Research on coupled thermo-hydro-mechanical dynamic response characteristics of saturated porous deep-sea sediments under vibration of mining vehicle*

Wei ZHU¹, Xinyu SHI², Rong HUANG³, Liyue HUANG⁴, Wenbo MA^{4,†}

1. Post-doctoral Research Station of Statistics, School of Mathematics and Computational Science, Xiangtan University, Xiangtan 411105, Hunan Province, China;
2. School of Mathematics and Computational Science, Xiangtan University, Xiangtan 411105, Hunan Province, China;
3. School of Mathematic and Computational Science, Hunan University of Science and Technology, Xiangtan 411201, Hunan Province, China;
4. College of Civil Engineering and Mechanics, Xiangtan University, Xiangtan 411105, Hunan Province, China

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Abstract The excessive deformation of deep-sea sediments caused by the vibration of the mining machine will adversely affect the efficiency and safety of mining. Combined with the deep-sea environment, the coupled thermo-hydro-mechanical problem for saturated porous deep-sea sediments subject to the vibration of the mining vehicle is investigated. Based on the Green-Lindsay (G-L) generalized thermoelastic theory and Darcy's law, the model of thermo-hydro-mechanical dynamic responses for saturated porous deep-sea sediments under the vibration of the mining vehicle is established. We obtain the analytical solutions of non-dimensional vertical displacement, excess pore water pressure, vertical stress, temperature, and change in the volume fraction field with the normal mode analysis method, and depict them graphically. The normal mode analysis method uses the canonical coordinate transformation to solve the equation, which can quickly decouple the equation by ignoring the modal coupling effect on the basis of the canonical mode. The results indicate that the vibration frequency has obvious influence on the vertical displacement, excess pore water pressure, vertical stress, and change in volume fraction field. The loading amplitude has a great effect on the physical quantities in the foundation, and the changes of the physical quantities increase with the increase in loading amplitude.

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† Corresponding author, E-mail: mawenbo@xtu.edu.cn

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1 Introduction

With the rapid development of the world economy, people have higher and higher requirements for the quality of material life, and the mineral resources on the land can no longer meet the growing needs of human beings^[1–4]. The deep sea contains abundant and unexploited mineral resources, such as polymetallic nodules, cobalt-rich crusts, and hydrothermal sulfides, and thus people began to turn their attention to the ocean^[5–6]. Furthermore, the mineral deposits of the Pacific Ocean are extremely rich. At the end of the 19th century, the United States, Mexico, Colombia, and other countries had already exploited mineral resources here. Since the 1990s, China has conducted more than 10 surveys in the Clarion-Clipperton (CC) area of the East Pacific Basin^[7]. Therefore, deep-sea resource extraction technology is one of the important topics of international research. In order to obtain deep-sea mineral resources efficiently, safely, and environmentally, the mining technology of seabed minerals needs to be continuously improved and perfected^[8–9]. At present, the typical mining systems include continuous chain bucket method, shuttle boat type and hydraulic, and pipeline lifting type, among which the pipeline lifting type is the most widely used^[10–11]. The deep-sea pipeline lifting mining system is shown in Fig. 1^[12–13]. The walking technology of the mining machine is one of the key technologies of the deep-sea mining system^[14]. However, the vibration of the mining vehicle will cause the displacement of soil particles, resulting in the sharp rise of pore water pressure and the decrease of effective stress in a short period of time, thus causing soil liquefaction. Soil liquefaction will cause the mining machine to sink and slip, which will have an adverse effect on the efficiency and safety of mining^[15–16]. Therefore, it is of great significance to study the influence of the vibration of the mining machine on the deep-sea sediments.

Fig. 1 Deep-sea pipeline lifting mining system (color online)

Since the 1970s, the mining activities of polymetallic ocean nodules have become more frequent, and the environmental problems that may arise from deep-sea mining have attracted widespread attention from the international community^[17–18]. Therefore, the mining of deep-

sea minerals not only needs to improve its safety and efficiency, but also needs to improve its environmental protection. The deep sea has an extremely rich variety of microorganisms, and each organism has its own suitable living temperature^[19–21]. In order to reduce the damage of mineral mining to the deep-sea environment, we should also consider the temperature changes produced during the mining process. In 1956, Biot^[22] proposed the classical thermoelastic theory. Lord and Shulman^[23] (L-S) and Green and Lindsay^[24] (G-L) modified it by introducing thermal relaxation time, and obtained the L-S generalized thermoelasticity theory and the G-L generalized thermoelasticity theory, respectively. Green and Naghdi^[25–27] established a G-N generalized thermoelastic theory without energy dissipation. In recent years, the problem of thermal-hydro-mechanical coupling has become a hot topic. Guo et al.^[28] studied the influence of the L-S generalized thermoelasticity theory and G-L generalized thermoelasticity theory on thermal-hydro-mechanical coupling problems. Liu et al.^[29] used the Fourier transform and inverse transform to analyze the problem of thermal-hydro-mechanical coupling dynamic response. Wang et al.^[30] proposed an algebraic multigrid iteration scheme for non-symmetric matrix equations to solve the thermal-hydro-mechanical coupling problem. Lu et al.^[31] studied the effects of temperature on the stress, displacement, and pore water pressure of saturated porous foundations on the basis of the generalized thermoelastic theory. Bai^[32] studied the wave response of saturated porous media under a cyclic thermal load by using the Laplace transform and its inverse transformation. Chen et al.^[33] proposed a new thermal-hydro-mechanical coupling model by using the finite element method. Xiong et al.^[34–35] studied the dynamic response of the porous saturated elastic foundation under the action of elliptic cosine wave, load, and porosity anisotropy. Qin et al.^[36] studied the thermal-hydro-mechanical coupling characteristics of unsaturated soils.

In this study, based on the G-L generalized thermoelastic theory and Darcy's law, the thermal-hydro-mechanical coupled dynamic response of saturated porous deep-sea sediments under the vibration of the mining machine is studied. The analytic solution to the problem is obtained by using the normal mode analysis method, and the influence of the load frequency and load amplitude of the mining machine on the vertical displacement, vertical stress, excess pore water pressure, temperature, and change in volume fraction field is analyzed. The solution provided in this paper is applicable in deep-sea mining and has guiding significance for optimizing the mining system.

2 Governing equation

2.1 Basic assumptions

In this paper, deep-sea sediments are used as saturated porous elastic foundations to study the influence of the thermal-hydro-mechanical coupling dynamic response of saturated porous foundations under the vibration of a mining machine. The basic assumptions are as follows.

- (i) The saturated porous deep-sea sediment is a fluid-solid coupled two-phase medium, the internal solid particles are incompressible, and the influence of solute is not considered.
- (ii) The foundation is a homogeneous and isotropic two-dimensional semi-infinite elastic body.
- (iii) The deformation of the foundation is small.
- (iv) Pore water seepage obeys Darcy's law.
- (v) It satisfies the G-L generalized thermoelasticity theory.

2.2 Basic governing equation^[34,37]

The strain-displacement relation is

$$\varepsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}), \quad (1)$$

where $i, j = x, z$; a comma followed by a suffix denotes a derivative with respect to the material coordinates; ε_{ij} are components of the strain tensor; u_i are the displacement vector components.

The constitutive equations are

$$\sigma_{ij} = 2G\varepsilon_{ij} + (\lambda e - \beta_1(\theta + \tau_1\dot{\theta}) - P + c\phi)\delta_{ij}, \quad (2)$$

$$s = -ce - \xi\phi + l\theta, \quad (3)$$

$$h_i = \gamma\phi_{,i}, \quad (4)$$

$$h_{i,i} + s = \rho\chi\ddot{\phi}, \quad (5)$$

where the superposed dot indicates the derivative with respect to time; σ_{ij} are components of the stress tensor; λ and G are Lamé's constants; e is the cubical dilatation; $\beta_1 = (3\lambda + 2G)\alpha_s$, in which α_s is the coefficient of linear thermal expansion of solid grains; $\theta = T - T_0$, in which T is the absolute temperature, and T_0 is the reference uniform temperature of the body; τ_1 is the thermal relaxation time; P is the excess pore water pressure; c, ξ , and γ are the pore material constants; χ is the equilibrated inertia; ϕ is the change in the volume fraction field; δ_{ij} are the Kronecker delta functions; s is the intrinsic equilibrated body force; l is the thermo-pore coefficient; h_i are the components of equilibrated stress vector; ρ is the density of the medium, $\rho = n_0\rho_w + (1 - n_0)\rho_s$, in which n_0 is the porosity, ρ_w is the density of water, and ρ_s is the density of solid grains.

The stress equation of motion is

$$\sigma_{ij,j} + F_i = \rho\ddot{u}_i, \quad (6)$$

where F_i are the body forces.

The motion equation is

$$Gu_{i,jj} + (\lambda + G)u_{j,ij} - \beta_1(\theta_{,i} + \tau_1\dot{\theta}_{,i}) - P_{,i} + c\phi_{,i} = \rho\ddot{u}_i. \quad (7)$$

The volume fraction field equation is

$$\gamma\nabla^2\phi - cu_{i,i} - \xi\phi + l\theta = \rho\chi\ddot{\phi}. \quad (8)$$

The heat conduction equation is

$$m(\dot{\theta} + \tau_2\ddot{\theta}) + \beta_1T_0\dot{e} = K\theta_{,ii}, \quad (9)$$

where $m = n_0\rho_w c_w + (1 - n_0)\rho_s c_s$, in which c_w is the specific heat with a constant strain of pore water, and c_s is the specific heat with a constant strain of solid grains; τ_2 is the thermal relaxation time; K is the coefficient of thermal conductivity.

The mass conservation equation of water is

$$b(\alpha_u\dot{\theta} - \dot{e}) - \rho_w\ddot{e} + P_{,ii} = 0, \quad (10)$$

where $b = \rho_w g/k_d$, in which g is the gravitational acceleration, and k_d is the coefficient of permeability; $\alpha_u = n_0\alpha_w + (1 - n_0)\alpha_s$, in which α_w is the coefficient of linear thermal expansion of pore water.

2.3 Formulation of the problem

In this paper, combined with the generalized thermoelastic theory and Darcy's law, the thermal-hydro-mechanical coupling problem is studied. Based on the assumption of a homogeneous, isotropic, and two-dimensional semi-infinite elastic body, the space variables x and z and the time variable t will be used to represent the foundation. As shown in Fig. 2, the x -axis is the horizontal direction, and the z -axis points vertically inwards. The Cartesian coordinate system (x, y, z) is introduced, and the displacement component is expressed as $u_i = (u, 0, w)$.

The displacement component can be written as

$$u_x = u(x, z, t), \quad u_y = 0, \quad u_z = u(x, z, t). \quad (11)$$

According to Eqs. (1) and (11), the strain tensor can be written as

$$e_{xx} = \frac{\partial u}{\partial x}, \quad e_{zz} = \frac{\partial w}{\partial z}, \quad e_{xz} = \frac{1}{2}\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right), \quad e_{xy} = e_{yy} = e_{yz} = 0, \quad (12)$$

Fig. 2 Model of problem

where the cubical dilatation e can be expressed as

$$e = \frac{\partial u}{\partial x} + \frac{\partial w}{\partial z}. \quad (13)$$

In order to make the calculation more convenient and universal, the following dimensionless quantities are introduced^[28]:

$$\begin{cases} (x', z', u', w') = V\eta(x, z, u, w), & (t', \tau'_1, \tau'_2) = V^2\eta(t, \tau_1, \tau_2), & \theta' = \frac{\beta_1\theta}{\lambda + 2G}, \\ P' = \frac{P}{\lambda + 2G}, & \sigma'_{ij} = \frac{\sigma_{ij}}{G}, & \phi' = V^2\eta^2\phi, & \eta = \frac{m}{K}, & V = \sqrt{\frac{\lambda + 2G}{\rho}}. \end{cases} \quad (14)$$

According to Eq. (14), Eqs. (2), (7), (8), (9), and (10) can be simplified to

$$\sigma_{xx} = 2\frac{\partial u}{\partial x} + \frac{\lambda}{G}e - \beta^2\left(\theta + \tau_1\frac{\partial\theta}{\partial t} + P\right) + \varphi_1\phi, \quad (15)$$

$$\sigma_{zz} = 2\frac{\partial w}{\partial z} + \frac{\lambda}{G}e - \beta^2\left(\theta + \tau_1\frac{\partial\theta}{\partial t} + P\right) + \varphi_1\phi, \quad (16)$$

$$\sigma_{xz} = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}, \quad (17)$$

$$\nabla^2 u + (\beta^2 - 1)\frac{\partial e}{\partial x} - \beta^2\left(\frac{\partial\theta}{\partial x} + \tau_1\frac{\partial^2\theta}{\partial x\partial t} + \frac{\partial P}{\partial x}\right) + \varphi_2\beta^2\frac{\partial\phi}{\partial x} = \beta^2\frac{\partial^2 u}{\partial t^2}, \quad (18)$$

$$\nabla^2 w + (\beta^2 - 1)\frac{\partial e}{\partial z} - \beta^2\left(\frac{\partial\theta}{\partial z} + \tau_1\frac{\partial^2\theta}{\partial z\partial t} + \frac{\partial P}{\partial z}\right) + \varphi_2\beta^2\frac{\partial\phi}{\partial z} = \beta^2\frac{\partial^2 w}{\partial t^2}, \quad (19)$$

$$\nabla^2\phi - \varphi_3e - \varphi_4\phi + \varphi_5\theta = \varphi_6\frac{\partial^2\phi}{\partial t^2}, \quad (20)$$

$$\nabla^2\theta = \left(\frac{\partial\theta}{\partial t} + \tau_2\frac{\partial^2\theta}{\partial t^2}\right) + \varphi_7\frac{\partial e}{\partial t}, \quad (21)$$

$$\nabla^2 P = \varphi_8\frac{\partial e}{\partial t} + \varphi_9\frac{\partial\theta}{\partial t} + \varphi_{10}\frac{\partial^2 e}{\partial t^2}, \quad (22)$$

where

$$\begin{aligned} \varphi_1 &= \frac{c\rho}{G\eta^2(\lambda + 2G)}, & \varphi_2 &= \frac{c\rho}{G\beta^2\eta^2(\lambda + 2G)}, & \varphi_3 &= \frac{c}{\gamma}, & \varphi_4 &= \frac{\xi\rho}{\gamma\eta^2(\lambda + 2G)}, \\ \varphi_5 &= \frac{l(\lambda + 2G)}{\gamma\beta_1}, & \varphi_6 &= \frac{\chi(\lambda + 2G)}{\gamma}, & \varphi_7 &= \frac{T_0\beta_1^2}{m(\lambda + 2G)}, & \varphi_8 &= \frac{b}{\eta(\lambda + 2G)}, \\ \varphi_9 &= -\frac{b\alpha_u}{\eta\beta_1}, & \varphi_{10} &= -\frac{\rho_w}{\rho}, & \beta^2 &= \frac{\lambda + 2G}{G}. \end{aligned}$$

Differentiating Eq. (18) with respect to x and Eq. (19) with respect to z and then adding

them together yield

$$\left(\nabla^2 - \frac{\partial^2}{\partial t^2}\right)e - \nabla^2 P - \left(1 + \tau_1 \frac{\partial}{\partial t}\right)\nabla^2 \theta + \varphi_2 \nabla^2 \phi = 0. \quad (23)$$

2.4 Boundary conditions

This paper studies the thermal-hydro-mechanical coupling problem when the surface of the saturated porous seabed sediment is subject to the vibration of the mining machine. When $z = 0$, the boundary conditions of the foundation surface are as follows.

(i) The surface of the foundation is subject to mechanical loads, i.e.,

$$\sigma_{xz} = 0, \quad \sigma_{zz} = -qQ(x, t), \quad (24)$$

$$Q(x, t) = Q^*(a, \omega)\exp(\omega t + iax), \quad (25)$$

where q is the magnitude of the applied mechanical force; $Q(x, t)$ is the distribution function of the load along the x -axis; Q^* is the load amplitude; $i = \sqrt{-1}$.

(ii) The temperature load is not considered on the foundation surface as

$$\theta = 0. \quad (26)$$

(iii) The foundation surface is permeable as

$$P = 0. \quad (27)$$

3 Normal mode analysis

The solution of each physical quantity can be decomposed in lights of normal mode analysis method in the following form:

$$(u, w, e, \theta, \sigma_{ij}, P, \phi) = (u^*(z), w^*(z), e^*(z), \theta^*(z), \sigma_{ij}^*(z), P^*(z), \phi^*(z))\exp(\omega t + iax), \quad (28)$$

where ω is the frequency; a is a constant number of waves in the x -direction; $u^*(z), w^*(z), e^*(z), \theta^*(z), \sigma_{ij}^*(z), P^*(z)$, and $\phi^*(z)$ are the amplitudes of the field quantities.

Substituting Eq. (28) into Eqs. (20)–(23), we can get

$$(D^2 - a^2 - \omega^2)e^*(z) = (D^2 - a^2)(P^*(z) + (1 + \tau_1\omega)\theta^*(z) - \varphi_2\phi^*(z)), \quad (29)$$

$$(D^2 - a^2 - \varphi_4 - \varphi_6\omega^2)\phi^*(z) = \varphi_3e^*(z) - \varphi_5\theta^*(z), \quad (30)$$

$$(D^2 - a^2 - \omega(1 + \tau_2\omega))\theta^*(z) = \varphi_7\omega e^*(z), \quad (31)$$

$$(D^2 - a^2)P^*(z) = (\varphi_8\omega + \varphi_{10}\omega^2)e^*(z) + \varphi_9\omega\theta^*(z), \quad (32)$$

where $D = \frac{\partial}{\partial z}$.

The partial differential equation satisfied by $e^*(z)$ is obtained by eliminating $\theta^*(z), P^*(z)$, and $\phi^*(z)$,

$$(D^6 - AD^4 + BD^2 - C)e^*(z) = 0. \quad (33)$$

Similarly, we can get the partial differential equations that $\theta^*(z), P^*(z)$, and $\phi^*(z)$ satisfy

$$(D^6 - AD^4 + BD^2 - C)\theta^*(z) = 0, \quad (34)$$

$$(D^6 - AD^4 + BD^2 - C)P^*(z) = 0, \quad (35)$$

$$(D^6 - AD^4 + BD^2 - C)\phi^*(z) = 0, \quad (36)$$

where $A = 3a^2 + \omega + \omega^2 + \tau_2\omega^2 + \varphi_4 + \varphi_6\omega^2 + \varphi_8\omega + \varphi_{10}\omega^2 + \varphi_7\omega + \varphi_7\tau_1\omega^2 - \varphi_2\varphi_3$, $B = (a^2 + \omega^2)(a^2 + \omega + \tau_2\omega^2) + (a^2 + \varphi_4 + \varphi_6\omega^2)(2a^2 + \omega + \omega^2 + \tau_2\omega^2) + (\varphi_8\omega + \varphi_{10}\omega^2)(2a^2 + \omega + \tau_2\omega^2 + \varphi_4 + \varphi_6\omega^2) - \varphi_7\varphi_9\omega^2 + (\varphi_7\omega + \varphi_7\tau_1\omega^2)(2a^2 + \varphi_4 + \varphi_6\omega^2) - \varphi_2\varphi_3(2a^2 + \omega + \tau_2\omega^2) - \varphi_2\varphi_5\varphi_7\omega$, and $C = (a^2 + \varphi_4 + \varphi_6\omega^2)(a^2 + \omega^2)(a^2 + \omega + \tau_2\omega^2) + (a^2 + \omega + \tau_2\omega^2)(a^2 + \varphi_4 + \varphi_6\omega^2)(\varphi_8\omega + \varphi_{10}\omega^2) - \varphi_7\varphi_9\omega^2(a^2 + \varphi_4 + \varphi_6\omega^2) + a^2(\varphi_7\omega + \varphi_7\tau_1\omega^2)(a^2 + \varphi_4 + \varphi_6\omega^2) - \varphi_2\varphi_3a^2(a^2 + \omega + \tau_2\omega^2) - \varphi_2\varphi_5\varphi_7a^2\omega$.

Equation (33) can be expressed as

$$(D^2 - k_1^2)(D^2 - k_2^2)(D^2 - k_3^2)e^*(z) = 0, \quad (37)$$

where k_i^2 ($i = 1, 2, 3$) are the roots of the characteristic equation $k^6 - Ak^4 + Bk^2 - C = 0$.

From the relationship between the root and the coefficient, we can get

$$k_1^2 + k_2^2 + k_3^2 = A, \quad (38)$$

$$k_1^2 k_2^2 + k_1^2 k_3^2 + k_2^2 k_3^2 = B, \quad (39)$$

$$k_1^2 k_2^2 k_3^2 = C. \quad (40)$$

The solution to Eq. (37) can be expressed as follows:

$$e^*(z) = \sum_{i=1}^3 e_i^*(z), \quad (41)$$

where $e_i^*(z)$ are the solutions to $(D^2 - k_i^2)e_i^* = 0$ ($i = 1, 2, 3$).

The solution to Eq. (37), which is bounded as $z \rightarrow \infty$, is provided by

$$e_i^*(z) = R_i(a, \omega)e^{-k_i z}, \quad (42)$$

where $R_i(a, \omega)$ is an undetermined parameter.

Similarly, we can get

$$\theta^*(z) = \sum_{i=1}^3 R_i'(a, \omega)e^{-k_i z}, \quad (43)$$

$$P^*(z) = \sum_{i=1}^3 R_i''(a, \omega)e^{-k_i z}, \quad (44)$$

$$\phi^*(z) = \sum_{i=1}^3 R_i'''(a, \omega)e^{-k_i z}, \quad (45)$$

where $R_i'(a, \omega)$, $R_i''(a, \omega)$, and $R_i'''(a, \omega)$ are undetermined parameters.

Substituting Eqs. (42)–(43) into Eqs. (29)–(32), we can get

$$R_i' = g_i R_i, \quad (46)$$

$$R_i'' = j_i R_i, \quad (47)$$

$$R_i''' = h_i R_i, \quad (48)$$

where

$$g_i = \frac{\varphi_7 \omega}{k_i^2 - a^2 - \omega(1 + \tau_2 \omega)}, \quad j_i = \frac{(\varphi_8 \omega + \varphi_{10} \omega^2)(k_i^2 - a^2 - \omega(1 + \tau_2 \omega)) + \varphi_7 \varphi_9 \omega^2}{(k_i^2 - a^2)(k_i^2 - a^2 - \omega(1 + \tau_2 \omega))},$$

$$h_i = \frac{\varphi_3(k_i^2 - a^2 - \omega(1 + \tau_2 \omega)) - \varphi_5 \varphi_7 \omega}{(k_i^2 - a^2 - \varphi_4 - \varphi_6 \omega^2)(k_i^2 - a^2 - \omega(1 + \tau_2 \omega))}.$$

According to Eqs. (19) and (28), we can get

$$(D^2 - n^2)w^* = \beta^2 \left((1 + \tau_1 \omega) \frac{\partial \theta}{\partial z} + \frac{\partial P}{\partial z} \right) - (\beta^2 - 1) \frac{\partial e}{\partial z} - \varphi_2 \beta^2 \frac{\partial \phi}{\partial z}, \quad (49)$$

where $n^2 = a^2 + \beta^2 \omega^2$.

According to Eq. (49), since the solution is bounded at $z \rightarrow \infty$, we can get

$$w^*(z) = F e^{-n z} + \sum_{i=1}^3 \left(\frac{-\beta^2 k_i ((1 + \tau_1 \omega) g_i + j_i) + (\beta^2 - 1) k_i + \varphi_2 \beta^2 h_i k_i}{(k_i^2 - n^2)} \right) R_i(a, \omega) e^{-k_i z}, \quad (50)$$

where $F(a, \omega)$ is an undetermined parameter.

According to Eqs. (13) and (28), we can get

$$u^*(z) = -\frac{i}{a} \left(e^* - \frac{\partial w^*}{\partial z} \right). \quad (51)$$

Substituting Eqs. (18) and (50) into Eq. (51), we can get

$$u^*(z) = -\frac{i}{a} \left(nF e^{-nz} + \sum_{i=1}^3 \left(\frac{-\beta^2 k_i^2 ((1 + \tau_1 \omega) g_i + j_i) + (\beta^2 - 1) k_i^2 + \varphi_2 \beta^2 h_i k_i^2}{(k_i^2 - n^2)} + 1 \right) R_i(a, \omega) e^{-k_i z} \right). \quad (52)$$

According to Eqs. (16), (17), and (28), we can get

$$\sigma_{zz}^* = 2 \frac{\partial w^*}{\partial z} + \frac{\lambda}{G} e^* - \beta^2 ((1 + \tau_1 \omega) \theta^* + P^*) + \varphi_1 \phi, \quad (53)$$

$$\sigma_{xz}^* = \frac{\partial u^*}{\partial z} + i a w^*. \quad (54)$$

Substituting Eqs. (41)–(52) into Eqs. (53) and (54), we can get

$$\begin{aligned} \sigma_{zz}^* = & - \sum_{i=1}^3 \left(\frac{-2\beta^2 k_i^2 ((1 + \tau_1 \omega) g_i + j_i) + 2(\beta^2 - 1) k_i^2 + 2\varphi_2 \beta^2 h_i k_i^2}{(k_i^2 - n^2)} \right) R_i(a, \omega) e^{-k_i z} \\ & - \sum_{i=1}^3 \left(-\frac{\lambda}{G} + \beta^2 ((1 + \tau_1 \omega) g_i + j_i) + \varphi_1 h_i \right) R_i(a, \omega) e^{-k_i z} + 2nF e^{-nz}, \end{aligned} \quad (55)$$

$$\begin{aligned} \sigma_{xz}^* = & \frac{i}{a} \sum_{i=1}^3 \left(\frac{-\beta^2 k_i ((1 + \tau_1 \omega) g_i + j_i) (k_i^2 + a^2) + (\beta^2 - 1) (k_i^2 + a^2) k_i}{(k_i^2 - n^2)} \right) R_i(a, \omega) e^{-k_i z} \\ & + \frac{i}{a} \sum_{i=1}^3 \left(\frac{\varphi_2 \beta^2 h_i k_i (k_i^2 + a^2)}{(k_i^2 - n^2)} + k_i \right) R_i(a, \omega) e^{-k_i z} + (a^2 + n^2) F e^{-nz}. \end{aligned} \quad (56)$$

In order to determine the undetermined parameters R_i ($i = 1, 2, 3$) and F , the boundary conditions ($z = 0$) need to be considered as

$$\begin{aligned} & \frac{i}{a} \sum_{i=1}^3 \left(\frac{-\beta^2 k_i ((1 + \tau_1 \omega) g_i + j_i) (k_i^2 + a^2) + (\beta^2 - 1) (k_i^2 + a^2) k_i + \varphi_2 \beta^2 h_i k_i (k_i^2 + a^2)}{(k_i^2 - n^2)} + k_i \right) R_i \\ & + (a^2 + n^2) F = 0, \end{aligned} \quad (57)$$

$$\begin{aligned} & - \sum_{i=1}^3 \left(\frac{-2\beta^2 k_i^2 ((1 + \tau_1 \omega) g_i + j_i) + 2(\beta^2 - 1) k_i^2 + 2\varphi_2 \beta^2 h_i k_i^2}{(k_i^2 - n^2)} - \frac{\lambda}{G} \right. \\ & \left. + \beta^2 ((1 + \tau_1 \omega) g_i + j_i) + \varphi_1 h_i \right) R_i + 2nF = -qQ, \end{aligned} \quad (58)$$

$$\sum_{i=1}^3 h_i R_i = 0, \quad (59)$$

$$\sum_{i=1}^3 j_i R_i = 0. \quad (60)$$

Solving Eqs. (57)–(60) can yield the expressions of R_1, R_2, R_3 , and F .

4 Numerical results and discussion

Taking saturated porous deep-sea sediments as an example, this paper applies the mechanical load of the mining machine on the upper surface of the foundation to study the influence of frequency and load amplitude on the vertical displacement, excess pore water pressure, vertical stress, temperature, and change in the volume fraction field. According to the obtained analytical solution, with the help of MATLAB, combined with the boundary conditions, the distributing regular diagram of each physical quantity is obtained.

Taking the deep-sea sediments of the Pacific CC mining area as the foundation and seawater as the pore water, the parameters are as follows^[38]:

$$\begin{aligned} E &= 2 \times 10^6 \text{ Pa}, \quad G = E/(2(1 + \mu)), \quad \lambda = E\mu/((1 + \mu)(1 - 2\mu)), \\ \alpha_s &= -3.315 \times 10^{-5} \text{ K}^{-1}, \quad \alpha_w = 2.57 \times 10^{-3} \text{ K}^{-1}, \quad k_d = 1.258 \times 10^{-7} \text{ m/s}, \\ c_s &= 1376 \text{ J/(kg} \cdot \text{K)}, \quad c_w = 3890 \text{ J/(kg} \cdot \text{K)}, \quad K_s = 0.327 \text{ W/(m} \cdot \text{K)}, \\ K_w &= 0.622 \text{ W/(m} \cdot \text{K)}, \quad \rho_s = 2.54 \times 10^3 \text{ kg/m}^3, \quad \rho_w = 1.025 \times 10^3 \text{ kg/m}^3, \\ \mu &= 0.3, \quad n_0 = 0.87, \quad \tau_1 = 0.03, \quad \tau_2 = 0.02, \quad a = 1.2. \end{aligned}$$

The linear thermal expansion coefficient of soil particles is negative, since MnO_2 , P_2O_5 , TiO_2 , Al_2O_3 , CaO , MnO , etc. are enriched in the sediments of the Pacific CC mining area, and the compounds of Mn, Mo, Zn, etc. have the characteristics of negative thermal expansion materials^[39–41]. Since ω is a complex time constant, it has the form of $\omega = \omega_0 + i\zeta$, where $i = \sqrt{-1}$, and $e^{\omega t} = e^{\omega_0 t}(\cos(\zeta t) + i\sin(\zeta t))$, when the time is extremely small, $\omega = \omega_0$.

The material constants are as follows^[42]:

$$\begin{aligned} \gamma &= 3.688 \times 10^{-5} \text{ N}, \quad \xi = 1.457 \times 10^{10} \text{ N/m}^2, \quad \chi = 1.753 \times 10^{-15} \text{ m}^2, \\ c &= 1.13849 \times 10^{10} \text{ N/m}^2, \quad l = 7.322 \times 10^3 \text{ N/(m}^2 \cdot \text{K)}. \end{aligned}$$

A mechanical load is applied to the surface of the foundation, to study the foundation response under the under amplitude of the load (simulating mining machine). Only the seabed sediment under the mining machine is considered here. The following 10 figures show the variation trend of each physical quantity when $t = 0.1$ on the $x = 0$ plane. The first group is shown in Figs. 3–7, which study the variation trends of vertical displacement w , excess pore water pressure P , vertical stress σ_{zz} , temperature T , and change in volume fraction field ϕ under different frequencies. The second group is shown in Figs. 8–12, which study the variation trends of vertical displacement w , excess pore water pressure P , vertical stress σ_{zz} , temperature T , and change in volume fraction field ϕ under different load amplitudes. The positive and negative signs in the figures only indicate that the foundation is under compression or tension.

In Figs. 3–7, the effects of frequency variation on vertical displacement, excess pore water pressure, vertical stress, temperature, and change in volume fraction field are respectively described. In Fig. 3, the response of the vertical displacement to the frequency change is reflected. The four curves intersect in the $0 < z < 0.5$ interval, and the vertical displacement increases as the frequency increases before the intersection point. And in this interval, the surface displacement will change the direction, since the deep-sea sediments are relatively soft and prone to shear failure. After the intersection and before the direction changes, the vertical displacement decreases with the increase in the frequency; after changing the direction, the vertical displacement increases with the increase in the frequency. In Fig. 4, the variation of excess pore water pressure with frequency is described. As the depth of the foundation increases, the excess pore water pressure first increases and then decreases, reaching a peak in the interval of depth $0 < z < 0.5$. As the frequency increases, the peak value increases. This is because the greater the vibration frequency of the mining machine, the greater the deformation of the deep-sea sediments, and the more sharply the excess pore water pressure will rise. However, the greater

Fig. 3 The distribution of non-dimensional vertical displacement at $Q^* = 10$ kN

Fig. 4 The distribution of non-dimensional excess pore water pressure at $Q^* = 10$ kN

Fig. 5 The distribution of non-dimensional vertical stress at $Q^* = 10$ kN

Fig. 6 The distribution of non-dimensional temperature at $Q^* = 10$ kN

Fig. 7 The distribution of non-dimensional change in the volume fraction field at $Q^* = 10$ kN

Fig. 8 The distribution of non-dimensional vertical displacement at $\omega = 20$ Hz

the excess pore water pressure is, the easier the deep-sea sediments are to liquefy, which may cause the sinking of the mining machine. This is very unfavorable for deep-sea mining, making mining more dangerous and reducing mining efficiency. Because it is assumed that the surface of the foundation is permeable, the excess pore water pressure is zero at $z = 0$. In Fig. 5, it shows that the vertical stress increases with the increase in frequency. This is because as the frequency increases, the time for the foundation to drain the pore water becomes shorter, and

Fig. 9 The distribution of non-dimensional excess pore water pressure at $\omega = 20$ Hz

Fig. 10 The distribution of non-dimensional vertical stress at $\omega = 20$ Hz

Fig. 11 The distribution of non-dimensional temperature at $\omega = 20$ Hz

Fig. 12 The distribution of non-dimensional change in the volume fraction field at $\omega = 20$ Hz

the pore water also bears a certain pressure. In Fig. 6, the influence of frequency changes on temperature is reflected, and the four curves intersect in the interval of $0 < z < 0.5$. Before the intersection point, the temperature increases with the increase in frequency. This is because the higher the frequency is, the more difficult it is to drain the pore water in the foundation, and the thermal conductivity of water is greater than that of soil. While the temperature after the intersection point decreases with the increase in frequency, since the pore water decreases with the increase in depth. And the temperature is very small at different frequencies. This is because the heat generated by the pure mechanical load is not much and the frequency has little influence on the temperature, which can be ignored. In Fig. 7, the influence of different frequencies on the change in volume fraction field is described. The change in volume fraction field increases with the increase in frequency, because the higher the frequency, the shorter the time for the foundation to discharge pore water, and the concentration of soil particles increases significantly. On the whole, the change of frequency has great influence on vertical displacement, excess pore water pressure, vertical stress, and change in volume fraction. As the frequency increases, the excess pore water pressure, vertical stress, and change in volume fraction field all increase. And when the depth is $0 < z < 0.5$, the change trend of each physical quantity is relatively large, and this depth interval is most prone to soil liquefaction. Therefore, when the depth is 0 m–0.5 m, the frequency of the mining machine should be as small as possible, and the mining should be more cautious to prevent the liquefaction of deep-sea sediments, which affects the safety and efficiency of the deep-sea mining. As the depth increases, the frequency

of the mining machine can be appropriately increased to make deep-sea mining more efficient.

In Figs. 8–12, the effects of load amplitude variation on vertical displacement, excess pore water pressure, vertical stress, temperature and change in volume fraction field are respectively described. In Fig. 8, the distribution of vertical displacement with load amplitude is described. As the load amplitude increases, the vertical displacement also increases. In Fig. 9, the influence of load amplitude changes on the excess pore water pressure is reflected. With the increase in load amplitude, the excess pore water pressure also increases obviously. And the greater the excess pore water pressure is, the easier it is to liquefy the soil. In Fig. 10, it shows that the vertical stress increases with the increase in load amplitude. Figure 11 shows the change in temperature, as the load amplitude increases, the greater the effect on temperature is. In Fig. 12, it shows that the change in volume fraction field increases with the increase in load amplitude. On the whole, the change of load amplitude has a great effect on the vertical displacement, excess pore water pressure, vertical stress, and change in volume fraction field. With the increase in load amplitude, the vertical displacement, excess pore water pressure, vertical stress, temperature, and change in volume fraction field increase. Therefore, while ensuring the efficiency of mining work, the load amplitude can be appropriately reduced to prevent the liquefaction of deep-sea sediments.

5 Conclusions

Based on the G-L generalized thermoelasticity theory and Darcy's law, this paper studies the thermal-hydro-mechanical coupled dynamic response of saturated porous deep-sea sediments under the action of mining mechanical loads. The normal mode analysis method is used to solve the problem, the analytical solution of each physical quantity is obtained, and the influence of frequency and load amplitude on each physical quantity in the foundation is analyzed. The main conclusions are as follows.

(i) The normal mode analysis method can well solve the thermal-hydro-mechanical coupling problem of semi-infinite saturated porous seabed sediments, and finally obtain the change pattern of each physical quantity.

(ii) The frequency of the mining machine has very obvious influence on the vertical displacement, excess pore water pressure, vertical stress, and change in volume fraction field, especially the influence of frequency on pore water pressure. With the increase in frequency, the greater the pore water pressure changes, the faster the foundation deformation occurs, and the easier it is to liquefy. Moreover, when the depth is $0 < z < 0.5$, the change trend of each physical quantity is relatively large, and this depth interval is most prone to soil liquefaction. Since the liquefaction of deep-sea sediments may cause the mining machine to sink, when the depth is 0 m–0.5 m, the frequency of the mining machine can be appropriately reduced to prevent the liquefaction of deep-sea sediments.

(iii) The load amplitude of the mining machine has great influence on the physical quantities in the foundation. With the increase in load amplitude, the vertical displacement, vertical stress, excess pore water pressure, temperature, and change in volume fraction field increase. Moreover, the change of excess pore water pressure has great influence on the liquefaction of the foundation. The greater the excess pore water pressure is, the easier the foundation is to liquefy. Therefore, while ensuring the efficiency of mining work, the load amplitude can be appropriately reduced to prevent the liquefaction of deep-sea sediments.

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