A new tunable elastic metamaterial structure for manipulating band gaps/wave propagation*

Zhenyu WANG, Zhaoyang MA, Xingming GUO[†], Dongsheng ZHANG

Shanghai Institute of Applied Mathematics and Mechanics, Shanghai Key Laboratory of
Mechanics in Energy Engineering, Shanghai University, Shanghai 200444, China

(Received Aug. 3, 2021 / Revised Sept. 10, 2021)

Abstract A one-dimensional mechanical lattice system with local resonators is proposed as an elastic metamaterial model, which shows negative mass and negative modulus under specific frequency ranges. The proposed representative units, consisting of accurately arranged rigid components, can generate controllable translational resonance and achieve negative mass and negative modulus by adjusting the local structural parameters. A shape memory polymer is adopted as a spring component, whose Young's modulus is obviously affected by temperature, and the proposed metamaterials' tunable ability is achieved by adjusting temperature. The effect of the shape memory polymer's stiffness variation on the band gaps is investigated detailedly, and the special phenomenon of intersecting dispersion curves is discussed, which can be designed and controlled by adjusting temperature. The dispersion relationship of the continuum metamaterial model affected by temperature is obtained, which shows great tunable ability to manipulate wave propagation.

Key words tunable elastic metamaterial, negative mass, negative modulus

Chinese Library Classification O421⁺.5

2010 Mathematics Subject Classification 74H45

1 Introduction

As significant advances in advanced materials and engineering structures, periodic structures, such as phononic crystals (PCs) and metamaterials^[1], have drawn more and more attention to applications in manipulating wave propagation and noise reduction. Advantages of these materials/structures come from their special properties, such as effective negative mass/elastic modulus and negative refraction^[2–5], which cannot be found in the natural world. The concept of metamaterials originates from the emergence of electromagnetic metamaterials^[6], and excited by electromagnetic metamaterials, acoustic/elastic metamaterials are proposed. Acoustic/elastic metamaterials have shown significant value in applying cloaking, imaging, and noise

* Citation: WANG, Z. Y., MA, Z. Y., GUO, X. M., and ZHANG, D. S. A new tunable elastic metamaterial structure for manipulating band gaps/wave propagation. *Applied Mathematics and Mechanics (English Edition)*, **42**(11), 1543–1554 (2021) <https://doi.org/10.1007/s10483-021-2787-8>

[†] Corresponding author, E-mail: xmguo@shu.edu.cn

Project supported by the National Natural Science Foundation of China (Nos. 11872233, 12102245, and 11727804)

©The Author(s) 2021

and vibration control^[7–13]. Liu et al.^[14] put forward a new theory that the mechanism of negative parameters is local resonance, which results in band gaps, and then many works^[15–23] were concentrated on designing or broadening band gaps. However, after a metamaterial structure is given, the band gaps are fixed. Expanding band gaps and designing a system automatically tuned with different external incentives are still a big challenge.

Researchers have designed different structures to generate multiple frequency band gaps and widen band gaps to improve acoustic materials' vibration isolation and noise reduction capabilities. Starkey et al.^[24] presented a thin acoustic metamaterial comprised of rigid metal and air that gives rise to near-unity absorption of airborne sound on resonance. Climente et al.^[25] proposed an acoustic black-hole acting like an omnidirectional broadband absorber with absorbing solid efficiency.

Although the design increases the number of frequency bands and widens band gaps, some frequencies still cannot prevent the wave's vibration, and only partial frequency waves' propagation can be controlled. To broaden band gaps further and satisfy vibration isolation in a different frequency environment, researchers seek to improve vibration isolation capabilities by introducing tunable metamaterials, which can adjust band gaps with the change of noise frequency.

Many mechanisms, such as circuit control^[26], magneto fluid^[27–28], and structural deformation^[29], are introduced to realize adjusting resonance frequency of local resonators, which results in tunable band gaps when subjected to different external environments. Zhang et al.^[30] experimentally demonstrated handedness switching in metamaterials in response to external optical stimuli. Chen et al.^[31] presented a tunable acoustic metamaterial in which an external direct-current voltage can control the band gaps. Ma et al.^[32] took a spatial sound modulator into an acoustic metasurface, consisting of unit cells with two states, switchable through programmed electronics. Although these structures broaden band gaps, they still have limitations. For example, controlling the deformation of the structure by turning the circuit on or off lacks tunability for specific frequencies, and replacing the elastomer with magnetic fluid is difficult to achieve. Therefore, designing a more comprehensive range of metamaterials tuned to adjust different frequencies has become a challenge.

A structure with multiple frequency band gaps is designed in this work, and a shape memory polymer^[33] material is used in this structure. The modulus of this material will decrease with temperature increasing in a specific range. The shape memory polymer replaces the elastomer to affect the structure's band gaps by adjusting temperature. The system can achieve vibration isolation and reduce noise in most different frequencies by tuning temperature.

The paper is organized as follows. Section 2 introduces a mass-in-mass lattice system, then calculates and analyzes the effective negative mass and negative modulus under different structural parameters. Section 3 illustrates the harmonic propagation through this system, discusses the consequences with other structural parameters, and observes the parameter's influence on the harmonic propagation. Section 4 introduces the characteristics of shape memory polymers, and shows different band gaps and harmonic propagations at different temperatures. Finally, the conclusions of this paper's highlights and achievements are given in Section 5.

2 Lattice model and formula derivation

2.1 Analysis of lattice model

Consider a metamaterial system, as shown in Fig. 1(a), whose cell consists of a big rigid body with M and four mass blocks, m_1 , m_2 , m_3 , and m_4 , connected by linear springs with elastic coefficients of k and k_1 . The unit cell n is shown in Fig. 1(b), and n is the horizontal order in the metamaterial system. The structural elements are considered to be deformed in the horizontal direction. Its displacements of different mass blocks are defined as u_L , u_1 , u_2 , u_3 , u_4 , and u_5 , as shown in Fig. 1(c).

Fig. 1 Illustrations of (a) infinite mass-in-mass lattice system, (b) its effective model, and (c) representative unit cell n (color online)

As shown in Fig. 1(c), the governing equations of wave propagation are expressed as

$$\begin{cases} f_L + k(u_1 - u_L) = 0, \\ M\ddot{u}_1 + ku_1 - ku_2 + k(u_1 - u_L) = f_R, \\ m_1\ddot{u}_2 + 2ku_2 + k_1u_2 - ku_1 - ku_3 + k_1u_5 = 0, \\ m_2\ddot{u}_3 + 2ku_3 - ku_2 - ku_4 = 0, \\ m_2\ddot{u}_4 + 2ku_4 - ku_3 - ku_5 = 0, \\ m_1\ddot{u}_5 + ku_5 + k_1u_5 - k_1u_2 - ku_4 = 0, \end{cases} \quad (1)$$

and Eq. (1) can be rewritten as

$$\mathbf{M}\ddot{\mathbf{u}} + \mathbf{K}\mathbf{u} = \mathbf{f}, \quad (2)$$

where the mass matrix is $\mathbf{M} = \text{diag}(0, M, m_1, m_2, m_2, m_1)$, the displacement vector is $\mathbf{u} = (u_L, u_1, u_2, u_3, u_4, u_5)^T$, the force vector is $\mathbf{f} = (f_L, f_R, 0, 0, 0, 0)^T$, the stiffness matrix is

$$\mathbf{K} = \begin{pmatrix} k & -k & 0 & 0 & 0 & 0 \\ -k & 2k & -k & 0 & 0 & 0 \\ 0 & -k & 2k + k_1 & -k & 0 & -k_1 \\ 0 & 0 & -k & 2k & -k & 0 \\ 0 & 0 & 0 & -k & 2k & -k \\ 0 & 0 & -k_1 & 0 & -k & k + k_1 \end{pmatrix}, \quad (3)$$

and the neighbor unit cells are connected by linear springs with an elastic coefficient of k .

It is assumed that these four rigid blocks are much smaller than the rigid frame, and the collisions between the rigid frame and blocks do not occur during the vibration process.

The displacement vector and the force vector can be postulated as

$$\mathbf{u} = \mathbf{U}e^{i\omega t}, \quad \mathbf{f} = \mathbf{F}e^{i\omega t}, \quad (4)$$

where $\mathbf{U} = (U_L, U_1, U_2, U_3, U_4, U_5)^T$ is the wave amplitude vector, $\mathbf{F} = (F_L, F_R, 0, 0, 0, 0)^T$ is the force received by each mass, and ω is the angular frequency.

According to the Bloch theorem, the periodic relationships of the left boundary and right boundary of the unit cell can be expressed as

$$U_R = U_L e^{-iKL}, \quad F_L = F_R e^{-iKL}, \quad (5)$$

where K is the wavenumber, and L is the length of the unit cell. Substitute Eq. (5) into Eq. (4), and then the following equations can be obtained:

$$\mathbf{U} = \mathbf{A}(K)\mathbf{U}_r, \quad \mathbf{F} = \mathbf{B}(K)\mathbf{F}_r, \quad (6)$$

where

$$\begin{cases} \mathbf{U}_r = (u_L, u_1, u_2, u_3, u_4, u_5)^T, & \mathbf{F}_r = (f_L, f_R, 0, 0, 0, 0)^T, \\ \mathbf{A}(K) = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ e^{-iKL} & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}, & \mathbf{B}(K) = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ -e^{-iKL} & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}. \end{cases} \quad (7)$$

Substitute Eq. (6) into Eq. (2). The dispersion equations of the proposed metamaterial model are obtained,

$$|\mathbf{K}_r(K) - \omega^2 \mathbf{M}_r(K)| = 0, \quad (8)$$

where $\mathbf{K}_r(K) = \mathbf{A}^H \mathbf{K} \mathbf{A}$, $\mathbf{M}_r(K) = \mathbf{A}^H \mathbf{M} \mathbf{A}$, and $\mathbf{A}^H$ is the conjugate transposition matrix of $\mathbf{A}$. The wave propagating characters can be obtained by solving this equation. This model can be regarded as a monatomic lattice system, as shown in Fig. 1(b), and then the effective mass expression with the wavenumber of the structure can be obtained,

$$\begin{aligned} \frac{m_{\text{eff}}}{M} &= 1 - \frac{1}{qf^2} + \frac{1}{qf^2} \frac{ab - 1}{(2 + \eta - pf^2)(ab - 1) - (\eta + a)(2 - f^2) - \eta(\eta b + 1)(2 - f^2)} \\ &= \frac{1 - \cos(KL)}{\frac{qf^2}{2}}, \end{aligned} \quad (9)$$

where $a = (1 + \eta - pf^2)(2 - f^2) - 1$, and $b = 3 - 4f^2 + f^4$.

According to the literature of Liu et al.^[34], the expression of effective modulus can be obtained,

$$\frac{E_{\text{eff}}}{E} = 1 - qf^2 \frac{m_{\text{eff}}}{4M}. \quad (10)$$

Define that ω_1 and ω_2 are the natural frequencies of the mass blocks 2 and 3, and f , p , q , and η are given by

$$\frac{k}{m_1} = \omega_1^2, \quad \frac{k}{m_2} = \omega_2^2, \quad \frac{\omega}{\omega_2} = f, \quad \frac{m_1}{m_2} = p, \quad \frac{M}{m_2} = q, \quad \frac{k_1}{k} = \eta.$$

2.2 Analysis of negative mass and negative modulus

As discussed in the literature^[1,7,34–35], frequency ranges of band gaps are identical with frequency ranges in which the single negative property appears. In this part, effective mass and modulus results at different η are obtained and discussed, and it is meaningful for designing metamaterials to deal with different vibration isolation frequencies. According to Eqs. (9) and (10), effective mass and modulus can be calculated. Figure 2 shows that effective mass curves are presented by solid blue lines, while solid orange lines represent effective modulus curves.

First, the lattice structures with mass ratios $p = q = 1$ are studied, and several stiffness ratios ($\eta = 0.5, 1$, and 3) are obtained by adjusting modulus. The results of effective mass and effective modulus with different stiffness ratios are calculated by using Eqs. (9) and (10),

Fig. 2 Comparison of effective mass and effective modulus with different η when $p = q = 1$: (a) $\eta = 0.5$; (b) $\eta = 1$; (c) $\eta = 3$ (color online)

as shown in Figs. 2(a)–2(c). The negative mass and modulus regions are presented by gray and blue areas, respectively. It is observed that frequency ranges of negative mass and negative modulus do not overlap, and they form several band gaps together.

In Figs. 2(a)–2(c), the frequency ranges of effective negative mass/modulus change along with the stiffness ratio η , which displays excellent potential applications in manipulating elastic wave propagation. In detail, with the increase of stiffness ratio η , the first ranges of negative mass/modulus become narrower, and the start frequency of this range moves to a high-frequency direction. This feature guarantees that it has the tunable ability of vibration isolation at a low frequency. It should be pointed out that a special phenomenon appears in these metamaterials. The results of $\eta = 0.5$, 1, and 3 are compared, a band gap disappears in the case of $\eta = 1$, which means that the on/off of this band gap can be controlled by adjusting the stiffness ratio η , and it is of great value in manipulating elastic wave propagation. This phenomenon can be explained by local resonance, which will be illustrated in the next section. In addition, when $\eta = 3$, the second and third band gaps tend to merge. With the increase of k_1 , the structure can be considered as a rigid body, which reduces the number of mass blocks and band gaps.

In summary, the proposed lattice system has many modes in manipulating elastic wave propagation, such as the on/off of the special band gaps and tunable band gaps. After the metastructure is designed, it still has a vast adjusting ability to manipulate elastic waves. The band gaps can be adjusted by changing the value of η , that is, changing the elastic coefficient of k_1 . There are many methods to change the modulus of elastomers, such as voltage regulation, magnetic field regulation, and temperature regulation.

3 Elastic wave propagation

In this section, the characters of elastic waves propagating in the proposed metamaterials are discussed. The dispersion curves of the metamaterials are calculated with Eq. (9), and Fig. 3 displays dispersion curves and wave propagation modes of $p = 1$, $q = 1$, and $\eta = 1, 3$. It can be observed that there are five elastic wave modes and five band gaps in total generally. The dispersion curves of different modes change when the stiffness ratio η changes, especially for the modes 2, 3, 4, and 5. The adjusting function of stiffness ratio η makes it easily realized to manipulate elastic waves and isolate vibration for different frequencies. However, elastic waves of mode 1 change slightly for different stiffness ratios η . This phenomenon can be explained by the wave propagation mode. The displacement curves of mass blocks 1–5 are displayed in Fig. 3(b). For the mode 1, the phase and amplitude of blocks 2, 3, 4, and 5 are almost the same, and they can be regarded as a whole block. The adjusting spring belongs to this big block. Therefore, the mode 1 is affected by the stiffness ratio η very slightly. Manipulating elastic waves of mode 1 can be realized if replacing the spring with the elastic coefficient k_1 with shape memory polymers.

It should be noted that a band gap disappears when $\eta = 1$, as shown in Fig. 3(a), and two dispersion curves (modes 2 and 3) are cut across each other. The dispersion curve of mode 3 (see solid blue line) becomes a horizontal straight line, and the corresponding frequency is $\omega/\omega_2 = 1.414$. The waves' propagation mode can illustrate this phenomenon. As shown in Fig. 3(b), for the dispersion curve of mode 3 (see solid blue line in Fig. 3(a)) in the case of $p = 1$, the corresponding displacement mode at the point C is shown in Fig. 3(b)- C . Only blocks 2 and 5 vibrate with the phase difference of π . The amplitudes of blocks 1, 3, and 4 are zero. In other words, when p , q , and η are all equal to 1, only blocks 2 and 5 vibrate, while other mass blocks remain stationary, which makes the blue dispersion curve a straight line in Fig. 3(a).

One of the tunable metamaterial advantages is tunable ability in service by adjusting the material parameters. After adjusting modulus, the stiffness ratio reaches 3, i.e., $\eta = 3$, and the dash lines show dispersion curves in Fig. 3(a). Propagation characters and band gaps become different. It is observed that the phenomenon of intersecting curves disappears, and the number of band gaps increases. Especially, by comparing Figs. 3(b)- C and 3(b)- C' , the displacement mode changes a lot, and all blocks vibrate in this case, which means traveling wave. Exploring the mechanism of intersecting curves is of great value for manipulating wave propagation, and further analysis is elaborated in the following.

Fig. 3 Comparison of results in the case of $p = 1$, $q = 1$, and $\eta = 1, 3$: (a) dispersion curves, where solid lines are $\eta = 1$, and dash lines are $\eta = 3$; (b) different waves propagating modes (color online)

As shown in Fig. 4, springs and blocks are redefined. From left to right, the elastic coefficients of the springs are k , k_0 , k_1 , k_2 , k_3 , and k_4 , respectively, and the mass blocks are m_1 , m_2 , m_3 , m_4 , and m_5 . According to the previous analysis, when the dispersion curve is a straight line, the frame or the mass block 1 remains stationary. By introducing this condition, the dynamic

Fig. 4 Unit cell with redefined number of blocks and springs (color online)

equations can be obtained,

$$\begin{cases} k_1 u_3 + k_4 u_5 = 0, \\ -k_1 u_3 + k_2(u_4 - u_3) = m_3 \ddot{u}_3, \\ -k_2(u_4 - u_3) + k_3(u_5 - u_4) = m_4 \ddot{u}_4, \\ -k_4 u_5 - k_3(u_5 - u_4) = m_5 \ddot{u}_5. \end{cases} \quad (11)$$

Equation (11) is over-determined equations, and the solution exists when two of the equations are constant. Then, the conditions that guarantee a straight dispersion curve existing can be obtained,

$$\begin{cases} k_1 k_3 - k_2 k_4 = 0, \\ m_5(k_1 + k_2) - m_3(k_3 + k_4) = 0. \end{cases} \quad (12)$$

When Eq. (12) is satisfied, one mode of the dispersion curves can be a straight line. Substitute Eq. (12) into Eq. (9). The frequency of the straight dispersion curve can be obtained. For $p = 1$, $q = 1$, and $\eta = 1$, the springs' stiffness and the mass of blocks satisfy Eq. (12), and the frequency of the straight dispersion curve is

$$\frac{\omega}{\omega_2} = \sqrt{\frac{k_4 + k_3}{m_5}} = \sqrt{\frac{k_1 + k_2}{m_3}}.$$

According to this formula, bring in these parameters from Fig. 3, and we can get that the characteristic frequency $\omega/\omega_2 = 1.414$, which agrees with the numerical results in Fig. 3(a). Through Eq. (12), the frequency of the straight dispersion curve can be designed, or the straight line can exist or disappear, to adjust band gaps.

In the following part, dispersion curves and their mode displacement shapes under a different case are verified. The metamaterial of $k_1 = k_2 = 3 = k_4$, $m_3 = 2$ is discussed, and the other parameters are equal to 1, which satisfies Eq. (12). The results of this case are shown in Fig. 5. It can be seen that the stiffness and mass of components in the unit cell conform to Eq. (12), and the crossing phenomenon is found on the dispersion curves (see solid lines) in Fig. 5(a), which verifies that satisfying Eq. (12) is the condition of this phenomenon. By adjusting the value of k_4 , the width and quantity of the frequency band can be controlled, and in this case, the frequency of the straight dispersion curve is $\omega/\omega_2 = 2$, which intersects with the mode 3 (see solid green line). For this case, when changing k_4 by adjusting temperature, the straight dispersion curve disappears, and a new band gap appears, as shown in Fig. 5(a) (see dash lines).

In addition, in the dispersion curve, the vibration modes of the points C and D' are close, resulting in the approximate overlap of the two curves.

By comparing the vibration modes of the point A in Figs. 3(b) and 5(b), it is found that, except for the vibration of the outer frame, the other mass blocks can be regarded as simple harmonic vibration with the same amplitude and the same phase, which can be considered as

Fig. 5 Comparison of results in the case of $k_1 = k_2 = 3$, $k_4 = 3, 1$: (a) dispersion curves, where solid lines are $k_4 = 3$, and dash lines are $k_4 = 1$; (b) different waves propagating modes (color online)

a whole. When changing the value of k_4 , the influence on the vibration mode of the point A is very weak. Therefore, the first mode of dispersion curves almost remains the same when k_4 changes, as shown in Figs. 3(a) and 5(a). Figure 4 shows that, when internal mass blocks are regarded as a whole, the band gaps' elastic moduli are k and k_0 . Therefore, if we want to adjust the range of the lowest frequency band, we can adjust the range by changing the modulus of the elastomer between the units or changing the modulus of the first spring connected internally.

4 Tunable structure model

A tunable structure model is proposed based on the lattice system above. As shown in Fig. 6(a), dark blue represents the mass block with high mass density and Young's modulus. Elastomer I takes elastic materials with the elastic modulus $E = 500$ MPa, and the yellow part is shape memory polymers^[33]. The elastic modulus of the shape memory polymer material will change with the influence of temperature. As shown in Fig. 6(b), for the convenience of calculation, the temperature-elastic modulus curve of the material is expressed as a line.

According to the model proposed in Ref. [33], Young's modulus of shape memory polymers can be expressed as

$$E_0 = -150T + 9000, \quad (13)$$

where E_0 is the modulus of shape memory polymers with the unit of MPa, and T is external temperature. Consider Elastomer I, and η can be described as

$$\eta = \frac{E_0 \cdot A_0 / L_0}{E \cdot A_1 / L_1} = -0.3T + 18, \quad (14)$$

Fig. 6 (a) Continuum metamaterial model and (b) modulus of shape memory polymers^[33] (color online)

where A_0 and A_1 are the areas, and L_0 and L_1 are the lengths of the elastomer. For the convenience of calculation, the ratio $\frac{A_0/L_0}{A_1/L_1}$ here is equal to 1.

Substitute Eq. (14) into Eq. (9), and then the formula expression of effective mass can be obtained,

$$\begin{aligned} \frac{m_{\text{eff}}}{M} &= 1 - \frac{1}{qf^2} + \frac{1}{qf^2} \\ &\cdot \frac{ab - 1}{(2 + 18 - 0.3T - pf^2)(ab - 1) - (18 - 0.3T + a)(2 - f^2) - (18 - 0.3T)((18 - 0.3T)b + 1)(2 - f^2)} \\ &= \frac{1 - \cos(KL)}{\frac{qf^2}{2}}, \end{aligned} \quad (15)$$

where $a = (1 + 18 - 0.3T - pf^2)(2 - f^2) - 1$, and $b = 3 - 4f^2 + f^4$. As we all know, the frequency ranges of band gaps are identical with frequency ranges, in which the single negative property appears. For the structure in this paper, band gaps' frequency ranges correspond to negative mass or negative modulus, and for Eq. (15), the solution of band gaps is transforming to that with KL being imaginary, i.e., the conditions of vibration isolation can be rewritten as, $|\cos(KL)| = 1 - \frac{qf^2 m_{\text{eff}}}{2M} > 1$. The conditions of sound isolation can be rewritten as

$$1 - \frac{qf^2}{2} \frac{m_{\text{eff}}}{M} < -1, \quad (16)$$

or

$$1 - \frac{qf^2}{2} \frac{m_{\text{eff}}}{M} > 1, \quad (17)$$

where Eq. (17) is true in the case of negative mass, while Eq. (16) is true in the case of negative modulus. Based on Eq. (15), dispersion curves can be calculated for different temperatures. As Fig. 7 shows, the horizontal axis and vertical axis are frequency and temperature, respectively, and the intensity of color stands for the value of KL/π , whose range is $[0, 1]$.

Figure 7 provides dispersion curves in the case of $p = 1$ and $q = 1, 3$, and 5 . In the case of $q = 1$ in Fig. 7(a), the white area exhibits that the wave cannot propagate. From Fig. 7(a), the variation of band gaps for different temperatures is expressed clearly, and band gaps will move towards the low-frequency direction with the increase of temperature. At the same time, the width of band gaps changes at different temperatures. Figure 7(a) directly displays the adjusting function of temperature for band gaps, which is meaningful for designing metamaterials that isolate vibration in the required frequency range. It should be noted that the second and third frequency bands are merged at 57°C . At 57°C , the stiffness ratio is $\eta = 1$, which satisfies Eq. (12), and two dispersion curves intersect each other. Therefore, the intersection dispersion curves can be realized by adjusting temperature after metamaterials are fixed.

Fig. 7 Wavenumbers at different temperatures and frequencies when $p = 1$: (a) $q = 1$; (b) $q = 3$; (c) $q = 5$ (color online)

According to the size of the structure during manufacturing, band gaps are adjusted mainly by changing the value of q . Compared with Figs. 7(a), 7(b), and 7(c), it is evident that the larger the value of q , the wider the vibration isolation band gap. However, the width of the vibration isolation band gap in the low-frequency region will become narrow with the increase of q . Different values of q can cause different areas of frequency band gap. For the frequency range of vibration isolation, the value of q is designed, and then temperature is adjusted to deal with the different frequency excitation, which can achieve vibration isolation and noise reduction for most frequency ranges. When q is large enough, compared with other blocks in the frame, its mass is large enough, and it is reasonable to ignore other mass blocks. Therefore, the number of propagation mode curves decreases to 2 in Fig. 7(c).

5 Conclusions

This paper proposes a new structure of one-dimensional acoustic metamaterials, which possesses negative mass or modulus, resulting in band gaps. By adjusting the structure's parameters, the band gaps' position and width can be changed. Furthermore, the shape memory polymer is introduced as a soft material (spring) in the designed structure, leading to the tunable ability of the new metamaterial, and by adjusting temperature, Young's modulus of shape memory polymers can be changed, which contributes to varied band gaps, and wave propagation is discussed by dispersion analysis and displacement mode. The special phenomenon of intersecting curves is investigated whose designing principle is obtained. Finally, a continuum metamaterial model is proposed, and band gaps with different environment temperatures are obtained.

Open Access This article is licensed under a Creative Commons Attribution 4.0 International License, which permits use, sharing, adaptation, distribution and reproduction in any medium or format, as long as you give appropriate credit to the original author(s) and the source, provide a link

to the Creative Commons licence, and indicate if changes were made. To view a copy of this licence, visit <http://creativecommons.org/licenses/by/4.0/>.

References

- [1] WALSER, R. M. Metamaterials: what are they? what are they good for? *APS March Meeting*, American Physical Society, Ridge (2000)
- [2] LU, L., RU, C. Q., and GUO, X. M. Negative effective mass of a filled carbon nanotube. *International Journal of Mechanical Sciences*, **134**, 174–181 (2017)
- [3] WU, B., GUO, X. M., and RU, C. Q. Reduced vibrational frequencies of multiwall carbon nanotubes due to interlayer degrees of freedom. *European Journal of Mechanics-A/Solids*, **47**, 206–210 (2014)
- [4] HU, Z. L., GUO, X. M., and RU, C. Q. Enhanced critical pressure for buckling of carbon nanotubes due to an inserted linear carbon chain. *Nanotechnology*, **19**(30), 305703 (2008)
- [5] HU, Z. L., GUO, X. M., and RU, C. Q. The effects of an inserted linear carbon chain on the vibration of a carbon nanotube. *Nanotechnology*, **18**(48), 485712 (2007)
- [6] VESELAGO, V. G. The electrodynamics of substances with simultaneously negative values of ϵ and μ . *Soviet Physics Uspekhi*, **10**(4), 509–514 (1968)
- [7] CRASTER, R. V. and GUENNEAU, S. *Acoustic Metamaterials: Negative Refraction, Imaging, Lensing and Cloaking*, Springer, the Netherlands (2013)
- [8] LI, Y., LIANG, B., GU, Z. M., ZOU, X. Y., and CHENG, J. C. Reflected wavefront manipulation based on ultrathin planar acoustic metasurfaces. *Scientific Reports*, **3**, 2546 (2013)
- [9] LI, Y., JIANG, X., LI, R. Q., LIANG, B., ZOU, X. Y., YIN, L. L., and CHENG, J. C. Experimental realizations of full control of reflected waves with subwavelength acoustic metasurfaces. *Physical Review Applied*, **2**(6), 064002 (2014)
- [10] ZHU, Y. F., ZOU, X. Y., LI, R. Q., JIANG, X., TU, J., LIANG, B., and CHENG, J. C. Dispersionless manipulation of reflected acoustic wavefront by subwavelength corrugated surface. *Scientific Reports*, **5**, 10966 (2015)
- [11] ZHAO, J. J., LI, B. W., CHEN, Z. N., and QIU, C. W. Redirection of sound waves using acoustic metasurface. *Applied Physics Letters*, **103**(15), 151604 (2013)
- [12] SHELBY, R. A., SMITH, D. R., and SCHULTZ, S. Experimental verification of a negative index of refraction. *Science*, **292**, 77–79 (2001)
- [13] XIE, Y. B., WANG, W. Q., CHEN, H. Y., KONNEKER, A., POPA, B. I., and CUMMER, S. A. Wavefront modulation and subwavelength diffractive acoustics with an acoustic metasurface. *Nature Communications*, **5**, 5553 (2014)
- [14] LIU, Z. Y., ZHANG, X. X., MAO, Y. W., ZHU, Y. Y., YANG, Z. Y., CHAN, C. T., and SHENG, P. Locally resonant sonic materials. *Science*, **289**, 1734–1736 (2000)
- [15] HUSSEIN, M. I., HULBERT, G. M., and SCOTT, R. A. Dispersive elastodynamics of 1D banded materials and structures: design. *Journal of Sound and Vibration*, **307**, 865–893 (2007)
- [16] KHELIF, A., CHOUJAA, A., and BENCHABANE, S. Guiding and bending of acoustic waves in highly confined photonic crystal waveguides. *Applied Physics Letters*, **84**(22), 4400–4402 (2004)
- [17] LUCKLUM, R. and LI, J. Phononic crystals for liquid sensor applications. *Measurement Science & Technology*, **20**(12), 124014 (2009)
- [18] LU, L., RU, C. Q., and GUO, X. M. Vibration isolation of few-layer graphene sheets. *International Journal of Solids and Structures*, **185–186**, 78–88 (2020)
- [19] MILTON, G. W. and WILLIS, J. R. On modifications of Newton's second law and linear continuum elastodynamics. *Proceedings of the Royal Society A*, **463**, 855–880 (2007)
- [20] CUMMER, S. A., CHRISTENSEN, J., and ALÙ, A. Controlling sound with acoustic metamaterials. *Nature Reviews Materials*, **1**(3), 16001 (2016)
- [21] AN, X. Y., LAI, C. L., FAN, H. L., and ZHANG, C. Z. 3D acoustic metamaterial-based mechanical metalattice structures for low-frequency and broadband vibration attenuation. *International Journal of Solids and Structures*, **191**, 293–306 (2020)

- [22] ZHANG, Y. Y., GAO, N. S., and WU, J. H. New mechanism of tunable broadband in local resonance structures. *Applied Acoustics*, **169**, 107482 (2020)
- [23] HU, X. H., HO, K. M., CHAN, C. T., and ZI, J. Homogenization of acoustic metamaterials of Helmholtz resonators in fluid. *Physical Review*, **77**, 21–24 (2008)
- [24] STARKEY, T. A., SMITH, J. D., HIBBINS, A. P., SAMBLES, J. R., and RANEE, H. J. Thin structured rigid body for acoustic absorption. *Applied Physics Letters*, **110**, 041902 (2017)
- [25] CLIMENTE, A., TORRENT, D., and SÁNCHEZ-DEHESA, J. Omnidirectional broadband acoustic absorber based on metamaterials. *Applied Physics Letters*, **100**, 144103 (2012)
- [26] ZHU, R., CHEN, Y. Y., BARNHART, M. V., HU, G. K., SUN, C. T., and HUANG, G. L. Experimental study of an adaptive elastic metamaterial controlled by electric circuits. *Applied Physics Letters*, **108**, 1734–1736 (2016)
- [27] SHIGA, T., OKADA, A., and KURAUCHI, T. Magnetorheological behavior of composite gels. *Journal of Applied Polymer Science*, **58**(4), 787–792 (1995)
- [28] YOON, J. Y., HONG, S. W., PARK, Y. J., KIM, S. H., KIM, G. W., and CHOI, S. B. Tunable Young's moduli of soft composites fabricated from magnetorheological materials containing micro-sized iron particles. *Materials*, **13**, 3378 (2020)
- [29] CHEN, H. J., ZHAO, W. X., and HAO, C. C. Tunable broadband sound absorbing metamaterial with negative modulus. *Chinese Journal of Liquid Crystals and Displays*, **30**, 234–239 (2015)
- [30] ZHANG, S., ZHOU, J. F., PARK, Y. S., RHO, J., SINGH, R., NAM, S., AZAD, A. K., CHEN, H. T., YIN, X. B., TAYLOR, A. J., and ZHANG, X. Photoinduced handedness switching in terahertz chiral metamolecules. *Nature Communications*, **3**, 942–949 (2012)
- [31] CHEN, Z., XUE, C., FAN, L., ZHANG, S. Y., LI, X. J., ZHANG, H., and DING, J. A tunable acoustic metamaterial with double-negativity driven by electromagnets. *Scientific Reports*, **6**, 30254 (2016)
- [32] MA, G., FAN, X., SHENG, P., and FINK, M. Shaping reverberating sound fields with an actively tunable metasurface. *Proceedings of the National Academy of Sciences*, **115**, 6638–6643 (2018)
- [33] WANG, X. J., GUO, X. G., YE, J. L., ZHENG, N., KOHLI, P., CHOI, D., ZHANG, Y., XIE, Z. Q., ZHANG, Q. H., LUAN, H. W., NAN, K. W., KIM, B. H., XU, Y. M., SHAN, X. W., BAI, W. B., SUN, R. J., WANG, Z. Z., JANG, H., ZHANG, F., MA, Y. J., XU, Z., FENG, X., XIE, T., HUANG, Y. G., ZHANG, Y. H., and ROGERS, J. A. Freestanding 3D mesostructures, functional devices, and shape-programmable systems based on mechanically induced assembly with shape memory polymers. *Advanced Materials*, **31**, 1805615 (2019)
- [34] LIU, Y. Q., SU, X. Y., and SUN, C. T. Broadband elastic metamaterial with single negativity by mimicking lattice systems. *Journal of the Mechanics and Physics of Solids*, **74**, 158–174 (2015)
- [35] YI, J. L., NEGAHBAN, M., LI, Z., SU, X. Y., and XIA, R. Y. Conditionally extraordinary transmission in periodic parity-time symmetric phononic crystals. *International Journal of Mechanical Sciences*, **163**, 105134 (2019)