New insight into the stability and dynamics of fluid-conveying supported pipes with small geometric imperfections*

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(Received Oct. 13, 2020 / Revised Feb. 25, 2021)

Abstract In several previous studies, it was reported that a supported pipe with small geometric imperfections would lose stability when the internal flow velocity became sufficiently high. Recently, however, it has become clear that this conclusion may be at best incomplete. A reevaluation of the problem is undertaken here by essentially considering the flow-induced static deformation of a pipe. With the aid of the absolute nodal coordinate formulation (ANCF) and the extended Lagrange equations for dynamical systems containing non-material volumes, the nonlinear governing equations of a pipe with three different geometric imperfections are introduced and formulated. Based on extensive numerical calculations, the static equilibrium configuration, the stability, and the nonlinear dynamics of the considered pipe system are determined and analyzed. The results show that for a supported pipe with the geometric imperfection of a half sinusoidal wave, the dynamical system could not lose stability even if the flow velocity reaches an extremely high value of 40. However, for a supported pipe with the geometric imperfection of one or one and a half sinusoidal waves, the first-mode buckling instability would take place at high flow velocity. Moreover, based on a further parametric analysis, the effects of the amplitude of the geometric imperfection and the aspect ratio of the pipe on the static deformation, the critical flow velocity for buckling instability, and the nonlinear responses of the supported pipes with geometric imperfections are analyzed.

Key words supported pipes conveying fluid, geometric imperfection, absolute nodal coordinate formulation (ANCF), static equilibrium configuration, critical flow velocity, nonlinear dynamics

* Citation: ZHOU, K., NI, Q., CHEN, W., DAI, H. L., PENG, Z. R., and WANG, L. New insight into the stability and dynamics of fluid-conveying supported pipes with small geometric imperfections. *Applied Mathematics and Mechanics (English Edition)*, 42(5), 703–720 (2021) <https://doi.org/10.1007/s10483-021-2729-6>

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Project supported by the National Natural Science Foundation of China (Nos.11972167 and 12072119) and the Alexander von Humboldt Foundation

Chinese Library Classification O322, O326
2010 Mathematics Subject Classification 37K45, 37K50

1 Introduction

In the past for a long time, the system of pipes conveying fluid has been a hot topic because of its applications in many engineering industries, e.g., nuclear industry, marine engineering, and aviation industry. In order to make pipes better and safer to serve engineering industries, the establishment of theoretical models is essential for predicting and suppressing the flow-induced vibration (FIV) of pipes conveying fluid. Under this consideration, a large number of articles devoted to this hot topic have emerged in the past decades. Most of the existing studies were concerned with straight pipes conveying fluid under cantilevered^[1–5] or supported^[6–13] boundary conditions, which are fundamentally different since the cantilevered pipe system is non-conservative while the pipe with both ends supported corresponds to a conservative system^[14]. It was also shown that the system of a straight pipe is capable of displaying rich dynamical behaviors when some other nonlinearities have been added to the pipe system.

In addition to straight pipes, circular-shaped curved pipes are also one of the typical components in engineering industries for fluid transport. Thus, the problem of circular pipes conveying fluid has also attracted the attention of many scholars^[15–27]. According to the discussion of Misra et al.^[20–21], the studies on circular pipes conveying fluid were mainly based on three different theories, i.e., the conventional inextensible theory, the extensible theory, and the modified inextensible theory. In order to compare these three theories in detail, Misra et al.^[20–21] considered three different semi-circular pipe models. Their numerical results indicated that a semi-circular pipe with both ends supported would lose stability via buckling when the conventional inextensible theory was considered. However, if the extensible theory or the modified inextensible theory is considered, the results will be quite different: buckling instability is impossible in the considered semi-circular pipe with both ends supported even if the internal flow velocity reaches an extremely large value. Up to date, the extensible theory and the modified inextensible theory have been accepted by most researchers, and most of the subsequent studies on semi-circular pipes conveying fluid are based on the extensible theory or the modified inextensible theory.

The centerlines of fluid-conveying pipes mentioned above are either straight or circularly curved. That is, the initial shapes of the centerlines are perfect and idealized. In engineering practice, however, initial geometric imperfections may always exist in pipes due to machining errors. Thus, the dynamics of fluid-conveying pipes with initial geometric imperfections needs to be explored in many cases^[28–39].

Sinir^[28] investigated the dynamics of a slightly curved supported pipes conveying steady flow, and indicated that periodic and chaotic motions could occur in the considered slightly curved pipe system. Wang et al.^[29] performed linear and nonlinear analyses to determine the critical flow velocities and post-buckling responses of the simply supported pipes conveying fluid with four different types of initial geometric imperfections. The results indicated that buckling instability would occur in all cases. Hu and Zhu^[30] proposed a novel linear theoretical model to deal with the supported pipes conveying fluid with an arbitrary initial curved shape, and demonstrated that the initial curved configurations could have great effects on the critical flow velocity for buckling instability. Orolu et al.^[31] explored the effects of the initial curvature, temperature, and longitudinal vibration on the linear and nonlinear dynamics of a slightly curved pipes conveying hot pressurized fluid. Oyelade and Oyediran^[32] investigated the linear and nonlinear dynamics of slightly curved pipes under thermal loading with different boundary conditions, including pinned-pinned, clamped-clamped, and clamped-pinned ones, considering both longitudinal and transverse vibrations. They showed that the critical flow velocity for buckling instability would decrease by increasing the thermal loading and increase by increasing

the amplitude of the initial curved configuration. Li and Yang^[33] explored the dynamics of a slightly curved pipes conveying pulsating fluid, mainly devoted to determining the curves of the resonance responses, and observed some dynamical behaviors, e.g., periodic, quasi-periodic, and chaotic vibrations. Based on a novel nonlinear dynamical model, Li et al.^[34] studied the nonlinear parametric vibrations of a slightly curved supported pipes conveying pulsating fluid in the pre- and post-buckling states. Czerwiński and Luczko^[35] developed a 3D dynamic model to investigate the non-planar parametric vibrations of slightly curved fluid-conveying pipes with both ends supported.

It must be mentioned that theoretical models for slightly curved supported pipes proposed in the above literature are mainly developed based on the small-deformation assumption. Most importantly, the static deformations due to the flowing fluid of slightly curved fluid-conveying pipes are not considered. However, since the pipe under consideration is initially curved, the static deformation does exist and may be expected to have great effects on the dynamics of the pipe. To the authors' best knowledge, unfortunately, this fact has not been taken into account in previous studies about supported pipes with small geometric imperfections. Bearing this in mind, the problem of fluid-conveying supported pipes with small geometric imperfections will be re-visited in this work by applying the large deformation finite element method, i.e., absolute nodal coordinate formulation (ANCF), originally proposed by Shabana^[40] for other problems.

The remaining parts of this paper are organized as follows. The nonlinear governing equations of the considered pipe system are introduced in Section 2 with the aid of the ANCF and the extended Lagrange equations. Based on the static equilibrium equation, the static equilibrium configurations due to the flowing fluid of the pipe are determined in Section 3. In Section 4, a linear analysis is performed to explore the stability of the pipe around the static equilibrium configuration. Then, the nonlinear responses of the pipe are predicted in Section 5. Finally, some important and interesting conclusions are summarized in Section 6.

2 Theoretical model of the pipe system

In this paper, the horizontally placed pinned-pinned pipes with the length L , the mass per unit length m , the flexural rigidity EI , and the cross-sectional area A_p are studied, conveying the fluid with the mass per unit length M and the mean axial velocity U . As shown in Fig. 1, three different small geometric imperfections are considered: (a) Model I, a half sinusoidal wave (the first-mode eigenfunction of a simply supported beam, $y_0(x) = a_0 \sin(\pi x)$); (b) Model II, one sinusoidal wave (the second-mode eigenfunction of a simply supported beam, $y_0(x) = a_0 \sin(2\pi x)$); and (c) Model III, one and a half sinusoidal waves (the third-mode eigenfunction of a simply supported beam, $y_0(x) = a_0 \sin(3\pi x)$).

In the existing studies on supported pipes with geometric imperfections, the nonlinear

Fig. 1 Schematic of the fluid-conveying supported pipes with three different geometric imperfections

governing equations of the pipes were always derived by employing the Hamilton principle based on the assumption of relatively small deformation. In fact, this method may not be accurate enough in many cases. In this paper, therefore, a more suitable ANCF will be used to develop the dynamical model of the supported pipes with geometric imperfections. Before introducing the nonlinear governing equations of the considered pipe system, it should be mentioned that some basic assumptions should be made to simplify the theoretical model: (i) the internal fluid is considered to be incompressible and non-viscous; (ii) the internal flow velocity is constant; (iii) the pipe is considered to behave like an Euler-Bernoulli beam, so that the rotary inertia and shear deformation of the pipe can be neglected; (iv) the pipe centerline is extensible; (v) the motions of the pipe are assumed to be planar; (vi) the initial length of the pipe is L for all pipe models. Based on these assumptions, the 2-node planar curved ANCF elements^[41] are chosen to discretize the pipe. Then, the extended Lagrange equation introduced by Irschik and Holl^[42] is adopted to derive the nonlinear governing equations of the pipe element^[39]. The final nonlinear governing equations can be written as

$$\mathbf{M}_e \ddot{\mathbf{q}} + \mathbf{C}_e \dot{\mathbf{q}} + \mathbf{K}_e \mathbf{q} + \mathbf{N}_e(\mathbf{q}) = \mathbf{0}, \quad (1)$$

where $\mathbf{q}$ represents the nodal coordinate vector of the pipe element. $\mathbf{M}_e$, $\mathbf{C}_e$, and $\mathbf{K}_e$ denote the linear mass, damping, and stiffness matrices of the pipe element, respectively. $\mathbf{N}_e(\mathbf{q})$ is the vector of the nonlinear terms. These matrices and vectors can be expressed as follows^[39]:

$$\left\{ \begin{array}{l} \mathbf{M}_e = (M + m) \int_0^L \mathbf{S}^T \mathbf{S} dx, \\ \mathbf{C}_e = MU \int_0^L \frac{\mathbf{S}^T \mathbf{S}' - \mathbf{S}'^T \mathbf{S}}{\sqrt{\mathbf{q}_0^T \mathbf{S}'^T \mathbf{S}' \mathbf{q}_0}} dx + MU (\mathbf{S}^T \mathbf{S}|_{x=L} - \mathbf{S}^T \mathbf{S}|_{x=0}), \\ \mathbf{K}_e = - \left(MU^2 + \frac{1}{2} EA_p \right) \int_0^L \frac{\mathbf{S}'^T \mathbf{S}'}{\mathbf{q}_0^T \mathbf{S}'^T \mathbf{S}' \mathbf{q}_0} dx \\ \quad + MU^2 \left(\frac{\mathbf{S}^T \mathbf{S}'}{\sqrt{\mathbf{q}_0^T \mathbf{S}'^T \mathbf{S}' \mathbf{q}_0}} \Big|_{x=L} - \frac{\mathbf{S}^T \mathbf{S}'}{\sqrt{\mathbf{q}_0^T \mathbf{S}'^T \mathbf{S}' \mathbf{q}_0}} \Big|_{x=0} \right), \\ \mathbf{N}_e(\mathbf{q}) = \frac{1}{2} EA_p \int_0^L \frac{\mathbf{S}'^T \mathbf{S}' \mathbf{q} \mathbf{q}^T \mathbf{S}'^T \mathbf{S}' \mathbf{q}}{(\mathbf{q}_0^T \mathbf{S}'^T \mathbf{S}' \mathbf{q}_0)^2} dx \\ \quad + EI \int_0^L \left(\frac{((\tilde{\mathbf{I}} \mathbf{S}')^T \mathbf{S}'' + \mathbf{S}''^T (\tilde{\mathbf{I}} \mathbf{S}')^T) \mathbf{q} \mathbf{q}^T (\tilde{\mathbf{I}} \mathbf{S}')^T \mathbf{S}'' \mathbf{q}}{(\mathbf{q}^T \mathbf{S}'^T \mathbf{S}' \mathbf{q})^3} \right. \\ \quad \left. - \frac{3 \mathbf{S}'^T \mathbf{S}' \mathbf{q} \mathbf{q}^T (\tilde{\mathbf{I}} \mathbf{S}')^T \mathbf{S}'' \mathbf{q} \mathbf{q}^T (\tilde{\mathbf{I}} \mathbf{S}')^T \mathbf{S}'' \mathbf{q}}{(\mathbf{q}^T \mathbf{S}'^T \mathbf{S}' \mathbf{q})^4} \right) dx \\ \quad - EI \int_0^L \frac{\mathbf{q}_0^T (\tilde{\mathbf{I}} \mathbf{S}')^T \mathbf{S}'' \mathbf{q}_0}{(\mathbf{q}_0^T \mathbf{S}'^T \mathbf{S}' \mathbf{q}_0)^{3/2}} \left(\frac{((\tilde{\mathbf{I}} \mathbf{S}')^T \mathbf{S}'' + \mathbf{S}''^T (\tilde{\mathbf{I}} \mathbf{S}')^T) \mathbf{q}}{(\mathbf{q}^T \mathbf{S}'^T \mathbf{S}' \mathbf{q})^{3/2}} \right. \\ \quad \left. - \frac{3 \mathbf{S}'^T \mathbf{S}' \mathbf{q} \mathbf{q}^T (\tilde{\mathbf{I}} \mathbf{S}')^T \mathbf{S}'' \mathbf{q}}{(\mathbf{q}^T \mathbf{S}'^T \mathbf{S}' \mathbf{q})^{5/2}} \right) dx, \end{array} \right. \quad (2)$$

where $\mathbf{q}_0$ is the nodal coordinate vector of the initial curved pipe element, and can be determined from the expression of $y_0(x)$. $\mathbf{S}$ is the global shape function of the curved pipe element, which

has been defined in Ref. [43]. The matrix $\tilde{\mathbf{I}}$ can be written as follows:

$$\tilde{\mathbf{I}} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}. \quad (3)$$

For the detailed derivations of Eqs. (1) and (2), one may refer to Ref. [39]. In order to make the following analysis more general, the following dimensionless quantities are defined:

$$\begin{cases} \tau = \left(\frac{EI}{M+m} \right)^{\frac{1}{2}} \frac{t}{L^2}, & \mathbf{q}^* = \frac{\mathbf{q}}{L}, & \mathbf{q}_0^* = \frac{\mathbf{q}_0}{L}, \\ u = \left(\frac{M}{EI} \right)^{\frac{1}{2}} UL, & \beta = \frac{M}{M+m}, & \Pi_0 = \frac{A_p L^2}{I}. \end{cases} \quad (4)$$

Then, by substituting Eq. (4) into Eq. (1), one can obtain the following equations:

$$\mathbf{M}_e^* \ddot{\mathbf{q}}^* + \mathbf{C}_e^* \dot{\mathbf{q}}^* + \mathbf{K}_e^* \mathbf{q}^* + \mathbf{N}_e^*(\mathbf{q}^*) = \mathbf{0}, \quad (5)$$

where

$$\left\{ \begin{aligned} \mathbf{M}_e^* &= \int_0^l \mathbf{S}^T \mathbf{S} dx, \\ \mathbf{C}_e^* &= u \sqrt{\beta} \int_0^1 \frac{\mathbf{S}^T \mathbf{S}' - \mathbf{S}'^T \mathbf{S}}{\sqrt{\mathbf{q}_0^{*T} \mathbf{S}'^T \mathbf{S}' \mathbf{q}_0^*}} dx + u \sqrt{\beta} L (\mathbf{S}^T \mathbf{S}|_{x=l} - \mathbf{S}^T \mathbf{S}|_{x=0}), \\ \mathbf{K}_e^* &= - \left(u^2 + \frac{1}{2} \Pi_0 \right) \int_0^l \frac{\mathbf{S}'^T \mathbf{S}'}{\mathbf{q}_0^{*T} \mathbf{S}'^T \mathbf{S}' \mathbf{q}_0^*} dx \\ &\quad + u^2 L \left(\frac{\mathbf{S}^T \mathbf{S}'}{\sqrt{\mathbf{q}_0^{*T} \mathbf{S}'^T \mathbf{S}' \mathbf{q}_0^*}} \Big|_{x=l} - \frac{\mathbf{S}^T \mathbf{S}'}{\sqrt{\mathbf{q}_0^{*T} \mathbf{S}'^T \mathbf{S}' \mathbf{q}_0^*}} \Big|_{x=0} \right), \\ \mathbf{N}_e^*(\mathbf{q}^*) &= \frac{1}{2} \Pi_0 \int_0^l \frac{\mathbf{S}'^T \mathbf{S}' \mathbf{q}^* \mathbf{q}^{*T} \mathbf{S}'^T \mathbf{S}' \mathbf{q}^*}{(\mathbf{q}_0^{*T} \mathbf{S}'^T \mathbf{S}' \mathbf{q}_0^*)^2} dx \\ &\quad + \int_0^l \left(\frac{((\tilde{\mathbf{I}} \mathbf{S}')^T \mathbf{S}'' + \mathbf{S}''^T (\tilde{\mathbf{I}} \mathbf{S}')^T) \mathbf{q} \mathbf{q}^{*T} (\tilde{\mathbf{I}} \mathbf{S}')^T \mathbf{S}'' \mathbf{q}}{(\mathbf{q}^{*T} \mathbf{S}'^T \mathbf{S}' \mathbf{q}^*)^3} \right. \\ &\quad \left. - \frac{3 \mathbf{S}'^T \mathbf{S}' \mathbf{q}^* \mathbf{q}^{*T} (\tilde{\mathbf{I}} \mathbf{S}')^T \mathbf{S}'' \mathbf{q}^* \mathbf{q}^{*T} (\tilde{\mathbf{I}} \mathbf{S}')^T \mathbf{S}'' \mathbf{q}^*}{(\mathbf{q}^{*T} \mathbf{S}'^T \mathbf{S}' \mathbf{q}^*)^4} \right) dx \\ &\quad - \int_0^l \frac{\mathbf{q}_0^{*T} (\tilde{\mathbf{I}} \mathbf{S}')^T \mathbf{S}'' \mathbf{q}_0^*}{(\mathbf{q}_0^{*T} \mathbf{S}'^T \mathbf{S}' \mathbf{q}_0^*)^{3/2}} \left(\frac{((\tilde{\mathbf{I}} \mathbf{S}')^T \mathbf{S}'' + \mathbf{S}''^T (\tilde{\mathbf{I}} \mathbf{S}')^T) \mathbf{q}^*}{(\mathbf{q}^{*T} \mathbf{S}'^T \mathbf{S}' \mathbf{q}^*)^{3/2}} \right. \\ &\quad \left. - \frac{3 \mathbf{S}'^T \mathbf{S}' \mathbf{q}^* \mathbf{q}^{*T} (\tilde{\mathbf{I}} \mathbf{S}')^T \mathbf{S}'' \mathbf{q}^*}{(\mathbf{q}^{*T} \mathbf{S}'^T \mathbf{S}' \mathbf{q}^*)^{5/2}} \right) dx. \end{aligned} \right. \quad (6)$$

After determining the dynamic equations of each pipe element, the nonlinear governing equations of the whole pipe system can be obtained by assembling all the element equations as follows:

$$\mathbf{M} \ddot{\mathbf{e}} + \mathbf{C} \dot{\mathbf{e}} + \mathbf{K} \mathbf{e} + \mathbf{N}(\mathbf{e}) = \mathbf{0}, \quad (7)$$

where $\mathbf{M}$, $\mathbf{C}$, and $\mathbf{K}$ denote the linear mass, damping, and stiffness matrices for the whole pipe system, respectively. $\mathbf{e}$, $\dot{\mathbf{e}}$, and $\ddot{\mathbf{e}}$ are the generalized coordinate, velocity, and acceleration vectors of the whole pipe system, respectively. $\mathbf{N}(\mathbf{e})$ represents the assembled vector of nonlinearities. It is noticed that Eq.(7) are ordinary differential equations, and hence the fourth-order Runge-Kutta integration algorithm can be directly used to solve these equations after applying the pinned-pinned boundary conditions.

Before embarking some numerical results, it should be noted that 8 planar curved ANCF pipe elements will be adopted to discretize the considered supported pipes with geometric imperfections, including Model I, Model II, and Model III. Appendix A is given to prove that the utilization of 8 pipe elements can obtain the convergent numerical results. In addition, some results of two typical examples reported in existing studies are reproduced in Appendix B to demonstrate the accuracy of the present ANCF pipe model on studying the dynamics of straight and circularly curved pipes with both ends supported.

3 Static equilibrium configuration of the geometrically imperfect pipe

Since the pipes conveying fluid considered in the present study have geometric imperfections and are initially curved, the pipes may have static deformations under the action of the internal flow, even though the initial geometric imperfections are quite small. Most importantly, the static deformations may have great effects on the stability of the curved pipe^[14]. It should be pointed out again that the static deformations and their effects on the stability of the pipe were not taken into account in several previous studies on fluid-conveying supported pipes with geometric imperfections^[28–29,31–32]. This is the main reason why the new results shown in this paper are quite different from those in the literature, which will be proven in the next section. In this section, the static equilibrium configurations of the fluid-conveying supported pipes with different geometric imperfections are explored in detail.

To determine the static deformations of pipes with geometric imperfections, the generalized coordinate vector $\mathbf{e}$ can be divided into two parts, i.e., the static part $\mathbf{e}_s$ and a perturbation $\Delta\mathbf{e}$ about the static part, i.e.,

$$\mathbf{e} = \mathbf{e}_s + \Delta\mathbf{e}. \quad (8)$$

Then, by substituting Eq. (8) into Eq. (7) and deleting all time-dependent terms, we can obtain the static equilibrium equations of the considered pipe system as

$$\mathbf{K}\mathbf{e}_s + \mathbf{N}(\mathbf{e}_s) = \mathbf{0}. \quad (9)$$

Therefore, the static equilibrium configurations can be determined by solving Eq. (9) with the aid of a Newton-Raphson method.

3.1 Effects of geometric imperfections on the static equilibrium configurations

The effects of geometric imperfections on the static equilibrium configurations of the pipes will be explored first. To this end, based on Eq. (9), the static equilibrium configurations of the supported pipes with three different geometric imperfections, including Model I, Mode II, and Model III, are, respectively, displayed in Figs. 2(a), 2(b), and 2(c) for various flow velocities. In these figures,

$$L = 1, \quad \beta = 0.47, \quad \Pi_0 = 10\,000, \quad a_0 = 0.01.$$

It should be pointed out that, in all the following static deformation diagrams, red lines denote the initial curved configurations of the pipe, while blue lines denote the static equilibrium configurations at different flow velocities. By inspecting Fig. 2, it is immediately found that the initial geometric imperfections have great effects on the static equilibrium configurations of the pipe systems under consideration.

Fig. 2 Static equilibrium configurations of the supported pipes conveying fluid with three different geometric imperfections, i.e., (a) Model I, (b) Model II, and (c) Model III, for $a_0 = 0.01$, $\beta = 0.47$, and $\Pi_0 = 10\,000$, and various flow velocities (color online)

The static equilibrium configurations for Model I are illustrated in Fig. 2(a) for various flow velocities. In this case, the geometric imperfection is in the form of a half sinusoidal wave (the first-mode eigenfunction of a simply supported beam). From this figure, it can be easily found that the amplitude of the static equilibrium configuration increases with increasing the flow velocity. When the flow velocity increases to 8, the amplitude of the static deformation almost reaches 0.05, which is 5 times that of the amplitude of the initial geometric imperfection. Most importantly, in the range of the flow velocity under consideration, the static equilibrium configurations for Model I are always in the form of the first-mode eigenfunction of a simply supported beam. In other words, for Model I, an increase in the flow velocity can only increase the amplitude of the static equilibrium configuration but cannot change its basic shape.

The static equilibrium configurations for Model II are illustrated in Fig. 2(b) for various flow velocities. By comparing the results shown in Fig. 2(b) with those shown in Fig. 2(a), three different features can be detected: (i) the midpoint vertical coordinate of the static equilibrium configuration may be either negative or positive, depending on the flow velocity chosen for calculations; (ii) the flow velocity changes not only the amplitude of the static equilibrium configuration but also its basic shape in the range of $4.2 < u < 8$, while only slightly changes the amplitude of the static equilibrium configuration in the range of $0 < u < 4.2$; (iii) with the increase in the flow velocity, the static equilibrium configuration is shaped like that of Model I, but not exactly the same. The first feature can be easily understood since this is a bi-stable problem. The numerical solution for such a pipe may be either positive or negative, depending on the initial conditions utilized in the numerical calculations. The second feature is actually related to the occurrence of buckling instability. In the range of $0 < u < 4.2$, the flow velocity has a small effect on the amplitude of the static equilibrium configuration, while has no effect on its basic shape. When the flow velocity exceeds 4.2, however, the static equilibrium configuration of the pipe deviates from the initial curved shape (the second-mode shape of a simply supported beam) since the buckling instability occurs. Thus, it is speculated that the flow velocity may have considerable effects on the amplitude and basic shape of the static equilibrium configuration in the range of $4.2 < u < 8$. This conjecture will be further confirmed by a linear stability analysis performed in Section 4. With regard to the foregoing discussion on the deformed configurations of Model II, something important should be stressed. When the flow velocity exceeds the critical value, the static equilibrium configuration of the pipe is just the buckling configuration, since the buckling is a static form of instability and is independent of time. As for the third feature, it can be explained by that, in this case, the buckling instability occurs first in the first mode. Thus, the static equilibrium configurations (buckling configurations) will approach the first-mode shape of a simply supported beam with the increase in the flow velocity. It is also noted that the buckling configurations of Model II are asymmetric due to the presence of geometric imperfection.

The static equilibrium configurations for Model III are shown in Fig. 2(c). Although the curves plotted in Fig. 2(c) look quite different from those given in Fig. 2(b), the laws that they change with the flow velocity are fundamentally the same. Based on previous discussion, two conclusions can be drawn from Fig. 2(c): (i) the first-mode buckling instability can occur in the case of Model III within the range of $0 < u < 10$; (ii) the critical flow velocity for buckling instability is about $u_{cr} = 4.0$. These two conclusions can also be demonstrated in Section 4, where the linear stability analysis for the pipe around the static equilibrium configuration will be performed.

3.2 Effects of a_0 on the static equilibrium configurations

After the effects of geometric imperfections on the static equilibrium configurations are determined, in this subsection, the effects of the values of a_0 will be investigated. For that purpose, three different values of a_0 , including 0.005, 0.01, and 0.05, are taken into account, and the other system parameters are selected to be

$$L = 1, \quad \beta = 0.2, \quad \Pi_0 = 10\,000.$$

Based on a large number of numerical calculations, the static equilibrium configurations of the fluid-conveying supported pipes for Model I, Model II, and Model III for different values of a_0 are given in Fig. 3.

The results for Model I with different values of a_0 can be found in Figs. 3(a1), 3(a2), and 3(a3). Comparing these figures, it can be easily found that with the increase in a_0 , the static deformation of the pipe relative to the initial curved configuration is reduced at the same flow velocity. In other words, for fluid-conveying supported pipes with a relatively large geometric imperfection, a higher flow velocity is required to produce a large static deformation. In addition, for all considered a_0 , the static equilibrium configurations for Model I are always shaped as the first-mode shape of a simply supported beam. This feature also means that the buckling instability does not occur in the system of Model I. This is, of course, unexpected.

The curves plotted in Figs. 3(b1), 3(b2), and 3(b3) are the results for Model II with different a_0 . From this figure, it is noted that when a_0 is small, such as 0.005 or 0.01, there is always a critical flow velocity that can make the static equilibrium configuration deviate from the initial curved shape, indicating that the buckling instability occurs in this situation. It is also found that the critical flow velocity of Model II with $a_0 = 0.01$ is larger than that of Model II with $a_0 = 0.005$. When the amplitude of the geometric imperfection is increased to 0.05, the static deformation relative to the initial curved configuration is quite small and is always the second-mode shape of a simply supported beam in the range of $0 < u < 10$, indicating that Model II with $a_0 = 0.05$ does not lose stability within the considered flow velocity range.

The results for Model III are shown in Figs. 3(c1), 3(c2), and 3(c3). It can be seen that the results are quite similar to those for Model II given in Figs. 3(b1), 3(b2), and 3(b3). Some similar features include: (i) when a_0 is small, such as 0.005 or 0.01, the buckling instability will occur in the considered system of Model III; (ii) the critical flow velocity for the case of $a_0 = 0.01$ is larger than that for the case of $a_0 = 0.005$; (iii) when the amplitude of the geometric imperfection becomes large, the pipe of Model III is not easy to lose stability.

3.3 Effects of Π_0 on the static equilibrium configurations

Finally, the effects of the values of Π_0 on the static deformations are explored. To achieve this goal, numerical calculations are progressed with three different values of Π_0 , including 1 000, 4 000, and 10 000. The corresponding numerical results are summarized in Fig. 4. Based on these plotted curves, it is indicated that the values of Π_0 have considerable effects on the static equilibrium configurations. Four features can be observed: (i) in all cases, when the value of Π_0 is increased, the amplitude of the static equilibrium configuration will decrease; (ii) for all values of Π_0 under consideration, the static equilibrium configuration of Model I is always the first-mode shape of a simply supported beam; (iii) when Π_0 is small, the static equilibrium

Fig. 3 Static equilibrium configurations of the supported pipes conveying fluid with three different geometric imperfections, i.e., (a) Model I, (b) Model II, and (c) Model III, for different values of a_0 with $\beta = 0.47$, $\Pi_0 = 10\,000$, and various flow velocities (color online)

configurations of Model II and Model III gradually approach the first-mode shape of a simply supported beam as the flow velocity increases; (iv) when Π_0 is large, the basic shapes of the static deformations for Model II and Model III are always identical to the initial curved shape. From these features, we can also conclude that the increase in Π_0 will increase the critical flow velocities of Model II and Model III for buckling instability. Actually, for a fluid-conveying supported pipe without geometric imperfections, the values of Π_0 will not affect the critical flow velocity, since Π_0 is related to the nonlinear terms. However, based on the results shown in Fig. 4, the situation is different for a supported pipe with geometric imperfections. This can be easily understood since the critical flow velocity of the supported pipe with geometric imperfections is closely related to the static equilibrium configuration, while the values of Π_0 can affect the static equilibrium configuration.

4 Stability analysis around the static equilibrium configuration of the pipe

The results shown in Section 3 indicate that the critical flow velocity and the buckling mode of the fluid-conveying supported pipe with geometric imperfections are strongly associated with the static equilibrium configuration. To further demonstrate this point, a linear stability

Fig. 4 Static equilibrium configurations of the supported pipes conveying fluid with three different geometric imperfections, i.e., (a) Model I, (b) Model II, and (c) Model III, for different values of Π_0 with $\beta = 0.47$, $a_0 = 0.05$, and various flow velocities (color online)

analysis around the static equilibrium configuration of the pipe will be performed in this section. For that purpose, the nonlinear equations of Eq. (7) are linearized. Then, one can obtain

$$\mathbf{M}\Delta\ddot{\mathbf{e}} + \mathbf{C}\Delta\dot{\mathbf{e}} + (\mathbf{K} + \mathbf{K}_T)\Delta\mathbf{e} = \mathbf{0}, \quad (10)$$

where $\mathbf{K}_T$ is the tangential stiffness matrix at the static equilibrium configuration, and

$$\mathbf{K}_T = \left. \frac{\partial \mathbf{N}(\mathbf{e})}{\partial \mathbf{e}} \right|_{\mathbf{e}=\mathbf{e}_s}. \quad (11)$$

After some transformations, Eq. (10) can be rewritten as the first-order form, and then the frequencies of the pipe around the static equilibrium configuration can be determined.

The lowest four dimensionless frequencies of the transverse motions as a function of the dimensionless flow velocity for Model I, Model II, and Model III with

$$a_0 = 0.01, \quad \beta = 0.47, \quad \Pi_0 = 10\,000$$

are displayed in Fig. 5. It should be mentioned that when the frequency drops to zero, the corresponding flow velocity is defined as the critical flow velocity for buckling instability.

Fig. 5 The first four-order dimensionless frequencies of the transverse motions as a function of the dimensionless flow velocity for (a) Model I, (b) Model II, and (c) Model III with $a_0 = 0.01$, $\beta = 0.47$, and $\Pi_0 = 10\,000$ (color online)

Combining this definition, three points can be observed: (i) the buckling instability does not occur in the pipe system of Model I within the range of $0 < u < 10$; (ii) for both Model II and Model III, the first-mode buckling instability can occur in the range of $0 < u < 10$; (iii) the critical flow velocity for Model II is about 4.2, while that for Model III is about 4.0. Obviously, these linear results are in agreement with the conclusions about the static equilibrium configurations shown in Subsection 3.1. This demonstrates that the discussions about the static equilibrium configurations of the pipes are correct.

It has been shown in Refs. [28], [29], [31], and [32] that the supported pipe with the geometric imperfection of a half sinusoidal wave would be subjected to buckling instability when the flow velocity becomes high, which is completely contrary to the result obtained in this study. There may be two possible reasons: (i) the effect of the static deformation on the stability of the pipe was not taken into account in all previous investigations, while in the present study it is considered; (ii) the flow velocity range considered in this paper is not high enough to cause buckling instability in the pipe system of Model I. To find out the real reason for the inconsistency between the present results and those reported in previous studies, additional calculations are performed, and the results are drawn in Fig. 6.

In Fig. 6(a), the effect of the static deformation on the stability of the pipe system is not taken into account in the ANCF pipe model. Interestingly, the results shown in Fig. 6(a) are identical to those reported in Refs. [28], [29], [31], and [32], i.e., buckling instability can occur

Fig. 6 The first four-order dimensionless frequencies of the transverse motions as a function of the dimensionless flow velocity for Model I with $a_0 = 0.01$, $\beta = 0.47$, and $\Pi_0 = 1\,000$ (a) without and (b) with the consideration of the effect of the static deformation (color online)

in the pipe system of Model I. This also means that the first possible reason is reasonable. In Fig. 6(b), the effect of the static deformation is considered, and the range of flow velocity is changed to be $0 < u < 40$. It can be found that buckling instability still does not occur in the pipe system of Model I, even if the flow velocity reaches an extremely large value of 40. Thus, combining Figs. 6(a) and 6(b), it is believed that the real reason for the inconsistency between the present results and those in Refs. [28], [29], [31], and [32] is whether the effect of the flow-induced static deformation is considered or not. Due to this fact, an important conclusion is carried out for the fluid-conveying supported pipes with geometric imperfections: the stability analysis must be performed around the static equilibrium configurations. Only in this way, the stability of the pipe system can be determined correctly. The reason why Model I cannot be destabilized while Model II and Model III can be destabilized may be explained by the fact that the initial curved shape of Model I is generally consistent with the instability mode shape of a straight pipe. In other words, when the initial curved shape of the supported pipe is identical to the instability mode shape of the straight pipe with the same system parameters, the pipe may not lose stability.

The results given in Subsections 3.2 and 3.3 indicate that the values of a_0 and Π_0 could have effects on the static deformations of the supported pipe with geometric imperfections. Thus, it is expected that these two parameters will also affect the critical flow velocities of Model II and Model III. To analyze this further, the critical flow velocities for Model II and Model III as functions of a_0 and Π_0 are shown in Fig. 7. From this figure, it can be found that the critical flow velocities of Model II and Model III increase exponentially with the increase in a_0 , while increase linearly with the increase in Π_0 . Moreover, the critical flow velocity of Model II is always slightly higher than that of Model III.

Fig. 7 Dimensionless critical flow velocities for Model II and Model III as functions of (a) a_0 with $\beta = 0.47$ and $\Pi_0 = 1000$ and (b) Π_0 with $\beta = 0.47$ and $a_0 = 0.01$ (color online)

5 Nonlinear analysis of the geometrically imperfect pipe

Finally, the nonlinear responses of the supported pipes with different geometric imperfections, denoted as Model I, Model II, and Model III, are examined for various values of a_0 and Π_0 . For numerical calculation purpose, the initial displacement in the Y -direction at the midpoint of the pipe is chosen as 0.001. Then, based on the nonlinear governing equations of Eq. (7), the nonlinear responses of the pipe can be determined.

5.1 Effects of a_0 on bifurcation diagrams

The bifurcation diagrams of the dimensionless midpoint displacement amplitudes of the pipes for different values of a_0 , including 0.005, 0.01, and 0.05, with $\beta = 0.47$ and $\Pi_0 = 1000$ are displayed in Figs. 8(a), 8(b), and 8(c), respectively.

Fig. 8 Bifurcation diagrams of the dimensionless midpoint displacement amplitudes of (a) Model I, (b) Model II, and (c) Model III for different values of a_0 with $\beta = 0.47$ and $\Pi_0 = 1000$ (color online)

Figure 8(a) indicates that the absolute value of the displacement amplitude of the pipe increases continuously with the increase in the flow velocity, demonstrating that buckling instability does not occur in the pipe system of Model I. Although the displacement amplitude of the pipe jumps from a positive value to a negative one at $u = 7.4$ for the case of $a_0 = 0.05$, it is due to the utilized initial conditions rather than the occurrence of buckling instability. In addition, the displacement amplitude of Model I with $a_0 = 0.05$ is the largest in the considered flow velocity range.

Figure 8(b) shows that there is an obvious amplitude jump in the bifurcation diagram and the flow velocity corresponding to the jumping point is exactly the critical flow velocity obtained by the linear analysis, demonstrating the occurrence of buckling instability. Different from the results shown in Fig. 8(a), the displacement amplitude of Model II with $a_0 = 0.05$ is the smallest. This is because the critical flow velocity of Model II with $a_0 = 0.05$ is the highest. Based on the discussion in Ref. [32], the later the bifurcation point appears, the smaller the buckling displacement of the pipe will be at the same flow velocity. Moreover, since the initial configuration of Model II is antisymmetric about the midpoint, the midpoint displacement of the pipe is always zero when the flow velocity is lower than the critical flow velocity.

The results shown in Fig. 8(c) are similar to those shown in Fig. 8(b). The similarities include: (i) there is an obvious amplitude jump in the bifurcation diagram; (ii) the critical flow velocity predicted by the nonlinear analysis is consistent with that obtained by the linear stability analysis; (iii) the midpoint displacement amplitude of the pipe decreases as a_0 increases at the same flow velocity.

5.2 Effects of Π_0 on bifurcation diagrams

The bifurcation diagrams of the considered pipes for different values of Π_0 , including 1000, 4000, and 10000, with

$$\beta = 0.47, \quad a_0 = 0.01$$

are shown in Fig. 9. According to this figure, two points can be concluded: (i) there is still no amplitude jump in the bifurcation diagram for Model I, while such a jump appears in the bifurcation diagrams for Model II and Model III; (ii) in all three models, the midpoint displacement amplitude of the pipe decreases as Π_0 increases for a given flow velocity.

6 Conclusions

To investigate the nonlinear static equilibrium configurations, the linear stability around the static deformations, and the nonlinear dynamics of the fluid-conveying supported pipes with three different geometric imperfections, the ANCF and the extended Lagrange equations written for systems containing non-material volumes are adopted to introduce the nonlinear governing

Fig. 9 Bifurcation diagrams of the dimensionless midpoint displacement amplitude of (a) Model I, (b) Model II, and (c) Model III for different values of Π_0 with $\beta = 0.47$ and $a_0 = 0.01$ (color online)

equations. Then, a set of numerical simulations are performed based on these equations and their variants to obtain the results for statics, linear dynamics, and nonlinear dynamics of the considered pipe systems. According to the obtained numerical results, some fresh and important conclusions can be drawn out as follows.

(i) For Model I (the geometric imperfection is in the form of a half-sinusoidal wave), increasing the flow velocity will only change the amplitude of the static equilibrium configuration without changing its basic shape. However, for Model II (the geometric imperfection is in the form of a one-sinusoidal wave) and Model III (the geometric imperfection is in the form of one and half sinusoidal wave), the increase in the flow velocity will not only change the amplitude of the static equilibrium configuration but also change its basic shape in the range of high flow velocity. In addition, the values of a_0 and Π_0 can have considerable effects on the static deformations.

(ii) Even if the flow velocity reaches an extremely large value of 40, the pipe of Model I could not lose stability by buckling type, which is completely different from the results reported in Refs. [28], [29], [31], and [32]. This new finding is explained by the fact that the stability analysis in Refs. [28], [29], [31], and [32] was not performed around the static equilibrium configurations but around the initial curved configurations of the pipe. However, different from Model I, the first-mode buckling instability will occur in the pipe systems of Model II and Model III. The corresponding critical flow velocities will increase exponentially with the increase in a_0 , and increase linearly with the increase in Π_0 .

(iii) The bifurcation diagrams for the deformations of the pipe indicate that the values of a_0 and Π_0 will also have considerable effects on the dynamic responses of the considered pipe systems.

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References

- [1] SEMLER, C., LI, G., and PAÏDOUSSIS, M. P. The non-linear equations of motion of pipes conveying fluid. *Journal of Sound and Vibration*, **169**, 577–599 (1994)
- [2] PAÏDOUSSIS, M. P. and SEMLER, C. Nonlinear and chaotic oscillations of a constrained cantilevered pipe conveying fluid: a full nonlinear analysis. *Nonlinear Dynamics*, **4**, 655–670 (1993)

-
- [3] CHEN, W., HU, Z. Y., DAI, H. L., and WANG, L. Extremely large-amplitude oscillation of soft pipes conveying fluid under gravity. *Applied Mathematics and Mechanics (English Edition)*, **41**(9), 1381–1400 (2020) <https://doi.org/10.1007/s10483-020-2646-6>
 - [4] ZHOU, K., NI, Q., WANG, L., and DAI, H. L. Planar and non-planar vibrations of a fluid-conveying cantilevered pipe subjected to axial base excitation. *Nonlinear Dynamics*, **99**, 2527–2549 (2020)
 - [5] LIU, Z. Y., WANG, L., and SUN, X. P. Nonlinear forced vibration of cantilevered pipes conveying fluid. *Acta Mechanica Solida Sinica*, **31**, 32–50 (2018)
 - [6] LI, Q., LIU, W., LU, K., and YUE, Z. Nonlinear parametric vibration of a fluid-conveying pipe flexibly restrained at the ends. *Acta Mechanica Solida Sinica*, **33**, 327–346 (2020)
 - [7] LIANG, F., GAO, A., and YANG, X. D. Dynamical analysis of spinning functionally graded pipes conveying fluid with multiple spans. *Applied Mathematical Modelling*, **83**, 454–469 (2020)
 - [8] YANG, T. Z., LIU, T., TANG, Y., HOU, S., and LV, X. F. Enhanced targeted energy transfer for adaptive vibration suppression of pipes conveying fluid. *Nonlinear Dynamics*, **97**, 1937–1944 (2019)
 - [9] LU, Z. Q., ZHANG, K. K., DING, H., and CHEN, L. Q. Nonlinear vibration effects on the fatigue life of fluid-conveying pipes composed of axially functionally graded materials. *Nonlinear Dynamics*, **100**, 1091–1104 (2020)
 - [10] MODARRES-SADEGHI, Y. and PAÏDOUSSIS, M. P. Nonlinear dynamics of extensible fluid-conveying pipes, supported at both ends. *Journal of Fluids and Structures*, **25**, 535–543 (2009)
 - [11] TAN, X., DING, H., and CHEN, L. Q. Nonlinear frequencies and forced responses of pipes conveying fluid via a coupled Timoshenko model. *Journal of Sound and Vibration*, **455**, 241–255 (2019)
 - [12] JIANG, T. L., DAI, H. L., ZHOU, K., and WANG, L. Nonplanar post-buckling analysis of simply supported pipes conveying fluid with an axially sliding downstream end. *Applied Mathematics and Mechanics (English Edition)*, **41**(1), 15–32 (2020) <https://doi.org/10.1007/s10483-020-2557-9>
 - [13] WANG, Y. J., ZHANG, Q. C., WANG, W., and YANG, T. Z. In-plane dynamics of a fluid-conveying corrugated pipe supported at both ends. *Applied Mathematics and Mechanics (English Edition)*, **40**(8), 1119–1134 (2019) <https://doi.org/10.1007/s10483-019-2511-6>
 - [14] PAÏDOUSSIS, M. P. *Fluid-Structure Interactions: Slender Structures and Axial Flow*, Academic Press, London (1998)
 - [15] CHEN, S. S. Flow-induced in-plane instabilities of curved pipes. *Nuclear Engineering and Design*, **23**, 29–38 (1972)
 - [16] CHEN, S. S. Vibration and stability of a uniformly curved tube conveying fluid. *The Journal of the Acoustical Society of America*, **51**, 223–232 (1972)
 - [17] CHEN, S. S. Out-of-plane vibration and stability of curved tubes conveying fluid. *Journal of Applied Mechanics*, **40**, 362–368 (1973)
 - [18] JUNG, D. and CHUNG, J. In-plane and out-of-plane motions of an extensible semi-circular pipe conveying fluid. *Journal of Sound and Vibration*, **311**, 408–420 (2008)
 - [19] JUNG, D. and CHUNG, J. A steady-state equilibrium configuration in the dynamic analysis of a curved pipe conveying fluid. *Journal of Sound and Vibration*, **294**, 410–417 (2006)
 - [20] MISRA, A., PAÏDOUSSIS, M. P., and VAN, K. On the dynamics of curved pipes transporting fluid, part I: inextensible theory. *Journal of Fluids and Structures*, **2**, 221–244 (1988)
 - [21] MISRA, A., PAÏDOUSSIS, M. P., and VAN, K. On the dynamics of curved pipes transporting fluid, part II: extensible theory. *Journal of Fluids and Structures*, **2**, 245–261 (1988)
 - [22] WANG, L. and NI, Q. In-plane vibration analyses of curved pipes conveying fluid using the generalized differential quadrature rule. *Computers and Structures*, **86**, 133–139 (2008)

-
- [23] NI, Q., TANG, M., WANG, Y. K., and WANG, L. In-plane and out-of-plane dynamics of a curved pipe conveying pulsating fluid. *Nonlinear Dynamics*, **75**, 603–619 (2013)
 - [24] ZARE, A., EGHTEHAD, M., and DANESHMAND, F. An isogeometric analysis approach to the stability of curved pipes conveying fluid. *Marine Structures*, **59**, 321–341 (2018)
 - [25] LI, F. Q., AN, C., DUAN, M. L., and SU, J. In-plane and out-of-plane dynamics of curved pipes conveying fluid by integral transform method. *Journal of the Brazilian Society of Mechanical Sciences and Engineering*, **41**, 542 (2019)
 - [26] ZHAO, Q. L. and SUN, Z. Flow-induced vibration of curved pipe conveying fluid by a new transfer matrix method. *Engineering Applications of Computational Fluid Mechanics*, **12**, 780–790 (2018)
 - [27] LUO, Y. Y., TANG, M., NI, Q., WANG, Y., and WANG, L. Nonlinear vibration of a loosely supported curved pipe conveying pulsating fluid under principal parametric resonance. *Acta Mechanica Sinica*, **29**, 468–478 (2016)
 - [28] SINIR, B. G. Bifurcation and chaos of slightly curved pipes. *Mathematical and Computational Applications*, **15**, 490–502 (2010)
 - [29] WANG, L., DAI, H. L., and QIAN, Q. Dynamics of simply supported fluid-conveying pipes with geometric imperfections. *Journal of Fluids and Structures*, **29**, 97–106 (2012)
 - [30] HU, Y. J. and ZHU, W. D. Vibration analysis of a fluid-conveying curved pipe with an arbitrary undeformed configuration. *Applied Mathematical Modelling*, **64**, 624–642 (2018)
 - [31] OROLU, K. O., FASHANU, T. A., and OYEDIRAN, A. A. Cusp bifurcation of slightly curved tensioned pipe conveying hot pressurized fluid. *Journal of Vibration and Control*, **25**, 1109–1121 (2019)
 - [32] OYELADE, A. O. and OYEDIRAN, A. A. The effect of various boundary conditions on the nonlinear dynamics of slightly curved pipes under thermal loading. *Applied Mathematical Modelling*, **87**, 332–350 (2020)
 - [33] LI, Y. D. and YANG, Y. R. Nonlinear vibration of slightly curved pipe with conveying pulsating fluid. *Nonlinear Dynamics*, **88**, 2513–2529 (2017)
 - [34] LI, Q., LIU, W., LU, K., and YUE, Z. F. Nonlinear parametric vibration of the geometrically imperfect pipe conveying pulsating fluid. *International Journal of Applied Mechanics*, **12**, 2050064 (2020)
 - [35] CZERWIŃSKI, A. and LUCZKO, J. Non-planar vibrations of slightly curved pipes conveying fluid in simple and combination parametric resonances. *Journal of Sound and Vibration*, **413**, 270–290 (2018)
 - [36] OWOSENI, O., OROLU, K., and OYEDIRAN, A. Dynamics of slightly curved pipe conveying hot pressurized fluid resting on linear and nonlinear viscoelastic foundations. *Journal of Vibration and Acoustics*, **140**, 021005 (2018)
 - [37] LUCZKO, J. and CZERWIŃSKI, A. Three-dimensional dynamics of curved pipes conveying fluid. *Journal of Fluids and Structures*, **91**, 102704 (2019)
 - [38] SHI, R., WANG, K., LONG, F., and LI, H. Vertical buckling of offshore pipelines with geometrical imperfection on soft seabed. *Electronic Journal of Geotechnical Engineering*, **18**, 5491–5508 (2013)
 - [39] ZHOU, K., NI, Q., CHEN, W., DAI, H. L., PETER, H., and WANG, L. Static equilibrium configuration and nonlinear dynamics of slightly curved cantilevered pipe conveying fluid. *Journal of Sound and Vibration*, **490**, 115711 (2021)
 - [40] SHABANA, A. A. Definition of the slopes and the finite element absolute nodal coordinate formulation. *Multibody System Dynamics*, **1**, 339–348 (1997)
 - [41] SUGIYAMA, H., KOYAMA, H., and YAMASHITA, H. Gradient deficient curved beam element using the absolute nodal coordinate formulation. *Journal of Computational and Nonlinear Dynamics*, **5**, 021001 (2010)
 - [42] IRSCHIK, H. and HOLL, H. The equations of Lagrange written for a non-material volume. *Acta Mechanica*, **153**, 231–248 (2002)

- [43] BERZERI, M. and SHABANA, A. A. Development of simple models for the elastic forces in the absolute nodal co-ordinate formulation. *Journal of Sound and Vibration*, **235**, 539–565 (2000)

Appendix A

It is well-known that the number of pipe elements plays a key role in the accuracy and convergence of the numerical results. Thus, a suitable number of elements needs to be determined. Model III will be adopted here to determine the suitable number of elements, since its geometric imperfection is the most complicated. By considering three different numbers of pipe elements, including 4, 8, and 12, the static equilibrium configurations of Model III are illustrated in Fig. A1 for different flow velocity values. The other parameters are

$$a_0 = 0.05, \quad \beta = 0.47, \quad \Pi_0 = 1000.$$

From Fig. A1, it is indicated that 8 pipe elements in the present ANCF pipe model are sufficient to predict the static equilibrium configurations of supported pipes with geometric imperfections. In this study, therefore, 8 pipe elements will be utilized to discretize all the pipes under consideration.

Fig. A1 Convergence analysis on the static equilibrium configurations for Model III with $a_0 = 0.05$, $\beta = 0.47$, and $\Pi_0 = 1000$ for (a) $u = 3$ and (b) $u = 9$ (color online)

Appendix B

In this appendix, some results of two typical pipe models reported in the existing literature are reproduced to prove the correctness of the present ANCF pipe model.

The first typical pipe model is a straight fluid-conveying pipe with pinned-pinned ends, which was previously investigated by Modarres-Sadeghi and Païdoussis^[10]. By considering the same system parameters utilized in the study of Modarres-Sadeghi and Païdoussis^[10]:

$$\gamma = 0.1, \quad \beta = 0.47, \quad \Pi_0 = 1000,$$

the bifurcation diagram of the dimensionless midpoint displacement amplitude of a simply supported straight pipe is displayed in Fig. B1(a). From Fig. B1(a), we can see that the results obtained by using the present ANCF pipe model are almost consistent with those reported in Ref. [10], indicating that the present ANCF pipe model is reliable.

Additionally, the linear results of a semi-circular pipe with pinned-pinned ends, which have been reported by Misra et al.^[21], are reproduced in Fig. B1(b) by applying the same system parameters $\beta = 0.5$ and $\Pi_0 = 10000$ with 12 pipe elements. From Fig. B1(b), it can be easily found that the present results are almost the same as those obtained by Misra et al.^[21] based on the extensible theory. This further demonstrates that the present ANCF model has the ability to deal with initially curved fluid-conveying pipes with pinned-pinned ends.

Fig. B1 (a) Bifurcation diagram of the dimensionless midpoint displacement amplitude of a straight pipe with pinned-pinned ends obtained by the present ANCF model and the Galerkin model of Modarres-Sadeghi and Païdoussis^[10] for $\gamma = 0.1$, $\beta = 0.47$, and $\Pi_0 = 1000$. (b) Dimensionless in-plane frequencies $\text{Im}(\omega^*)$ of a semi-circular pipe with pinned-pinned ends, as a function of the dimensionless flow velocity u^* , obtained by the present ANCF model and the extensible theory of Misra et al.^[21] for $\beta = 0.5$ and $\Pi_0 = 10\,000$ (color online)