Electrokinetic energy conversion of electro-magneto-hydro-dynamic nanofluids through a microannulus under the time-periodic excitation*

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Abstract In this work, the effects of externally applied axial pressure gradients and transverse magnetic fields on the electrokinetic energy conversion (EKEC) efficiency and the streaming potential of nanofluids through a microannulus are studied. The analytical solution for electro-magneto-hydro-dynamic (EMHD) flow is obtained under the condition of the Debye-Hückel linearization. Especially, Green's function method is used to obtain the analytical solutions of the velocity field. The result shows that the velocity distribution is characterized by the dimensionless frequency Ω , the Hartmann number Ha , the volume fraction of the nanoparticles ϕ , the geometric radius ratio a , and the wall ζ potential ratio b . Moreover, the effects of three kinds of periodic excitations are compared and discussed. The results also show that the periodic excitation of the square waveform is more effective in increasing the streaming potential and the EKEC efficiency. It is worth noting that adjusting the wall ζ potential ratio and the geometric radius ratio can affect the streaming potential and the EKEC efficiency.

Key words electrokinetic energy conversion (EKEC) efficiency, nanofluid, streaming potential, magnetic field, time-periodic excitation

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1 Introduction

With the tremendous progress of micro-electro-mechanical-systems (MEMS) and microfluidic technology, the research on it has become very hot, and a series of efforts have

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been made in many fields, such as chemical separation devices, heat exchange, micropumps, biomedical diagnosis, and lab-on-a-chip systems^[1–3]. In these applications, electric field, pressure gradient, magnetic field or their combination are usually used to drive fluid flow. As a basic electrokinetic phenomenon, electroosmotic flow (EOF) requires the intervention of an external electric field to drive fluid flow. A lot of basic research on EOF has been carried out in hydrodynamics and heat transfer phenomena^[4–8]. The interaction of ions in the electric double layer (EDL) with the externally applied electric field produces EOF. Unlike traditional macrosized channels, flow analysis in microchannels must consider the existence of EDL. This is due to the interaction between the charged wall and the ionized solution^[9]. If the channel surface is negatively charged, the ions with the same charge will be attracted to the surface, while the ions with opposite charge will be repulsed by the surface. This rebalances the positive and negative ions in the solution and maintains the neutral state of the entire system. Under the action of strong electrostatic force, the positively charged ions gathered near the wall of the microchannel form a fixed Stern layer, a typical thick layer of ion diameter. The diffusion layer is adjacent to the Stern layer and contains positive and negative ions. The variation of ion density follows Boltzmann distribution. The Stern layer and the diffusion layer formed at the solid-liquid interface constitute an EDL.

As another electrodynamic phenomenon, the streaming potential can also trigger EOF in microfluidic systems. Different from the ordinary EOF induction mechanism, this EOF does not need the intervention of an external electric field, but forces the liquid to pass under the pressure gradient according to its own streaming potential. This is because the liquid carries excess charges close to the wall, and due to their accumulation downstream, an electric field is formed, which drives the current in the opposite direction (through the ionic conduction of the liquid). The ζ potential in colloidal science is opposite to the direction of fluid flow. The steady state is established quickly, and the measured potential difference at both ends of the microchannel is called streaming potential. Moreover, the research of streaming potential focuses on the electroviscosity effect^[10–13] and the energy conversion efficiency^[14–16] of the fluid in the microchannel. In many related studies, the process of converting the mechanical energy transmitted by pressure-driven and the chemical energy of EDL into the streaming potential electric energy is called the electrokinetic energy conversion (EKEC)^[17–19]. Many researchers have studied the EKEC of fluids through microchannels and nano-channels. Bandopadhyay and Chakraborty^[16] and Bandopadhyay et al.^[17] studied the EKEC efficiency of viscoelastic fluid passing through two parallel plates under the condition of no-slip boundary. Jian et al.^[20] researched the EKEC efficiency of viscoelastic fluid through a polyelectrolyte-grafted nanochannel under the condition of no-slip boundary. Ren and Stein^[21] analyzed the energy conversion problem in micropipes based on slip theory. Liu et al.^[22] investigated the electrochemical mechanical energy conversion efficiency in curved rectangular nanochannels. The results show that within a certain range of parameters, the EKEC efficiency of the curved rectangular nanochannel is 1.17 times larger than that of the straight rectangular nanochannel. Therefore, the electrochemical mechanical energy conversion efficiency can be improved by using curved rectangular nanochannels. Xie and Jian^[23] took the lead in studying the EKEC of nanofluids. They found that, when the radius of the microtube is much larger than the thickness of the EDL, the energy conversion efficiency was improved when nanoparticles were introduced into the working electrolyte at a given ionic molar concentration. In addition, the maximum output power of the electrolyte containing 5% nanoparticles can be increased by more than 16% compared with that of the electrolyte containing no nanoparticles.

In recent years, with the rapid development of nanomaterials and technology, a new type of fluid, namely nanofluid, has emerged. The so-called nanofluid is to add nano-scale metal or non-metal oxide particles into the liquid in a certain manner and a certain proportion to

form a heat transfer cooling fluid. This new type of functional fluid that has unique heat transfer properties can greatly improve the thermal conductivity of liquid, strengthen the heat transfer characteristics of the liquid, uniformity, and stability, and is not easy to cause wear and blockage to the pipeline. Therefore, it has broad application prospects in the fields of vehicles, energy, electric power, chemical industry, and aviation. In addition, many publications have reported the applications of nanofluids in various advanced systems and micro/nano mechanical equipments^[24–35].

Most of the studies on streaming potential in the literature focus on steady state conditions, while the unsteady state condition is rarely emphasized. Green's function method is widely used to solve inhomogeneous partial differential equations. Therefore, Green's function method is effective for solving problems under unsteady state conditions. Kang et al.^[36] used Green's function method to solve the problem of EOF in a cylindrical channel with the sinusoidal waveform. Moghadam^[37] used Green's function method to get the exact solution of the velocity distribution of alternating current electroosmotic flows in a circular microchannel. The time-periodic EOF in a microannulus was investigated by Moghadam^[38–39], where the exact solution of velocity distributions was obtained by using Green's function method. The effects of various periodic waveforms on the velocity field have also been discussed. It is possible to solve non-homogeneous problems without applying Green's function method, and Jian et al.^[40] studied the time-periodic electroosmotic flow in a microannulus based on the linearized Poisson-Boltzmann equation. The analytical solution of their velocity distribution is to substitute the complex forms of the electric field and velocity field into the governing equation. Since the initial conditions are not considered in their solution process, their solutions are only suitable for steady-state time periodic conditions. Tang et al.^[41] studied the EOF with the axisymmetric lattice Boltzmann method in a microannulus.

In this study, the changes of the streaming potential and the EKEC efficiency of nanofluids through the microannulus under the combined action of magnetic field and different periodic pressure gradients are discussed. Zhao et al.^[42] studied the influence of the streaming potential for heat transfer under the combined action of pressure gradient and the magnetic field with steady state conditions. In the present work, we consider the hydrodynamic behavior under unsteady state conditions and excitations of different time periodic functions (cosine waveform, square waveform, and triangle waveform). We hope that these research results can provide ideas for the study of microfluidic systems under different cyclical excitation forms and will be valuable for future research.

2 Mathematical formulation

2.1 Problem definition

Figure 1 describes the flow of incompressible viscous nanofluids through a microannulus with an inner radius of $a\Re$ ($0 < a < 1$) and an outer radius of $\Re$ under the combined action

Fig. 1 Schematic theme of the problem geometry

of the Y -direction uniform magnetic field B_0 and the X -direction pressure-gradient, where a is the geometric radius ratio between 0 and 1. The inner and outer walls of the microannulus are evenly charged with ζ potentials $b\zeta_0$ and ζ_0 , respectively. The flow is assumed to be unsteady, fully developed, and hydrodynamical. In addition, we do not consider the EDL formed by nanoparticles, and assume that the EDLs formed on the channel wall in this study do not overlap.

2.2 Electrical potential equation and approximate solution

For a symmetric ($z : z$) electrolyte solution, the potential $\Psi(R)$ of the EDL and the net charge density $\rho_e(R)$ can be described by the Poisson-Boltzmann equations as follows^[23]:

$$\frac{d^2\Psi}{dR^2} + \frac{1}{R} \frac{d\Psi}{dR} = -\frac{\rho_e}{\varepsilon}, \quad (1)$$

$$\rho_e(R) = ze(n_+ - n_-), \quad (2)$$

where ε is the permittivity of the fluid, z is the valence of ions, e is the elementary charge, and n_+ and n_- are the number densities of the electrolyte cations and anions, respectively, given by the Boltzmann distribution, i.e.,

$$n_{\pm} = n_0 \exp\left(\mp \frac{ez\Psi(R)}{\kappa_B T_{av}}\right), \quad (3)$$

in which n_0 is the ion density of the bulk liquid, κ_B is the Boltzmann constant, and T_{av} is the absolute temperature. Under the Debye-Hückel approximation, Eq. (3) can be expressed as

$$\exp\left(\mp \frac{ez\Psi(R)}{\kappa_B T_{av}}\right) \approx 1 \mp \frac{ez\Psi(R)}{\kappa_B T_{av}}. \quad (4)$$

Substituting Eqs. (3)–(4) into Eq. (2), the net charge density is obtained as

$$\rho_e(R) = -2zen_0 \sinh\left(\frac{ze\Psi(R)}{\kappa_B T_{av}}\right). \quad (5)$$

The boundary conditions of the electrical potential are

$$\begin{cases} \Psi = b\zeta_0 & \text{at } R = a\Re, \\ \Psi = \zeta_0 & \text{at } R = \Re. \end{cases} \quad (6)$$

First, introduce several dimensionless variables as follows:

$$r = \frac{R}{\Re}, \quad K = \kappa\Re, \quad \psi = \frac{ze}{\kappa_B T_{av}}\Psi. \quad (7)$$

Equation (1) can be linearized by the Debye-Hückel approximation, and we write Eq. (1) in a dimensionless form as follows:

$$\frac{d^2\psi}{dr^2} + \frac{1}{r} \frac{d\psi}{dr} = K^2\psi. \quad (8)$$

The boundary conditions are

$$\begin{cases} \psi = bZ_0 & \text{at } r = a, \\ \psi = Z_0 & \text{at } r = 1, \end{cases} \quad (9)$$

where $\kappa = (2n_0z^2e^2/(\varepsilon\kappa_B T_{av}))^{1/2}$ is the Debye-Hückel parameter, $1/\kappa$ represents the thickness of the EDL, and Z_0 is the dimensionless potential at the outer walls. The solution to Eq. (8)

subject to the boundary conditions (9) is^[38]

$$\psi(r) = \frac{Z_0 (I_0(Kr)(bK_0(K) - K_0(Ka)) - K_0(Kr)(bI_0(K) - I_0(Ka)))}{K_0(K)I_0(Ka) - K_0(Ka)I_0(K)}, \quad (10)$$

where I_0 is a zeroth-order modified Bessel function of the first type, and K_0 is a zeroth-order modified Bessel function of the second type.

2.3 Analytical solutions of the velocity field

There is only one velocity component in the X -direction, which depends on Y and T . Therefore, the corresponding Navier-Stokes equation governing the flow can be given as^[40]

$$\rho_{\text{eff}} \frac{\partial U}{\partial T} = \mu_{\text{eff}} \left(\frac{\partial^2 U}{\partial R^2} + \frac{1}{R} \frac{\partial U}{\partial R} \right) - \frac{dP}{dX} + \rho_e(R)E_s - \sigma_{\text{eff}} B_0^2 U, \quad (11)$$

where $-\frac{dP}{dX} = \Omega' F(\omega T)$, Ω' is the amplitude, U is the axial velocity, P is the periodic pressure, E_s is the streaming potential, and B_0 is the magnetic field strength. ρ_{eff} is the effective density of the nanofluid, which is given by

$$\rho_{\text{eff}} = \phi \rho_s + (1 - \phi) \rho_f, \quad (12)$$

where ϕ is the volume fraction of the nanoparticles, ρ_s is the density of a solid, and ρ_f is the density of fluid. μ_{eff} is the effective viscosity of the nanofluid, which is given by

$$\mu_{\text{eff}} = \frac{\mu_f}{(1 - \phi)^{2.5}}, \quad (13)$$

where μ_f is the viscosity of the base fluid. In addition, σ_{eff} is the effective electrical conductivity of the nanofluid, which is given by

$$\sigma_{\text{eff}} = \sigma_f \left(1 + \frac{3(\sigma_s - \sigma_f)\phi}{(\sigma_s + 2\sigma_f) - (\sigma_s - \sigma_f)\phi} \right), \quad (14)$$

where σ_s is the conductivity of the nanoparticles, and σ_f is the conductivity of the base fluids. The two-wall no-slip boundary conditions and initial conditions of the microannulus are

$$\begin{cases} U = 0 & \text{at } R = a\Re, \quad 0 < a < 1, \\ U = 0 & \text{at } R = \Re, \\ U = 0 & \text{at } T = 0. \end{cases} \quad (15)$$

Introduce the following dimensionless group:

$$\begin{cases} u = \frac{U}{U_p}, \quad U_p = \frac{\Omega' \Re^2}{\mu_f}, \quad U_e = \frac{\kappa_B T_{\text{av}} \varepsilon E_x}{ez \mu_f}, \quad \bar{E}_s = \frac{E_s}{E_x}, \\ Ha = B_0 \Re \sqrt{\frac{\sigma_f}{\mu_f}}, \quad t = \frac{\mu_f}{\rho_f \Re^2} T, \quad \Omega = \frac{\rho_f \Re^2}{\mu_f} \omega, \quad u_r = \frac{U_e}{U_p}, \end{cases} \quad (16)$$

where $\Omega' = -\frac{dP}{dX}$, U_p is a reference pressure-driven flow velocity, U_e is the Helmholtz-Smoluchowski electroosmotic velocity, and Ω is the dimensionless frequency. E_x is the characteristic electric field, and K is called the electrokinetic radius (the length scale ratio). Ha is the Hartmann number. After the dimensionless parameters in Eqs. (7) and (16) are inserted into Eq. (11), the dimensionless momentum equation, relevant boundary conditions,

and initial conditions have the following forms:

$$\frac{\partial u}{\partial t} = \alpha \left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} \right) - \beta H a^2 u + \underbrace{\eta (F(\Omega t) - u_r K^2 \psi(r) \bar{E}_s)}_{Q(r,t)}, \quad (17)$$

$$u|_{r=a} = 0, \quad u|_{r=1} = 0, \quad u|_{t=0} = 0, \quad (18)$$

where $\alpha = \frac{\mu_{\text{eff}}}{\mu_f} \cdot \frac{\rho_f}{\rho_{\text{eff}}}$, $\beta = \frac{\sigma_{\text{eff}}}{\sigma_f} \cdot \frac{\rho_f}{\rho_{\text{eff}}}$, and $\eta = \frac{\rho_f}{\rho_{\text{eff}}}$.

2.4 Solution procedure

The non-dimensional equation (17) is constrained by the boundary conditions and initial conditions (18). Now, Green's function method is used to solve it. Green's function satisfies

$$\frac{\partial g}{\partial t} = \alpha \left(\frac{\partial^2 g}{\partial r^2} + \frac{1}{r} \frac{\partial g}{\partial r} \right) - \beta H a^2 g + \frac{\delta(r-l)\delta(t-\tau)}{2\pi r}, \quad a < r, l < 1, \quad t, \tau > 0, \quad (19)$$

which is subject to the following homogeneous boundary conditions and initial conditions:

$$g(a, t|l, \tau) = g(1, t|l, \tau) = 0, \quad (20a)$$

$$g(r, 0|l, \tau) = 0, \quad (20b)$$

where $\delta(x)$ is the Dirac delta function. We first apply the Laplace transform to Eq. (19),

$$\left(\frac{\partial^2 \bar{g}}{\partial r^2} + \frac{1}{r} \frac{\partial \bar{g}}{\partial r} \right) - \frac{\beta H a^2 + s}{\alpha} \bar{g} = -\frac{e^{-s\tau}}{\alpha} \frac{\delta(r-l)}{2\pi r}. \quad (21)$$

Next, the regular Sturm-Liouville problem is considered,

$$\frac{1}{r} \frac{d}{dr} \left(r \frac{d\varphi}{dr} \right) + k^2 \varphi = 0, \quad \varphi(a) = \varphi(1) = 0. \quad (22)$$

The eigenfunctions that satisfy Eq. (22) are

$$\varphi_i(r) = Y_0(k_i a) J_0(k_i r) - J_0(k_i a) Y_0(k_i r), \quad (23)$$

where k_i is the i th root of $J_0(ka)Y_0(k) - J_0(k)Y_0(ka) = 0$. J_0 is a zeroth-order Bessel function of the first type, and Y_0 is a zeroth-order Bessel function of the second type. Then, the Hankel transformation for Eq. (21) can be written as

$$\tilde{\bar{g}}(k_i, s|l, \tau) = \frac{1}{s - (-\alpha k_i^2 + \beta H a^2)} \cdot e^{-s\tau} \cdot \frac{\varphi_i(l)}{2\pi}. \quad (24)$$

The inverse Laplace transform for Eq. (24) can be written as

$$\tilde{g}(k_i, t|l, \tau) = H(t - \tau) \cdot e^{-(\alpha k_i^2 + \beta H a^2)(t - \tau)} \cdot \frac{\varphi_i(l)}{2\pi}. \quad (25)$$

The Hankel inverse transformation for Eq. (25) yields

$$g(r, t|l, \tau) = \frac{\pi}{4} H(t - \tau) \cdot \sum_{i=1}^{\infty} \frac{(k_i J_0(k_i))^2 \varphi_i(l) \varphi_i(r)}{J_0^2(a k_i) - J_0^2(k_i)} \cdot e^{-(\alpha k_i^2 + \beta H a^2)(t - \tau)}. \quad (26)$$

Therefore, the dimensionless velocity can be obtained by the following simplified formula:

$$u(r, t) = \int_0^t \int_a^1 Q(l, \tau) g(r, t|l, \tau) dl d\tau, \quad (27)$$

where $Q(l, \tau)$ is defined in Eq. (17). Next, three different kinds of periodic functions are introduced. If the periodic function is cosine waveform,

$$F(\Omega t) = \cos(\Omega t), \quad (28)$$

the velocity profile will be given by Eq. (27) and can be obtained as

$$u(r, t) = \frac{\pi\eta}{4} \sum_{i=1}^{\infty} \frac{(k_i J_0(k_i))^2 \varphi_i(r)}{J_0^2(k_i a) - J_0^2(k_i)} \int_a^1 \varphi_i(l) dl \cdot \frac{(\alpha k_i^2 + \beta H a^2) \cos(\Omega t) + \Omega \sin(\Omega t) - (\alpha k_i^2 + \beta H a^2) e^{-(\alpha k_i^2 + \beta H a^2)t}}{(\alpha k_i^2 + \beta H a^2)^2 + \Omega^2} - R_i, \quad (29)$$

where

$$R_i = \frac{\pi\eta u_r K^2 \bar{E}_s}{4} \sum_{i=1}^{\infty} \frac{(k_i J_0(k_i))^2 \varphi_i(r)}{J_0^2(k_i a) - J_0^2(k_i)} \int_a^1 \psi(l) \varphi_i(l) dl \frac{1 - e^{-(\alpha k_i^2 + \beta H a^2)t}}{\alpha k_i^2 + \beta H a^2}. \quad (30)$$

If the periodic function is square waveform,

$$F(\Omega t) = \frac{2}{\pi} \sum_{m=1}^{\infty} \frac{1 - \cos(m\pi)}{m} \sin(m\Omega t), \quad (31)$$

the velocity distribution will be given by Eq. (27) and can be obtained as

$$u(r, t) = \frac{\eta}{2} \sum_{m=1}^{\infty} \sum_{i=1}^{\infty} \frac{1 - \cos(m\pi)}{2} Z_{mi} - R_i, \quad (32)$$

where

$$Z_{mi} = \frac{(k_i J_0(k_i))^2 \varphi_i(r)}{J_0^2(k_i a) - J_0^2(k_i)} \int_a^1 \varphi_i(l) dl \cdot \frac{(\alpha k_i^2 + \beta H a^2) \sin(m\Omega t) - m\Omega \cos(m\Omega t) + m\Omega e^{-(\alpha k_i^2 + \beta H a^2)t}}{(\alpha k_i^2 + \beta H a^2)^2 + (m\Omega)^2}. \quad (33)$$

If the periodic function is triangular waveform,

$$F(\Omega t) = \frac{8}{\pi^2} \sum_{m=1}^{\infty} \frac{\sin(m\pi/2)}{m^2} \sin(m\Omega t), \quad (34)$$

the velocity distribution will be given by Eq. (27) and can be obtained as

$$u(y, t) = \frac{2\eta}{\pi} \sum_{m=1}^{\infty} \sum_{i=0}^{\infty} \frac{\sin(m\pi/2)}{m^2} Z_{mi} - R_i. \quad (35)$$

It is obvious that the streaming potential (E_s) which appears in the velocity field is unknown. It can be determined by setting the following electro-neutrality conditions.

2.5 Calculation of the streaming potential

To maintain the electroneutrality of fluid system, the conduction current and the streaming maintain equilibrium are in a steady state, i.e., $I_c + I_s = 0$,

$$I = 2\pi e z \int_{aR}^R (n_+ u_+ - n_- u_-) R dR = I_s + I_c = 0, \quad (36)$$

where I_s is the streaming current, I_c is the conduction current, and $u_{\pm}$ are the combination of nanofluid advection velocity (U) and electromigrative velocity ($\pm ezE_s/f$), and thus they can be written as

$$u_{\pm} = U \pm \frac{ezE_s}{f}, \quad (37)$$

where f is the ionic friction coefficient. Substituting Eqs. (3), (29), and (37) into Eq. (36), we get the dimensionless streaming potential of cosine waveform as

$$\overline{E}_s = \frac{A}{B_1 + u_r B_2}, \quad (38)$$

where

$$A = \frac{\pi\eta}{4} \sum_{i=1}^{\infty} \frac{(k_i J_0(k_i))^2}{J_0^2(k_i a) - J_0^2(k_i)} \cdot \int_a^1 \varphi_i(l) dl \cdot \int_a^1 \varphi_i(r) \psi(r) r dr$$

$$\cdot \frac{(\alpha k_i^2 + \beta H a^2) \cos(\Omega t) + \Omega \sin(\Omega t) - (\alpha k_i^2 + \beta H a^2) e^{-(\alpha k_i^2 + \beta H a^2)t}}{(\alpha k_i^2 + \beta H a^2)^2 + \Omega^2}, \quad (39)$$

$$B_1 = \frac{ezE_x}{2fU_p} (1 - a^2), \quad (40)$$

$$B_2 = \frac{\pi\eta K^2}{4} \sum_{i=1}^{\infty} \frac{(k_i J_0(k_i))^2}{J_0^2(k_i a) - J_0^2(k_i)} \frac{1 - e^{-(\alpha k_i^2 + \beta H a^2)t}}{\alpha k_i^2 + \beta H a^2} \int_a^1 \psi(l) \varphi_i(l) dl \int_a^1 \psi(r) \varphi_i(r) r dr. \quad (41)$$

Substituting Eqs. (3), (32), and (37) into Eq. (36), we get the dimensionless streaming potential of the square waveform as

$$\overline{E}_s = \frac{C}{B_1 + u_r B_2}, \quad (42)$$

where

$$C = \frac{\eta}{2} \sum_{m=1}^{\infty} \sum_{i=1}^{\infty} \frac{1 - \cos(m\pi)}{m} \cdot \frac{(k_i J_0(k_i))^2}{J_0^2(k_i a) - J_0^2(k_i)} \int_a^1 \varphi_i(l) dl \cdot \int_a^1 \varphi_i(r) \psi(r) r dr$$

$$\cdot \frac{(\alpha k_i^2 + \beta H a^2) \sin(m\Omega t) - m\Omega \cos(m\Omega t) + m\Omega e^{-(\alpha k_i^2 + \beta H a^2)t}}{(\alpha k_i^2 + \beta H a^2)^2 + (m\Omega)^2}. \quad (43)$$

Substituting Eqs. (3), (35), and (37) into Eq. (36), we get the dimensionless streaming potential of triangular waveform as

$$\overline{E}_s = \frac{D}{B_1 + u_r B_2}, \quad (44)$$

where

$$D = \frac{2\eta}{\pi} \sum_{m=1}^{\infty} \sum_{i=1}^{\infty} \frac{\sin(m\pi/2)}{m^2} \cdot \frac{(k_i J_0(k_i))^2}{J_0^2(k_i a) - J_0^2(k_i)} \int_a^1 \varphi_i(l) dl \cdot \int_a^1 \varphi_i(r) \psi(r) r dr$$

$$\cdot \frac{(\alpha k_i^2 + \beta H a^2) \sin(m\Omega t) - m\Omega \cos(m\Omega t) + m\Omega e^{-(\alpha k_i^2 + \beta H a^2)t}}{(\alpha k_i^2 + \beta H a^2)^2 + (m\Omega)^2}. \quad (45)$$

2.6 Efficiency of the electrokinetic energy conversion

Since the mechanical energy of the pressure-driven flow is converted into electrical energy in the process of producing the streaming current (I_s) and the streaming electric field (E_s), we need to consider the energy conversion. The definition of the EKEC efficiency ξ is as follows^[16]:

$$\xi = \left| \frac{P_{\text{out}}}{P_{\text{in}}} \right|, \quad (46)$$

where P_{in} and P_{out} respectively represent the input and output powers, and the expressions are as follows:

$$P_{\text{out}} = \left| \left(\frac{I_s}{2} \right) \left(\frac{E_s}{2} \right) \right|, \quad (47)$$

$$P_{\text{in}} = \left| -\frac{dP}{dX} Q_{\text{in}} \right|. \quad (48)$$

Here, Q_{in} represents the flow rate for this pressure-driven flow, which can be expressed as

$$Q_{\text{in}} = 2\pi \int_{a\Re}^{\Re} -\frac{1}{4\mu_f} \frac{dP}{dX} (\Re^2 - R^2) R dR, \quad (49)$$

$$I_s = 2\pi e z \int_{a\Re}^{\Re} U(n_+ - n_-) R dR. \quad (50)$$

Thus, we can get ξ from Eqs. (43)–(46) as

$$\xi = \frac{2K^2 \bar{E}_s^2 u_r^2}{1 - a^2} \chi, \quad (51)$$

where $\chi = \frac{e^2 z^2 \mu_f}{\varepsilon \kappa_B T_{\text{av}} f}$.

3 Results and discussion

We derive the analytical solutions of the velocity, the streaming potential, and the EKEC efficiency of nanofluids passing through a microannulus. They vary depending on many of the dimensionless parameters defined above. In order to obtain meaningful results, the actual range of these dimensionless parameters should be provided according to the relevant physical variables, as shown below^[22,38,40,43]: $\Re = 100 \mu\text{m}$, $T_{\text{av}} = 298 \text{ K}$, $\kappa_B = 1.381 \times 10^{-23} \text{ J} \cdot \text{K}^{-1}$, $\varepsilon = 7 \times 10^{-10} \text{ C}^2 \cdot \text{N}^{-1} \cdot \text{m}^{-2}$, $e = 1.6 \times 10^{-19} \text{ C}$, $z = 1$, $\mu_f = 8.91 \times 10^{-3} \text{ kg} \cdot \text{m}^{-1} \cdot \text{s}^{-1}$, $\rho_s = 3600 \text{ kg} \cdot \text{m}^{-3}$, $\rho_f = 997.1 \text{ kg} \cdot \text{m}^{-3}$, $\sigma_f = 0.05 \text{ S} \cdot \text{m}^{-1}$, $\sigma_s = 10^{-12} \text{ S} \cdot \text{m}^{-1}$, $Z_0 = 0.5$, and $u_r = 1.0$. The range of E_x changes from $1 \text{ K} \cdot \text{V} \cdot \text{m}^{-1}$ to $10^2 \text{ K} \cdot \text{V} \cdot \text{m}^{-1}$, f is taken as $10^{-12} \text{ N} \cdot \text{s} \cdot \text{m}^{-1}$, the EDL thickness K ranges from 10 to 60, Ha varies from 0 to 3.0, the range of Ω changes from 30 to 120, and the range of ϕ changes from 0 to 0.05. a is set to 0.4 to 0.6, and b is set to -2 to 2 .

In the present work, the average EKEC efficiency of three waveforms in one period is defined as $\bar{\xi} = (\int_0^T \xi dt)/T$, where $T = 2\pi/\Omega$ for comparison with the study of Xie and Jian^[23]. If the geometric radius ratio a , the wall ζ potential ratio b , and Ha tend to 0, our research can be extended to be consistent with the research background of Xie and Jian^[23]. Figure 2 shows the change of EKEC efficiency with the electrokinetic radius K when other parameters are constant. From the results of Fig. 2, it can be seen that the average EKEC efficiencies of square waveform and triangular waveform are significantly higher than those studied by Xie and Jian^[23] under steady state conditions. This can be explained that square waveform and

triangular waveform can be superimposed to provide large amounts of mechanical energy to be converted into electrical energy, which makes the streaming potential increase, thus making the EKEC efficiency increase significantly. We can also see that the EKEC efficiency of cosine waveform is relatively small. When K is greater than 1.6, its EKEC efficiency will be less than that studied by Xie and Jian^[23]. This is because the cosine waveform is relatively single, and there is no multiple superposition of mechanical energy converted to electrical energy. The generated streaming potential is smaller, and the EKEC efficiency is also smaller.

Fig. 2 Comparisons of EKEC efficiency between the present result and that of Ref. [23], where $Ha \approx 0$, $a \approx 0$, $b \approx 0$, $\Omega = 60$, and $\phi = 0.02$ (color online)

Figure 3 depicts the dimensionless velocity distribution of the square waveform as a function of the volume fraction of nanoparticles when other parameters are fixed, and we study it in one period. The volume fraction of nanoparticles in Fig.3 is set to be 0, 0.02, 0.03, and 0.05, respectively. From Fig.3, it can be clearly seen that the dimensionless velocity gradually decreases with the increase in the volume fraction, and the change in the microannulus takes the form of simple harmonics. It can be explained that the effective viscosity of nanofluids increases with the increase in the shear rate, and increases with the increase in nanoparticle volume fraction, which leads to the decrease in the dimensionless velocity. In addition, we can also observe that the dimensionless velocity changes with time in the form of simple harmonics.

Figure 4 illustrates the change of dimensionless velocity in triangular waveform with the dimensionless frequency in time $0 \sim 0.6$. The dimensionless frequency in Fig.4 is set to be 30, 60, 90, and 120, respectively. It is obvious that the increase in the dimensionless frequency leads to the overall decrease in the dimensionless velocity. The reason is that the larger dimensionless frequency results in smaller time period, and at the same time, the velocity propagation time becomes shorter and the oscillation becomes faster, which in turn makes the fluid flow unable to fully develop. Therefore, the overall dimensionless velocity is decreasing. Moreover, it can be observed that the dimensionless velocity at the wall of the microannulus is lower than the dimensionless velocity at the middle of the microannulus. It can also be seen that the dimensionless velocity is in simple harmonic motion with time.

Figure 5(a) depicts the variations of the dimensionless streaming potential of cosine waveform with the dimensionless time at different dimensionless frequencies Ω . The dimensionless frequency in Fig.5(a) is set to be 30, 60, 90, and 120, respectively. It is clearly seen that the period and amplitude of the dimensionless streaming potential decrease as the dimensionless frequency increases. This is because the larger the dimensionless frequency, the shorter the oscillation period of the EKEC efficiency, which leads to the incomplete conversion of energy and the smaller the EKEC efficiency. In addition, with the increase in time, the variation

Fig. 3 Dimensionless velocity distributions of the square waveform for one period with (a) $\phi = 0$, (b) $\phi = 0.02$, (c) $\phi = 0.03$, and (d) $\phi = 0.05$ ($K = 50$, $Ha = 2$, $\Omega = 30$, $E_x = 10^5$, $a = 0.5$, and $b = 1$) (color online)

of dimensionless streaming potential presents a simple harmonic shape. In the case of cosine waveform, the streaming potential of the fluid will have a buffer phase at the initial moment, and finally the overall trend will change from an unsteady state to a steady state as time increases.

Figure 5(b) shows the variations of the dimensionless streaming potential of square waveform with the dimensionless time at different values of the wall ζ potential ratio b . The wall ζ potential ratio in Fig. 5(b) is set to be -2 , -1 , 1 , and 2 , respectively. It can be found that the streaming potential increases with the increase in the absolute value of the wall ζ potential ratio. The reason is that the larger the b is, the greater the potential difference between the two walls of the annulus is, which causes larger streaming potentials. In addition, the direction of the streaming potential with negative values of b is opposite to that of the streaming potential with positive values of b . Moreover, the change of the streaming potential is a simple harmonic shape with the increase in time. In the case of square waveform, the streaming potential change is steady with time overall.

Figure 5(c) illustrates the variations of dimensionless streaming potential of triangular waveform with dimensionless time under different geometric radius ratio values of a . The geometric radius ratio in Fig. 5(c) is set to be 0.4 , 0.5 , and 0.6 , respectively. It is found that the dimensionless streaming potential decreases with the increase in the geometric radius ratio.

Fig. 4 Dimensionless velocity distributions of the triangular waveform with (a) $\Omega = 30$, (b) $\Omega = 60$, (c) $\Omega = 90$, and (d) $\Omega = 120$ ($K = 53$, $Ha = 2$, $\phi = 0.02$, $E_x = 10^5$, $a = 0.5$, and $b = 1$) (color online)

This is because larger geometric radius ratio leads to a narrower microannulus, and the same number of ions will be denser in the microannulus, which hinders the flow of fluid, thereby reducing the streaming potential. Besides, with the increase in time, in the case of triangular waveform, the change of the streaming potential shows a simple harmonic shape and is steady.

In Fig. 5(d), the dimensionless streaming potential of three waveforms with the dimensionless time in the case of the same parameters is compared. The results show that the square waveform excitation produces higher streaming potentials, while the triangular waveform produces slightly smaller streaming potentials, and the cosine waveform produces much smaller streaming potentials than the first two waveforms. This is because square waveform and triangular waveform have the mechanical energy multiple overlaps to convert into electric energy, and then increase the streaming potential, while the cosine waveform is relatively single. Moreover, we can see that the streaming potentials of the square waveform and the triangle waveform are steady with time, and the cosine waveform gradually changes from an unsteady state to a steady state. Furthermore, it is clearly seen that square waveform is more effective in increasing the streaming potential.

Figure 6(a) shows the variation of the EKEC efficiency ξ with dimensionless time under different values of the geometric radius ratio a of the cosine waveform. It can be observed that the EKEC efficiency decreases as the geometric radius ratio increases. The reason is

Fig. 5 Dimensionless streaming potential changing with the dimensionless time at different values of (a) dimensionless frequency Ω of the cosine waveform when $a = 0.5$, $b = 1$, $K = 10$, $Ha = 2$, $E_x = 10^2$, and $\phi = 0.02$; (b) b of the square waveform when $a = 0.5$, $K = 10$, $Ha = 2$, $E_x = 10^3$, $\Omega = 30$, and $\phi = 0.02$; (c) a of the triangular waveform when $b = 1$, $K = 10$, $Ha = 2$, $E_x = 10^2$, $\Omega = 30$, and $\phi = 0.02$ in the microchannel; (d) comparison of three waveforms when $a = 0.5$, $b = 1$, $K = 10$, $Ha = 2$, $E_x = 10^2$, $\Omega = 30$, and $\phi = 0.02$ (color online)

that a larger geometric radius ratio means a narrower microannulus, which makes the fluid flow slower, and thus the streaming potential decreases, which further leads to a lower EKEC efficiency. Therefore, in this case, adjusting the geometric radius ratio can have an impact on the EKEC efficiency. As a result, a smaller geometric radius ratio leads to a more effective improvement of the EKEC efficiency. Furthermore, in the case of cosine waveform, the EKEC efficiency will have a buffer stage at the initial moment. However, with the increase in time, the variation of the EKEC efficiency will change from an unsteady state to a steady state.

Figure 6(b) depicts the variation of the EKEC efficiency ξ with dimensionless time for different values of the wall ζ potential ratio b of the square waveform. The wall ζ potential ratio b in Fig. 6(b) is set to be 0.5, 1.0, 1.5, and 2.0, respectively. It can be seen from Fig. 6(b) that the EKEC efficiency increases as the wall ζ potential ratio b increases. This is because a greater wall ζ potential ratio means a greater potential difference between the two walls of the annulus, namely, the greater the streaming potential, the greater the EKEC efficiency. Hence, in this case, adjusting the wall ζ potential ratio can change the EKEC efficiency. Results show that the larger the wall ζ potential ratio b , the more effective the improvement of the EKEC

efficiency. Besides, it can be observed that the EKEC efficiency changing with time present a wave shape.

Figure 6(c) shows the variation of the EKEC efficiency ξ of square waveform with respect to the volume fraction of nanoparticles with different values over dimensionless time. The volume fraction of nanoparticles in Fig. 6(c) is set to be 0, 0.02, 0.03, and 0.05, respectively. It can be observed that the EKEC efficiency decreases as the volume fraction of nanoparticles increases. The reason is that as the volume fraction of nanoparticles increases, the increased number of nanoparticles weakens the velocity of the nanofluid, which in turn causes the streaming potential to decrease and further causes the EKEC efficiency to decrease. In addition, with the increase in time, the EKEC efficiency shows a decreasing trend and changes in a waveform.

Figure 6(d) illustrates the variations of EKEC efficiency ξ of triangular waveform with dimensionless time at dimensionless frequency with different values. The dimensionless frequency in Fig. 6(d) is set to be 30, 60, 90, and 120, respectively. From Fig. 6(d), with the increase in the dimensionless frequency, the EKEC efficiency gradually decreases. This is because the larger the dimensionless frequency is, the shorter the oscillation period of the EKEC efficiency is, and thus the energy cannot be completely converted, resulting in a smaller

Fig. 6 The EKEC efficiency ξ changing with the dimensionless time for different values (a) of a of the cosine waveform when $b = 1$, $K = 10$, $Ha = 2$, $E_x = 10^3$, $\Omega = 30$, and $\phi = 0.02$; (b) of b of the square waveform when $a = 0.5$, $K = 10$, $Ha = 2$, $E_x = 10^4$, $\Omega = 30$, and $\phi = 0.02$; (c) of ϕ of the square waveform when $a = 0.5$, $b = 1$, $K = 10$, $Ha = 2$, $E_x = 5 \times 10^3$, and $\Omega = 30$; (d) of Ω of the triangular waveform when $a = 0.5$, $b = 1$, $K = 25$, $Ha = 2$, $E_x = 10^3$, and $\phi = 0.02$ (color online)

EKEC efficiency. At the same time, the amplitude and period of the EKEC efficiency change gradually decrease. Similarly, we can clearly see that in the case of triangular waveform, the EKEC efficiency changes with time in a waveform and shows a decreasing trend.

Figure 7 shows a comparison of the variations of the EKEC efficiency for three waveforms versus the dimensionless time with the same parameters. It is obvious from Fig. 7 that the EKEC efficiency of the square waveform is the largest. In these three cases, their changes are waveforms and not monotonous. At the same time, in the case of cosine waveform, the change of the EKEC efficiency will have a buffer stage at the initial moment, and with the increase in time, it will change from an unsteady state to a steady state. For the other two waveforms, the buffer phase at the initial moment is not obvious. Furthermore, the maximum EKEC efficiencies for square waveform, triangle waveform, and cosine waveform are 1.618×10^{-2} , 6.566×10^{-3} , and 8.697×10^{-4} , respectively. Therefore, we can get the result that the EKEC efficiency of square waveform is 146.42% higher than that of triangle waveform, and the EKEC efficiency of triangle waveform is 654.97% higher than that of cosine waveform. Especially, compared with triangle waveform and cosine waveform, square waveform is more effective to improve the EKEC efficiency.

Fig. 7 Comparison of the EKEC efficiency ξ for three waveforms changing with the dimensionless time when $a = 0.5$, $b = 1$, $K = 10$, $Ha = 2$, $E_x = 5 \times 10^3$, $\phi = 0.02$, and $\Omega = 30$ (color online)

4 Conclusions

In this paper, under the condition of unsteady state, the flow and EKEC of nanofluids through a microannulus under the combined action of applied pressure gradient and magnetic field are analyzed. The analytical solutions of velocity fields under three different periodic excitations (cosine waveform, square waveform, and triangle waveform) are obtained by using Green's function method. Moreover, the variations of their streaming potential and EKEC efficiency with time under the influence of different parameters, such as the electrokinetic radius K , the dimensionless frequency Ω , the Hartmann number Ha , the volume fraction of the nanoparticles ϕ , the geometric radius ratio a , and the wall ζ potential ratio b , are further discussed. The results show that the velocity distribution decreases with the increase in the volume fraction and frequency, and thus the streaming potential and the EKEC efficiency decrease. It is observed in this study that, the larger the geometric radius ratio is, the smaller the streaming potential and the EKEC efficiency are. Besides, a larger wall ζ potential ratio leads to a larger streaming potential and a larger EKEC efficiency, and at the same time, the sign of the wall ζ potential ratio can affect the direction of the streaming potential. Therefore,

adjusting the geometric radius ratio and the wall ζ potential ratio can have an impact on the streaming potential and the EKEC efficiency. In addition, the comparisons of the streaming potential and EKEC efficiency under the excitation of three different time periodic functions with the same parameters are also discussed. Through comparison and discussion, it is found that the square waveform is more effective in improving the streaming potential and the EKEC efficiency.

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