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ORIGINAL RESEARCH

On the binary formulation of air traffic flow management problems

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Abstract
We discuss a widely used air traffic flow management formulation. We show that this formulation can lead to a solution where air delays are assigned to flights during their take-off which is prohibited in practice. Although air delay is more expensive than ground delay, the model may assign air delay to a few flights during their take-off to save more on not having as much ground delay. We present a modified formulation and verify its functionality in avoiding incorrect solutions.

Keywords Transportation · Air traffic flow management (ATFM) · Departure capacity constraint · Ground delay · Network optimization

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1 Introduction

Aviation management has received great attention in the literature to enhance operations efficiency and reduce associated costs. Several management areas have been investigated, such as ground holding Andreatta et al. (1993); Brunetta et al. (1998); Mukherjee and Hansen (2007), gate assignment Bihr (1990); Cheng et al. (2012); Ding et al. (2005), runway sequencing and scheduling Bennell et al. (2013); Atkin et al. (2007); Ikli et al. (2021); Sölveling and Clarke (2014), conflict resolution Alonso-Ayuso et al. (2014); Menon et al. (1999); Pallottino et al. (2002); Peyronne et al. (2015), airspace capacity management Barnhart et al. (2012); Liu and Hu (2009); Sherali and Hill (2013), and air traffic flow management Boujarif et al. (2021b). An air traffic flow management problem (ATFM) involves optimizing flight schedules to match schedules with the available airport and airspace capacities Bertsimas and Patterson (1994). It considers a network of airports and airspace sectors, where flights fly between these airports passing through the airspace sectors during a specific planning horizon. In the ATFM problem, flights are controlled through ground delays, air delays, rerouting or cancellation decisions Hamdan et al. (2021); Boujarif et al. (2021a).

The pioneering work of Bertsimas and Patterson (1994) gave the first binary formulation for the ATFM problem with airspace sector capacities. This is an NP-hard problem for which the binary formulation has succeeded in solving large-scale practical size instances with thousands of flights. The proof of NP-hardness lays in the fact that the problem can be reduced to the job-shop scheduling problem, which is in turn NP-hard Bertsimas and Patterson (1994); Diao and Chen (2018). Several algorithms and approaches were proposed and used to solve large-scale instances, such as Lagrangian relaxation Zhang and Mahadevan (2017), fix-and-relax Agustín et al. (2012b); Hamdan et al. (2021), sequential strategy (Akgunduz and Kazerooni (2018), heuristic-repair Junker (2012) and hierarchical heuristic Zhang et al. (2018).

The formulation has been then used widely in the related literature Agustín et al. (2012a, b); Alonso et al. (2000); Bertsimas et al. (2008, 2011, 2012); Boujarif et al. (2021b); Churchill et al. (2009); Hamdan et al. (2018); Dal Sasso et al. (2018, 2019); Hamdan et al. (2019, 2020, 2021); Vossen et al. (2012). Although the binary formulation is widely used in the literature, other formulations and approaches exist, such as the graph coloring Barnier and Brisset (2004), the real-time holding and rerouting Chen et al. (2020), the non-time segmented Akgunduz and Kazerooni (2018) and the shortest path with common capacity constraint Garcćа-Heredia et al. (2019). This paper contributes to the literature by discussing the widely used binary formulation and highlighting circumstances where this formulation will give incorrect solutions. The widely used binary formulation in the literature can lead to a solution where air delays are assigned to flights during their take-off, which is prohibited in practice. We present a modified formulation to prevent this issue. Unless otherwise specified, the *current formulation* denotes the binary formulation widely used in the literature, and the *enhanced formulation* denotes the modified formulation presented in this work. For consistency with the literature, we follow the notations and terminologies used in Bertsimas and Patterson (1994). It is worth noting that despite the advanced features and aspects presented in the wide ATFM literature, this possible modeling weakness appears in following works that used this formulation.

The ATFM model considers a set of flights ($f \in \mathscr{F}$), a network of airports ($k \in \mathscr{K}$) and airspace sectors ($j \in \mathscr{J}$), under a discrete finite planning horizon ($t \in \mathscr{T}$). The model uses a set $P_f = \{P(f, i),\ 1 \le i \le N_f\}$ that contains the flight path, with $P(f, i)$ denoting the i^{th} sector in this path. The path starts with the departure airport $P(f, 1)$, goes through certain airspace sectors, and ends with the arrival airport $P(f, N_f)$. Flight f must spend a minimum time l_{fj} in each sector j. This helps in defining the set of feasible times to reach sector j denoted by T_f^j. The scheduled departure and arrival times of a flight f are defined as d_f and r_f, respectively. The airport departure, airport arrival and airspace sector capacities at time t are given by $D_k(t)$, $A_k(t)$, $S_j(t)$, respectively. The model uses the binary decision variable $w_{f,t}^j$ that is equal to 1 if flight f enters sector j by time t, and 0 otherwise. The definition "by time t" means that if $w_{f,t}^j$ is equal to 1 for period t, then it will be equal to 1 for all later periods.

The model optimizes flight schedules under limited available airspace capacities in order to minimize the total flight delay costs as given in Eq. (1). Although we present an objective function with only ground and air delays, the issue discussed appears in all extensions that include aspects such as cancellation, rerouting, early and late arrivals if they consider a ground and air delays calculation with the same formulation.

$$Min \sum_{f \in \mathscr{F}} \Bigg[c_f^g \Bigg(\sum_{\substack{t \in T_f^k, \\ k=P(f,1)}} (t - d_f) \left(w_{f,t}^k - w_{f,t-1}^k \right) \Bigg) + c_f^a \Bigg(\sum_{\substack{t \in T_f^k, \\ k=P(f,N_f)}} (t - r_f) \left(w_{f,t}^k - w_{f,t-1}^k \right) - \Bigg(\sum_{\substack{t \in T_f^k, \\ k=P(f,1)}} (t - d_f) \left(w_{f,t}^k - w_{f,t-1}^k \right) \Bigg) \Bigg) \Bigg]. \tag{1}$$

The first part in (1) gives the ground delay cost by multiplying the ground delay unit cost c_f^g by the amount of ground delay calculated at the departure airport. The second part calculates the air delay cost by multiplying the air delay unit cost c_f^a by the amount of air delay, which is the time difference between the actual and the scheduled arrival times, minus the ground delay.

As we focus on the ground delay calculation, we provide its expression separately. It is given by

$$\sum_{f \in \mathscr{F}} c_f^g \Bigg(\sum_{t \in T_f^k, k=P(f,1)} t \left(w_{f,t}^k - w_{f,t-1}^k \right) - d_f \Bigg). \tag{2}$$

The departure capacity constraint ensures that the number of flights which may take-off from airport k at time-period t does not exceed the departure capacity $D_k(t)$. It is given by

$$\sum_{f \in \mathscr{F}: P(f,1)=k} (w_{f,t}^k - w_{f,t-1}^k) \leq D_k(t), \quad \forall\, k \in \mathscr{K},\ t \in \mathscr{T}. \tag{3}$$

The path connectivity constraint ensures that a flight cannot enter its next resource (airport or sector) in its path unless it has spent at least the minimum time needed in the previous resource. It is given by

$$w_{f,t+l_{fj}}^{j'} - w_{f,t}^j \leq 0, \quad \forall f \in \mathscr{F},\ t \in T_f^j,\ j = P(f,i),\ j' = P(f,i+1),\ i < N_f. \tag{4}$$

Equation (1) and Constraints (3) and (4) here correspond to the objective function, Constraint (2) and Constraint (5) in Bertsimas and Patterson (1994), respectively. The remaining constraints in the problem formulation are related to the airport arrival capacity, the sector capacity, time connectivity, and continuing flights.

We show how this current formulation can be exploited leading to an incorrect solution for the ATFM problem. The identified shortfall does not reduce the novelty of previous works. The correction presented in this paper ensures correct solutions and thus helps keeping correct advancement in the ATFM research field.

The organization of this paper is as follows. Section 2 discusses the potential weakness in the current formulation. Section 3 presents an enhanced formulation. Section 4 gives two detailed examples illustrating the exploitation using the current formulation and how it is corrected using the enhanced formulation. Finally, Sect. 5 concludes the paper.

2 Potential weakness in the current ATFM formulation

Constraint (3) counts the number of flights that took off during period t from airport k and compares it with the departure capacity. Flights are counted by computing the difference between $w^k_{f,t}$ and $w^k_{f,t-1}$. If the difference equals to 1, then flight f took off at period t ($w^k_{f,t} = 1$ and $w^k_{f,t-1} = 0$). If the difference equals to 0, it means that either flight f has not taken off yet ($w^k_{f,t} = 0$ and $w^k_{f,t-1} = 0$) or flight f took off in a previous period ($w^k_{f,t} = 1$ and $w^k_{f,t-1} = 1$).

Constraint (4) allows flight f to enter its next resource (j') after it has spent at least l_{fj} in its previous resource (j). The use of "at least" allows imposing speed control. For example, if flight f needs a minimum of 1 period ($l_{fj} = 1,\ j = P(f, 1)$) during the take-off and takes off at t, then Constraint (4) will allow it to enter its first sector (j') anytime from $t + l_{fj}$ till its last possible time.

The definition of the path connectivity constraint using "at least" along with the flight counting method in the departure capacity constraint and the ground delay calculation expression lead to potentially incorrect results. In practice and logically speaking, a flight should enter its first sector immediately after spending the required time at the departure airport. However, in the current formulation, a flight may take off to benefit from the available departure airport capacity at a certain time-period, where one period after the take-off, the flight is not counted in the departure capacity. Then, the flight may be assigned air delay for some periods during its take-off. After that, it enters its first airspace sector at a later time due to capacity-related issues. During the take-off and the appearance in the first sector times, other flights may exploit the available airport and airspace sector capacities.

Consider the solution scenario in Fig. 1. It provides the optimal schedule for one flight, where the rows are the resources of the flight path, the columns are the time-periods, and the values inside the cells are example solutions of the decision variable $w^j_{f,t}$. The leading 1's, in the grey cells, give the time of departure, the time of arrival at each sector, and the time of arrival at the destination airport. Assume that flight f requires zero time-periods for its take-off, then a prohibited solution is obtained if flight f takes off, say, at $t = 2$ and enters its first modeled sector (Sector A in Fig. 1) at a later time, say, $t = 5$. This situation means that flight f is not detected in any capacity constraint at $t = 3$ and $t = 4$. For instance at $t = 4$, flight f already took off from Airport 1 in a previous period since $w^{Airport1}_{f,4} = w^{Airport1}_{f,3} = 1$ from Constraint (3), but it did not enter its first sector (Sector A) since $w^{SectorA}_{f,4} = w^{SectorB}_{f,4} = 0$, see Constraint (4) in Bertsimas and Patterson (1994). Although this solution results in three

Fig. 1 Illustration of the deficiencies in Case 1 and Case 2 using the solution of one flight

	Time							
Flight path	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
Airport 1		*1*	1	1	1	1	1	1
Sector A					*1*	1	1	1
Sector B						*1*	1	1
Sector C							*1*	1
Airport 3								*1*

periods of air delay for flight f ($t = 2, 3, 4$), it allows other flights to benefit from the available airport and sector capacities as flight f is not counted and results in less total delay.

One might argue that the model is unlikely to prefer assigning air delay over ground delay if both options are feasible since $c_f^g < c_f^a$. This is true for one flight. However, in the case of a network with many flights, assigning air delay to a small group of flights might be cheaper than assigning ground delay to a larger number of flights. In other words, the total air delay needed becomes cheaper than the total ground delay alternative. Thus, in the previous example, the savings from other flights should be more than the three air delay periods of flight f to make the model exploit this possibility. Note that the same holds if the required time at the departure airport is greater than zero. Examples illustrating this will be given in Sect. 4. This issue may be further apparent when other decisions, such as reroutings and cancellations, are accounted for, when several objective functions are used or when stochastic aspects are considered. Note also that the departure capacity constraint will not function as intended if the take-off requires more than one time-period (although this case is unpractical) as this case is not restricted in the modeling.

3 Enhanced formulation

To prevent the exploitation discussed in Sect. 2, we need to ensure that airport departure capacity and ground delay are calculated correctly and that each flight enters its first sector at the right time.

In the case when the required time at the departure airport is zero, then if the departure occurs at t, flight f needs to be counted in the airport departure capacity and counted in its first sector at t. At the same time, it should immediately enter its first sector after the departure. Thus, we propose the following modification to the path connectivity constraint.

$$\begin{cases} w^{j'}_{f,t+l_{fj}} - w^{j}_{f,t} = 0, \ \ \forall f \in \mathscr{F}, \ t \in T^j_f, \ j = P(f,i), \\ \qquad\qquad j' = P(f,i+1), \ i < N_f & \textbf{if } i = 1 \\ w^{j'}_{f,t+l_{fj}} - w^{j}_{f,t} \leq 0, \ \ \forall f \in \mathscr{F}, \ t \in T^j_f, \ j = P(f,i), \\ \qquad\qquad j' = P(f,i+1), \ i < N_f & \textbf{if } i > 1. \end{cases} \tag{5}$$

Constraint (5) ensures that each flight enters its first modeled sector immediately after spending the required time at the departure airport ($i = 1$). It also ensures that each flight enters its next sector after spending at least the minimum time needed in the previous one ($i > 1$). This result is summarized in Proposition 1.

Table 1 Example of an infeasible solution that can be prevented

t	1	2	3	4	5	6	7	8	9	10	11	12
$w_{f,t}^{j=P(f,1)=k}$	0	0	0	1	1	1	1	1	1	1	1	1
$w_{f,t}^{j=P(f,2)}$	0	0	0	0	0	0	1	1	1	1	1	1

Proposition 1 *Constraint (5) and the time window of each resource (T_f^j) that specifies the earliest and the latest entry times prevent any violations.*

Proof Assume that flight f is scheduled to depart at $t = 2$ and let us say that flight f has a maximum delay of 4 periods. This means that the latest departure time is $t = 2 + 4 = 6$. Consequently, $T_f^j = 2, 3, 4, 5, 6, \;\; j = P(f, 1) = k$. If the minimum time spent at the departure airport is 5 ($l_{fj} = 5, \;\; j = P(f, 1)$), then flight f can enter its next sector within a time window $T_f^j = 2 + 5, 3 + 5, ..., 6 + 5 = 7, 8, ..., 11, \;\; j = P(f, 2)$. Therefore, flight f cannot enter sector j earlier (e.g. before $t = 7$) due to the time window $T_f^j, \;\; j = P(f, 2)$.

Now, assume that flight f receives 2 periods of delay, then the take-off occurs at $t = 4$. We illustrate how the flight cannot enter its first sector earlier than $t = 4 + 5 = 9$, say at $t = 7$ as shown in Table 1. If flight f can enter its first sector earlier, then due to the time connectivity "that ensures if $w_{f,t}^j = 1$ at any period, it will be one for all later periods", the sector connectivity (Constraint (5)) will not be violated as $w_{f,t}^j = 1$ at $t = 9$.

However, if we look at Constraint (5), it is checked at each $T_f^{j=P(f,1)}$ "departure time window": from 2 to 6 - in our example. This means that: at $t = 2$, $w_{f,t=2+5}^{j=P(f,2)} - w_{f,t=2}^k = 0 \rightarrow$ since $w_{f,t=2}^k = 0$, $w_{f,t=2+5}^{j=P(f,2)}$ should be zero. Hence, $w_{f,t=7}^{j=P(f,2)} = 1$, is infeasible and is detected by Constraint (5). Alternatively, for $w_{f,t=7}^{j=P(f,2)}$ to be equal to 1, $w_{f,t=7-l_{fk}}^k$ should equal to 1.

At $t = 3$, $w_{f,t=3+5}^{j=P(f,2)} - w_{f,t=3}^k = 0 \rightarrow$ since $w_{f,t=3}^k = 0$, $w_{f,t=3+5}^{j=P(f,2)}$ will be zero. Similarly at $t = 4$, $w_{f,t=4+5}^{j=P(f,2)} - w_{f,t=4}^k = 0 \rightarrow$ since $w_{f,t=4}^k = 1$, $w_{f,t=4+5}^{j=P(f,2)} = 1$.

Thus, if the flight departs at $t = 4$, it cannot enter its first sector earlier than $t = 4 + 5$. This makes the only case where $w_{f,t=2+5}^{j=P(f,2)} = 1$ is when $w_{f,t=2}^k = 1$, due to checking all the time periods in $T_f^{j=P(f,1)}$. Consequently, the flight cannot enter its first sector earlier, which completes the proof of the proposition. □

In the case when the required time at the departure airport is greater than zero, then we need to ensure that the airport departure capacity is correctly functioning if the required time at the airport is more than one period. Thus, for the airport departure capacity, i.e., Constraint (3), flights are counted if they did enter their first sector by comparing the value of the decision variable at the departure airport with the value in the first sector after the departure airport. This leads to

$$\sum_{f \in \mathscr{F}: P(f,1)=k, j=P(f,2)} (w_{f,t}^k - w_{f,t}^j) \leq D_k(t), \quad \forall\, k \in \mathscr{K},\; t \in \mathscr{T}. \tag{6}$$

Constraint (6) ensures that a flight is counted as long as it is still at its departure airport. It stops considering flight f at its departure airport once it enters its first modeled sector. One may prefer to calculate ground delays using the first sector's information as no delay is allowed between the departure and the first sector entry. Thus, checking the delay at the departure airport ($k = P(f, 1)$) in Expression (2) is replaced by the first sector after the departure airport ($j = P(f, 2)$). This modification requires subtracting the scheduled departure time and the minimum time to be spent at the departure airport ($d_f + l_{fj'}$ with $j' = P(f, 1)$) from the time a flight enters its first sector after the departure airport as given in Expression (7).

$$\sum_{f \in \mathscr{F}} c_f^g \left(\sum_{t \in T_f^j, j=P(f,2), j'=P(f,1)} t \left(w_{f,t}^j - w_{f,t-1}^j \right) - (d_f + l_{fj'}) \right). \tag{7}$$

Although the terms "$c_f^g \times d_f$" in Expression (2) and "$c_f^g \times (d_f + l_{fj'})$" in Expression (7) are constants and can be removed, they are used to facilitate interpreting ground delay costs. Note also that in the case of allowing cancellations, Expressions (2) and (7) can be rewritten as $\sum_{f \in \mathscr{F}} c_f^g \left(\sum_{t \in T_f^k, k=P(f,1)} (t - d_f) \left(w_{f,t}^k - w_{f,t-1}^k \right) \right)$ and $\sum_{f \in \mathscr{F}} c_f^g \left(\sum_{t \in T_f^j, j=P(f,2), j'=P(f,1)} (t - (d_f + l_{fj'})) \left(w_{f,t}^j - w_{f,t-1}^j \right) \right)$, respectively.

In addition, the enhancement for the case when the required time at the departure airport is zero applies when the time required is one period. Note also that we consider the case where flights may take more than one period. By doing so, we provide a general and comprehensive enhancement independent of the length of the time period or the number of periods required for take-off. In previous models in the literature, it is rarely stated how much time an aircraft requires to take off, but it is implicitly assumed that it will require either zero or one time period. However, if a short period is used, an aircraft may require more than one period during the take-off.

4 Examples

In this section, we provide two detailed illustrations for the potential exploitation of the current formulation discussed in Sect. 2. In the first example, the departure airport capacity of one airport varies over time. In the second example, the departure airport capacity of one airport and the capacity of one sector vary over time. In both examples, the network consists of three airports and four airspace sectors. The planning horizon is ten periods. The minimum time to be spent in each sector is one period. The arrival capacity is equal to 5 flights for each airport at each period. The network and the flight path from each airport are given in Fig. 2. The ground delay and air delay unit costs are c_f^g =€120 and c_f^a =€200. The time to be spend at the departure airport for each flight is assumed to be zero in Case 1 and one time-period in Case 2. Note that we also report the impact on large networks in Appendix A. The model was implemented and solved using the Julia programming language on a Jupyter notebook with an Intel(R) Core(TM) i7-9750H 2.6 GHz CPU, 16 Gb of RAM, and 64-bit Windows 10 Home operating system. CPLEX 20.1.0 was used as the solver.

4.1 Example 1: varying the airport capacity

There are five flights scheduled to depart at t = 2. Flights 1, 2 and 3 depart from Airport 1, while Flights 4 and 5 depart from Airport 2. All flights will land at Airport 3. The departure capacity for each airport at each time-period is three flights per period, except for Airport 2.

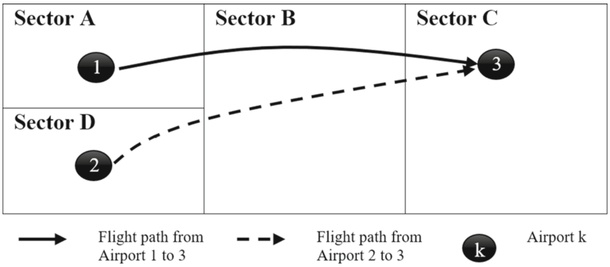

Fig. 2 The location of the airports, airspace sector arrangements and flight paths used in the examples

(a)

	Current formulation										Enhanced formulation									
t	1	2	3	4	5	6	7	8	9	10	1	2	3	4	5	6	7	8	9	10
Flight 1																				
Airport 1			1	1	1	1	1	1	1	1			1	1	1	1	1	1	1	1
Sector A			1	1	1	1	1	1	1	1			1	1	1	1	1	1	1	1
Sector B				1	1	1	1	1	1	1				1	1	1	1	1	1	1
Sector C					1	1	1	1	1	1					1	1	1	1	1	1
Airport 3						1	1	1	1	1						1	1	1	1	1
Flight 2																				
Airport 1		1	1	1	1	1	1	1	1	1		1	1	1	1	1	1	1	1	1
Sector A		1	1	1	1	1	1	1	1	1		1	1	1	1	1	1	1	1	1
Sector B			1	1	1	1	1	1	1	1			1	1	1	1	1	1	1	1
Sector C				1	1	1	1	1	1	1				1	1	1	1	1	1	1
Airport 3					1	1	1	1	1	1					1	1	1	1	1	1
Flight 3																				
Airport 1				1	1	1	1	1	1	1			1	1	1	1	1	1	1	1
Sector A				1	1	1	1	1	1	1			1	1	1	1	1	1	1	1
Sector B					1	1	1	1	1	1				1	1	1	1	1	1	1
Sector C						1	1	1	1	1					1	1	1	1	1	1
Airport 3							1	1	1	1						1	1	1	1	1
Flight 4																				
Airport 2		1	1	1	1	1	1	1	1	1					1	1	1	1	1	1
Sector D		1	1	1	1	1	1	1	1	1					1	1	1	1	1	1
Sector B			1	1	1	1	1	1	1	1						1	1	1	1	1
Sector C				1	1	1	1	1	1	1							1	1	1	1
Airport 3					1	1	1	1	1	1								1	1	1
Flight 5																				
Airport 2		1	1	1	1	1	1	1	1	1		1	1	1	1	1	1	1	1	1
Sector D			1	1	1	1	1	1	1	1		1	1	1	1	1	1	1	1	1
Sector B				1	1	1	1	1	1	1			1	1	1	1	1	1	1	1
Sector C					1	1	1	1	1	1				1	1	1	1	1	1	1
Airport 3						1	1	1	1	1					1	1	1	1	1	1

(b)

	Current formulation										Enhanced formulation									
t	1	2	3	4	5	6	7	8	9	10	1	2	3	4	5	6	7	8	9	10
Flight 1																				
Airport 1				1	1	1	1	1	1	1			1	1	1	1	1	1	1	1
Sector A					1	1	1	1	1	1				1	1	1	1	1	1	1
Sector B						1	1	1	1	1					1	1	1	1	1	1
Sector C							1	1	1	1						1	1	1	1	1
Airport 3								1	1	1							1	1	1	1
Flight 2																				
Airport 1		1	1	1	1	1	1	1	1	1		1	1	1	1	1	1	1	1	1
Sector A			1	1	1	1	1	1	1	1			1	1	1	1	1	1	1	1
Sector B				1	1	1	1	1	1	1				1	1	1	1	1	1	1
Sector C					1	1	1	1	1	1					1	1	1	1	1	1
Airport 3						1	1	1	1	1						1	1	1	1	1
Flight 3																				
Airport 1			1	1	1	1	1	1	1	1			1	1	1	1	1	1	1	1
Sector A				1	1	1	1	1	1	1				1	1	1	1	1	1	1
Sector B					1	1	1	1	1	1					1	1	1	1	1	1
Sector C						1	1	1	1	1						1	1	1	1	1
Airport 3							1	1	1	1							1	1	1	1
Flight 4																				
Airport 2		1	1	1	1	1	1	1	1	1					1	1	1	1	1	1
Sector D				1	1	1	1	1	1	1						1	1	1	1	1
Sector B					1	1	1	1	1	1							1	1	1	1
Sector C						1	1	1	1	1								1	1	1
Airport 3							1	1	1	1									1	1
Flight 5																				
Airport 2		1	1	1	1	1	1	1	1	1		1	1	1	1	1	1	1	1	1
Sector D			1	1	1	1	1	1	1	1			1	1	1	1	1	1	1	1
Sector B				1	1	1	1	1	1	1				1	1	1	1	1	1	1
Sector C					1	1	1	1	1	1					1	1	1	1	1	1
Airport 3						1	1	1	1	1						1	1	1	1	1

Fig. 3 Optimal solution of example 1 using the current and the enhanced formulations under (**a**) Case 1 and (**b**) Case 2

It is set to be zero for periods 3 and 4. Note that the drop in the capacity can be substituted with situations in which there are more flights in the network. The sector capacity is limited to two flights per period in each sector, except for Sector D, where the capacity is one flight per period.

Figure 3 provides the optimal solutions obtained using the current formulation as well as the enhanced one for Cases 1 and 2. The leading 1's (in the grey cells) indicate the time when a flight takes off from an airport, enters a sector, or arrives at an airport. Red cells indicate location where the solution given by the current formulation becomes incorrect.

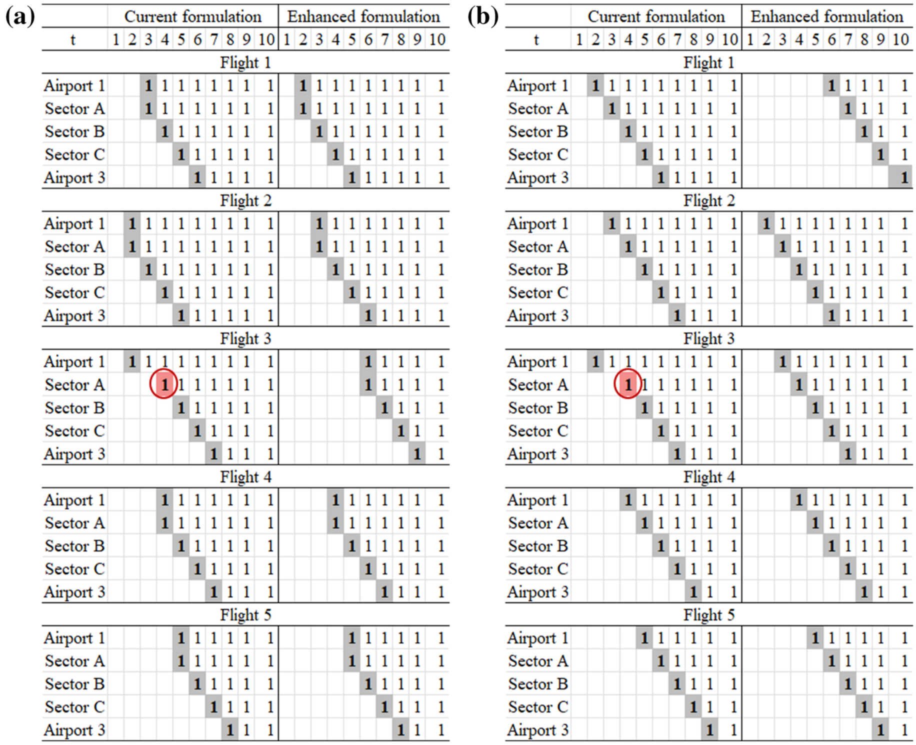

Fig. 4 Optimal solution of example 2 using the current and the corrected formulations under (**a**) Case 1 and (**b**) Case 2

Since Sector D allows one flight per period, it is cheaper to assign at least one unit of ground delay to either flight 4 or 5. However, as the departure capacity drops to zero in periods 3 and 4, either flight 4 or 5 will be held in the ground for at least three periods. In case 1 and using the current formulation, flight 5 leaves its departure airport at $t = 2$ but appears in Sector D at $t = 3$ (Fig. 3a). This situation means that as flight 5 requires zero time-periods, flight 5 is assigned one air delay period during its take-off phase, which is prohibited in practice. In Case 2, flight 4 utilizes the glitch in the current formulation and takes off at $t = 2$, assigns one time-period of air delay during the take-off (Fig. 3b). In this case and since the time to be spent at the airport is one time-period, the departure capacity constraint does not detect flight 4 during its delayed take-off at $t = 3$ ($w^k_{4,3} - w^k_{4,2} = 1 - 1 = 0$). As a result, the solution remains feasible, although it is incorrect. In this situation, delaying flight 4 results in an air delay of one period (€200). On the other hand, flight 4 in the enhanced formulation takes off at $t = 5$, and the resulting delay is three periods ($120 \times 3 =$ €360). This enhanced solution is less efficient than the exploited solution in the current formulation but satisfies the departure capacity requirement correctly. Note that the optimal cost for the current and the enhanced formulations in Case 1 are €560 and €600, respectively. Case 2 results in the same costs as Case 1.

4.2 Example 2: varying the sector capacity

In this example, all five flights take off from Airport 1 and land at Airport 3, and they are scheduled to depart at $t = 2$ except flights 4 and 5, which are scheduled at $t = 3$ and 4, respectively. The departure capacity of Airport 1 is three flights at $t = 2$, and then it drops to one flight from $t = 3$ till $t = 10$ due to low visibility. Sector A has a capacity of three flights per period, and due to some military activities, the available capacity at $t = 1$ till $t = 3$ is one flight per period. The capacity of the remaining sectors is two flights per period.

Figure 4 illustrates the optimal solutions under both cases using the current and the enhanced formulations. The departure capacity of Airport 1 allows three flights to take off. Then flights need to enter its first sector (Sector A) immediately after the departure in Case 1 and after one time-period in Case 2. Although the departure capacity is high, the reduced sector capacity of Sector A limits the number of departures.

In Case 1 (Fig. 4a), the optimal costs using the current and the enhanced formulations are €760 (two periods of ground delay and two periods of air delay) and €840 (seven periods of ground delay), respectively. In the current formulation, flight 3 takes off at $t = 2$ along with flight 2 as the departure capacity allows. However, only flight 2 enters Sector A at $t = 2$ (after its take-off) due to the sector capacity. Flight 3 enters at $t = 4$ when the sector capacity increases. The early take-off of flight 3 with the assigned two periods of air delay during its take-off allows the solution to avoid delaying other flights by a total of four periods of ground delay due to capacity restrictions.

In Case 2 (Fig. 4b), the optimal costs using the current and the enhanced formulations are €560 (two periods of ground delay and one period of air delay) and €840 (seven periods of ground delay), respectively. In the current formulation, flight 1 takes off from Airport 1 at $t = 2$ along with flight 3 while flight 2 takes off at $t = 3$. Flight 3 receives air delay during its take-off and enters Sector A at $t = 4$. Consequently, it is not counted in the departure capacity and its first modeled sector's capacity at $t = 3$ although it took off at $t = 2$.

5 Conclusion

The current formulation used in the literature might lead to incorrect results as the formulation can be exploited under certain circumstances. The shortfall can be detected through the analysis of the optimal flight schedule. The cause of the shortfall is that the current formulation may assign air delay to flights during their take-off, which allows other flights to benefit from the available airport and sector capacities. In this paper, we proposed an enhanced formulation to resolve this issue. We also provided two examples to illustrate the discussed flaw.

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Code Availability The code that support the findings of this study are available from the corresponding author, S.H., upon reasonable request.

Declarations

Conflict of interest No potential competing interest was reported by the authors.

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Appendix A Impact on large networks

We consider more complex networks than the one presented in Sect. 4. Table 2 shows the percentage difference in the total cost between the current formulation and the enhanced one for several instances, calculated as

$$100 \times \frac{\text{Total cost of the enhanced formulation} - \text{Total cost of the current formulation}}{\text{Total cost of the enhanced formulation}}.$$

We observe that the current formulation, where delays and departure capacity are exploited, leads to a lower bound for the total cost. Table 2 shows also the number of undetected violations in the capacity constraint. Note that the minimum time at the departure airport is set to one for all instances. The current formulation can result in violations ranging from 10 to 27% and errors in the total cost that can reach 26%. These gaps depend on the network configuration, capacity values, flight paths and other factors.

Table 2 Cost errors and capacity violations in the current formulation

#	$\|\mathscr{F}\|/\|\mathscr{K}\|/\|\mathscr{J}\|$	Number of capacity violations [flights] (Percentage violations [%])	Total cost of the current formulation [€]	Total cost of the enhanced formulation [€]	Relative cost error [%]
1	50/6/15	7 (14)	282, 538	326, 872	13.56
2	50/6/15	7 (14)	145, 793	152, 485	4.39
3	150/6/15	33 (22)	1, 097, 868	1, 471, 912	25.41
4	150/6/15	30 (20)	1, 124, 717	1, 500, 241	25.03
5	150/6/15	32 (21.33)	1, 446, 270	1, 788, 273	19.12
6	200/6/15	53 (26.5)	3, 074, 313	3, 624, 407	15.18
7	200/6/15	43 (21.5)	2, 328, 591	2, 774, 520	16.07
8	300/6/15	63 (21)	5, 516, 408	6, 092, 468	9.46
9	500/10/25	50 (10)	2, 147, 598	2, 383, 893	9.91

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