Displacement of surrounding rock in a deep circular hole considering double moduli and strength-stiffness degradation*

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Abstract The problem of cavity stability widely exists in deep underground engineering and energy exploitation. First, the stress field of the surrounding rock under the uniform stress field is deduced based on a post-peak strength drop model considering the rock's characteristics of constant modulus and double moduli. Then, the orthogonal non-associative flow rule is used to establish the displacement of the surrounding rock under constant modulus and double moduli, respectively, considering the stiffness degradation and dilatancy effects in the plastic region and assuming that the elastic strain in the plastic region satisfies the elastic constitutive relationship. Finally, the evolution of the displacement in the surrounding rock is analyzed under the effects of the double modulus characteristics, the strength drop, the stiffness degradation, and the dilatancy. The results show that the displacement solutions of the surrounding rock under constant modulus and double moduli have a unified expression. The coefficients of the expression are related to the stress field of the original rock, the elastic constant of the surrounding rock, the strength parameters, and the dilatancy angle. The strength drop, the stiffness degradation, and the dilatancy effects all have effects on the displacement. The effects can be characterized by quantitative relationships.

Key words deep rock, double moduli, strength-stiffness degradation, circular hole, displacement solution

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1 Introduction

Circular holes are widely involved in deep underground energy exploitation, nuclear waste storage, and underground space development, such as circular chambers in mines, water conveyance tunnels, mine borehole pressure relief, oil and gas development, and columnar holes in coalbed methane mining^[1–5]. In addition to the original rock stress, this type of holes is also subject to different internal pressures. The instability of the chamber is manifested as shrinkage or expansion, which is related to the plastic failure and displacement of the surrounding rock. The instability of the surrounding rock of the hole has brought many challenges to the excavation of deep underground engineering, such as drill hole inclination, hole collapse in soft rock, and jamming of a drilling tool. Therefore, understanding the mechanical response of the surrounding rock in deep circular holes under different combinations of pressures is of great significance for maintaining the stability of deep engineering.

Researchers have done a lot of work in predicting the stress and displacement fields of circular holes. For example, based on the Hoek-Brown failure criterion, Brown et al.^[6] analyzed the theoretical solutions for the stress and displacement of circular holes in two models, i.e., elastic-plastic and elastic-brittle-plastic. Sharan^[7–8] analyzed the displacement of the cavern based on the Hoek-Brown failure criterion in an elastoplastic model. Jiang and Shen^[9] studied the elastic, brittle, and plastic expansion of cylindrical holes based on the Drucker-Prager (D-P) criterion and Mohr-Column (M-C) criterion. Wen et al.^[10] used the twin-shear strength theory to obtain analytical solutions for the stress and displacement of the failure zone, the plastic zone, and the elastic zone of the circular chamber. For deep well seepage problems, some scholars considered the seepage force as radial force and provided analytical solutions for underwater chambers^[11] and porous elastic-brittle-plastic rock^[12–13] based on the elastic-brittle-plastic model and M-C criterion. The tunnel boring machine (TBM) method or mine blasting method causes initial damage to the surrounding rock of a chamber. Thus, in many reports, the analytical solutions of the stress and displacement of a circular hole were obtained considering the initial damage caused by blasting loads^[14–17]. From the existing results, researchers have established analytical solutions for various circular holes based on different working conditions using different constitutive models. In these solutions, the displacement in the plastic zone was developed based on the following three assumptions. First, the elastic strain in the plastic zone was assumed to be constant and equal to the strain at the elastoplastic interface^[18–20]. Second, the plastic region was considered as an elastic thick-walled cylinder to obtain the solution of the strain^[7]. Third, the elastic displacement of the plastic region was assumed to meet the law of elasticity, and thus the elastic constitutive equation and the stress in the plastic region can be used to obtain the elastic strains^[21–22]. Park and Kim^[23] compared the calculated displacement in the plastic zone using the above three methods. Based on the comparison results, it was found that the first method would underestimate the displacement value. When the dilatancy angle was zero, the results from the second and third methods were similar, but the difference between the two methods increased as the dilatancy angle increased.

The relevant conclusions in the above reports are of great significance for understanding the instability of deep circular holes. However, rock is an inhomogeneous material with internal cracks. Therefore, it has double moduli in tension and compression. After the chamber was excavated, the surrounding rock in the plastic zone experiences not only the strength drop but also the modulus damage, which was rarely studied in the present achievements. Therefore, this paper focuses on the solution of the displacement field of surrounding rocks under the dual-modulus properties and strength-stiffness degradation, instead of the influence of multi-field coupling or the selection of strength criteria.

2 Definition of problem

2.1 Description of problem

As shown in Fig. 1(a), the deep circular chamber with a radius a is subject to the hydrostatic pressure p_0 . The inner wall of the chamber is subject to an internal pressure p_i . At this time, three stresses of the surrounding rock σ_r , σ_θ , and σ_z are all principal stresses. p_i can be regarded as the virtual internal supporting force provided by the tunnel face during the excavation of the roadway. As the tunnel face moves away from the section, p_i gradually decreases. When it decreases to a certain value, the surrounding rock close to the free face of the roadway enters the plastic zone, and then strength drop and modulus damage occur. At this time, the three principal stresses are all compressive stresses. In addition, p_i can also be regarded as the internal pressure, the drilling pressure, or the high internal water pressure of the water and oil pipelines. When the force is greater than a certain value, the surrounding rock of the roadway also undergoes the plastic deformation, and the strength and modulus of the plastic zone drop. At this time, σ_r and σ_z are still compressive stresses, while the tangential stress σ_θ changes from the tensile force to the compressive force from the inner boundary to the outer boundary of the hole.

Let the compressive stress be positive and the tensile stress be negative. The dimensionless radial coordinates are defined as $t = r/a$ ($t \geq 1$), the boundary of the plastic zone of the surrounding rock is defined as $t = t_p$, and the position for the tangential stress to change sign is $t = t_s$. The deformation of the surrounding rock can be divided into the following zones (see Figs. 1(b) and 1(c)).

Case A Ω_I^1 zone: $1 \leq t \leq t_p$, $\sigma_r > 0$, $\sigma_\theta > 0$, $\sigma_z > 0$, the plastic damage zone (PDZ).

Ω_I^2 zone: $t > t_p$, $\sigma_r > 0$, $\sigma_\theta > 0$, $\sigma_z > 0$, the elastic compression zone (ECZ).

Case B Ω_{II}^1 zone: $1 \leq t \leq t_p$, $\sigma_r > 0$, $\sigma_\theta < 0$, $\sigma_z > 0$, the PDZ.

Ω_{II}^2 zone: $t_p \leq t \leq t_s$, $\sigma_r > 0$, $\sigma_\theta < 0$, $\sigma_z > 0$, the elastic tension and compression zone (ETCZ).

Ω_{II}^3 zone: $t \geq t_s$, $\sigma_r > 0$, $\sigma_\theta > 0$, $\sigma_z > 0$, the ECZ.

According to the theory of different elastic moduli in tension and compression^[24], Ω_{II}^1 zone and Ω_{II}^2 zone should be analyzed by the dual-modulus theory, and the other zones can be analyzed by the constant modulus.

Fig. 1 Analytical model for circular hole under deep hydrostatic pressure (color online)

2.2 Elastic constitutive and deterministic equations of rock considering double moduli

Assume that the elastic moduli in compression and tension of the surrounding rock are E^+ and E^- , respectively, and the transverse deformation coefficients are μ^+ and μ^- , respectively. According to the theory of different elastic moduli in tension and compression^[24–25], the plane

strain elasticity constitutive is

$$\begin{cases} \varepsilon_r = b_{11}\sigma_r + b_{12}\sigma_\theta, \\ \varepsilon_\theta = b_{21}\sigma_r + b_{22}\sigma_\theta, \end{cases} \quad (1)$$

where

$$\begin{aligned} b_{11} &= \frac{a_{11}a_{33} - a_{13}a_{31}}{a_{33}}, & b_{12} &= \frac{a_{12}a_{33} - a_{13}a_{32}}{a_{33}}, \\ b_{21} &= \frac{a_{21}a_{33} - a_{23}a_{31}}{a_{33}}, & b_{22} &= \frac{a_{22}a_{33} - a_{23}a_{32}}{a_{33}}. \end{aligned}$$

For deep surrounding rocks, $\sigma_r > 0$, and $\sigma_z > 0$. Therefore, $a_{11} = a_{33} = 1/E^+$, and $a_{21} = a_{31} = a_{13} = a_{23} = -\mu^+/E^+$. a_{12} , a_{22} , and a_{32} are set based on the compressive and tensile signs of the tangential stress σ_θ . The plane differential equation for axisymmetric problems can be simplified as

$$\frac{d\sigma_r}{dr} + \frac{\sigma_r - \sigma_\theta}{r} = 0. \quad (2)$$

The geometric equation is

$$\varepsilon_r = \frac{du}{dr}, \quad \varepsilon_\theta = \frac{u}{r}. \quad (3)$$

Let $\sigma_r = \Phi/r$, and $\sigma_\theta = d\Phi/dr$. The deterministic control equations of stress can be obtained by combining Eqs. (2) and (3),

$$t^2 \frac{d^2\Phi}{dt^2} + t \frac{d\Phi}{dt} - \lambda^2 \Phi = 0, \quad (4)$$

where $\lambda = \sqrt{\frac{b_{11}+b_{12}-b_{21}}{b_{22}}}$, which is related to the double moduli of the rock.

The general solution of elastic stress can be obtained using Eq. (4),

$$\begin{cases} \sigma_r = C_1 t^{\lambda-1} + C_2 t^{-\lambda-1}, \\ \sigma_\theta = C_1 \lambda t^{\lambda-1} - C_2 \lambda t^{-\lambda-1}, \end{cases} \quad (5)$$

where C_1 and C_2 can be determined by the stress boundary conditions.

2.3 Yield and damage behaviors of surrounding rock

The M-C criterion is used to describe the strength characteristics of the surrounding rocks. Set the initial yield criterion to

$$\sigma_1 = \eta\sigma_3 + \xi. \quad (6)$$

After the strength drop, the subsequent yield criterion is

$$\sigma_1 = \eta_r\sigma_3 + \xi_r, \quad (7)$$

where

$$\eta = \frac{1 + \sin \varphi}{1 - \sin \varphi}, \quad \xi = \frac{2c \cos \varphi}{1 - \sin \varphi}, \quad \eta_r = \frac{1 + \sin \varphi_r}{1 - \sin \varphi_r}, \quad \xi_r = \frac{2c_r \cos \varphi_r}{1 - \sin \varphi_r},$$

in which c and φ are the initial cohesion and internal friction angle of the surrounding rock, respectively, and c_r and φ_r are the cohesion and internal friction angle of the surrounding rock in the residual stage after the strength drop, respectively.

Assume that the elastic strain is smaller than the plastic strain and that the strain in the plastic region obeys the non-associated flow rule, as shown in Fig. 2. Then, the following equation can be satisfied:

$$\varepsilon_{3p}^p + \Theta \varepsilon_{1p}^p = 0, \quad (8)$$

where the superscript p represents the plastic region, and the subscript p represents the plastic strain. Considering the modulus damage of the surrounding rock in the plastic zone, and assuming that the elastic strain in the plastic zone satisfies the law of elasticity, the elastic strain in the plastic zone can satisfy the following equation:

$$\begin{cases} \varepsilon_r = b_{11}^r \sigma_r + b_{12}^r \sigma_\theta, \\ \varepsilon_\theta = b_{21}^r \sigma_r + b_{22}^r \sigma_\theta, \end{cases} \quad (9)$$

where the superscript r indicates the modulus after damage.

Fig. 2 Elastic-brittle-ideal plastic behavior of rocks and non-associated flow laws (color online)

3 Analytical solutions of surrounding rock in circle opening

3.1 Analytical solutions in Case A

3.1.1 Stress solutions

In this case, in the general solution of the elastic stress, i.e., Eq. (5), $\lambda = 1$. Assume that the stress in the axial direction of the chamber is the intermediate principal stress. Then, $\sigma_1 = \sigma_\theta$, and $\sigma_3 = \sigma_r$. Combining Eqs. (2) and (7) and considering the boundary conditions of $t = 1$ and $\sigma_r = p_i$, the stress solutions of the plastic zone can be obtained as follows:

$$\begin{cases} \sigma_r^p = \left(p_i + \frac{\xi_r}{\eta_r - 1} \right) t^{\eta_r - 1} - \frac{\xi_r}{\eta_r - 1}, \\ \sigma_\theta^p = \eta_r \left(p_i + \frac{\xi_r}{\eta_r - 1} \right) t^{\eta_r - 1} - \frac{\xi_r}{\eta_r - 1}. \end{cases} \quad (10)$$

The elastoplastic interface stress satisfies $\sigma_r + \sigma_\theta = 2p_0$. Substituting this equation into Eq. (6), the radial stress at the elastoplastic interface can be obtained as follows:

$$\sigma_{r^p}^{t_p} = \frac{2p_0 - \xi}{\eta + 1}. \quad (11)$$

Based on $\sigma_r^p(t_p) = \sigma_{r^p}^{t_p}$, the radius of the plastic zone can be obtained as follows:

$$t_p = \left(\frac{(\eta_r - 1)(2p_0 - \xi) + (\eta + 1)\xi_r}{(\eta + 1)((\eta_r - 1)p_i + \xi_r)} \right)^{\frac{1}{\eta_r - 1}}. \quad (12)$$

Using Eq. (5) and the boundary conditions of $t = t_p$, $\sigma_r^e(t_p) = \sigma_r^{t_p}$ and $t \rightarrow \infty$, $\sigma_r^e = p_0$, the stress solutions of the elastic zone can be obtained as follows:

$$\begin{cases} \sigma_r^e = \left(1 - \left(\frac{t_p}{t}\right)^2\right)p_0 + \left(\frac{t_p}{t}\right)^2 \sigma_r^{t_p}, \\ \sigma_\theta^e = \left(1 + \left(\frac{t_p}{t}\right)^2\right)p_0 - \left(\frac{t_p}{t}\right)^2 \sigma_r^{t_p}. \end{cases} \quad (13)$$

3.1.2 Displacement solutions

Both elastic and plastic deformations exist in the plastic zone, and the total strains can be expressed as follows:

$$\begin{cases} \varepsilon_r^p = \varepsilon_{re}^p + \varepsilon_{rp}^p, \\ \varepsilon_\theta^p = \varepsilon_{\theta e}^p + \varepsilon_{\theta p}^p. \end{cases} \quad (14)$$

According to Eq. (8), the following equation can be obtained for this condition:

$$\varepsilon_{rp}^p + \Theta \varepsilon_{\theta p}^p = 0, \quad (15)$$

where $\Theta = (1 + \sin \psi)/(1 - \sin \psi)$, in which ψ is the dilatancy angle of the surrounding rock.

If only the displacement caused by excavation is considered, the deformation caused by the original rock stress before excavation should be removed, and the elastic strains should meet the following equation:

$$\begin{cases} \varepsilon_{re}^p = b_{11}^r(\sigma_r^p - p_0) + b_{12}^r(\sigma_\theta^p - p_0), \\ \varepsilon_{\theta e}^p = b_{21}^r(\sigma_r^p - p_0) + b_{22}^r(\sigma_\theta^p - p_0). \end{cases} \quad (16)$$

For plane axisymmetric problems, the surrounding rock has only the radial displacement u . Combining Eqs. (3), (15), and (16), the differential equations for the displacement of the plastic zone can be obtained as follows^[23]:

$$\frac{du_p}{dt} + \Theta \frac{u_p}{t} = af(t), \quad (17)$$

where $f(t) = (b_{11}^r + \Theta b_{21}^r)\sigma_r^p + (b_{12}^r + \Theta b_{22}^r)\sigma_\theta^p - (b_{11}^r + b_{12}^r + \Theta b_{21}^r + \Theta b_{22}^r)p_0$.

The displacement at the elastoplastic interface is

$$u^{t_p} = at_p \varepsilon_\theta = at_p (b_{22} - b_{21})(p_0 - \sigma_r^{t_p}). \quad (18)$$

Combine Eqs. (17) and (18). Then, the displacement of the plastic zone can be obtained as follows:

$$u_p = \frac{a}{t^\Theta} \int_{t_p}^t t^\Theta f(t) dt + u^{t_p} \left(\frac{t_p}{t}\right)^\Theta. \quad (19)$$

The displacement of the plastic zone can be obtained by integrating the above equation,

$$\begin{aligned} \frac{u_p}{r} = \frac{1}{t^{\Theta+1}} & ((A_1^r B_1^r + A_2^r B_3^r)(t^{\Theta+\eta_r} - t_p^{\Theta+\eta_r}) \\ & - (A_1^r B_2^r + A_2^r B_2^r + A_3^r B_0)(t^{\Theta+1} - t_p^{\Theta+1}) + u^{t_p} t_p^\Theta), \end{aligned} \quad (20)$$

where

$$\begin{cases} A_1^r = (b_{11}^r + \Theta b_{21}^r), & A_2^r = (b_{12}^r + \Theta b_{22}^r), & A_3^r = b_{11}^r + b_{12}^r + \Theta b_{21}^r + \Theta b_{22}^r, \\ B_1^r = \frac{(\eta_r - 1)p_i + \xi_r}{(\Theta + \eta_r)(\eta_r - 1)}, & B_2^r = \frac{\xi_r}{(\Theta + 1)(\eta_r - 1)}, \\ B_3^r = \frac{\eta_r((\eta_r - 1)p_i + \xi_r)}{(\Theta + \eta_r)(\eta_r - 1)}, & B_0 = \frac{p_0}{\Theta + 1}. \end{cases}$$

Combining Eqs. (1) and (13), the displacement of the elastic zone can be obtained as follows:

$$u_e = a(b_{22} - b_{21})(p_0 - \sigma_r^{t_p}) \frac{t_p^2}{t}. \quad (21)$$

3.2 Analytical solutions in Case B

3.2.1 Stress solutions

(i) Stress in Ω_{II}^1 zone

Since $\sigma_r > 0$, $\sigma_\theta < 0$, $\sigma_1 = \sigma_r$, and $\sigma_3 = \sigma_\theta$, combining Eqs. (2) and (6), the stress solutions of the plastic zone can be obtained,

$$\begin{cases} \sigma_r^p = \left(p_i + \frac{\xi_r}{\eta_r - 1}\right) t^{\frac{1-\eta_r}{\eta_r}} - \frac{\xi_r}{\eta_r - 1}, \\ \sigma_\theta^p = -\frac{1}{\eta_r} \left(p_i + \frac{\xi_r}{\eta_r - 1}\right) t^{\frac{1-\eta_r}{\eta_r}} - \frac{\xi_r}{\eta_r - 1}. \end{cases} \quad (22)$$

The stress at the elastoplastic interface is

$$\sigma_r^{t_p} = \frac{2p_0\eta + \xi}{1 + \eta}. \quad (23)$$

The radius of the plastic zone can be obtained by $\sigma_r^p(t_p) = \sigma_r^{t_p}$ and Eq. (12),

$$t_p = \left(\frac{(\eta_r - 1)(2p_0\eta + \xi) + (1 + \eta)\xi_r}{(1 + \eta)((\eta_r - 1)p_i + \xi_r)} \right)^{\frac{\eta_r}{1-\eta_r}}. \quad (24)$$

(ii) Stress in Ω_{II}^2 zone

The boundary conditions of the Ω_{II}^2 zone are $\sigma_r = \sigma_r^{t_p}$ when $t = t_p$, and $\sigma_\theta = 0$ when $t = t_s$. The stress solutions can be obtained by Eq. (5),

$$\begin{cases} \sigma_r = \left(t^{\lambda-1} + \frac{t_s^{2\lambda}}{t^{\lambda+1}}\right) \frac{t_p^{\lambda+1}}{t_p^{2\lambda} + t_s^{2\lambda}} \sigma_r^{t_p}, \\ \sigma_\theta = \left(t^{\lambda-1} - \frac{t_s^{2\lambda}}{t^{\lambda+1}}\right) \frac{t_p^{\lambda+1}}{t_p^{2\lambda} + t_s^{2\lambda}} \lambda \sigma_r^{t_p}. \end{cases} \quad (25)$$

(iii) Stress in Ω_{II}^3 zone

In this zone, $a_{11} = a_{22}$, and $b_{11} = b_{22}$. Thus, $\lambda = 1$, and the boundary conditions are $\sigma_\theta = 0$ when $t = t_s$, and $\sigma_r = p_0$ when $t \rightarrow \infty$.

The stress solutions can be obtained by Eq. (5),

$$\begin{cases} \sigma_r = \left(1 + \frac{t_s^2}{t^2}\right) p_0, \\ \sigma_\theta = \left(1 - \frac{t_s^2}{t^2}\right) p_0. \end{cases} \quad (26)$$

According to the stress continuity condition $(\sigma_r)_{t=s^-} = (\sigma_r)_{t=s^+}$, the transcendental equation of s can be obtained,

$$t_s^{2\lambda} - \frac{\sigma_r^{t_p}}{p_0} t_p^{\lambda+1} t_s^{\lambda-1} + t_p^{2\lambda} = 0. \quad (27)$$

3.2.2 Displacement solutions

(i) Displacement in Ω_{Π}^1 zone In the plastic zone, the following equation is obtained:

$$\varepsilon_{\theta p}^p + \Theta \varepsilon_{rp}^p = 0. \quad (28)$$

The elastic strain still satisfies Eq. (9), and the differential equation for displacement control is rewritten as

$$\Theta \frac{du_p}{dt} + \frac{u_p}{t} = ag(t), \quad (29)$$

where $g(t) = (b_{21}^r + \Theta b_{11}^r)\sigma_r^p + (b_{22}^r + \Theta b_{12}^r)\sigma_\theta^p - (b_{21}^r + b_{22}^r + \Theta b_{11}^r + \Theta b_{12}^r)p_0$.

The displacement at the elastoplastic interface can still be obtained by Eq. (19),

$$u_p = \frac{a}{\Theta t^{\frac{1}{\Theta}}} \int_{t_p}^t g(t) t^{\frac{1}{\Theta}} dt + u^{t_p} \left(\frac{t_p}{t} \right)^{\frac{1}{\Theta}}. \quad (30)$$

The displacement of the plastic zone can be obtained by integrating the above equation,

$$\begin{aligned} \frac{u_p}{r} = \frac{1}{t^{\frac{1}{\Theta}+1}} & \left((C_1^r D_1^r + C_2^r D_2^r) \left(t^{\frac{1}{\Theta} + \frac{1}{\eta_r}} - t_p^{\frac{1}{\Theta} + \frac{1}{\eta_r}} \right) \right. \\ & \left. - (C_1^r D_3^r - C_2^r D_3^r + C_3^r D_0) \left(t^{\frac{\Theta+1}{\Theta}} - t_p^{\frac{\Theta+1}{\Theta}} \right) + u^{t_p} (t_p)^{\frac{1}{\Theta}} \right), \end{aligned} \quad (31)$$

where

$$\begin{cases} C_1^r = (b_{21}^r + \Theta b_{11}^r), & \Theta_2^r = (b_{22}^r + \Theta b_{12}^r), & C_3^r = b_{21}^r + b_{22}^r + \Theta b_{11}^r + \Theta b_{12}^r, \\ D_1^r = B_1^r \eta_r, & D_2^r = B_1^r, & D_3^r = B_2^r, & D_0 = B_0. \end{cases}$$

(ii) Displacement in Ω_{Π}^2 zone

The displacement in this zone can be obtained by combining Eqs. (1) and (2),

$$u_e^{\Omega_{\Pi}^2} = a \left(\left((b_{21} + \lambda b_{22}) t^\lambda + (b_{21} - \lambda b_{22}) \frac{t_s^{2\lambda}}{t^\lambda} \right) \frac{t_p^{\lambda+1}}{t_p^{2\lambda} + t_s^{2\lambda}} \sigma_r^{t_p} - t(b_{21} + b_{22})p_0 \right). \quad (32)$$

(iii) Displacement in Ω_{Π}^3 zone

The displacement in this zone can be obtained by combining Eqs. (1) and (26),

$$u_e^{\Omega_{\Pi}^3} = a(b_{21} - b_{22}) \frac{t_s^2}{t} p_0. \quad (33)$$

3.3 Unified solution of displacement in two cases

From the above analysis, the displacement of the plastic zone can be written in a unified form for both cases,

$$\frac{u_p}{r} = \frac{1}{t^{\alpha_1+1}} \left(K_1(t^{\alpha_2} - t_p^{\alpha_2}) + K_2(t^{\alpha_3} - t_p^{\alpha_3}) + u^{t_p} (t_p)^{\alpha_1} \right). \quad (34)$$

The expression of each parameter is summarized in Table 1.

4 Analysis and comparison of solution

The tensile and compressive elastic modulus ratio is defined as $\varpi_E = E^+/E^-$, the ratio of Poisson's ratios is defined as $\varpi_\mu = \mu^+/\mu^-$, and the residual strength coefficient in the plastic strength drop zone is defined as $\lambda_c = c_r/c$ and $\lambda_\varphi = \varphi_r/\varphi$. Considering the modulus damage in the strength drop zone, the residual modulus coefficient is set to be $D_E = E_r^+/E^+ = E_r^-/E^-$; that is, the tensile and compressive moduli are considered to be degraded at the same level, and the transverse deformation coefficient damage is not considered. The values of the parameter in the two cases in Eq. (34) are shown in Table 2.

Table 1 Values of variables and constants in unified solution of displacement

Variable	Case A	Case B
K_1	$A_1^r B_1^r + A_2^r B_3^r$	$C_1^r D_1^r + C_2^r D_2^r$
K_2	$-(A_1^r B_2^r + A_2^r B_2^r + A_3^r B_0)$	$-(C_1^r D_3^r - C_2^r D_3^r + C_3^r D_0)$
u^{t_p}	$at_p(b_{22} - b_{21})(p_0 - \sigma_r^{t_p})$	$a\left(\left((b_{21} + \lambda b_{22})t_p^\lambda + (b_{21} - \lambda b_{22})\frac{t_p^{2\lambda}}{t_p^\lambda}\right)\frac{t_p^{\lambda+1}}{t_p^{2\lambda} + t_s^{2\lambda}}\sigma_r^{t_p} - t_p(b_{21} + b_{22})p_0\right)$
$\sigma_r^{t_p}$	$\frac{2p_0 - \xi}{\eta + 1}$	$\frac{2p_0\eta + \xi}{1 + \eta}$
t_p	$\left(\frac{(\eta_r - 1)(2p_0 - \xi) + (\eta + 1)\xi_r}{(\eta + 1)((\eta_r - 1)p_1 + \xi_r)}\right)^{\frac{1}{\eta_r - 1}}$	$\left(\frac{(\eta_r - 1)(2p_0\eta + \xi) + (1 + \eta)\xi_r}{(1 + \eta)((\eta_r - 1)p_1 + \xi_r)}\right)^{\frac{\eta_r}{1 - \eta_r}}$
α_1	Θ	$\frac{1}{\Theta}$
α_2	$\Theta + \eta_r$	$\frac{1}{\Theta} + \frac{1}{\eta_r}$
α_3	$\Theta + 1$	$\frac{\Theta + 1}{\Theta}$

Table 2 Values of each modulus in unified solution of displacement

Modulus	Case A	Case B
b_{11}^r	$\frac{1}{E_r^+}(1 - \mu^+ \mu^+)$	$\frac{1}{E_r^+}(1 - \mu^+ \mu^+)$
b_{21}^r	$\frac{\mu^+}{E_r^+}(-1 - \mu^+)$	$\frac{\mu^+}{E_r^+}(-1 - \mu^+)$
b_{12}^r	$\frac{\mu^+}{E_r^+}(-1 - \mu^+)$	$\frac{\mu^-}{E_r^-}(-1 - \mu^+)$
b_{22}^r	$\frac{1}{E_r^+}(1 - \mu^+ \mu^+)$	$\frac{1}{E_r^-}(1 - \mu^+ \mu^-)$

The values of the basic calculation parameters in both cases are shown in Table 3.

Table 3 Values of basic calculation parameters

Type	Original stress p_0/MPa	Internal stress p_i/MPa	Initial strength c/MPa	Initial strength $\varphi/(\circ)$	Residual strength λ_c	Residual strength λ_φ	Modulus E^-/GPa	Modulus μ^-	Modulus ratio ϖ_E	Modulus ratio ϖ_μ	Damage coefficient D_E	Dilatancy angle ψ
Case A	60	3	30	25	0.3	0.3	30	0.25	0.8	0.6	0.6	5
Case B	30	80	30	25	0.3	0.3	30	0.25	0.8	0.6	0.6	5

4.1 Influence of dual-modulus characteristics on displacement

Figure 3 shows the influence of compressive and tensile dual-modulus characteristics on the displacement of the surrounding rock. In Case A, the elastic constants in tension and compression of the surrounding rock are the same; that is, the tensile and compressive elastic modulus ratio $\varpi_E = 1.0$, and the ratio of Poisson's ratios $\varpi_\mu = 1.0$. The displacement of the surrounding rock is not affected by the difference of modulus. In Case B, the shear stresses, radial stresses, and axial stresses in Ω_{II}^1 and Ω_{II}^2 have opposite signs. Therefore, the difference between compressive and tensile moduli has a great impact on the displacement. Figure 3(a) shows the evolution law of the displacement of the surrounding rock when the tensile and compressive elastic modulus ratio ϖ_E is different. At smaller ϖ_E , the displacement of the

surrounding rock in the plastic zone is larger. As ϖ_E increases, the displacement of the plastic zone decreases. When $\varpi_E < 1.0$, the effect of the difference between tensile and compressive elastic moduli on the displacement of the plastic zone is more significant than that at $\varpi_E > 1.0$. The influence of ϖ_E on the displacement in the Ω_{II}^3 elastic zone exhibits the same trend as that in the Ω_{II}^1 zone. In comparison, the effect of ϖ_E on the displacement in the Ω_{II}^3 elastic zone is less significant. In addition, ϖ_E has no effect on the relative radius t_p of the plastic zone, but it has a greater effect on the relative radius t_s of the Ω_{II}^2 zone. As ϖ_E increases, t_s also continues to expand outward. From Fig. 3(b), the influence of ϖ_μ on the displacement of the surrounding rock is just the opposite to the influence of ϖ_E . With the increase in ϖ_μ , the displacements of the surrounding rock in Ω_{II}^1 and Ω_{II}^2 zones increase continuously and have the same trend. ϖ_μ has no effect either on the relative radius of the plastic zone t_p . The relative radius of the Ω_{II}^2 zone decreases with the increase in ϖ_μ , but the change is not significant.

Fig. 3 Influence of double modulus characteristics on displacement of surrounding rock (Case B) (color online)

4.2 Influence of strength drop on surrounding rocks

Figures 4 and 5 show the influence of the strength drop characteristics of the plastic zone on the displacement of the surrounding rock in two cases. With the increase in the strength drop, the roadway displacements are continuously increased. In the case of constant modulus (Case A), the radius of the plastic zone of the surrounding rock and the displacement at the elastoplastic interface both increase with the increase in the strength drop. In comparison, the effect of cohesive drop is more significant. When λ_c is dropped to 10% of the original value, the maximum displacement of the surrounding rock is increased by 223%. On the other hand,

Fig. 4 Influence of strength drop characteristics on displacement of surrounding rock (Case A) (color online)

Fig. 5 Influence of strength drop characteristics on displacement of surrounding rock (Case B) (color online)

when λ_ϕ drops to 10% of its original value, the maximum displacement of the surrounding rock is increased by only 32.3%.

In the case of double moduli (Case B), with the drop of cohesion and friction angle, the displacement of the surrounding rock increases significantly, especially in the plastic zone. Unlike Case A, the radius of the surrounding plastic zone increases with the drop of the strength parameter, but the displacement at the elastoplastic interface is not changed. In addition, the friction angle drop has a more significant effect on the displacement of the surrounding rock than the cohesive force drop. When λ_ϕ is dropped to 10% of the original value, the maximum displacement of the surrounding rock increases by 148%. With the same drop of cohesion, the maximum displacement of the surrounding rock increases by only 63%.

4.3 Influence of dilatancy effect

The dilatancy effect has great influence on the displacement of the surrounding rock in the plastic zone as shown in Fig. 6. Under different dilatancy angles, the displacements of the surrounding rock in Case A and Case B show different characteristics. In Case A, the displacement of the surrounding rock shrinks inward. As the dilatancy angle increases, the convergent displacement of the surrounding rock increases significantly, showing a dilatation effect. In Case B, the displacement of the surrounding rock expands outward. As the dilatancy angle increases, the displacement of the surrounding rock tends to decrease, showing a compacting effect on the plastic zone due to the large pressure in the roadway. In comparison, the dilatancy effect has more significant influence in Case A.

Fig. 6 Influence of dilatancy angle on displacement of surrounding rock (color online)

4.4 Influence of stiffness degradation on displacement

After excavation of the chamber, in addition to the strength drop of the surrounding rock in the plastic zone, the stiffness is also significantly degraded. Figure 7 shows the effect of the residual modulus coefficient on the displacement of the surrounding rock in both cases. For the convenience of analysis, it is assumed that the elastic moduli of tension and compression are attenuated by the same degree. Under the conditions of constant modulus and double moduli, the displacement of the surrounding rock increases obviously with the deterioration of stiffness. As the modulus gets more degraded, the displacement of the surrounding rock in the plastic zone is increased faster.

Fig. 7 Influence of modulus damage on displacement of surrounding rock (color online)

The above analysis considers the influence of the tension-compression double-modulus characteristics of the surrounding rock, the strength of the plastic zone, the stiffness degradation, and the dilatancy effect on the displacement of the surrounding rock. From the perspective of the displacement of the surrounding rock, Park and Kim^[23] analyzed in detail the validity of three different calculation methods of elastic strain in the plastic zone and the equivalent replacement of M-C and Hoek-Brown parameters. If the double-modulus characteristics of the rock and the stiffness degradation of the plastic zone are not considered, the solution in the study is consistent with the solution proposed by Park and Kim.

5 Conclusions

In this paper, a unified analytical solution for the displacement of the surrounding rock is obtained for the problem of circular holes in deep underground engineering, considering different characteristics of the tensile and compressive elastic moduli of the surrounding rock, the strength-stiffness degradation, and the dilatancy behavior in the plastic zone. In addition, the effects of the tensile and compressive elastic modulus ratio, the residual strength, the residual stiffness, and the dilatancy angle on the displacement of the surrounding rock are analyzed. The main conclusions are as follows:

(i) According to the sign change of tangential stress, the surrounding rocks under a constant modulus can be divided into two zones, i.e., the PDZ and the ECZ. On the other hand, the surrounding rocks under double moduli can be divided into three zones, i.e., the PDZ, the ETCZ, and the ECZ.

(ii) Considering the strength drop, the stiffness degradation, and the dilatancy effect, the analytical solutions of displacement of the surrounding rock in the plastic zone under the conditions of constant modulus and dual moduli using non-associated flow laws and the M-C yield criterion can adopt a unified expression.

(iii) Under the dual moduli, the tensile and compressive elastic constant ratio has a significant effect on the displacement of the surrounding rock. With the increase in the tensile and compressive elastic modulus ratio ϖ_E , the displacement of the plastic zone is decreased. However, with the continuous increase in the ratio of Poisson's ratio ϖ_μ , the displacement of the surrounding rock appears to continuously increase. In comparison, the impact of ϖ_E is more significant. Although ϖ_E has no effect on the relative radius of the plastic zone, it has a great effect on the boundary between the two zones of the ETCZ and the ECZ.

(iv) The strength drop, the stiffness degradation, and the dilatancy effect in the plastic zone affect the displacement of the surrounding rock in both cases. As the strength drop of the surrounding rock is greater and the stiffness degradation is more severe, the displacement in the plastic zone is greater. However, the dilatancy characteristics have a different effect. The dilatancy angles have opposite influence on the displacement of the surrounding rock under the constant modulus and double moduli.

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