$\widetilde{\gamma}_i = \omega_1\gamma_{i1} \oplus \omega_2\gamma_{i2} \oplus \omega_3\gamma_{i3} \oplus \omega_4\gamma_{i4} \quad i = 1, 2, \cdots, 5.$$

Through calculation, it can be obtained that the comprehensive evaluation values $\widetilde{\gamma}_i$ are

$$\begin{cases} \widetilde{\gamma}_1 = \{0.522/0.048, 0.523/0.112, 0.528/0.072, 0.528/0.168, 0.539/0.072, 0.539/0.168, 0.544/0.108, 0.544/0.252\}, \\ \quad \{0.375/0.120, 0.405/0.280, 0.376/0.180, 0.406/0.420\} >; \\ \widetilde{\gamma}_2 = \{0.429/0.048, 0.435/0.072, 0.440/0.112, 0.446/0.168, 0.441/0.072, 0.447/0.108, 0.451/0.168, 0.457/0.252\}, \\ \quad \{0.282/0.160, 0.347/0.240, 0.282/0.240, 0.348/0.360\} >; \\ \widetilde{\gamma}_3 = \{0.269/0.064, 0.317/0.096, 0.275/0.096, 0.322/0.144, 0.272/0.096, 0.319/0.144, 0.278/0.144, 0.324/0.216\}, \\ \quad \{0.435/0.400, 0.436/0.600\} >; \\ \widetilde{\gamma}_4 = \{0.519/0.048, 0.530/0.112, 0.540/0.072, 0.550/0.168, 0.521/0.072, 0.532/0.168, 0.541/0.108, 0.552/0.252\}, \\ \quad \{0.446/0.300, 0.453/0.700\} >; \\ \widetilde{\gamma}_5 = \{0.569/0.012, 0.586/0.108, 0.587/0.028, 0.603/0.252, 0.570/0.018, 0.588/0.162, 0.588/0.042, 0.604/0.378\}, \\ \quad \{0.424/0.060, 0.431/0.240, 0.430/0.140, 0.437/0.560\} > . \end{cases}$$

According to the comprehensive evaluation value $\widetilde{\gamma}_i$ and Definition 3.4, the scores $\rho(\widetilde{\gamma}_i)$ of $\widetilde{\gamma}_i$ are calculated as follows:

$$\rho\left(\widetilde{\gamma}_1\right) = -0.0035, \rho\left(\widetilde{\gamma}_2\right) = -0.0652, \rho\left(\widetilde{\gamma}_3\right) = -0.0833, \rho\left(\widetilde{\gamma}_4\right) = -0.0011, \rho\left(\widetilde{\gamma}_5\right) = -0.0024.$$

Since $\rho(\widetilde{\gamma}_4) > \rho(\widetilde{\gamma}_5) > \rho(\widetilde{\gamma}_1) > \rho(\widetilde{\gamma}_2) > \rho(\widetilde{\gamma}_3)$, we know that the comprehensive ranking of the five cities in disease prevention emergency management is $Y_4 \succ Y_5 \succ Y_1 \succ Y_2 \succ Y_3$. That is to say, the interactive group evaluation according to the attribute indexes $\{c_1, c_2, c_3, c_4\}$ shows that the emergency management ability of the second city Y_4 is the best.

In addition, we further observe the influence of adjusting the weight value of experts on the value of score function through simulation experiments, the simulation results are given in Table 6, where i,$\lambda^{(i)}$, $S^{(i)}$ and $\rho^{(i)}$ are corresponding number of iteration, value of expert weight vector, valve of similarity and score valves as follows:

$\lambda^{(i)} = (\lambda_1, \lambda_2, \lambda_3)$, $S^{(i)} = (S(R^{(1)}, R), S(R^{(2)}, R), S(R^{(3)}, R))$, $i = 1, 2, 3, 4, 5$, and they are the weight vector and PrHPFM similarity vector corresponding to the three experts, respectively, $\rho^{(i)} = (\rho(\widetilde{\gamma}_1), \rho(\widetilde{\gamma}_2), \rho(\widetilde{\gamma}_3), \rho(\widetilde{\gamma}_4), \rho(\widetilde{\gamma}_5))$ is the obtained score vector corresponding to the five alternatives. Their specific values and ranking are shown in Table 6.

It can be seen from Table 6 that the similarities can be continuously modified to meet the index requirements by iteratively adjusting expert weights. It should be noted that different similarity index requirements may lead to different ranking results. For example, when the similarity index is required to be less than 0.95, the ranking result is $Y_5 \succ Y_2 \succ Y_1 \succ Y_4 \succ Y_3$; when the similarity index is required to be greater than or equal to 0.95, the ranking result is $Y_4 \succ Y_5 \succ Y_1 \succ Y_2 \succ Y_3$.

The similarities and score values corresponding to different expert weights are shown in Figs. 4 and 5.

From Figs. 4 and 5, it can be seen that expert weights can be adjusted to make each similarity meet the index requirements. Therefore, we can rank the alternatives according to the score values, so as to give the best alternative. In addition, the choice of different attribute weights $\omega = (\omega_1, \omega_2, \ldots, \omega_n)$,

Table 5 Group evaluation matrix R' after modifying expert weight

Schemes	Attribute indices			
	c_1	c_2	c_3	c_4
Y_1	<{0.526/1}, {0.495/0.4,0.506/0.6}>	<{0.211/0.4,0.318/0.6}, {0.518/0.3,0.628/0.7}>	<{0.693/0.4,0.701/0.6}, {0.242/1}>	<{0.243/0.3,0.248/0.7}, {0.451/1}>
Y_2	<{0.315/0.4,0.455/0.6}, {0.525/0.4,0.533/0.6}>	<{0.429/0.3,0.455/0.7}, {0.216/1}>	<{0.484/1}, {0.274/0.4,0.463/0.6}>	<{0.214/0.4,0.326/0.6}, {0.489/1}>
Y_3	<{0.487/0.4,0.501/0.6}, {0.528/1}>	<{0.235/0.4,0.252/0.6}, {0.445/1}>	<{0.211/0.4,0.339/0.6}, {0.365/1}>	<{0.262/1}, {0.654/0.4,0.675/0.6}>
Y_4	<{0.601/0.4,0.617/0.6}, {0.283/1}>	<{0.324/0.4,0.413/0.6}, {0.505/1}>	<{0.650/1}, {0.450/0.3,0.466/0.7}>	<{0.213/0.3,0.430/0.7}, {0.417/1}>
Y_5	<{0.547/0.4,0.561/0.6}, {0.493/0.3,0.566/0.7}>	<{0.579/0.3,0.620/0.7}, {0.464/1}>	<{0.550/0.1,0.594/0.9}, {0.354/1}>	<{0.620/1}, {0.518/0.2,0.616/0.8}>

Table 6 Expert weights and corresponding PrHPFM similarity, scores and ranking results

i	$\lambda^{(i)}$	$S^{(i)}$	$\rho^{(i)}$	ranking
1	(0.33,0.34,0.33)	(0.9612,0.9519,0.9486)	(−0.0176,-0.0003,-0.1244,-0.0510,0.0113)	$Y_5 \succ Y_2 \succ Y_1 \succ Y_4 \succ Y_3$
2	(0.29,0.34,0.37)	(0.9605,0.9521,0.9490)	(−0.0148,-0.0004,-0.1165,-0.0533,0.0092)	$Y_5 \succ Y_2 \succ Y_1 \succ Y_4 \succ Y_3$
3	(0.25,0.34,0.41)	(0.9598,0.9523,0.9493)	(−0.0124,-0.0006,-0.1091,-0.0556,0.0071)	$Y_5 \succ Y_2 \succ Y_1 \succ Y_4 \succ Y_3$
4	(0.21,0.34,0.45)	(0.9591,0.9524,0.9495)	(−0.0101,-0.0007,-0.1021,-0.0580,0.0048)	$Y_5 \succ Y_2 \succ Y_1 \succ Y_4 \succ Y_3$
5	(0.17,0.34,0.49)	(0.9585,0.9525,0.9497)	(−0.0078,-0.0008,-0.0955,-0.0603,0.0025)	$Y_5 \succ Y_2 \succ Y_1 \succ Y_4 \succ Y_3$
6	(0.13,0.34,0.53)	(0.9578,0.9526,0.9499)	(−0.0056,-0.0010,-0.0892,-0.0627,0.0000)	$Y_5 \succ Y_2 \succ Y_1 \succ Y_4 \succ Y_3$
7	(0.09,0.34,0.57)	(0.9572,0.9527,0.9500)	(−0.0035,-0.0652,-0.0833,-0.0011,-0.0024)	$Y_4 \succ Y_5 \succ Y_1 \succ Y_2 \succ Y_3$

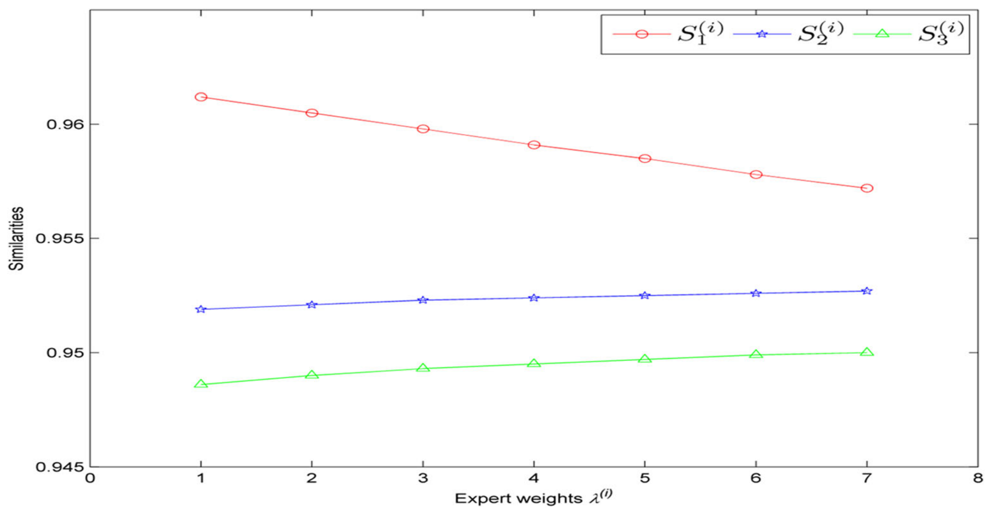

Fig. 4 Influence of adjusting expert weights to PrHPFM similarities

the distance measure coefficient η in Formula (4) will also affect the values of similarity, and then affect the final ranking result. At this point, the appropriate parameter values and index values can be selected according to different situations. In practical problems, it is difficult to obtain a satisfactory decision-making scheme by integrating the initial evaluation value of experts only once. It often needs to continuously modify the weight vector in the evaluation process and integrate the information several times before it can finally obtain a satisfactory decision-making scheme. The main difference between these different methods is that they use different integration operators, distance measures and similarity formulas to calculate the score function or relative closeness of the alternatives in the same fuzzy information environment, and

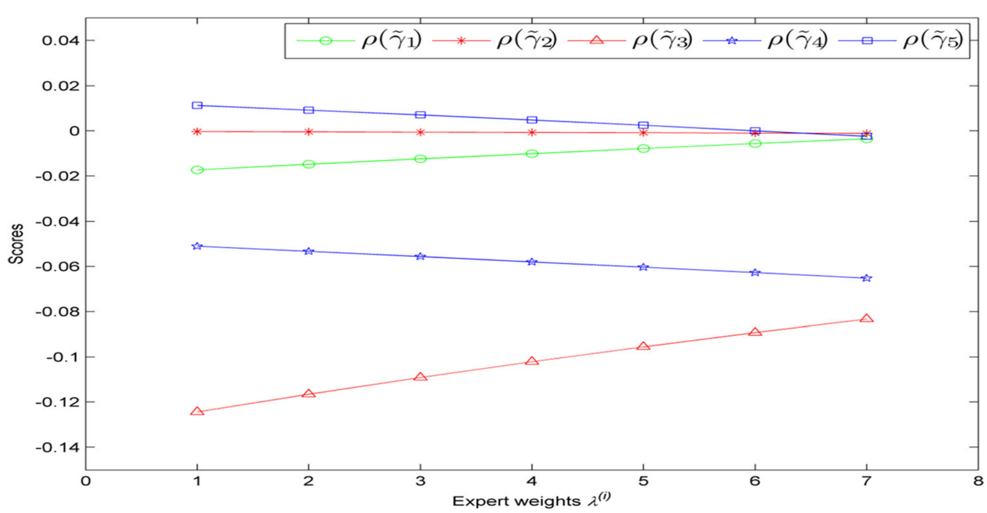

Fig. 5 Influence of adjusting expert weights to score values

sort the alternatives according to the rules from large to small according to the corresponding values, so as to determine the optimal scheme.

In order to illustrate the advantages of the interactive group decision-making method proposed in this paper, we will compare this method with the methods in [19, 21, 25]. In [19], the Hausdorff distance measure and PHFWA aggregation operator are used to obtain the score function of alternative schemes in the probability hesitation fuzzy environment, and then a new decision method was put forward by considering the probability distribution of membership set members. In [21], the TOPSIS method is proposed based on the hesitant Pythagorean fuzzy distance measure and the relative closeness of the positive and negative ideal solutions of the decision matrix obtained by HPFWA aggregation operator. In [25], under the hesitant Pythagorean fuzzy environment, the score functions of the alternative schemes is obtained through the arithmetic operations of PFNs and Maclaurin symmetric average aggregation operator, and a decision-making method is given according to the ranking of the score functions.

Next, we will separately compare the score function $\rho(\widetilde{\gamma}_i)$ of the comprehensive value $\widetilde{\gamma}_i$ corresponding to each scheme Y_i with the score functions $s(\widetilde{\gamma}_i)$ and $S(\widetilde{\gamma}_i)$ in [19, 25] and the relative closeness $RC(\widetilde{\gamma}_i)$ in [21], and the formula of relative closeness $RC(\widetilde{\gamma}_i)$ is

$$RC\left(\widetilde{\gamma}_i\right) = \frac{d\left(\widetilde{\gamma}_i, \gamma^-\right)}{d\left(\widetilde{\gamma}_i, \gamma^-\right) + d\left(\widetilde{\gamma}_i, \gamma^+\right)},$$

where d is a Hamming distance on hesitant Pythagorean fuzzy space, the γ^+ and γ^- are positive and negative ideal solutions, respectively. The calculation results are shown in Table 7.

It can be seen from the data in Table 7 that the methods proposed in [19, 25] and this paper are maked decision by calculating the score values of alternative schemes $\{Y_1, Y_2, Y_3, Y_4, Y_5\}$and their ranking. Although their score values are different, according to their ranking results the optimal scheme Y_4 is the same as the second scheme Y_5, but the ranking of the other three schemes is slightly different. This is because although Ref. [19] considers the probability distribution of membership degree of hesitant fuzzy set, it is based on the most primitive fuzzy set space and does not consider the influence of non-membership degree and hesitant degree on decision results, so its application scope is relatively narrow. In [21, 25], although the decision-making methods are proposed in the extended hesitation Pythagorean fuzzy environment, they ignore the influence of probability value on membership degree, non membership degree and hesitation degree. That is to say, the TOPSIS method is mainly put forward by ranking the relative closeness, positive and negative ideal solutions of the decision matrix obtained by HPFWA aggregation operator in [21]. The decision-.making method in [25] is given by the score function and its ranking of alternatives obtained by HPFMSM aggregation operator. In this paper, a more perfect decision-making method is proposed for some defects in [19, 21, 25], we not only expand the scope of application to the hesitant Pythagorean fuzzy environment, but also consider the probability distribution of membership and non membership, so the proposed decision-making method is more extensive and comprehensive.

In this paper, the interactive group decision-making method is used to modify the expert weight to finally reach the group consensus decision-making scheme. Because their

Table 7 Comparison and analysis between the proposed method and the other three methods

Different aggregation operators	Ranking criteria and score values of alternatives $\widetilde{\gamma}_1$ to $\widetilde{\gamma}_5$			Ranking of alternatives
	Ranking indices	Score values	Relative closeness	
PHFWA of Ref. [19] in hesitant probabilistic fuzzy environment	Score values	$s(\widetilde{\gamma}_1)=0.183,\ s(\widetilde{\gamma}_2)=0.126,\ s(\widetilde{\gamma}_3)=0.171,\ s(\widetilde{\gamma}_4)=0.331,\ s(\widetilde{\gamma}_5)=0.276$	NO	$Y_4 \succ Y_5 \succ Y_1 \succ Y_3 \succ Y_2$
HPFWA of Ref. [21] in hesitant Pythagorean fuzzy environment	Relative closeness	NO	$RC(\widetilde{\gamma}_1)=0.558,\ RC(\widetilde{\gamma}_2)=0.352,\ RC(\widetilde{\gamma}_3)=0.367,\ RC(\widetilde{\gamma}_4)=0.005,\ RC(\widetilde{\gamma}_5)=0.107$	$Y_4 \succ Y_5 \succ Y_2 \succ Y_3 \succ Y_1$
HPFMSM of Ref. [25] in hesitant Pythagorean fuzzy environment	Score values	$S(\widetilde{\gamma}_1)=0.207,\ S(\widetilde{\gamma}_2)=0.267,\ S(\widetilde{\gamma}_3)=0.121,\ S(\widetilde{\gamma}_4)=0.558,\ S(\widetilde{\gamma}_5)=0.382$	NO	$Y_4 \succ Y_5 \succ Y_2 \succ Y_1 \succ Y_3$
The proposed PrHPFWA in probabilistic hesitant Pythagorean fuzzy environment	Score values	$\rho(\widetilde{\gamma}_1)=-0.0035,\ \rho(\widetilde{\gamma}_2)=-0.0652,\ \rho(\widetilde{\gamma}_3)=-0.0833,\ \rho(\widetilde{\gamma}_4)=-0.0011,\ \rho(\widetilde{\gamma}_5)=-0.0024$	NO	$Y_4 \succ Y_5 \succ Y_1 \succ Y_2 \succ Y_3$

methods are different according to the score function formulas, this may lead to different ranking results, but it does not affect the final decision-making result of alternatives in multi-attribute index information. However, the proposed method in this paper is to overcome some defects of Refs. [19, 21, 25] at the same time, and obtain the optimal ranking of alternatives as $Y_4 \succ Y_5 \succ Y_1 \succ Y_2 \succ Y_3$. Therefore, the alternative scheme Y_4 is the optimal decision scheme.

In fact, the essence of decision-making is to select the best by relying on the ranking of alternatives. Ref. [19] can be obtained the weight of the expert group by maximizing the score deviation in the probability hesitation fuzzy environment, the proposed method is given by fusing probability hesitation fuzzy information. In the hesitant Pythagorean fuzzy environment, the TOPSIS method is proposed by the distances and relative closeness between each alternative and the ideal solution in [21], the multi-attribute group decision-making method is mainly proposed by HPFMSM operator in [25]. Their common weakness is that they do not consider probability value information. Under this background, this paper overcomes these weaknesses by introducing the distance measure and similarity of Pythagorean probability hesitation fuzzy numbers, and adds non-membership and probability information in the Pythagorean probability hesitation fuzzy environment (the sum of the squares of membership and non membership is less than or equal to 1). Besides, the proposed method can also eliminate the influence of too high or too low evaluation information of experts on the decision-making results at any time, so that the decision-maker can obtain a satisfactory scheme. Therefore, it has better advantages than the other three methods.

7 Conclusions

The main purpose of this paper is to establish a new multi-attribute interactive group decision-making method by hesitating the probabilistic representation and similarity of Pythagorean fuzzy numbers (HPFNs). PrHPFNs is added corresponding probability values to each membership and non-membership degree of HPFNs to describe fuzzy information, the interactive group decision-making method is realized by introducing the negotiation mechanism and constantly modifying the expert evaluation value to achieve the consensus of group preference. Hence, the interactive group decision-making method established by combining them in the environment of PrHPFNs has more advantages. This paper mainly uses probability hesitation Pythagorean fuzzy number distance measure and similarity measure are used to describe the fuzzy information of multi-attribute indicators, so as to make the expression of expert comprehensive evaluation more accurate. It not only reduces the loss of useful information in expert evaluation, but also fully reflects the psychological state of hesitation when experts evaluate objective things. In addition, in the hesitant intuitionistic fuzzy number environment, a new interactive group evaluation method is proposed by introducing PrHPFWA integration operator and PrHPFM similarity. This method extends the traditional interactive group decision-making method to the hesitant Pythagorean fuzzy information environment by adding probability information, and puts forward new PrHPFNs Hamming distance, matrix similarity and decision-making method. Besides, how to optimize and select the modified weight vector of experts according to PrHPFMs similarity or other indicators is our next focus.

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Declarations

Conflict of interest The authors declare that they have no conflict of interest.

Ethical approval This article does not contain any studies with human participants or animals performed by any of the authors.

References

1. Atanassov KT (1986) Intuitionistic fuzzy sets. Fuzzy Sets Syst 20(1):87–96
2. Yager RR (2013) Pythagorean fuzzy subsets. Proceeding Joint IFSA World Congress and NAFIPS Annual Meeting, June 24-28, 2013, Edmonton, Canada, pp: 57–61
3. Yager RR, Abbasov AM (2013) Pythagorean membership grades, complex numbers, and decision making. Int J Intell Syst 28(5):436–452
4. Yager RR (2013) Pythagorean membership grades in multicriteria decision making. IEEE Trans Fuzzy Syst 22(4):958–965
5. Torra V (2010) Hesitant fuzzy sets. Int J Intell Syst 25:529–539
6. Ren PJ, Xu ZS, Gou XJ (2016) Pythagorean fuzzy TODIM approach to multi-criteria decision making. Appl Soft Comput 42: 246–259
7. Zhang XL (2016) Multicriteria Pythagorean fuzzy decision analysis: a hierarchical QUALIFLEX approach with the closeness index-based ranking methods. Inf Sci 330:104–124
8. Garg H (2016) A novel correlation coefficients between Pythagorean fuzzy sets and its applications to decision-making processes. Int J Intell Syst 31(12):1234–1252
9. Liang DC, Xu ZS, Darko AP (2017) Projection model for fusing the information of Pythagorean fuzzy multicriteria group decision making based on geometric bonferroni mean. Int J Intell Syst 32(9): 966–987
10. Wei GW, Mao L (2018) Pythagorean fuzzy Maclaurin symmetric mean operators in multiple attribute decision making. Int J Intell Syst 33(5):1043–1070
11. Garg H (2020) Neutrality operations-based Pythagorean fuzzy aggregation operators and its applications to multiple attribute group decision-making process. J Ambient Intell Humaniz Comput 11(2): 3021–3041

12. Wang L, Li N (2020) Pythagorean fuzzy interaction power Bonferroni mean aggregation operators in multiple attribute decision making. Int J Intell Syst 35(1):150–183
13. Wei GW, Wei Y (2018) Similarity measures of Pythagorean fuzzy sets based on the cosine function and their applications. Int J Intell Syst 33(3):634–652
14. Li DQ, Zeng WY (2018) Distance measure of Pythagorean fuzzy sets. Int J Intell Syst 33(2):348–361
15. Zhu B, Xu ZS, Xia MM (2012) Dual hesitant fuzzy sets. J Appl Math 2012:2607–2645
16. Zhu B, Xu ZS (2014) Some results for dual hesitant fuzzy sets. Journal of Intelligent & Fuzzy Systems 26(4):1657–1668
17. Xu ZS, Zhou W (2017) Consensus building with a group of decision makers under the hesitant probabilistic fuzzy environment. Fuzzy Optim Decis Making 16(4):481–503
18. Xu ZS (2008) Group decision making based on multiple types of linguistic preference relations. Inf Sci 178(2):452–467
19. Gao J, Xu ZS, Liao H (2017) A dynamic reference point method for emergency response under hesitant probabilistic fuzzy environment. Int J Fuzzy Syst 19(5):1261–1278
20. Su Z, Xu ZS, Zhao H et al (2019) Entropy measures for probabilistic hesitant fuzzy information. IEEE Access 7:65714–65727
21. Liang DC, Xu ZS (2017) The new extension of TOPSIS method for multiple criteria decision making with hesitant Pythagorean fuzzy sets. Appl Soft Comput 60:167–179
22. Garg H (2018) Hesitant Pythagorean fuzzy sets and their aggregation operators in multiple attribute decision making. International Journal for Uncertainty Quantifications 8(3):267–289
23. Wang L, Hai W, Xu ZS et al (2019) The interval-valued hesitant Pythagorean fuzzy set and its applications with extended TOPSIS and Choquet integral-based method. Int J Intell Syst 34(6):1063–1085
24. Akram M, Luqman A, Kahraman C (2021) Hesitant Pythagorean fuzzy ELECTRE-II method for multi-criteria decision-making problems. Appl Soft Comput 108(3):107479
25. Garg H (2019) Hesitant Pythagorean fuzzy Maclaurin symmetric mean operators and its applications to multi-attribute decision-making process. Int J Intell Syst 34(4):601–626
26. Lu M, Wei GW, Alsaadi FE et al (2017) Hesitant Pythagorean fuzzy Hamacher aggregation operators and their application to multiple attribute decision making. Journal of Intelligent & Fuzzy Systems 33(2):1105–1117
27. Hao ZN, Xu ZS, Zhao H et al (2017) Probabilistic dual hesitant fuzzy set and its application in risk evaluation. Knowledge Based Systems 127(7):16–28
28. Garg H, Kaur G (2018) Algorithm for probabilistic dual hesitant fuzzy multi-criteria decision-making based on aggregation operators with new distance measures. Mathematics 6(12):1–30
29. Krishankumar R, Ravichandran KS, Liu PD et al (2021) A decision framework under probabilistic hesitant fuzzy environment with probability estimation for multi-criteria decision making. Neural Comput Applic 33(14):8417–8433
30. Luo SH, Liu J (2019) The probabilistic interval-valued hesitant pythagorean fuzzy set and its application in selecting processes of project private partner. IEEE Access 7(1):170304–170321
31. Gao Q, Liu XD, Wang ZW et al (2020) Recommendation decision-making algorithm for sharing accmmodation using probabilistic hesitant fuzzy sets and bipartite network projection. Complex & Intelligence Systems 6(2):431–445
32. Song HF, Chen ZC (2021) Multi-attribute decision-making method based distance and COPRAS method with probabilistic hesitant fuzzy environment. International Journal of Computational Intelligence Systems 14(1):2229–1241
33. Batool B, Abdullah S, Ashraf S, Ahmad M (2021) Pythagorean probabilistic hesitant fuzzy aggregation operators and their application in decision-making. Kybernetes 51(4):1626–1652
34. Liu PD, Wu YF, Li Y (2022) Probabilistic hesitant fuzzy taxonomy method based on best-worst-method (BWM) and indifference threshold-based attribute ratio analysis (ITARA) for multi-attribute decision-making. International Journal of Fuzzy Systems 24:1301–1317. https://doi.org/10.1007/s40815-021-01206-7
35. Lin Z M, Huang C, Lin M W (2022) Probabilistic hesitant fuzzy methods for prioritizing distributed stream processing frameworks for IoT applications. Mathematical Problems in Engineering. https://doi.org/10.1155/2021/6655477
36. Xu ZS (2012) Intuitionistic fuzzy multi-attribute decision making: an interactive method. IEEE Trans Fuzzy Syst 20(3):514–525
37. Zeng SZ, Su WH, Sun LR (2013) A method based on similarity measures for interactive group decision-making with intuitionistic fuzzy preference relations. Appl Math Model 37(10–11):6909–6917
38. Liao HC, Xu ZS (2014) Satisfaction degree based interactive decision making under hesitant fuzzy environment with incompleteweights. Int J Uncertainty Fuzziness Knowledge Based Syst 22:553–572
39. Liao HC, Xu ZS (2014) Distance and similarity measures for hesitant fuzzy linguistic term sets and their application in multi-criteria decision making. Inf Sci 271:125–142
40. Ding J, Xu ZS, Zhao N (2017) An interactive approach to probabilistic hesitant fuzzy multi-attribute group decision making with incomplete weight information. Journal of Intelligent & Fuzzy Systems 32(3):2523–2536
41. Wang GJ, Duan Y (2018) TOPSIS approach for multi-attribute decision making problems based on n-intuitionistic polygonal fuzzy sets description. Comput Ind Eng 124(10):573–581
42. Wang GJ, Tao YJ, Li YH (2020) TOPSIS evaluation system of logistics transportation based on an ordered representation of the polygonal fuzzy set. International Journal of Fuzzy systems 22(5):1565–1581
43. Li XP, Tao YJ, Li YH (2020) Decision making method for evaluating logistics companies based on the ordered representation of the polygonal fuzziness. Journal of Intelligent & Fuzzy Systems 9(3):3151–3166
44. Li XP, Li YH, Tao YJ (2021) Representation and aggregation of multi-source information of modern smart cities based on the intuitionistic polygonal fuzzy set. International Journal of Fuzzy systems 23(4):967–983
45. Sun G, Li XP, Chen DG (2022) Ranking defects and solving countermeasures for Pythagorean fuzzy sets with hesitant degree. Int J Mach Learn Cybern 13(5):1265–1281
46. Sun G, Wang MX, Li XP (2022) Centroid coordinate ranking of Pythagorean fuzzy numbers and its application in group decision making. Cogn Comput 14(2):602–623

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