Bandgaps and vibration isolation of local resonance sandwich-like plate with simply supported overhanging beam*

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Abstract The concept of local resonance phononic crystals proposed in recent years provides a new chance for theoretical and technical breakthroughs in the structural vibration reduction. In this paper, a novel sandwich-like plate model with local resonator to acquire specific low-frequency bandgaps is proposed. The core layer of the present local resonator is composed by the simply supported overhanging beam, linear spring and mass block, and well connected with the upper and lower surface panels. The simply supported overhanging beam is free at right end, and an additional linear spring is added at the left end. The wave equation is established based on the Hamilton principle, and the bending wave bandgap is further obtained. The theoretical results are verified by the COMSOL finite element software. The bandgaps and vibration characteristics of the local resonance sandwich-like plate are studied in detail. The factors which could have effects on the bandgap characteristics, such as the structural damping, mass of vibrator, position of vibrator, bending stiffness of the beam, and the boundary conditions of the sandwich-like plates, are analyzed. The result shows that the stopband is determined by the natural frequency of the resonator, the mass ratio of the resonator, and the surface panel. It shows that the width of bandgap is greatly affected by the damping ratio of the resonator. Finally, it can also be found that the boundary conditions can affect the isolation efficiency.

Key words local resonance, sandwich-like plate, elastic wave bandgap, vibration

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isolation

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1 Introduction

Recently, at the leading edge of the development of acoustics the new concepts such as phononic crystals and acoustic metamaterial have opened a new chapter of the study of wave propagation in artificially controlled elastic media and structures^[1–4]. Meanwhile, it provides a new chance for the theoretical and technical breakthroughs in structural vibration reduction. The bandgap characteristics of phononic crystals make the elastic wave propagation in the bandgap frequency range restrained, while the elastic wave propagation in other frequency ranges would be lossless under the effect of dispersion relation^[5]. According to the mechanism of bandgap formation, it can be divided into the local resonance type and the Bragg dispersion type. The concept of local resonance phononic crystals was first put forward in 2000 by Liu et al.^[6]. The mechanism of local resonance emphasizes the interaction between the resonance characteristics of a single dispersion and the long wave traveling wave in the matrix, which breaks through the limitation of Bragg scattering and provides a lower bandgap for smaller size. The ideal bandgap characteristics of the local resonance phononic crystal have attracted many scholars to study it^[7–10].

Wang^[11] deeply studied the relationship between the forming of phononic crystal bandgap and the vibration type of phononic crystal in his doctoral dissertation, and pointed out the direction for exploring the formation principle of phononic crystal bandgaps. Pennec et al.^[12], Yu^[13], and Wang and Wang^[14] applied phononic crystals to rod, beam, plate, and other structures, and designed a variety of phononic crystal structures with low bandgaps. Errico et al.^[15] adopted the wave finite element method to carry out the experimental and numerical research on the sound transmission loss of the complex curved plane excited by the diffuse reflection sound field. Oudich et al.^[16] directly used the commercial finite element software COMSOL multiphysics to accomplish the calculation and analysis of bandgaps of the local resonance plate. Ma et al.^[17] put forward the general design concept of phononic metamaterial. They introduced the common cooperative behavior into the design process and proposed a modal displacement method to extract the bending wave bandgaps of acoustic metamaterials. The structure they designed was driven by the need of functions. Mi et al.^[18] presented a variational method for bandgap analysis using the energy functional variation principle and Chebyshev orthogonal polynomials. Huang et al.^[19] studied the limits, applications, and challenges of phononic metamaterial of plates and films.

Due to the wide applications in practical engineering, resonance plate structure has been widely studied in sound insulation and vibration reduction in the last decades^[20–24]. In some studies, resonant elements or scatters were embedded in the plate. Under this circumstance, the periodic change of material parameters will affect the surface structure and performance of plates. It is known that the sandwich-like plate has a smooth surface and multilayer structure, i.e., two face sheets and a core layer. It is convenient for the scholars to put resonators inside the sandwich-like plate to achieve vibration and sound insulation on the base of the research of phononic characteristics of sandwich-like plate. Song et al.^[25] designed a periodic sandwich-like plate consisting of panels and additional oscillators, and compared and analyzed the sound transmission of the periodic and bare plates. The developed method can systematically tune the resonance frequency of local resonator which overcame the coincidence phenomenon and improved the mass ratio of the metamaterial sandwich-like structure. Lu et al.^[26] proposed a metamaterial structure with the arrayed energy harvesters to isolate the low frequency vibration and harvest electrical energy. Liu et al.^[27] proposed a new design method of locally resonant sandwich-like plates for the noise insulation in engineering applications. And it was found for the

first time that laminated metamaterials have wider stopbands than traditional metamaterials. He et al.^[28] studied the stopband of laminate acoustic metamaterials composed of carbon-fiber-reinforced polymer (CFRP) and a periodic array of mass-spring-damper subsystems integrated with the laminates. Qin et al.^[29] studied low-frequency vibration and radiation performance of an inclined plate, and established a theoretical model of infinite local resonant plate and strip structure with cycles. Li et al.^[1] presented a design and computation for a sandwich-like plate containing mass-beam resonators to obtain a specific and low-frequency stopband. It changes the situation that the traditional sandwich-like structure cannot be used to control or isolate the vibration precisely in a specific frequency range. Full frequency domain vibration attenuation can be done by adjusting the geometry of the cantilever beam and the mass of the block. But its substructure should not be adjusted, which is not conducive to the application in many occasions.

Based on the existing studies of the structural design of the double-layer plate composing local resonance phononic crystals and bandgap, a novel structure of sandwich-like plate composing local resonators is presented in this paper to acquire specific low-frequency bandgaps. The simply supported overhanging beam is free at the right end, and an additional linear spring is added at the left end. A mass block is attached to the right end of the beam. It is used as the core layer of the local resonance sub unit, and it is well connected with the upper and lower surface plates. The dispersion equation of frequency and wave number is derived by using the energy method, and two boundaries of stopband are obtained by solving the dispersion equation. The vibration attenuation and isolation performance are analyzed by using the finite element method. The bandgap attenuation characteristics of bending wave is acquired. The advantage of the structure is that it has lower frequency stopband, and it is suitable for practical engineering. It can also adjust the bandgap without changing the functions of main structure by changing its substructure.

2 Theoretical model and calculation method

The local resonance sandwich-like plate studied in this paper consists of two surface panels and a resonator between them as shown in Fig. 1. The length direction, depth direction, and height direction of local resonance sandwich-like plate are along the coordinate axes x , y , and z , respectively. Assuming that the two plates are fixed by supporting rods, the unit cell of the local resonance sandwich-like plate is shown in Fig. 2. It can be seen that x_1 , y_1 , and z_1 are the length, depth, and height of a unit cell, respectively. It is assumed that the rigidity of the supporting rod is infinity. The mass of the supporting rod is added to the panel. Only transverse vibration of the sandwich-like plate is considered.

Fig. 1 Model of local resonance sandwich-like plate (color online)

Fig. 2 Cell element of local resonance sandwich-like plate (color online)

For periodic structures, only the unit cell is considered to calculate the dispersion relation of the infinite structure. First, the energy of the panels is calculated. For the surface panel in the unit cell, by according to the thin plate theory, the displacement-strain relationship of any

point on the surface plates can be shown as follows:

$$\varepsilon_{xx} = -z \frac{\partial w^2}{\partial x^2}, \quad \varepsilon_{yy} = -z \frac{\partial w^2}{\partial y^2}, \quad \varepsilon_{xy} = -2z \frac{\partial w^2}{\partial x \partial y}, \quad (1)$$

where ε_{xx} , ε_{yy} , and ε_{xy} are the strains of the plate, and w is the transverse displacement. We only study curved waves because they carry more vibrational energy. The constitutive equation of the plate is

$$\begin{pmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \varepsilon_{xy} \end{pmatrix} = \begin{pmatrix} Q_{11} & Q_{12} & 0 \\ Q_{21} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{pmatrix} \begin{pmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{xy} \end{pmatrix}, \quad (2)$$

where Q_{ij} ($i, j = 1, 2, 6$) is the stiffness coefficient,

$$Q_{11} = Q_{22} = \frac{E}{1 - \nu^2}, \quad Q_{12} = Q_{21} = \frac{\nu E}{1 - \nu^2}, \quad Q_{66} = \frac{E}{2(1 + \nu)}. \quad (3)$$

The expressions of potential energy U and kinetic energy T of the plates are as follows:

$$U = \frac{1}{2} \int_V (\varepsilon)^T (\sigma) dV, \quad (4a)$$

$$T = \frac{1}{2} \int_V \rho \left(\frac{\partial w}{\partial t} \right)^2 dV, \quad (4b)$$

where ρ and V are the density and the volume of the plate, respectively.

For the resonator, the mechanical model is shown in Fig. 3, where Point C is the center of the mass block. And the displacement of arbitrary point for the simply supported overhanging beam can be given by

$$w_{x1} = \frac{Flx(a^2 - x^2)}{6E_b I a}, \quad 0 \leq x \leq a, \quad (5a)$$

$$w_{x2} = -\frac{F(l+a)(3x^2 + a^2)}{6E_b I} + \frac{Fx^3}{6E_b I} + \frac{Fa(4l+3a)x}{6E_b I}, \quad a \leq x \leq a+l, \quad (5b)$$

where w_{x1} and w_{x2} are the deflections of the beam in AB and BC regions, respectively (see Fig. 3). F is the force of gravity of the mass block.

Fig. 3 Mechanical model of resonator (color online)

The displacement at Point C is

$$w_{2c} = -\frac{F(a+l)^3}{3E_b I} + \frac{Fa(4l+3a)(a+l)}{6E_b I} - \frac{Fa^2(l+a)}{6E_b I}. \quad (6)$$

When the simply supported left end is added a linear spring additionally, the displacement at Point A caused by the elastic support can be written as follows:

$$\Delta l_A = \frac{F}{k}. \quad (7)$$

And the displacement of Point C is

$$w_1 - w_0 = -\Delta l_1 + w_{2c}, \quad (8)$$

where Δl_1 denotes the displacement of Point C caused by the deformation of the spring, and can be obtained by $\Delta l_1 = l\Delta l_A/a$.

From Eq. (8), the relationship between F and displacement of resonator (Point C) is obtained as follows:

$$F = -\frac{3(w_1 - w_0)akE_b I}{l(la^2k + l^2ak + 3E_b I)}. \quad (9)$$

By substituting Eq. (9) into Eqs. (5a) and (5b), Eqs. (5a) and (5b) can be rewritten as

$$w_{x1} = -\frac{(w_1 - w_0)kx(a^2 - x^2)}{2(la^2k + l^2ak + 3E_b I)}, \quad 0 \leq x \leq a, \quad (10a)$$

$$w_{x2} = \frac{3(w_1 - w_0)ak(l + a)(a^2 + 3x^2)}{2l(la^2k + l^2ak + 3E_b I)} - \frac{(w_1 - w_0)akx^3}{2l(la^2k + l^2ak + 3E_b I)} - \frac{(w_1 - w_0)a^2k(4l + 3a)x}{2l(la^2k + l^2ak + 3E_b I)}, \quad x \leq a, \quad (10b)$$

$$I = \frac{b_b h_b^3}{12}, \quad (10c)$$

where k is the stiffness coefficient of the linear restrain spring. a , l , b_b , and h_b are the left side length, right side length, width, and thickness of a simple support beam. E_b and I are the elastic modulus and moment of inertia of the beam. w_1 and w_0 are the displacement of the mass block and the displacement of the supporting point, respectively.

It is assumed that the mass of the mass block is far bigger than the beam, and thus the bending deformation of beam is far bigger than the mass block. Therefore, the kinetic energy of the simply supported overhanging beam and the bending potential energy of the mass can be neglected. The potential energy U_m and kinetic energy T_m of the resonator system are

$$T_m = \frac{1}{2}m\dot{w}_1^2, \quad (11a)$$

$$U_m = U_1 + U_2 + U_3, \quad (11b)$$

where U_1 , U_2 , and U_3 are the potential energy in the parts of AB and BC of the beam and the linear spring, while m is the mass of the resonator. And they can be derived by

$$U_1 = \frac{1}{2} \int_0^a E_b I \left(\frac{\partial^2}{\partial x^2} w_1 \right)^2 dx, \quad U_2 = \frac{1}{2} \int_a^{a+l} E_b I \left(\frac{\partial^2}{\partial x^2} w_{x2} \right)^2 dx, \quad U_3 = \frac{1}{2} \left(\frac{Fl}{a} \right)^2 k. \quad (11c)$$

Considering that the bending wave travels in the infinite sandwich-like plate with periodic cell, the displacements of plate w , the rigid support displacement w_0 , and the mass displacement w_1 can be given by

$$w = Ae^{i(\alpha x + \beta y - \omega t)}, \quad w_0 = Ae^{i(\alpha x - \omega t)}, \quad w_1 = Be^{i(\alpha x - \omega t)}. \quad (12)$$

In Eq. (12), α and β are wave numbers in the directions of x and y , respectively, and $\alpha = \frac{2\pi}{\gamma_1}$, $\beta = \frac{2\pi}{\gamma_2}$. γ_1 and γ_2 are the wave lengths in the directions of x and y , respectively. ω is the wave frequency. A and B are the undetermined amplitudes of vibration. Since only the low frequency vibration is considered in this paper, it is reasonable for us to assume that the wave length is far bigger than the length of the beam. Therefore, the phase difference between w_0 and w_1 is neglected. The transverse displacement of the support $w_0 = 0$, when only the system of spring, beam, and mass block is taken into consideration. From the conservation of mechanical energy, it can be concluded that

$$U_{m\max} - T_{m\max} = 0. \quad (13)$$

By substituting Eqs. (11a) and (11b) into the above equation, the natural frequency of the

resonator system is obtained as follows:

$$\omega_1 = \frac{1}{2\pi} \left(\frac{\sqrt{3mE_b I k(a^3 k + l a^2 k + 3E_b I)}}{m(l a^2 k + l^2 a k + 3E_b I)} \right). \quad (14)$$

To acquire the wave equation of the local resonance sandwich-like plate, Hamilton's principle is employed as follows:

$$\int_0^t \delta T - \delta U + \delta T_m - \delta U_m = 0. \quad (15)$$

Substitute Eqs. (4a), (4b), (11a), and (11b) into the above equation to acquire such algebraic equations,

$$\begin{aligned} & \frac{\sin(2\alpha p_a) \sin(2\beta p_b)}{3\alpha\beta} (3\rho h \omega^2 - (H^3 - (H-h)^3)(Q_{11}\alpha^4 \\ & + 2Q_{12}\alpha^2\beta^2 + 4Q_{66}\alpha^2\beta^2) - K) \cdot A + K \cdot B = 0, \end{aligned} \quad (16a)$$

$$K \cdot A + \left(\frac{m \cdot \omega^2}{2} - K \right) B = 0, \quad (16b)$$

where

$$K = \frac{3E_b I k(a^3 k + l a^2 k + 3E_b I)}{2(l a^2 k + l^2 a k + 3E_b I)^2}. \quad (16c)$$

If A and B have non-zero solutions, the coefficients of Eqs. (16a) and (16b) must be zero. The wave equation of ω connecting with A and B can be acquired as follows:

$$\begin{aligned} & \left(\frac{m\omega^2}{2} - K \right) \left(\left(\frac{\sin(2\alpha p_a) \sin(2\beta p_b)}{3\alpha\beta} (3\rho h \omega^2 - (H^3 - (H-h)^3) \right. \right. \\ & \left. \left. (Q_{11}\alpha^4 + 2Q_{12}\alpha^2\beta^2 + 4Q_{66}\alpha^2\beta^2) - K \right) \right) - K^2 = 0. \end{aligned} \quad (17)$$

The solution of ω is obtained by solving Eq. (17), using α and β to express them. Wave frequency ω is assumed to be a positive real number. Then, the dispersion surface of frequency ω is given by α and β . The upper bound and lower bound of the bandgap are obtained with α and $\beta \rightarrow 0$, α and $\beta \rightarrow \infty$ (see Ref. [25]). The bandgap can be shown as follows:

$$\left(\frac{1}{2\pi} \sqrt{\frac{2K}{m}}, \frac{1}{2\pi} \sqrt{\frac{2K}{m} + \frac{m}{8p_a p_b h \rho_1}} \right). \quad (18)$$

Substituting Eq. (16c) into Eq. (14) yields

$$\omega_1 = \frac{1}{2\pi} \sqrt{\frac{2K}{m}}. \quad (19)$$

Equation (18) can be shown as

$$(\omega_1, \omega_1 \sqrt{1 + \varphi}), \quad (20)$$

where $\varphi = \frac{m}{8p_a p_b h \rho_1}$ is the mass ratio of the resonator and the thin plates.

Equation (20) shows that the bandgap can be adjusted by changing the natural frequency of the resonator and the mass ratio of the resonator and the plates. Obviously, with the reduction of the natural frequency of the resonator, the bandgap moves to a low frequency range. When the mass ratio of resonator and plate φ remain unchanged, the width and the natural frequency of bandgap are proportional. When the natural frequency ω_1 is fixed, the width of bandgap can be increased by increasing the mass ratio of resonator and thin plate.

3 Results and discussion

3.1 Comparison study

One comparison study is performed to ensure the validation and accuracy of the present theoretical model and the COMSOL results.

Theoretical results: considering that the system is a sandwich-like plate built by 10×5 units of cells, the geometric and material parameters of the sandwich-like plate in this paper are shown in Table 1 and Table 2, respectively. In order to simplify the models, linear restrain spring is replaced by an elastomer. Stiffness coefficient k of the linear restrain spring is equivalent to elastic modulus of elastomer E_t . And $E_t = kl_t/A_t$, where l_t and A_t are the length and cross-sectional area of the elastomer, respectively. It can be assumed that the mass of the rigid support is distributed on the two surface panels uniformly. The stiffness matrix Q of local resonance sandwich-like plate and the mass m of mass block-beam system are acquired by using the cell geometric and material parameters in Table 1 and Table 2. By substituting material parameters E_b , Q , m and geometric parameters into Eq. (19), the dispersion relation of wave numbers α , and frequency ω can be obtained as shown in Fig. 4(a). Figure 4(b) is the front view of Fig. 4(a). As shown in Fig. 4(b), there is a bandgap that wave cannot travel through in the range of this bandgap 85.4 Hz–160.9 Hz. There is only one bandgap in Fig. 4. It is because we mainly study the local resonance bandgap, which is calculated by using the Hamilton principle.

Table 1 Geometric parameters of sandwich-like plate (m)

Structure	Geometric parameters (length, width, and height)	Structure	Geometric parameters (length, width, and height)
Plate	$0.03 \times 0.04 \times 0.001$	Supporting rod	$0.001 \times 0.001 \times 0.019$
Mass block	$0.01 \times 0.02 \times 0.01$	Elastomer support	$0.002 \times 0.002 \times 0.009$
Beam	$0.004 \times 0.001 \times 0.025$	Cylindrical pin	0.0004×0.005

Table 2 Material parameters of sandwich-like plate

Structure	Material	Elasticity modulus/Pa	Density /($\text{kg} \cdot \text{m}^{-3}$)	Poisson's ratio
Plate	Aluminum	70×10^9	2 700	0.28
Mass block, beam, supporting rod	Copper	110×10^9	8 700	0.35
Elastomer support	Rubber	2×10^7	1 300	0.47

Fig. 4 Theoretical results of dispersion relation for sandwich-like plate: (a) dispersion diagram and (b) front view of dispersion diagram (color online)

Finite element method results: the dispersion surface and bandgap of the local resonance sandwich-like plate are also acquired by using COMSOL software. First, the resonator is replaced by the spring-mass system as shown in Fig. 5(a). The equivalent stiffness is $k_d = (\sqrt{2K})^2$ from Eq. (19), and the equivalent mass is $m_d = m$. Thus, the final equivalent model is shown in Fig. 5(a).

Fig. 5 (a) Equivalent finite element model, (b) cell element of equivalent finite element model, and (c) finite element results of sandwich-like plate by COMSOL (color online)

The geometric and material parameters of the local resonance sandwich-like plate are the same as above. The finite element method adopts solid element to explore the vibration reduction characteristics of plate under the physical field of solid mechanics. Here, the mass block adopts the default free tetrahedron mesh, and the upper and lower panels and rubber bodies adopt the refined free tetrahedron mesh. Local resonance sandwich-like plate is clamped on both two ends in the direction of x by the red edges while the other two ends are free as shown in Fig. 5(a). On the black point ($x = 30$ mm, $y = 75$ mm), an excitation with an amplitude of 100 N is applied. On the red point shown in Fig. 5(a) ($x = 350$ mm, $y = 75$ mm), an acceleration response is detected. The acceleration response under the harmonic excitation on the local resonance sandwich-like plate is analyzed using the finite element model.

The acceleration response of the sandwich-like plate without resonator is also simulated in order to verify the vibration attenuation characteristics of the local resonance sandwich-like plate. Under the frequency of the harmonic excitation varying in the range of 0–400 Hz, acceleration responses of the sandwich-like plate on the red point ($x = 30$ mm, $y = 350$ mm) are shown in Fig. 5(c); two cases, i.e., with and without resonator are considered.

The finite element results show that the acceleration of sandwich plate without resonator does not decrease obviously. The reason is that bending wave can travel freely in the sandwich-like plate without a resonator. In the range of 85 Hz–160 Hz, the acceleration response of local

resonance sandwich-like plate with a resonator is clearly lower than those without a resonator, which means that the resonator does successfully suppress the vibration of the local resonator plate. The result is in good agreement with the location of the bandgap shown in Fig. 4(b). The result demonstrates that both the present theoretical method to calculate the bandgaps width and the COMSOL model are correct and effective.

3.2 Forced vibration of system

It is well known that when the frequency of external excitation is moving away from the natural frequency of the resonator, there is no effect on the propagation of the bending wave on the plate. When the frequency of excitation gets close to the natural frequency of the resonator, the energy of vibration is concentrated on the resonator, and the vibration occurs only on the resonator near the excitation. To prove it and explain the mechanism of the bandgap shown in Fig. 5(c), the dynamic responses of the system under the excitation with four typical frequencies are shown in Fig. 6. First, when the frequency of excitation is 50 Hz, which is less than the lower limit of bandgap (85 Hz), it does not play the role of vibration isolation, and the sandwich-like plate vibrates with the resonator together. It can be seen that in this case the bending tension coupling vibration is found for the plate. And the resonators in the region of the bending tension coupling vibration have larger amplitude than others. Figure 6(b) shows that when the excitation frequency gets close to the natural frequency of the resonator, the vibration of the resonator only occurs around the point of excitation. Interestingly, while the excitation frequency is around the stop bandgap frequency, the farther away from the excitation point, the smaller the vibration amplitude of the system, as demonstrated in Fig. 6(c). Hence, one can see that although the frequency of the external excitation is within this range of the bandgap, it is much larger than the natural frequency of the resonator which leads to weakening of the ability to absorb energy. From Fig. 6(d), one can see that when the frequency of the excitation is bigger than the stop bandgap, the sandwich-like plate exhibits the complex dynamic response and the resonators are almost in the static states. It shows that the resonator completely losses the ability to absorb energy.

Fig. 6 Dynamic responses of plate and resonator under different external excitation frequencies (left is mode of plate, and right is mode of resonator) (color online)

3.3 Effect of various parameters on bandgap

The effect of the geometric and material: from Eq. (20), without damping, the bandgap range of local resonance sandwich-like plate depends on the resonator's natural frequency and mass ratio of mass block and plate. Furthermore, the natural frequency of the resonator is dependent on E_t , E_b , m , and l , respectively. Hence, the effects of the elastic modulus of elastic support E_t and those of beam E_b , mass of the block m , and the position of the block l on the bandgap of the sandwich-like plate are analyzed, where l has the same meaning as that in Eq. (10).

For convenience purpose, Eq. (16c) can be simplified as

$$K = \frac{ax^2 + bx + c}{dx^2 + ex + f}, \quad (21)$$

where x is variable (E_t or E_b), and a, b, c, d, e , and f are coefficients (other invariants). Thus, E_t and E_b have the same functional form as ω_1 . From Fig. 7(a) and Fig. 7(b), it shows that the start and stop frequencies have the same trends when the values of E_t and E_b change. The bigger the elastic modulus of elastic support and beam, the bigger the start and stop frequencies of bandgap. That is because increasing the elastic modulus can cause the increase in the stiffness of the system. Also, one can see that it has more effects on the stop frequency and the bandgap gets wider. When the elastic modulus is bigger than a specific number $E_t \geq 4 \times 10^7$ or $E_b \geq 100 \times 10^9$, it will have fewer effects on bandgap. From Fig. 7(c), when the mass of the resonator increases, the start and stop frequencies get reduced. It has a bigger effect on start frequency and the bandgap gets wider. When the mass of resonator is bigger than a specific value, it has fewer effects on bandgap. It is shown from Fig. 7(d) that when the mass block gets away from the simply support point, the start and stop frequencies decrease linearly. It has a bigger effect on stop frequency and the bandgap gets narrower. Obviously, with the mass block getting away from the simply support point the length of the beam is increasing, and it makes the decrease in the stiffness of the system which brings about the results plotted in Fig. 7(d).

Fig. 7 Effects of parameters on frequency of sandwich-like plate (a) E_t , (b) E_b , (c) m , and (d) l (color online)

As stated above, the elastic moduli E_t and E_b and the mass of the resonator m can make bandgap wider. The mass of the resonator is essential for decreasing the start frequency. Increasing the mass of the resonator and the mass ratio of the resonator to the thin plate can

increase the stop frequency of bandgap.

The influence of damping: for the real vibration systems, it cannot avoid the factors of damping such as friction on surfaces, air resistance, elastic materials' inner damping, etc. In order to assess the influence of damping on the bandgap of local resonance sandwich-like plate, finite element method is adopted to study different response of the system under various kinds of damping. For the system of local resonance sandwich-like plate with simply supported overhanging beam, the damping ratio of the resonator can be adjusted by attaching damping material on the beam or springs. In the finite element simulation, the elastic modulus of the resonator involving the structural loss factor $\tilde{C}$ can be expressed as $\tilde{C} = C(1 + i\eta_s)$, of which C denotes elastic modulus of the resonator without damping, η_s is structural loss factor.

The local resonance sandwich-like plate with damping's acceleration response is shown in Fig. 8. Because of the existence of damping, it makes the mechanical system dissipate energy in vibration. Thus, with the increasing value of damping of the resonator, the vibration intensity decreases, the curve shows little fluctuation, and the width of bandgap increases.

Fig. 8 Damped frequency response curve of sandwich-like plate (color online)

Figure 9 illustrates the vibration of local resonance sandwich-like plate under three representative excitations when $\eta_s = 0.2$. One can see that, for the sandwich-like plate with damping, when the excitation frequency is 80 Hz, which is less than the lower limit of bandgap (85 Hz), vibrations occur only near the excitation point, as shown in Fig. 9(a). The amplitude of the vibration of the system without damping is an order of magnitude larger than that with $\eta_s = 0.2$. When frequency of the excitation is 90 Hz, which is in the range of bandgap, the dynamic responses of sandwich-like plates with damping are almost the same as those without damping, as demonstrated in Fig. 9(b). It shows that the vibration isolation effect is very obvious in this case. In contrast, the damping does not affect the vibration of the sandwich-like plate in the original bandgap range. For the sandwich-like plate with damping, when the excitation frequency is 180 Hz, which is higher than the upper limit of bandgap (160 Hz), the farther away from the excitation point, the smaller the vibration amplitude of the system, as demonstrated in Fig. 9(c). But the vibrations of the counterpart without damping are more intense and complex.

Influence of different boundary conditions: in this subsection, the influence of different boundary conditions on the bandgap and the vibration isolation effect are studied through finite element method. Figures 10(a)–10(d) show that the bandgap ranges are identical no matter what boundary conditions are applied to the sandwich-like plate. These findings demonstrate that the bandgap only relates to the natural frequency of the resonator and the ratio of the mass between the resonator and the plates.

The vibrations of the sandwich-like plate with resonator or without resonator under four

Fig. 9 Effects of vibration of sandwich-like plate with and without damping (left is $\eta_s = 0.2$, right is without damping) (color online)

Fig. 10 (a) Short edges clamped and long edges free (CFCE), (b) simply support at four corners (SSSS), (c) a short edge clamped and other edges free (CFFF), and (d) fully clamped boundary conditions (CCCC), where C is the abbreviation of clamped, F is the abbreviation of free, and S is the abbreviation of simply-supported (color online)

boundary conditions are shown in Tables 3–5. Tables 3–5 are the dynamic responses of the sandwich-like plate without resonator, the sandwich-like plate with resonator and resonators in the sandwich-like plate, respectively.

It can be seen from Table 3 that, under the given frequency of the external excitation, the dynamic response of the faceplate has obvious differences. It is noteworthy that for the faceplate with the CCCC boundary, the vibration of it can be found only near the excitation. It follows

Table 3 Dynamic responses of sandwich-like plate without resonator under different boundary conditions (color online)

Frequency/Hz	Boundary condition			
	CFCF	SSSS	CFFF	CCCC
50
90
120
160
240

Table 4 Dynamic responses of sandwich-like plate with resonator under different boundary conditions (color online)

Frequency/Hz	Boundary condition			
	CFCF	SSSS	CFFF	CCCC
50
90
120
160
240

Table 5 Dynamic responses of resonators in sandwich-like plate under different boundary conditions (color online)

Frequency/Hz	Boundary condition			
	CFCF	SSSS	CFFF	CCCC
50
90
120
160
240

that the boundary condition is an important factor. Compared with the faceplate, the dynamic responses of the sandwich-like plate with the resonator shown in Table 4 are quite different

especially for those in the bandgap (85 Hz and 160 Hz), in which the vibration is isolated or the vibration occurs only near the excitation point. This is because the energy of the external excitation is completely or partly absorbed by the resonator. Thus, the bending wave cannot travel to the boundary of the sandwich-like plate. For the case that the frequency of the excitation is outside the bandgap, there is far different between faceplate and the counterpart with resonator because of the coupling effect of them.

Furthermore, the energies absorbed by the resonator of the sandwich-like plate with the resonator and different boundary conditions are shown in Fig. 11. It is assumed that Point A is the excitation point, and the amplitude and frequency are the same as above. Vibration isolation efficiency is calculated by $\xi = \lg(a_1/a_2)$, where a_1 and a_2 are the accelerations of A_1 ($x = 15$ mm, $y = 75$ mm) and A_2 ($x = 35$ mm, $y = 75$ mm), respectively, as shown in Fig. 11. The relationship between vibration isolation loss rate and frequency is plotted in Fig. 12. It is pretty obvious that in the bandgap range, the more restrictive the condition is, the higher the isolation efficiency will be, as shown in Fig. 12(b). One can also see that under the CCCC boundary conditions, the system has the stronger energy absorption capability than other three boundary conditions. It shows that in order to isolate the vibration, both the natural vibration of the resonator and the boundary condition are all considered for a certain sandwich-like plate with the resonator.

Fig. 11 Incentive point and test points of finite element model (color online)

Fig. 12 Relationships between vibration isolation efficiency and frequency of sandwich-like plate (a) vibration efficiency of different boundary conditions and (b) detailed view of (a) within bandgap (color online)

4 Conclusions

In this paper, a novel structural design of a sandwich-like plate with local resonator is presented to acquire specific low-frequency bandgap. On the basis of the Hamilton principle,

the wave equation is established, and the bending wave bandgap is computed. The vibration attenuation and isolation performance of the sandwich-like plate are analyzed by using the finite element method. The effects of parameters to bandgap are also discussed, and the conclusion is as follows.

This structure can acquire a low start frequency and the width of bandgap is 75 Hz. It can acquire good vibration attenuation by adjusting the damping ratio of the local resonance sandwich-like plate. Damping does not influence the vibration characteristics of the plate in the range of bandgap, and it can widen the bandgap. The width of bandgap increases with the damping ratio.

Elastic modulus, mass of the resonator, position of resonator, mass ratio of the resonator, and thin plate φ can make a remarkable effect on the characteristics of the bandgap. Increasing the mass of the resonator can widen the bandgap, decrease the start frequency, and increase the stop frequency. It can also cause problems, such as, the mass of the sandwich-like plate being too big. Thus, it is especially important to choose or design the equivalent mass of the resonator.

The sandwich-like plate has different vibration isolation efficiencies under different boundary conditions. And the CCCC boundary condition has unique vibration isolation characteristics, worthy of further study. Therefore, we can choose different installation methods to obtain better vibration isolation effects according to the actual engineering situation.

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