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S.I. : SCALABLE OPTIMIZATION AND DECISION MAKING IN OR

Cooperation and decision making in a two-sided market motivated by the externality of a third-party social media platform

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Abstract
In recent years, with the rapid development of Internet technology, the integration of platform economy and e-commerce has become a popular business model. Two-sided platforms have a specific impact on sales, customer experience and transaction efficiency of both sides. In the current severe situation caused by the coronavirus pandemic, both the traditional unilateral market platform and the emerging two-sided market platform are in urgent need of a change in operation mode to reduce the marketing cost. Inspired by the cooperation between Meituan, a two-sided platform, and WeChat, a social media platform, this paper investigates the two-sided platform's scalable decisions on when to cooperate and how to optimize the pricing and investment decisions. We analyze how the two-sided platform makes decisions by considering the changes of network externalities from the cooperation with the social network platform. Compared with the scenario of non-cooperation, we derive the conditions under which platform cooperation can increase demands and increase platforms' profits, and analyze how cooperation affects the optimal pricing strategies. We find that the cooperation leads to a larger demand and a higher total profit, but might lead to higher registration prices for the platform users. Furthermore, we adopt the Nash bargaining framework and introduce platform bargaining power parameters to obtain the optimal cooperation and sharing strategy. Finally, we show how to adjust the investment strategy of the two-sided platform under platform cooperation.

Keywords Two-sided market · Social media platform · Pricing · Cooperation · Investment

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1 Introduction

With the rapid development of network technology, the combination of Internet and traditional industries is fast deepening. The phenomenon of "platform economy" has gradually emerged, and it has been rapidly integrated into manufacturing industries and people's life everywhere, such as online shopping platform, social media platform, life service platform, third-party payment and sharing platform (Qin et al. 2016; Tian and Jiang 2018; Zhao and Chen 2019; Pei et al. 2021). The platform economy is influencing the goods/service providers and consumers in a new way. These platforms do not produce products, but they can facilitate transactions between supply and demand of two or more parties, and obtain revenue by charging appropriate fees or earning the difference. Two-sided market is a kind of intermediary market with indirect network effect, and in such a market, the utility of one group of users (or agents) will be affected by that of another group of users. The number of users on one side will affect the number of users and transaction volume on the other side (Armstrong 2006; Chellappa and Mukherjee 2021). For typical two-sided platforms, such as Uber and Airbnb, when more car owners and house sharers join the platform, the utility of the customer group will be improved. The two sets of users interact through such intermediary platforms to attain their profits (Rysman 2009; Bernstein et al. 2020). In this context, the two-sided market can be captured as cross network externalities (Armstrong 2006; Hagiu and Halaburda 2014; Kung and Zhong 2017; Dou et al. 2018). Meanwhile, with the technology development of mobile Internet, social network begins to influence people's life from information dissemination, consumption and other aspects. The emergence of Facebook, WeChat and other platforms with social attributes constitutes a new ecosystem of social media in the era of mobile Internet. The existence of a two-sided platform can provide both buyers and sellers with transaction information, business security and service guaranties that both parties need (Athey et al. 2018; Jung et al. 2019); while social media platforms tend to be prominent in network traffic and user vitality, resulting in tigher connections among consumers (Huang et al. 2019; Uratnik 2016). Can these two types of platforms take advantage of their respective strengths to cooperate, and will the cooperation bring about overall benefits?

Inspired by the cooperation between Meituan, a two-sided platform, and WeChat, a social media platform, this paper investigates when the two-sided platform should choose to cooperate with the social network platform and how to optimize their pricing and investment decisions. Similar to this case, Airbnb has opened the social network connection function, allowing users to access their Facebook accounts.[1] Users can share and exchange their perceptions of consumption on a social media platform (Adamopoulos et al. 2018; Martínez-López et al. 2020), which has a positive impact on enhancing potential consumers' understanding of the products or services of the sellers on the two-sided platform. It is worth noting that the incorporation of social network platform will not only affect the operation decision of the two-sided platform in terms of cross network strength, but will also affect both of the platforms in terms of profit acquisition. This brings challenges to the decision makers. However, the effects of social media's impacts on a conventional two-sided platform's operational decisions, such as pricing strategies and investment level, have received little attention. Therefore, we raise the following questions.

(a) What is the impact on optimal prices under platforms cooperation? Is it worthwhile to invest since the third party social media platform brings extra externality?
(b) When could platforms cooperation improve the profit of the two-sided platform? How can the two kinds of platforms share the profit increment (if it exists)?

[1] Available at: https://www.lbbonline.com/news/airbnb-launches-trips-on-facebook-live.

In this paper, we build a decision optimization model for a two-sided platform to investigate the pricing and other decision-making in the case of cooperation with a social media platform. By solving the optimal pricing decisions of the two-sided platform under the cooperation, we found that the cooperation leads to larger demands and higher total profit, but may lead to higher prices on the users at both ends of the two-sided platform. Then, with respect to profit bargaining, by comparing with the non-cooperation scenario, we derive the conditions under which platforms cooperation will be maintained. A Nash negotiation profit sharing scheme is proposed. For the two-sided platform's investing decisions, we show that under platform cooperation, when the cost co-efficient of investment is low enough, it is not necessary to invest on consumers. To the best of our knowledge, the decision-making approach and the design of a coordination scheme under the cooperation of a two-sided platform and a social platform has not yet been studied before. This paper aims to fill this gap by incorporating the externality enhancement brought by platforms' cooperation and optimizing the platform decisions with consideration of the participation constraint of the two-sided platform. Our work also extends the decision-making of a traditional two-sided platform. The topic is close to the emerging practice in the social media era and is a new contribution to the literature related with platform operations management. The findings are instructive for the managers of such platforms by showing how to utilize cooperation to increase their profits.

The rest of this paper is organized as follows. Section 2 reviews the related literature. Section 3 presents the description of the problem and develops the mathematical models. Through analytical studies, we derive the equilibrium results under platform cooperation and compares the results with a benchmark scenario. In Sect. 4, we adopt a Nash negotiation framework to coordinate the cooperation. In Sect. 5, we derive the conditions under which it is necessary or unnecessary to invest for the two-sided platform. Finally, concluding remarks are summarized in Sect. 6.

2 Literature review

In this section, we focus on two streams of the literature: (1) Operational research on two-sided market, and (2) the impacts of social media platform's network externality on decision-making. The first topic (Sect. 2.1) reflects the relationship between this work and previous studies on two-sided markets. The second topic (Sect. 2.2) points out the enhancement of externalities brought by social media platforms, which supports our key assumptions.

2.1 Operational research on two-sided market

The two-sided platform has its specific business model. Rochet and Tirole (2003) pointed out that the two-sided platform should pay attention to its own business model, that is, how to coordinate all parties while making overall profits. They set up a platform competition model with two-sided market to reveal the determinants of price distribution and end-user earnings in different governance structures (profit maximization platform and non-profit joint enterprises). Armstrong (2006) constructed two-sided marketing models under monopoly and competition, and gave the determinants of equilibrium price including cross market, namely, network effect, cost structure and whether the agent is single or multiple. Hagiu and Halaburda (2014) studied the impact of different levels of information on the profits of two-sided platforms under the conditions of monopoly and competition. They found that platforms with more market power (monopoly) tend to face more informed users. In contrast, platforms with less market power (i.e. facing more fierce competition) have the opposite preference: when users have less information, they will get higher profits.

The two-sided platforms connect two different user groups, and create profits/surplus for them through the interaction or transaction between the two user groups (Schiff 2007; Hagiu and Spulber 2013; Dou et al. 2018; Chellappa and Mukherjee 2021). Anderson et al. (2014) built a strategic model to study the trade-off between investing in high platform performance and reducing investment. The results showed that, contrary to the traditional view of "winner takes all" market, a large amount of investment in the core performance of the platform does not always produce a competitive advantage. Due to the network externality across the market, two-sided platforms can affect the demand and profit by investing on value-added services (VASs) for users. Dou et al. (2016) studied the unilateral investment and pricing strategies of two-sided platforms. Results showed that when the marginal investment cost is lower than a certain critical value, the investment is optimal at the maximum level, and when the marginal investment cost is higher than the critical value, the investment reduction is optimal. With the development of e-commerce technology, online and offline (O2O) services have penetrated into our daily life and consumption. Kung and Zhong (2017) studied the optimal prices of two-sided platforms under different pricing strategies, mainly including member based pricing, transaction based pricing and cross subsidy. Results showed that these three strategies can balance the same number of shoppers, consumers and profits. Taking taxi rental market as an example, Wang et al. (2017b) analyzed the impact of government regulation on competition in the two-sided market characterized by network externalities in the O2O era. They found that the impact of price adjustment largely depends on the relative size of the network externalities of both sides. In many cases, platform providers need to invest heavily to get more consumers involved.

2.2 Network externality of social media platform

Consumers often refer to other consumers' use evaluation before making their first consumption decision (Boonlong and Wongsurawat 2015). Li and Wu (2014) found that when product information is transmitted through social media, Word of mouth (WOM) can improve product popularity, directly increase demand, and enlarge existing quality signals such as past sales. In the era of network economy, social media platform is an important place for consumers to exchange consumption experience (Adamopoulos et al. 2018), and it plays an increasingly important role in consumers' decision making. Adamopoulos et al. (2018) found that WOM plays an increasingly important role in shaping consumer behavior and preferences. Their research showed that easygoing, serious and open social media users are more effective communicators of WOM. On the introduction of green products, Hong et al. (2020) demonstrated that under specific conditions, WOM has no effect on the pricing strategies of green products and traditional products, but has a significant impact on the market share of both products.

As a means of communication, social media platform greatly influences consumers' preferences and potential purchasing behaviors. Nadeem et al. (2015) found that consumers' trust in e-retailers was directly affected by consumers' intention to use Facebook for shopping and the quality of service provided by the website. Moreover, peer recommendations directly affect the purchase results, and compared with men, women consumers' purchase attitudes are more likely to be affected by peer recommendations. Sarmah et al. (2018) studied the relationship among customer innovation, social media's attitude towards co-creative service innovation, subjective norms and perceived behavior control. The results showed that the key drivers of co-creation intention are valuable for social media as a platform for service innovation. Gruner and Power (2018) found that a wide range of social media investment is not conducive to the company's marketing. On the contrary, focusing on a specific social media platform helps the company form a successful organizational relationship. Martínez-López et al. (2020) pointed out that the integration between key social media platforms and external commercial websites can provide an integrated and seamless purchasing path for consumers (platform users) and enhance the value position of social media platforms in the current ecosystem.

2.3 Research gaps

Considering all the above research on green product promotion, our work fills the following gaps in the literature:

(1) In the existing literature on two-sided markets, it can be found that most of the research focuses on platform operations, such as registration fee pricing. In today's widely active social media platforms, the behavior of consumers is not only affected by the two-sided platforms themselves, but also by the social media field;
(2) When two-sided platforms cooperate with social media platforms, brought by the externality of the social media platform, it raises challenges on the operational decisions such as user pricing, revenue sharing and platform investing decision makings. Existing literature have studied how to integrate social media into product marketing strategy, new product release and recruitment strategy (Avinash 2017; Gruner et al. 2019; Villeda 2019), and to the best of our knowledge, our paper is the first to study the decision making of a two-sided platform who cooperates with a social media platform.

3 The model

3.1 Model description

Suppose there is a monopolistic two-sided platform (L_1) in the market. Platform L_1 connects two types of users (buyers b and sellers s) with different demands. The platform provides a series of services such as trading information disclosure, product advertising and promotion, and click promotion to buyers and sellers in the market. To make profit, the platform respectively charges the users with registration fees, i.e., p_b and p_s. In addition, there exists a positive cross network externality between the two groups of users. The primary externalities are denoted by μ_b and μ_s ($\mu_i > 0, i = b, s$). $\mu_b n_s$ is the external benefit obtained by the buyer from the unit user increase of the seller on the platform, and $\mu_s n_b$ is the external benefit obtained by the seller from the unit user increase of the buyer on the platform. Here, n_i ($i = b, s$) is the number of users actually participating on the platform. This assumption implies that the external utility of one side users is positively related to the number of users on the other end of the platform. For example, if there are a considerable number of service providers available on the platform, consumers will have more chances to get satisfactory services, and the utility of consumers can thus be improved. Similar assumptions can be found in previous work (Anderson et al. 2014; Dou et al. 2016; Chellappa and Mukherjee 2021).

When the two-sided platform choose to cooperate with a third party social media platform (denoted by L_2), the introduction of platform L_2 will enhance the network externality of consumers to sellers. Figure 1 provides a graphical explanation. We use parameter f ($f > 1$)

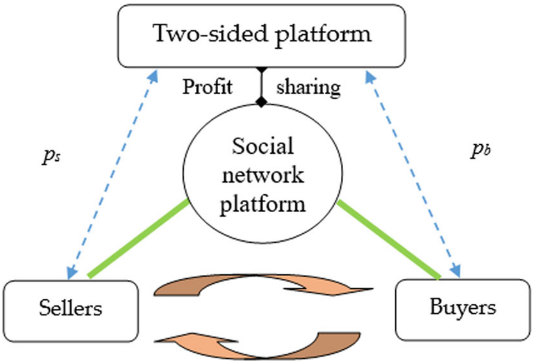

Fig. 1 Cooperation between the two-sided platform L_1 and the social media platform L_2

to show the enhancement on externality brought by the cooperation of platforms (Li and Wu 2014; Adamopoulos et al. 2018), and $f\mu_b n_s$ is the external benefit obtained by the buyer from the unit user increase of the seller on the platform. For simplicity, we assume the two groups' reservation utility of using the platform is equal as v. We use x_i, $i = b, s$ to capture the service evaluation of the platform provided for the two types of users. Then, the net utilities of the buyers and the sellers are given as $U_b = f\mu_b n_s - p_b + vx_b$ and $U_s = n_b\mu_s - p_s + vx_s$. It is reasonable to assume that the users will participate in the platform only if their net utilities are non-negative, i.e., $U_i \geq 0, i = b, s$.

In order to increase the participation utility to attract more users to participate in the platform, two-sided platforms often adopt investment strategies to provide better services or technical support (Athey et al. 2018; Li et al. 2020). For example, a sharing platform might adopt a unilateral investment strategy to invest in the buyer. If the investment level is x, then the consumer obtain a utility of $U_b = f\mu_b n_s - p_b + vx_b + \gamma x$. The buyers' utility will be increased by the platform's investment. Parameter $\gamma(\gamma > 0)$ denotes the sensitivity of investment on consumer utility. We summarize the descriptions of notation in Table 1. In the rest of the paper, we use subscript "TS" to express the case when two platforms cooperate, and subscript "TS,I" to denote the case when two platforms cooperate and platform L_1 invests on the consumers.

3.2 Equilibriums and analysis

In this section, we first solve the optimal pricing decisions of the two-sided platform L_1 under the cooperation with social media platform L_2. Then, compared with the benchmark scenario (with no cooperation), we analyze the impacts of platform cooperation on pricing, demand and profit. In addition, we derive the conditions under which the two-sided platform is willing/voluntary to cooperate with the third-party social media platform. The two-sided platform needs to make optimal prices to maximize the profit, and the two groups of users make decisions whether to participate in the transaction of the platform according to the utility obtained. We first derive the demands of the two sides as: $n_s = Pr(U_s \geq 0) = M_s\left(1 - \frac{p_s - n_b\mu_s}{v}\right)$ and $n_b = Pr(U_b \geq 0) = M_b\left(1 - \frac{p_b - f\mu_b n_s}{v}\right)$. With algebras, the closed forms of demands can be transformed into:

$$n_b^{TS}(p_b, p_s) = \frac{M_b\left(v^2 - f\mu_b M_s p_s + fv\mu_b M_s - vp_b\right)}{v^2 - f\mu_b M_b M_s \mu_s} \quad (1)$$

Table 1 Notations

Parameters	Definitions
$M_i, i = b, s$	The market potential of group i
$n_i, i = b, s$	The number of i who participate in the platform
v	Reservation utility of the users on both sides of the platform
μ_b	The externality obtained by a buyer from the unit user increase of the seller on the two-sided platform
μ_s	The externality obtained by a seller from the unit user increase of the buyer on the two-sided platform
γ	Unit increment of utility on the platform's investment
k	The cost coefficient on the lump sum investment cost
$L_j, j = 1, 2$	The two-sided platform and the third party social media platform
f	The enhancement on externality brought by platform L_2
β	The ratio of total profit shared by platform L_2 after cooperation
Decision variables	
$p_i, i = b, s$	The fee charged by Platform L_1 on the group i
x	Platform L_1's investment decision
ϕ	The two-sided platform's negotiation power, and $1 - \phi$ denotes the third-party social media platform's negotiation power ($0 < \phi < 1$)
Subscripts	
TS	Platform L_1 cooperates with L_2
TS'	There is no cooperation between platforms L_1 and L_2
TS, I	Platform L_1 invests on consumers under cooperation
TS, I'	Platform L_1 invests on consumers under no cooperation

$$n_s^{TS}(p_b, p_s) = \frac{M_s\left(v^2 - M_b p_b \mu_s + v M_b \mu_s - v p_s\right)}{v^2 - f\mu_b M_b M_s \mu_s} \tag{2}$$

With the demand functions derived above, we model the two-sided platform's profit function as:

$$\begin{aligned}\max_{p_b, p_s} \Pi^{TS} &= n_b^{TS}(p_b, p_s) \cdot p_b + n_s^{TS}(p_b, p_s) \cdot p_s \\ &= \frac{M_b\left(p_b\left(f\mu_b M_s (v - p_s) - v p_b + v^2\right) + M_s p_s \mu_s (v - p_b)\right) + v M_s p_s (v - p_s)}{v^2 - f\mu_b M_b M_s \mu_s}\end{aligned} \tag{3}$$

When $v > \frac{f\mu_b + \mu_s}{2}\sqrt{M_b M_s}$ is satisfied, profit Π^{TS} is jointly concave on (p_b, p_s), then we have:

Proposition 1 *When the two-sided platform (L_1) cooperates with the third party social media platform (L_2), platform L_1's optimal equilibriums are given as:*

$$p_s^{TS*} = \frac{v\left(M_b\left(f\mu_b\left(f\mu_b M_s + v\right) + \mu_s\left(f\mu_b M_s - v\right)\right) - 2v^2\right)}{M_b M_s\left(f\mu_b + \mu_s\right)^2 - 4v^2}$$

$$p_b^{TS*} = \frac{v\left(M_s\left(f\mu_b\left(M_b\mu_s - v\right) + \mu_s\left(M_b\mu_s + v\right)\right) - 2v^2\right)}{M_b M_s\left(f\mu_b + \mu_s\right)^2 - 4v^2}$$

The demands and the total profit are solved as: $n_b^{TS*} = \frac{vM_b(M_s(f\mu_b+\mu_s)+2v)}{4v^2-M_bM_s(f\mu_b+\mu_s)^2}$, $n_s^{TS*} = \frac{vM_s(M_b(f\mu_b+\mu_s)+2v)}{4v^2-M_bM_s(f\mu_b+\mu_s)^2}$, *and* $\Pi^{TS*} = \frac{v^2(M_b(M_s(f\mu_b+\mu_s)+v)+vM_s)}{4v^2-M_bM_s(f\mu_b+\mu_s)^2}$.

Proof By solving the equation of demands from utilities, i.e., $n_s^{TS} = M_s\left(1 - \frac{p_s - n_b\mu_s}{v}\right)$ and $n_b^{TS} = M_b\left(1 - \frac{p_b - f\mu_b n_s}{v}\right)$, we can derive the closed form of demands as follows:

$$n_b^{TS} = -\frac{f\mu_b M_b M_s p_s - fv\mu_b M_b M_s + vM_b p_b - v^2 M_b}{v^2 - f\mu_b M_b M_s\mu_s}$$
$$n_s^{TS} = \frac{M_s\left(-M_b p_b\mu_s + vM_b\mu_s - vp_s + v^2\right)}{v^2 - f\mu_b M_b M_s\mu_s}$$

Then we derive the profit function of the platform as:

$$\Pi^{TS} = \frac{M_s p_s\left(vM_b\mu_s - M_b p_b\mu_s - vp_s + v^2\right)}{v^2 - f\mu_b M_b M_s\mu_s} - \frac{p_b\left(f\mu_b M_b M_s p_s - fv\mu_b M_b M_s + vM_b p_b - v^2 M_b\right)}{v^2 - f\mu_b M_b M_s\mu_s}$$

To ensure the optimal results can be obtained, the following must hold:

$$\frac{\partial^2\Pi^{TS}}{\partial p_s^2} = -\frac{2vM_s}{v^2 - f\mu_b M_b M_s\mu_s} < 0$$

$$\frac{\partial^2\Pi^{TS}}{\partial p_b^2} = -\frac{2vM_b}{v^2 - f\mu_b M_b M_s\mu_s} < 0$$

With $\frac{\partial^2\Pi^{TS}}{\partial p_s\partial p_b} = \frac{M_bM_s(f\mu_b+\mu_s)}{f\mu_bM_bM_s\mu_s - v^2}$, the Hessian matrix

$$H^{TS} = \begin{pmatrix} -\frac{2vM_s}{v^2-f\mu_bM_bM_s\mu_s} & \frac{M_bM_s(f\mu_b+\mu_s)}{f\mu_bM_bM_s\mu_s-v^2} \\ \frac{M_bM_s(f\mu_b+\mu_s)}{f\mu_bM_bM_s\mu_s-v^2} & -\frac{2vM_b}{v^2-f\mu_bM_bM_s\mu_s} \end{pmatrix}$$

should be negatively definite, which requires that $-\frac{M_bM_s\left(M_bM_s(f\mu_b+\mu_s)^2-4v^2\right)}{\left(v^2-f\mu_bM_bM_s\mu_s\right)^2} > 0$, or equivalently, $4v^2 > M_bM_s\left(f\mu_b + \mu_s\right)^2$. Then, the first order conditions give the optimal prices as:

$$p_s^{TS*} = \frac{v\left(M_b\left(f\mu_b\left(f\mu_b M_s + v\right) + \mu_s\left(f\mu_b M_s - v\right)\right) - 2v^2\right)}{M_b M_s\left(f\mu_b + \mu_s\right)^2 - 4v^2}$$

$$p_b^{TS*} = \frac{v\left(M_s\left(f\mu_b\left(M_b\mu_s - v\right) + \mu_s\left(M_b\mu_s + v\right)\right) - 2v^2\right)}{M_b M_s\left(f\mu_b + \mu_s\right)^2 - 4v^2}$$

Finally, the demands and optimal profit are therefore given as:

$$n_b^{TS*} = \frac{vM_b\left(M_s\left(f\mu_b+\mu_s\right)+2v\right)}{4v^2-M_bM_s\left(f\mu_b+\mu_s\right)^2}$$

$$n_s^{TS*} = \frac{vM_s\left(M_b\left(f\mu_b+\mu_s\right)+2v\right)}{4v^2-M_bM_s\left(f\mu_b+\mu_s\right)^2}$$

and

$$\Pi^{TS*} = \frac{v^2\left(M_b\left(M_s\left(f\mu_b+\mu_s\right)+v\right)+vM_s\right)}{4v^2-M_bM_s\left(f\mu_b+\mu_s\right)^2}$$

From Proposition 1, it can be found that the optimal prices, demands and profit are all impacted by the platforms' cooperation as compared to the results of the benchmark scenario TS' (The optimal results in scenario TS' can be found in Appendix A). With the impacts of the extra externality brought by the social media platform L_2, we derive the following findings on the changes on demands and total profit:

Proposition 2 *As compared to the benchmark case when there is no cooperation, we have:*

(1) *The cooperation with the social media platform increases the demands and the profit, i.e.,* $\frac{\partial n_i^{TS*}}{\partial f} > 0$ *or* $n_b^{TS*} > n_b^{TS'*}$, $n_s^{TS*} > n_s^{TS'*}$ *and* $\frac{\partial \Pi^{TS*}}{\partial f} > 0$ *or* $\Pi^{TS*} > \Pi^{TS'*}$*;*

(2) *The cooperation's impacts on the price are not direct, we have* $\frac{\partial p_b^{TS*}}{\partial f} > 0$ *if* $0 < M_b < M_{b,1}$*, otherwise,* $\frac{\partial p_b^{TS*}}{\partial f} \le 0$*; and* $\frac{\partial p_s^{TS*}}{\partial f} > 0$ *if* $M_s > M_{s,1}$*, otherwise,* $\frac{\partial p_s^{TS*}}{\partial f} \le 0$.

Here,

$$M_{s,1} = \frac{\sqrt{v^2\left(16vM_b\mu_s(f\mu_b+\mu_s)^2+\left(f\mu_b(f\mu_bM_b+4v)-3M_b\mu_s^2\right)^2-2f\mu_bM_b\mu_s\right)}+vM_b(f\mu_b+\mu_s)(f\mu_b-3\mu_s)+4fv^2\mu_b}{2M_b\mu_s(f\mu_b+\mu_s)^2}$$

and

$$M_{b,1} = \frac{\sqrt{v^2\left(16vM_s\mu_s(f\mu_b+\mu_s)^2+\left(f\mu_b(f\mu_bM_s+4v)-3M_s\mu_s^2\right)^2-2f\mu_bM_s\mu_s\right)}+vM_s(f\mu_b+\mu_s)(f\mu_b-3\mu_s)+4fv^2\mu_b}{2M_s\mu_s(f\mu_b+\mu_s)^2}.$$

Proof For the demands and profit, by solving the first order conditions with respect to f, we have:

$$\frac{\partial n_b^{TS*}}{\partial f} = \frac{v\mu_bM_bM_s\left(M_b\left(f\mu_b+\mu_s\right)\left(M_s\left(f\mu_b+\mu_s\right)+4v\right)+4v^2\right)}{\left(M_bM_s\left(f\mu_b+\mu_s\right)^2-4v^2\right)^2}$$

$$\frac{\partial n_s^{TS*}}{\partial f} = \frac{v\mu_bM_bM_s\left(M_s\left(f\mu_b+\mu_s\right)\left(M_b\left(f\mu_b+\mu_s\right)+4v\right)+4v^2\right)}{\left(M_bM_s\left(f\mu_b+\mu_s\right)^2-4v^2\right)^2}$$

and

$$\frac{\partial \Pi^{TS*}}{\partial f} = \frac{v^2\left(M_b\left(M_s\left(f\mu_b+\mu_s\right)+v\right)+vM_s\right)}{4v^2-M_bM_s\left(f\mu_b+\mu_s\right)^2}$$

Because $4v^2 > M_bM_s\left(f\mu_b+\mu_s\right)^2$ is assumed, then, we have $\frac{\partial n_i^{TS*}}{\partial f} > 0$ and $\frac{\partial \Pi^{TS*}}{\partial f} > 0$ $(i = b, s)$. When $f = 1$, we have $n_i^{TS*} = n_i^{TS'*}$ and $\Pi^{TS*} = \Pi^{TS'*}$, therefore, we can obtain that $[n_i^{TS*} - n_i^{TS'*}]|_{f>1} > [n_i^{TS*} - n_i^{TS'*}]|_{f=1} = 0$ and $[\Pi^{TS*} - \Pi^{TS'*}]|_{f>1} > [\Pi^{TS*} - \Pi^{TS'*}]|_{f=1} = 0$ since $f > 1$. For the prices, the first order conditions give that

$$\frac{\partial p_b^{TS*}}{\partial f} = \frac{v\mu_b M_s \left(4v^3 - M_b^2 M_s\mu_s (f\mu_b + \mu_s)^2 + vM_b \left(f\mu_b (f\mu_b M_s + 4v) - 2f\mu_b M_s\mu_s - 3M_s\mu_s^2\right)\right)}{\left(M_b M_s (f\mu_b + \mu_s)^2 - 4v^2\right)^2}$$

$$\frac{\partial p_s^{TS*}}{\partial f} = \frac{v\mu_b M_b \left(M_b M_s^2\mu_s (f\mu_b + \mu_s)^2 + vM_s \left(2f\mu_b M_b\mu_s - f\mu_b (f\mu_b M_b + 4v) + 3M_b\mu_s^2\right) - 4v^3\right)}{\left(M_b M_s (f\mu_b + \mu_s)^2 - 4v^2\right)^2}$$

It can be observed that the denominators are all positive, and therefore, whether $\frac{\partial p_b^{TS*}}{\partial f}(\frac{\partial p_s^{TS*}}{\partial f})$ is positive or negative depends on its numerators. Take $\frac{\partial p_b^{TS*}}{\partial f}$ for example. Denote the numerator as $\delta_0 = -M_b^2 M_s\mu_s (f\mu_b + \mu_s)^2 + vM_b\big(-2f\mu_b M_s\mu_s + f\mu_b (f\mu_b M_s + 4v) - 3M_s\mu_s^2\big) + 4v^3$. Then, we have $\frac{\partial^2\delta_0}{\partial M_b^2} = -2M_s\mu_s (f\mu_b + \mu_s)^2 < 0$, which means $\frac{\partial p_b^{TS*}}{\partial f}$ is concave on M_b. By solving $\delta_0 = 0$ on M_b, we have two roots as follows,

$$M_{b,1} = \frac{\sqrt{v^2 \left(16vM_s\mu_s (f\mu_b + \mu_s)^2 + \left(f\mu_b (f\mu_b M_s + 4v) - 2f\mu_b M_s\mu_s - 3M_s\mu_s^2\right)^2\right)} + vM_s (f\mu_b + \mu_s)(f\mu_b - 3\mu_s) + 4fv^2\mu_b}{2M_s\mu_s (f\mu_b + \mu_s)^2}$$

$$M_{b,2} = \frac{-\sqrt{v^2 \left(16vM_s\mu_s (f\mu_b + \mu_s)^2 + \left(f\mu_b (f\mu_b M_s + 4v) - 2f\mu_b M_s\mu_s - 3M_s\mu_s^2\right)^2\right)} + vM_s (f\mu_b + \mu_s)(f\mu_b - 3\mu_s) + 4fv^2\mu_b}{2M_s\mu_s (f\mu_b + \mu_s)^2}$$

It is direct to have $M_{b,1} - M_{b,2} = \frac{\sqrt{v^2\left(16vM_s\mu_s(f\mu_b+\mu_s)^2 + \left(-2f\mu_b M_s\mu_s + f\mu_b(f\mu_b M_s+4v) - 3M_s\mu_s^2\right)^2\right)}}{M_s\mu_s(f\mu_b+\mu_s)^2} > 0$ and we further have

$$\sqrt{v^2 \left(16vM_s\mu_s (f\mu_b + \mu_s)^2 + \left(f\mu_b (f\mu_b M_s + 4v) - 2f\mu_b M_s\mu_s - 3M_s\mu_s^2\right)^2\right)} > |vM_b (f\mu_b + \mu_s)(f\mu_b - 3\mu_s) + 4fv^2\mu_b|$$

which yields that $M_{b,2} < 0$. Then we can conclude that we have $\frac{\partial p_b^{TS*}}{\partial f} > 0$ if $0 < M_b < M_{b,1}$, otherwise, $\frac{\partial p_b^{TS*}}{\partial f} \leq 0$. By solving the roots of $\frac{\partial p_s^{TS*}}{\partial f} = 0$ on M_s, we also have two roots, i.e., $M_{s,1}$ and $M_{s,2}$ with $M_{s,1} > M_{s,2}$ and $0 > M_{s,2}$. Similarly, we have $\frac{\partial p_s^{TS*}}{\partial f} > 0$ if $M_s > M_{s,1}$, otherwise, $\frac{\partial p_s^{TS*}}{\partial f} \leq 0$. Here,

$$M_{s,1} = \frac{\sqrt{v^2 \left(16vM_b\mu_s (f\mu_b + \mu_s)^2 + \left(f\mu_b (f\mu_b M_b + 4v) - 3M_b\mu_s^2\right)^2 - 2f\mu_b M_b\mu_s\right)} + vM_b (f\mu_b + \mu_s)(f\mu_b - 3\mu_s) + 4fv^2\mu_b}{2M_b\mu_s (f\mu_b + \mu_s)^2}$$

$$M_{s,2} = \frac{-\sqrt{v^2 \left(16vM_b\mu_s (f\mu_b + \mu_s)^2 + \left(f\mu_b (f\mu_b M_b + 4v) - 3M_b\mu_s^2\right)^2 - 2f\mu_b M_b\mu_s\right)} + vM_b (f\mu_b + \mu_s)(f\mu_b - 3\mu_s) + 4fv^2\mu_b}{2M_b\mu_s (f\mu_b + \mu_s)^2}$$

In this proposition, we derive the impacts of the network externalities brought by the social platform on the pricing strategies, demands and profits of the two-sided platform. It can be found that the joining of the social platform increases the number of users participating in platform (L_1). This result can be explained that the increase of network externalities strengthen the communications and information acquisitions between consumers on the services provided by the sellers on platform L_1. In other words, with the intervention of social media platform, the communication and information sharing between consumers can not only make the buyers get more utility from the products, but also make the sellers known by more consumers who have not bought. Therefore, the profit of the two-sided platform will be improved due to the increase of "information stream". By cooperating with the social media platform, the two-sided platform can effectively increase the number of platform users and enjoy a higher total revenue. However, whether platform users will be charged higher or lower registration fees depends on the number of potential users.

However, for the platform registration fee, we find that the improvement of this externality brought by platform L_2 might lead to higher prices on users at both ends of platform L_1, i.e., $p_b^{TS*} > p_b^{TS'*}$ and $p_s^{TS*} > p_s^{TS'*}$. In addition, we also find that for the fee charged by the platform to consumers, we find that when the market capacity of consumer groups is large enough, i.e., $M_b > M_{b,1}$, the positive externalities brought by the social media platform will induce the platform to lower the price charged on consumers. For the optimal fee charged by the platform to the sellers, we observe an opposite conclusion. That is, when the number of sellers is large enough, i.e., $M_s > M_{s,1}$, the positive externalities brought by platform L_2 will induce the platform to increase the pricing on the sellers. With further algebras, we can derive that $\frac{\partial n_b^{TS*}}{\partial f} > \frac{\partial n_s^{TS*}}{\partial f}$ if $M_b > M_s$ is satisfied, otherwise we have $\frac{\partial n_s^{TS*}}{\partial f} > \frac{\partial n_b^{TS*}}{\partial f}$. This shows that the realized demands of group $i, i = b, s$ is more sensitive to the cooperation if group i has a larger initial market size.

On platform L_1's pricing strategies on the two types of users, we further have:

Proposition 3 (1) *On pricing the sellers, we have $p_s^{TS*} > 0$ if $0 < f < f_1$; $p_s^{TS*} \leq 0$ if $f \geq f_1$;*

(2) *On pricing the buyers, we have two cases:*

- *When $M_b > \frac{v}{\mu_s}$, we have $p_b^{TS*} > 0$ if $0 < f < f_2$; we have $p_b^{TS*} \leq 0$ if $f \geq f_2$;*
- *When $M_b < \frac{v}{\mu_s}$, we have $p_b^{TS*} > 0$ if $f > f_2$; we have $p_b^{TS*} \leq 0$ if $0 < f \leq f_2$.*

Here, $f_1 = \frac{\sqrt{(M_b(M_s\mu_s(M_s\mu_s+6v)+v^2)+8v^2M_s)(M_b)^{-1}}-M_s\mu_s-v}{2\mu_b M_s}$ and $f_2 = \frac{M_s\mu_s(M_b\mu_s+v)-2v^2}{\mu_b M_s(v-M_b\mu_s)}$.

Proof (1) Because $p_s^{TS*} = \frac{v\left(M_b(f\mu_b(f\mu_b M_s+v)+\mu_s(f\mu_b M_s-v))-2v^2\right)}{M_b M_s(f\mu_b+\mu_s)^2-4v^2}$ and $M_b M_s\left(f\mu_b+\mu_s\right)^2 -4v^2 < 0$, then we have $p_s^{TS*} > 0$ if $v\left(M_b\left(f\mu_b\left(f\mu_b M_s+v\right)+\mu_s\left(f\mu_b M_s-v\right)\right)-2v^2\right) < 0$.

Let $\delta_2 = v\left(M_b\left(f\mu_b\left(f\mu_b M_s+v\right)+\mu_s\left(f\mu_b M_s-v\right)\right)-2v^2\right)$. We then have $\frac{\partial^2\delta_2}{\partial f^2} = 2\mu_b^2 M_b M_s > 0$, which means $\delta_2(f)$ is convex on f. By solving $\delta_2(f) = 0$ on f, we have:

$$f_1 = -\frac{-\frac{\sqrt{M_b\left(M_s^2\mu_s^2+6vM_s\mu_s+v^2\right)+8v^2M_s}}{\sqrt{M_b}} + M_s\mu_s + v}{2\mu_b M_s}$$

$$f_{11} = -\frac{\frac{\sqrt{M_b\left(M_s^2\mu_s^2+6vM_s\mu_s+v^2\right)+8v^2M_s}}{\sqrt{M_b}} + M_s\mu_s + v}{2\mu_b M_s}$$

Table 2 Platform L_1's pricing strategies

Cases	Optimal prices
Cases I: $f > \frac{\mu_s}{\mu_b}$	$p_s^{TS*} > 0$ if $M_b > \frac{2v^2}{f\mu_b(f\mu_b M_s+v)+\mu_s(f\mu_b M_s-v)}$, otherwise $p_s^{TS*} \le 0$;
	$p_b^{TS*} > 0$ if $M_b < \frac{v(M_s(f\mu_b-\mu_s)+2v)}{M_s\mu_s(f\mu_b+\mu_s)}$, otherwise $p_b^{TS*} \le 0$
Cases II:$f \le \frac{\mu_s}{\mu_b}$	$p_s^{TS*} \ge 0$ if $M_b \le \frac{2v^2}{f\mu_b(f\mu_b M_s+v)+\mu_s(f\mu_b M_s-v)}$, otherwise $p_s^{TS*} < 0$;
	$p_b^{TS*} \ge 0$ if $M_b \ge \frac{v(M_s(f\mu_b-\mu_s)+2v)}{M_s\mu_s(f\mu_b+\mu_s)}$, otherwise $p_b^{TS*} < 0$

It is straightforward to have $f_1 - f_{11} = \frac{\sqrt{M_b\left(M_s\mu_s(M_s\mu_s+6v)+v^2\right)+8v^2M_s}}{\mu_b\sqrt{M_b}M_s} > 0$ and $f_{11} < 0$. Further, because $f > 0$, therefore, we have $p_s^{TS*} > 0$ if $0 < f < f_1$; $p_s^{TS*} \le 0$ if $f \ge f_1$;

(2) For p_b^{TS*}, because $p_b^{TS*} = \frac{v\left(M_s(f\mu_b(M_b\mu_s - v) + \mu_s(M_b\mu_s+v)) - 2v^2\right)}{M_bM_s(f\mu_b+\mu_s)^2-4v^2}$ and $M_bM_s\left(f\mu_b+\mu_s\right)^2 - 4v^2 < 0$, then we have $p_b^{TS*} > 0$ if $M_s\left(f\mu_b\left(M_b\mu_s - v\right) + \mu_s\left(M_b\mu_s + v\right)\right) - 2v^2 < 0$. Let $\delta_3 = M_s\left(f\mu_b\left(M_b\mu_s - v\right) + \mu_s\left(M_b\mu_s + v\right)\right) - 2v^2$. We have two cases: If $M_b > \frac{v}{\mu_s}$, $\frac{\partial\delta_3(f)}{\partial f} > 0$, then we have $p_b^{TS*} > 0$ if $0 < f < f_2$; $p_b^{TS*} \le 0$ if $f \ge f_2$; If $M_b < \frac{v}{\mu_s}$, $\frac{\partial\delta_3(f)}{\partial f} < 0$, then we have $p_b^{TS*} > 0$ if $f > f_2$; $p_b^{TS*} \le 0$ if $0 < f \le f_2$. Here, $f_2 = \frac{M_s\mu_s(M_b\mu_s+v)-2v^2}{\mu_bM_s(v-M_b\mu_s)}$.

In practice, two-sided platforms often treat users on both sides of the platform differently according to the information advantages they have mastered, that is, they might charge the buyers and the sellers in different ways. For example, many platform enterprises that are powerful in controlling the sellers, in order to attract customers, while charging, the platforms often subsidize consumers. In many cases, the consumption subsidy provided by a take out platform is even close to the commodity price itself. In this proposition, we analyze how the cooperation with social media platform (parameter f) affects the differential pricing of platform L_1 to the two group of users. We show that when the externality enhancement brought by platform L_2 is large enough, platform L_1 is more willing to subsidize the sellers (set a negative registration fee, i.e., $p_s^{TS*} < 0$) and similar result can be found on consumers when the buyers' market potential M_b is large enough ($M_b > \frac{v}{\mu_s}$). This finding suggests that, when making pricing decisions, the manager of the two-sided platform should not only consider the externality enhancement but also consider the market potential and the original externality coefficient.

The analytical results on prices can also be characterized with market potential M_b. See the following Table 2.

In Table 2, we show that when f parameter is larger than a threshold ($f > \frac{\mu_s}{\mu_b}$), the price charged on the sellers (p_s^{TS*}) is increasing with buyers' market size M_b; while the price charged on the buyers p_b^{TS*} is decreasing with M_b. However, there is a reverse observation if f parameter is less than the threshold ($f \le \frac{\mu_s}{\mu_b}$). That is, the price charged on the sellers (p_s^{TS*}) is decreasing with buyers' market size M_b; while the price charged on the buyers p_b^{TS*} is increasing with M_b. This indicates that a two-sided platform will provide more subsidies to the sellers when both parameter f and buyers' segment size are small. This finding is close to practice. For example, in order to attract more taxi drivers to use the platform, the car sharing platform DiDi spent a large amount of money as subsidies. By comparing the optimal prices

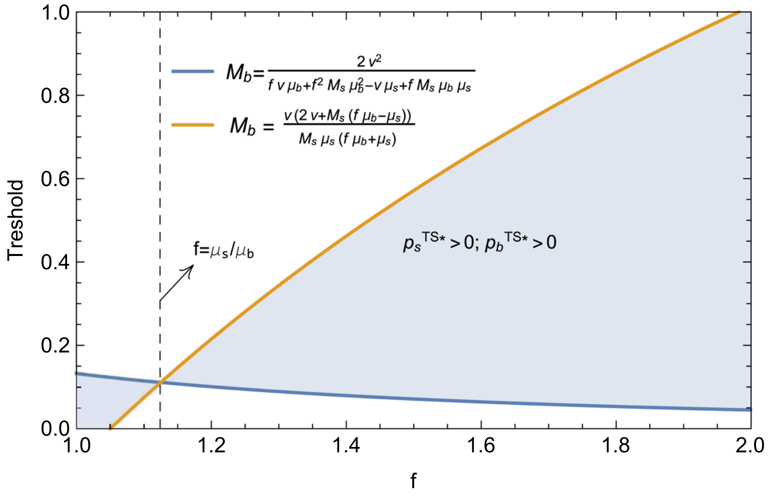

Fig. 2 Platform L_1's pricing strategies when f varies ($v = 0.6$, $\mu_b = 0.16$, $M_s = 100$ and $\mu_s = 0.18$)

on sellers and buyers, we can also find that in Case I ($f > \frac{\mu_s}{\mu_b}$), we have $p_b^{TS*} - p_s^{TS*} > 0$; and in Case II ($f \leq \frac{\mu_s}{\mu_b}$) we have $p_b^{TS*} - p_s^{TS*} \leq 0$. This finding indicates that the two-sided platform will set a higher price for the sellers than the buyers when the externality parameter f is less than the threshold $\frac{\mu_s}{\mu_b}$. The comparison also implies that $p_s^{TS*} > 0$ and $p_b^{TS*} < 0$ (resp. $p_s^{TS*} < 0$ and $p_b^{TS*} > 0$) cannot co-exist in Case I (resp. Case II). Since we have assumed that $f > 1$, it is interesting to find that platform L_1 will always set a larger price on the buyers if $\mu_s \leq \mu_b$ ($\frac{\mu_s}{\mu_b} \leq 1$) is satisfied.

In Figs. 2 and 3, we use a numerical example to show this phenomenon. The dashed areas denote the scenario that the prices charged on both the sellers and the buyers are positive ($p_b^{TS*} > 0$; $p_s^{TS*} > 0$). It can be observed that when the market potential of the sellers decreases ($M_s = 100 \rightarrow M_s = 18$), both the thresholds move upwards, and it is more likely for the two-sided platform to charge both the sellers with negative price, i.e., $p_s^{TS*} < 0$. This finding can be explained that when the sellers on the platform L_1 are relatively scarce, the platform is more inclined to provide the seller with exemptions or subsidies.

In Proposition 2, we demonstrate that the participation of platform L_2 promotes platform L_1's profit, and it is reasonable to study the case that platform L_1 will share part of profit to platform L_2 for its contribution. Suppose that the two-sided platform shares $(1-\beta)$ percent of the total profit to platform L_2 and keeps β percent for itself. Then, the participation constraint of the cooperation with the social media platform L_2 is that a larger profit will be obtained as compared to the benchmark scenario (Model TS') when there is no cooperation with the social media platform (The equilibriums are provided in Appendix A). Define

$$A_{L_1}(\beta) = \beta \cdot \Pi^{TS*} - \Pi^{TS'*} \tag{4}$$

$$A_{L_2}(\beta) = (1-\beta) \cdot \Pi^{TS*} \tag{5}$$

to denote the two platforms' profit after cooperation. Therefore, it is required that $A_{L_1} > 0$ to make sure that platform L_1 will cooperate with platform L_2. Here, $A_{L_2} > 0$ all the time. With algebra comparisons, we have the following proposition:

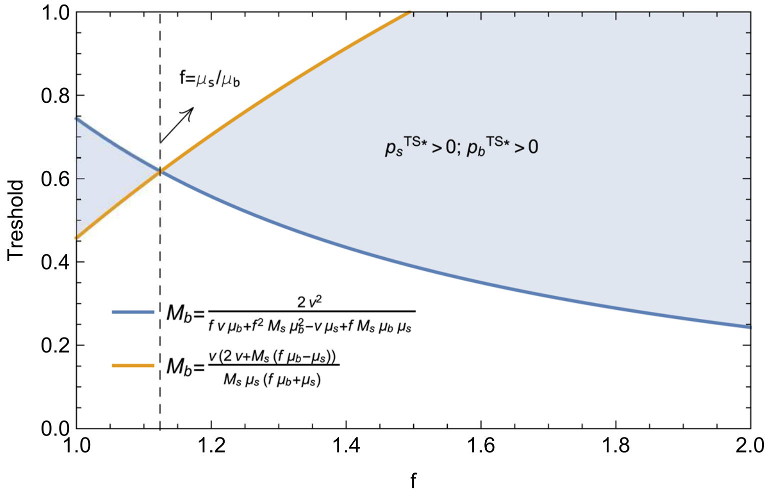

Fig. 3 Platform L_1's pricing strategies when f varies ($v = 0.6$, $\mu_b = 0.16$, $M_s = 18$ and $\mu_s = 0.18$)

Proposition 4 *When* $f > \max\{f_3, 1\}$, *we have* $A_{L_1} > 0$, *and the two-sided platform will cooperate with the third-party platform. Otherwise, the two-sided platform will not cooperate with the third-party platform.*

Here,

$$f_3 = \frac{\sqrt{\varphi_1\left(M_b\left(M_s\left(\mu_b+\mu_s\right)\left(\beta M_s\left(\mu_b+\mu_s\right)+4v\right)+4v^2\right)-4(\beta-1)v^2M_s\right)}+\varphi_2}{2\mu_b^2M_bM_s\left(M_b\left(M_s\left(\mu_b+\mu_s\right)+v\right)+vM_s\right)},$$

$$\varphi_1 = \mu_b^2M_bM_s\left(\beta M_b^2M_s\left(\mu_b+\mu_s\right)^2+4vM_b\left(M_s\left(\mu_b+\mu_s\right)-\beta v+v\right)+4v^2M_s\right) \text{ and}$$

$$\varphi_2 = \mu_bM_b^2M_s\left(M_s\left(\mu_b+\mu_s\right)\left(\beta\mu_b+(\beta-2)\mu_s\right)-2v\mu_s\right)-2v\mu_bM_bM_s\left(M_s\mu_s+2\beta v\right).$$

Proof When the two-sided platform shares its profit to the social platform (with $1-\beta$), the two-sided platform will cooperate with the third part social platform if and only if the remaining profit after sharing is greater than the initial profit before cooperation, or equivalently,

$$A_{L_1} = \frac{\beta\left(v^2\left(M_b\left(M_s\left(f\mu_b+\mu_s\right)+v\right)+vM_s\right)\right)}{4v^2-M_bM_s\left(f\mu_b+\mu_s\right)^2} - \frac{v^2\left(M_b\left(M_s\left(\mu_b+\mu_s\right)+v\right)+vM_s\right)}{4v^2-M_bM_s\left(\mu_b+\mu_s\right)^2} > 0$$

By deriving the second order derivatives of A_{L_1} on f, we have:

$$\frac{\partial^2 A_{L_1}}{\partial f^2} = \frac{2\beta v^2\mu_b^2M_bM_s(vM_b\left(3M_s\left(f\mu_b+\mu_s\right)\left(M_s\left(f\mu_b+\mu_s\right)+4v\right)+4v^2\right) + M_b^2M_s\left(f\mu_b+\mu_s\right)^2\left(M_s\left(f\mu_b+\mu_s\right)+3v\right)+4v^3M_s)}{\left(4v^2-M_bM_s\left(f\mu_b+\mu_s\right)^2\right)^3} > 0$$

This indicates that A_{L_1} is convex on f, the two roots of $A_{L_1}=0$ can be solved as:

$$f_3 = \frac{\sqrt{\varphi_1\left(M_b\left(M_s\left(\mu_b+\mu_s\right)\left(\beta M_s\left(\mu_b+\mu_s\right)+4v\right)+4v^2\right)-4(\beta-1)v^2M_s\right)} + \mu_bM_b^2M_s\left(M_s\left(\mu_b+\mu_s\right)\left(\beta\mu_b+(\beta-2)\mu_s\right)-2v\mu_s\right)-2v\mu_bM_bM_s\left(M_s\mu_s+2\beta v\right)}{2\mu_b^2M_bM_s\left(M_b\left(M_s\left(\mu_b+\mu_s\right)+v\right)+vM_s\right)}$$

$$f_4 = -\frac{\sqrt{\varphi_1 \left(M_b \left(M_s (\mu_b + \mu_s)(\beta M_s (\mu_b + \mu_s) + 4v) + 4v^2\right) - 4(\beta - 1)v^2 M_s\right)} - \mu_b M_s M_b^2 (M_s (\mu_b + \mu_s)(\beta \mu_b + (\beta - 2)\mu_s) - 2v\mu_s) + 2v\mu_b M_b M_s (M_s \mu_s + 2\beta v)}{2\mu_b^2 M_b M_s (M_b (M_s (\mu_b + \mu_s) + v) + vM_s)}$$

Here, $f_3 > f_4$ and $\varphi_1 = \mu_b^2 M_b M_s \Big(\beta M_b^2 M_s (\mu_b + \mu_s)^2 + 4vM_b (M_s (\mu_b + \mu_s) - \beta v + v) + 4v^2 M_s\Big)$. Because $f > 1$, it is straightforward to have that

$$\begin{aligned} f_4 - 1 &= \frac{\mu_b M_b M_s \left((\mu_b + \mu_s)(M_b ((\beta - 2)M_s (\mu_b + \mu_s) - 2v) - 2vM_s) - 4\beta v^2\right) - \sqrt{\varphi_1 \left(M_b \left(M_s (\mu_b + \mu_s)(\beta M_s (\mu_b + \mu_s) + 4v) + 4v^2\right) - 4(\beta - 1)v^2 M_s\right)}}{2\mu_b^2 M_b M_s (M_b (M_s (\mu_b + \mu_s) + v) + vM_s)} \\ &< -\frac{\sqrt{\varphi_1 \left(M_b \left(M_s (\mu_b + \mu_s)(\beta M_s (\mu_b + \mu_s) + 4v) + 4v^2\right) - 4(\beta - 1)v^2 M_s\right)}}{2\mu_b^2 M_b M_s (M_b (M_s (\mu_b + \mu_s) + v) + vM_s)} \\ &< 0 \end{aligned}$$

Then, we can conclude that when $f > \max\{f_3, 1\}$, we have $A_{L_1} > 0$, and the two -sided platform will cooperate with the third-party platform. Otherwise, the two-sided platform will not cooperate with the third-party platform.

In Proposition 4, we show that when the amplification factor of network externality brought by platform L_2 is greater than the threshold f_3, two-sided platform L_1 will cooperate with social media platform L_2. If $f_3 < 1$, the two-sided platform will cooperate with the third-party platform all the time. It can be observed that the profit sharing index β plays an important role on affecting platform L_1's decision on cooperation. In this section, we are working under the premise that parameter β is exogenously given. However, partners with business connections often determine the distribution of the total profit through negotiation (Hu et al. 2013; Car18; Cher20). This motivates our next work. In the next section, we will investigate when parameter β can be viewed as a decision variable.

4 Profit negotiation between the platforms

In this section, we will introduce how to establish a cooperative relationship between platforms L_1 and L_2 through consultation. Without Pareto improvement, the two platforms will not establish a cooperative relationship. We adopt the Nash negotiation framework (Myerson 1997), in which all parties involved can get any benefits from non cooperation and share any benefits brought by cooperation. Let ϕ $(0 \leq \phi \leq 1)$ denote the normalized bargaining power for two-sided platform L_1 and thus $(1 - \phi)$ represents the same for the third-party social media platform L_2. Let $(1 - \beta)\Pi^{TS*}$ be the shared profit of platform L_2 after the cooperation (We assume the profit of platform L_2 is 0 before cooperation with L_1). Along with Wang et al. (2017a), we assume that the two platforms bargain as follows. That is

$$(1 - \beta)\phi\Pi^{TS*} = A_{L_1}(1 - \phi) \tag{6}$$

Equation (5) can be reorganized as $\frac{\phi}{1-\phi} = \frac{A_{L_1}}{(1-\beta)\Pi^{TS*}}$. It can be found that the profit sharing ratio depends on the bargaining power of each platform. By solving the above functions, we have the following results:

Proposition 5 *When the negotiation power of the social media platform is ϕ, then it is optimal for the two-sided platform to share $\beta^*\Pi^{TS*}$ to the social media platform. Here,*

$$\beta^* = \frac{(\phi_1 - \phi_2)\left(M_b M_s (f\mu_b + \mu_s)^2 - 4v^2\right)}{M_b\left(M_s\left(f\mu_b + \mu_s\right) + v\right) + vM_s}$$

where $\phi_1 = \frac{(\phi-1)(M_b(M_s(\mu_b+\mu_s)+v)+vM_s)}{4v^2-M_bM_s(\mu_b+\mu_s)^2}$ *and* $\phi_2 = \frac{\phi(M_b(M_s(f\mu_b+\mu_s)+v)+vM_s)}{4v^2-M_bM_s(f\mu_b+\mu_s)^2}$.

Proof Because $1-\phi$ and ϕ respectively denotes the negotiation power of the third-party platform and the two-sided platform. The added profit of the two platforms can be expressed as

$$A_{L_1} = \frac{\beta\left(v^2\left(M_b\left(M_s\left(f\mu_b + \mu_s\right) + v\right) + vM_s\right)\right)}{4v^2 - M_bM_s\left(f\mu_b + \mu_s\right)^2} - \frac{v^2\left(M_b\left(M_s\left(\mu_b + \mu_s\right) + v\right) + vM_s\right)}{4v^2 - M_bM_s\left(\mu_b + \mu_s\right)^2}$$

$$A_{L_2} = \frac{(1-\beta)\left(v^2\left(M_b\left(M_s\left(f\mu_b + \mu_s\right) + v\right) + vM_s\right)\right)}{4v^2 - M_bM_s\left(f\mu_b + \mu_s\right)^2} - 0$$

The second term of the above two equations represents the initial profit of the two platforms when the third-party platform has not joined the cooperation. Based on the negotiation powers of the platforms, we have:

$$\frac{(1-\beta)\phi\left(v^2\left(M_b\left(M_s\left(f\mu_b + \mu_s\right) + v\right) + vM_s\right)\right)}{4v^2 - M_bM_s\left(f\mu_b + \mu_s\right)^2} = A_{L_1}(1-\phi)$$

By solving the equation above, we have that

$$\beta^* = \frac{(\phi_1 - \phi_2)\left(M_bM_s\left(f\mu_b + \mu_s\right)^2 - 4v^2\right)}{M_b\left(M_s\left(f\mu_b + \mu_s\right) + v\right) + vM_s}$$

Here, $\phi_1 = \frac{(\phi-1)(M_b(M_s(\mu_b+\mu_s)+v)+vM_s)}{4v^2-M_bM_s(\mu_b+\mu_s)^2}$ and $\phi_2 = \frac{\phi(M_b(M_s(f\mu_b+\mu_s)+v)+vM_s)}{4v^2-M_bM_s(f\mu_b+\mu_s)^2}$. Thus, the realized profits of the two platforms can be solved as:

$$\Pi_{L_1}^{TS*} = \frac{v^2\left(\phi_1 - \phi_2\right)\left(M_bM_s\left(f\mu_b + \mu_s\right)^2 - 4v^2\right)}{4v^2 - M_bM_s\left(f\mu_b + \mu_s\right)^2}$$

$$\Pi_{L_2}^{TS*} = \frac{v^2\left(M_b\left(M_s\left(f\mu_b + \mu_s\right) + v\right) + vM_s\right)\left(1 - \frac{(\phi_1-\phi_2)\left(M_bM_s(f\mu_b+\mu_s)^2-4v^2\right)}{M_b(M_s(f\mu_b+\mu_s)+v)+vM_s}\right)}{4v^2 - M_bM_s\left(f\mu_b + \mu_s\right)^2}$$

In Proposition 5, we present the optimal profit sharing scheme for the two-sided platform L_1 and social media platform L_2. The optimal sharing parameter β^* is derived by considering the two platforms' negotiation power. In the proposed Nash negotiation framework, both of the two platforms are benefited. The first order condition tells that β^* is positively related with platform L_1's negotiation power. Note that without this negotiation framework, we show that there exists a lower bound for β to ensure that platform L_1 is willing to participate the cooperation with platform L_2. That is, in order to have $A_{L_1} > 0$, β needs to be larger than $\beta = \frac{\Pi^{TS'*}}{\Pi^{TS*}}$. By deriving the difference of β^* with $\frac{\Pi^{TS'*}}{\Pi^{TS*}}$, we have:

$$\beta^* - \frac{\Pi^{TS'*}}{\Pi^{TS*}} = \frac{\begin{array}{l}(f-1)\phi\mu_bM_bM_s(v(M_s((f+1)\mu_b+2\mu_s)+4v)\\ +M_b(f\mu_b^2M_s+(f+1)\mu_b(M_s\mu_s+v)+\mu_s(M_s\mu_s+2v)))\end{array}}{(4v^2-M_bM_s(\mu_b+\mu_s)^2)(M_b(M_s(f\mu_b+\mu_s)+v)+vM_s)} > 0 \quad (7)$$

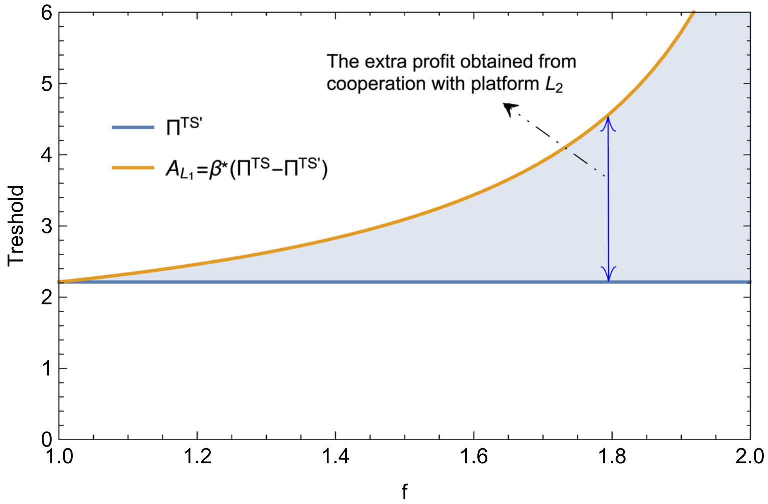

Fig. 4 Platform L_1's negotiated profit (with $\phi = 0.6$)

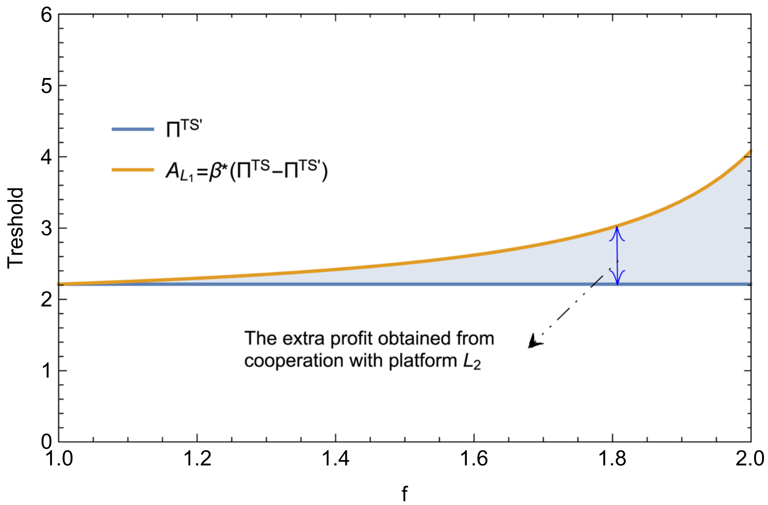

Fig. 5 Platform L_1's negotiated profit (with $\phi = 0.2$)

Because $4v^2 - M_b M_s(\mu_b + \mu_s)^2 > 0$, $\beta^* - \frac{\Pi^{TS'*}}{\Pi^{TS*}} > 0$ holds all the time. This indicates that the optimal profit sharing scheme decided by the proposed Nash negotiation framework benefits both platforms L_1 and L_2. It is not straightforward to find how parameter f affects β^*, and we study the impacts with the following numerical example ($v = 0.6$, $\mu_b = 0.16$, $M_s = 1$, $M_b = 5$ and $\mu_s = 0.18$). In Figs. 4 and 5, it can be observed that (1) When f increases, platform L_1's negotiated profit increases; (2) When the negotiation power of platform L_1 is low ($\phi = 0.2$), the marginal profit increase from increasing f will be reduced. This finding suggests that the manager of the two-sided platform with a larger negotiation power will benefit more from the platforms' cooperation.

Table 3 Consumer utilities with platform investment

Cases	Consumer utilities
No cooperation with L_2	$U_b = \mu_b n_s - p_b + v x_b + \gamma x$ $U_s = n_b \mu_s - p_s + v x_s$
Cooperation with L_2	$U_b = f \mu_b n_s - p_b + v x_b + \gamma x$ $U_s = n_b \mu_s - p_s + v x_s$

5 Is it necessary to invest on consumers?

In order to get more consumer participation, value-added service investment is adopted by many two-sided platforms (Dou et al. 2016; Tan et al. 2020). This kind of investments can increase the utility of consumers on the platform. Here we denote x to be the investment level, and we assume that the maximum value of x is 1 ($x \in [0, 1]$). Similarly, γ ($0 < \gamma < 1$) measures consumers' utility from an additional unit of investment. The consumers' utilities under platform investment are summarized in the following Table 3.

Based on the utilities in the Table 3, we derive the demand functions of the second case by solving the following systems of equations: $n_s = Pr(U_s \geq 0) = M_s \left(1 - \frac{p_s - n_b \mu_s}{v}\right)$ and $n_b = Pr(U_b \geq 0) = M_b \left(1 - \frac{p_b - f \mu_b n_s - \gamma x}{v}\right)$ and yields that:

$$n_b^{TS,I}(p_b, p_s, x) = \frac{M_b \left(f v \mu_b M_s - f \mu_b M_s p_s - v p_b + v^2 + \gamma v x\right)}{v^2 - f \mu_b M_b M_s \mu_s} \tag{8}$$

$$n_s^{TS,I}(p_b, p_s, x) = \frac{M_s \left(v M_b \mu_s - M_b p_b \mu_s + \gamma x M_b \mu_s - v p_s + v^2\right)}{v^2 - f \mu_b M_b M_s \mu_s} \tag{9}$$

With the demand functions derived above, we can model the two-sided platform's profit function as follows:

$$\max_{p_b, p_s, x} \Pi^{TS,I} = n_b^{TS,I}(p_b, p_s, x) \cdot p_b + n_s^{TS,I}(p_b, p_s, x) \cdot p_s - \frac{1}{2} k x^2 \tag{10}$$

The first term denotes platform L_1's gross revenue from the markets and the second term depicts the lump-sum fee that platform L_1 has to pay for the investment. The quadratic cost structure is widely used to capture the R&D investment cost (Zhu and He 2017 and Giri et al. 2019). Here, parameter k depicts the cost coefficient on the lump sum investment cost. We first investigates the impact of investment in the general case, that is, the profit function is a concave function without considering the decision boundary. The following findings are derived:

Proposition 6 *By comparing scenarios TS, I and TS, I',*

(1) *For the profits, we have $\Pi^{TS,I*} - \Pi^{TS',I*} > 0$;*
(2) *The optimal investment level in scenario TS, I is larger than that in scenario TS, I', i.e., $x^{TS,I*} - x^{TS',I*} > 0$.*

Proof With the demands derived from utilities functions, i.e., $n_b^{TS,I} = M_b \left(1 - \frac{p_b - f \mu_b n_s - \gamma x}{v}\right)$ and $n_s^{TS,I} = M_s \left(1 - \frac{p_s - n_b \mu_s}{v}\right)$, we can derive the closed form of demands as follows:

$$n_b^{TS,I} = \frac{M_b \left(f v \mu_b M_s - f \mu_b M_s p_s - v p_b + v^2 + \gamma v x\right)}{v^2 - f \mu_b M_b M_s \mu_s}$$

$$n_s^{TS,I} = \frac{M_s\left(\gamma x M_b\mu_s - M_b p_b\mu_s + vM_b\mu_s - vp_s + v^2\right)}{v^2 - f\mu_b M_b M_s\mu_s}$$

Then we derive the profit function of platform L_1 as:

$$\Pi^{TS,I} = -\frac{p_b\left(f\mu_b M_b M_s p_s - fv\mu_b M_b M_s + vM_b p_b - v^2 M_b - \gamma vxM_b\right)}{v^2 - f\mu_b M_b M_s\mu_s} + \frac{M_s p_s\left(-M_b p_b\mu_s + vM_b\mu_s + \gamma xM_b\mu_s - vp_s + v^2\right)}{v^2 - f\mu_b M_b M_s\mu_s} - \frac{kx^2}{2}$$

To ensure the optimal results can be obtained, the following should hold.

$$\frac{\partial^2\Pi^{TS,I}}{\partial p_s^2} = -\frac{2vM_s}{v^2 - f\mu_b M_b M_s\mu_s} < 0$$

$$\frac{\partial^2\Pi^{TS,I}}{\partial p_b^2} = -\frac{2vM_b}{v^2 - f\mu_b M_b M_s\mu_s} < 0$$

$$\frac{\partial^2\Pi^{TS,I}}{\partial x^2} = -k < 0$$

and the Hessian matrix $H^{TS,I} = \begin{pmatrix} \frac{\partial^2\Pi^{TS,I}}{\partial p_s^2} & \frac{\partial^2\hat{\Pi}^{ST}}{\partial p_s\partial p_b} & \frac{\partial^2\hat{\Pi}^{ST}}{\partial p_s\partial x} \\ \frac{\partial^2\Pi^{TS,I}}{\partial p_b\partial p_s} & \frac{\partial^2\hat{\Pi}^{ST}}{\partial p_b^2} & \frac{\partial^2\hat{\Pi}^{ST}}{\partial p_b\partial p_x} \\ \frac{\partial^2\Pi^{TS,I}}{\partial x\partial p_s} & \frac{\partial^2\hat{\Pi}^{ST}}{\partial x\partial p_b} & \frac{\partial^2\hat{\Pi}^{ST}}{\partial x^2} \end{pmatrix}$ should be negatively definite, which requires $(-1)^3\frac{M_b M_s\left(M_b\left(kM_s(f\mu_b+\mu_s)^2+2\gamma^2 v\right)-4kv^2\right)}{(v^2-f\mu_b M_b M_s\mu_s)^2} > 0$ or equivalently, $k > \frac{2\gamma^2 vM_b}{4v^2-M_b M_s(f\mu_b+\mu_s)^2}$. Then, the first order conditions give the optimal prices as:

$$p_s^{TS,I*} = \frac{v\left(M_b\left(fk\mu_b\left(f\mu_b M_s + v\right) + k\mu_s\left(f\mu_b M_s - v\right) + \gamma^2 v\right) - 2kv^2\right)}{M_b\left(kM_s\left(f\mu_b + \mu_s\right)^2 + 2\gamma^2 v\right) - 4kv^2}$$

$$p_b^{TS,I*} = \frac{v\left(M_s\left(fk\mu_b\left(M_b\mu_s - v\right) + \mu_s\left(M_b\left(k\mu_s - \gamma^2\right) + kv\right)\right) - 2kv^2\right)}{M_b\left(kM_s\left(f\mu_b + \mu_s\right)^2 + 2\gamma^2 v\right) - 4kv^2}$$

$$x^{TS,I*} = \frac{\gamma vM_b\left(M_s\left(f\mu_b + \mu_s\right) + 2v\right)}{4kv^2 - M_b\left(kM_s\left(f\mu_b + \mu_s\right)^2 + 2\gamma^2 v\right)}$$

With these, the demands and the optimal profit are therefore given as:

$$n_b^{TS,I*} = \frac{kvM_b\left(M_s\left(f\mu_b + \mu_s\right) + 2v\right)}{4kv^2 - M_b\left(kM_s\left(f\mu_b + \mu_s\right)^2 + 2\gamma^2 v\right)}$$

$$n_s^{TS,I*} = \frac{vM_s\left(M_b\left(fk\mu_b - \gamma^2 + k\mu_s\right) + 2kv\right)}{4kv^2 - M_b\left(kM_s\left(f\mu_b + \mu_s\right)^2 + 2\gamma^2 v\right)}$$

$$\Pi^{TS,I*} = -\frac{v^2\left(M_b\left(M_s\left(2k\left(f\mu_b + \mu_s\right) - \gamma^2\right) + 2kv\right) + 2kvM_s\right)}{2\left(M_b\left(kM_s\left(f\mu_b + \mu_s\right)^2 + 2\gamma^2 v\right) - 4kv^2\right)}$$

Regarding the comparing of the optimal profit with benchmark model, we have:

$$\Pi^{TS,I*} - \Pi^{TS',I*} = \frac{v^2\left(M_b\left(M_s\left(2k\left(f\mu_b+\mu_s\right)-\gamma^2\right)+2kv\right)+2kvM_s\right)}{2\left(4kv^2-M_b\left(kM_s\left(f\mu_b+\mu_s\right)^2+2\gamma^2 v\right)\right)} - \frac{v^2\left(M_b\left(M_s\left(2k\left(\mu_b+\mu_s\right)-\gamma^2\right)+2kv\right)+2kvM_s\right)}{8kv^2-2M_b\left(kM_s\left(\mu_b+\mu_s\right)^2+2\gamma^2 v\right)}$$
$$> -\frac{(f-1)kv^2\mu_b M_b M_s}{M_b\left(kM_s\left(f\mu_b+\mu_s\right)^2+2\gamma^2 v\right)-4kv^2} > 0$$

On comparing the optimal results on investment level, we have:

$$x^{TS,I*} - x^{TS',I*} = \frac{\gamma v M_b\left(M_s\left(f\mu_b+\mu_s\right)+2v\right)}{4kv^2-M_b\left(kM_s\left(f\mu_b+\mu_s\right)^2+2\gamma^2 v\right)} - \frac{\gamma v M_b\left(M_s\left(\mu_b+\mu_s\right)+2v\right)}{4kv^2-M_b\left(kM_s\left(\mu_b+\mu_s\right)^2+2\gamma^2 v\right)}$$
$$> \frac{\gamma v M_b\left(M_s\left(f\mu_b+\mu_s\right)+2v\right)}{4kv^2-M_b\left(kM_s\left(\mu_b+\mu_s\right)^2+2\gamma^2 v\right)} - \frac{\gamma v M_b\left(M_s\left(\mu_b+\mu_s\right)+2v\right)}{4kv^2-M_b\left(kM_s\left(\mu_b+\mu_s\right)^2+2\gamma^2 v\right)}$$
$$= \frac{\gamma(f-1)v\mu_b M_b M_s}{4kv^2-M_b\left(kM_s\left(\mu_b+\mu_s\right)^2+2\gamma^2 v\right)} > 0$$

This proves that the cooperation with the third-party social media platform will induce the decision maker to invest more on consumers if $4kv^2 - M_b\left(kM_s\left(f\mu_b+\mu_s\right)^2+2\gamma^2 v\right) > 0$ and $4kv^2 - M_b\left(kM_s\left(\mu_b+\mu_s\right)^2+2\gamma^2 v\right)$ are satisfied simultaneously.

In Proposition 6, when platform L_1 chooses to invest on the consumers, we show that the improvement of externality brought by platforms cooperation has a positive impact on the gross profit of platform L_1. This is in line with the findings in Proposition 2. At the same time, the optimal investment is also greater than that of platform L_1 before cooperation. This finding suggests that with the reinforcement of network externalities brought by cooperation, the manager of the two-sided platform should increase the investment level on the consumers. The optimal decisions of the first case when there is no cooperation with platform L_2 are summarized in Appendix A.

Note that we do not consider bound for the investment in Proposition 6 and the profit functions are assumed to be concave, and in practice, there might exist decision bounds of x and the profit function could also be convex. It is necessary to survey how the optimal investment decision should be made in such scenarios. In line with the work of Dou et al. (2016), we assume that the investment is located in $x \in [0, 1]$. By considering the special scenarios (when the profit function is jointly convex for example), we summarize the optimal investment strategies in the following proposition.

Proposition 7 *Define* $k_1 = \frac{\gamma v M_b(M_s(f\mu_b+\mu_s)+2(\gamma+v))}{4v^2-M_bM_s(f\mu_b+\mu_s)^2}$ *and* $k_2 = \frac{2\gamma^2 v M_b}{4v^2-M_bM_s(f\mu_b+\mu_s)^2}$, *the optimal investment strategies of platform* L_1 *can be derived as follows:*

Case (I). When $k \le k_2$, *we have* $x^{TS,I*} = 1$ *all the time.*

Case (II). When $k > k_2$, *we have*

$$x^{TS,I*} = \begin{cases} \frac{\gamma v M_b(M_s(f\mu_b+\mu_s)+2v)}{4kv^2-M_b\left(kM_s(f\mu_b+\mu_s)^2+2\gamma^2 v\right)}, & \text{if } k \ge k_1; \\ 1, & \text{if } k_2 < k < k_1; \end{cases}$$

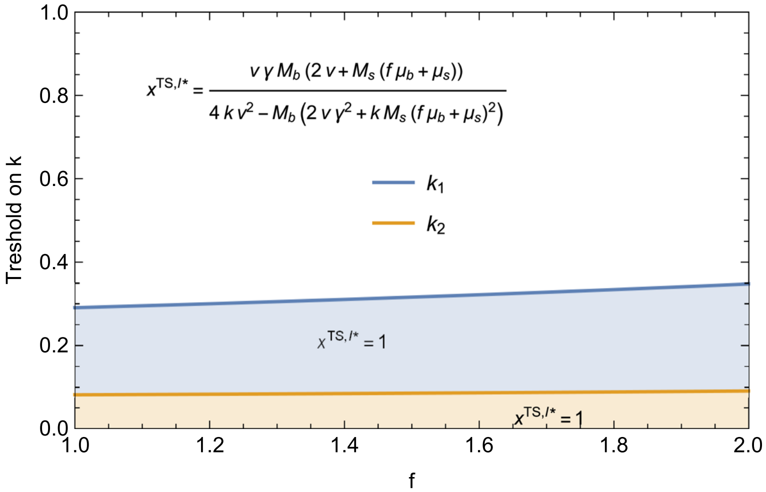

Fig. 6 Platform L_1's investment strategies when f varies ($v = 0.6$, $\gamma = 0.3$, $\mu_b = 0.16$, $M_b = 1$, $M_s = 1$ and $\mu_s = 0.18$)

Proof The proof can be further derived from the proof of Proposition 6, and we choose to omit it.

In Proposition 7, we show that there are two thresholds on parameter k, i.e., k_1 and k_2, that define platform L_1's investment decisions. It can be observed that when k is greater than k_1, the optimal investment level lies at the extreme point obtained according to the first derivative condition; when k is between the two thresholds k_1 and k_2, the optimal investment level is the investment upper bound ($x^{TS,I*} = 1$). This shows that with the increase of cost coefficient k, the optimal investment amount decreases. Interestingly, when k is less than the minimum threshold k_2 (Case I), the optimal service investment level is 1 all the time, that is, the investment should be at the highest level. Because $k_1 - k_2 = \frac{\gamma v M_b(M_s(f\mu_b+\mu_s)+2v)}{4v^2-M_bM_s(f\mu_b+\mu_s)^2} > 0$, we can conclude that when $k < k_1$ is satisfied, $x^{TS,I*} = 1$ always holds. This finding suggests that platform L_1 should always invest on consumers under the cooperation with platform L_2. This is consistent with the finding obtained when the investment bound is not considered. In other words, with an enhancement of the network externalities, the two-sided platform should invest more on the buyers.

We use Figs. 6 and 7 to show the decision zones of the two-sided platform's investment strategies. It can be observed that when parameter k is small enough (the light orange area ($k \le k_2$) and the light blue area ($k_2 < k < k_1$)), the optimal decision is to invest at the highest level. Correspondingly, when k is large enough (in the blue area), the platform should invest at the highest level. By comparing the two figures, it can be found that when the buyers' market potential increases, it is more likely for platform L_1 to set $x^* = 1$ as compared to $x^* = \frac{\gamma v M_b(M_s(f\mu_b+\mu_s)+2v)}{4kv^2-M_b(kM_s(f\mu_b+\mu_s)^2+2\gamma^2 v)}$. A practical implication is that when there are more buyers on the two-sided platform, a higher investment level should be made.

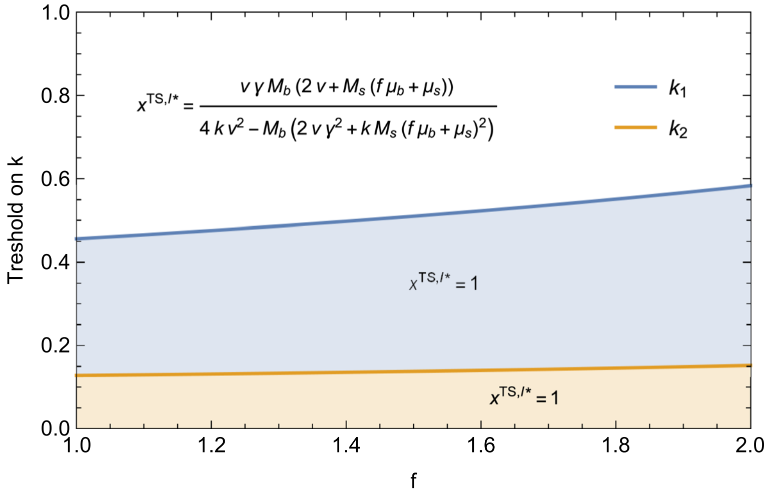

Fig. 7 Platform L_1's investment strategies when f varies ($v = 0.6$, $\gamma = 0.3$, $\mu_b = 0.16$, $M_b = 1.5$, $M_s = 1$ and $\mu_s = 0.18$)

6 Discussion and conclusion

In recent decades, with the rapid development of internet technology, the integration of platform economy and e-commerce has become a new hot business model. It is known that two-sided platforms have specific impacts on the product distribution, customer experience and transaction efficiency of both sides. Through this kind of platform, consumers and goods (or service) providers can better match each other. In the current severe situation of new coronavirus, online consumption become more popular. In this context, the emerging two-sided market platforms such as food delivery and takeout platforms are playing more important roles.

Inspired by the cooperation between Meituan, a two-sided platform, and WeChat, a social media platform, this paper investigates when should the two-sided platform choose to cooperate and how to make optimal pricing and investment decisions. We analyze how the two-sided platform make decisions by considering the changes of network externalities brought by the cooperation with the social media platform. The two-sided platform is scalable to make the cooperation based on the realized profit. Compared with non cooperation, we derive the conditions under which platforms cooperation can increase demands and increase platform profits, and analyze how cooperation affects the optimal pricing strategies. Results show that although the cooperation with a social media platform can increase the total profit of the two-sided platform, it also might bring higher registration fees for users. In other words, the increased price might offset the positive impact of network externality on consumer utility. Furthermore, we adopt Nash negotiation framework by introducing a bargaining power based profit sharing scheme to coordinate the cooperation. Finally, we show how to adjust the investment strategy of the two-sided platform under platforms cooperation. The main findings and managerial implications are summarized as follows:

- By cooperating with the social media platform, the two-sided platform can effectively reach a higher profit with an increased number of platform users. However, whether the platform users will be charged with a lower registration fee depends on the number of users.

- When pricing the buyers, the manager of the two-sided platform should not only consider the externality enhancement but also consider the market potential of the buyers segment and the original externality coefficient.
- Under cooperation, the two-sided platform with a larger negotiation power will benefit more from the platforms' cooperation.
- Under cooperation, the manager of the two-sided platform should increase the investment level for the consumers on the platform.

There are some limitations in our work that deserve further research in the future. This article studies a monopoly two-sided platform in a local market. However, duopoly competition is also commonly seen in practice. Therefore, further research can consider the situation when there exist competing two-sided platforms. In addition, we assume that the market size of the buyers is unchanged after the platforms' cooperation. It is worthwhile to study the case when the buyers' market size is affected by the cooperation. We leave these issues to our future study.

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Appendix A. The optimums of two benchmark cases.

Here, (1) Benchmark 1 stands for the scenario when there is no cooperation with the third-party social media platform and no invest on consumers. (2) Benchmark 2 stands for the scenario when there is no cooperation with the third-party social media platform and the two-sided platform chooses to invest on consumers.

Table 4 Optimal equilibriums of the two benchmark cases

Cases	Optimal equilibriums
Benchmark 1: Platform L_1 does not invest without cooperation with L_2	$p_s^{TS'*}=\frac{v\left(M_b(\mu_s(v-\mu_b M_s)-\mu_b(\mu_b M_s+v))+2v^2\right)}{4v^2-M_b M_s(\mu_b+\mu_s)^2}$
	$p_b^{TS'*}=\frac{v\left(M_s(\mu_b(v-M_b\mu_s)-\mu_s(M_b\mu_s+v))+2v^2\right)}{4v^2-M_b M_s(\mu_b+\mu_s)^2}$
	$n_s^{TS'*}=\frac{vM_s(M_b(\mu_b+\mu_s)+2v)}{4v^2-M_b M_s(\mu_b+\mu_s)^2}$
	$n_b^{TS'*}=\frac{vM_b(M_s(\mu_b+\mu_s)+2v)}{4v^2-M_b M_s(\mu_b+\mu_s)^2}$
	$\Pi^{TS'*}=\frac{v^2(M_b(M_s(\mu_b+\mu_s)+v)+vM_s)}{4v^2-M_b M_s(\mu_b+\mu_s)^2}$
Benchmark 2: Platform L_1 invests without cooperation with L_2	$p_s^{TS,I'*}=\frac{v\left(2kv^2-M_b\left(k\mu_b(\mu_b M_s+v)+k\mu_s(\mu_b M_s-v)+v\gamma^2\right)\right)}{4kv^2-M_b\left(kM_s(\mu_b+\mu_s)^2+2v\gamma^2\right)}$
	$p_b^{TS,I'*}=\frac{v\left(M_s\left(k\mu_b(v-M_b\mu_s)-\mu_s\left(M_b\left(k\mu_s-\gamma^2\right)+kv\right)\right)+2kv^2\right)}{4kv^2-M_b\left(kM_s(\mu_b+\mu_s)^2+2v\gamma^2\right)}$
	$x^{TS,I'*}=\frac{v\gamma M_b(M_s(\mu_b+\mu_s)+2v)}{4kv^2-M_b\left(kM_s(\mu_b+\mu_s)^2+2v\gamma^2\right)}$
	$n_s^{TS,I'*}=\frac{vM_s\left(M_b\left(k(\mu_b+\mu_s)-\gamma^2\right)+2kv\right)}{4kv^2-M_b\left(kM_s(\mu_b+\mu_s)^2+2v\gamma^2\right)}$
	$n_b^{TS,I'*}=\frac{kvM_b(M_s(\mu_b+\mu_s)+2v)}{4kv^2-M_b\left(kM_s(\mu_b+\mu_s)^2+2v\gamma^2\right)}$
	$\Pi^{TS,I'*}=\frac{v^2\left(M_b\left(M_s\left(2k(\mu_b+\mu_s)-\gamma^2\right)+2kv\right)+2kvM_s\right)}{8kv^2-2M_b\left(kM_s(\mu_b+\mu_s)^2+2v\gamma^2\right)}$

References

Adamopoulos, P., Ghose, A., & Todri, V. (2018). The impact of user personality traits on word of mouth: text-mining social media platforms. *Information Systems Research, 29*(3), 612–640.

Anderson, E. G., Parker, G. G., & Tan, B. (2014). Platform performance investment in the presence of network externalities. *Information Systems Research, 25*(1), 152–172.

Armstrong, M. (2006). Competition in two-sided markets. *The RAND Journal of Economics, 37*(3), 668–691.

Athey, S., Calvano, E., & Gans, J. (2018). The impact of consumer multi-homing on advertising markets and media competition. *Management science, 64*(4), 1574–1590.

Avinash, B. M. (2017). Social media as a marketing tool: its effectiveness in promoting airline business. *European Journal of Business and Management, 9*(16), 49–56.

Bernstein, F., DeCroix, G., & Keskin, B. (2020). Competition between two-sided platforms under demand and supply congestion effects. *Manufacturing & Service Operations Management*. https://doi.org/10.1287/msom.2020.0866.

Boonlong, S., & Wongsurawat, W. (2015). Social media marketing evaluation using social network comments as an indicator for identifying consumer purchasing decision effectiveness. *Journal of Direct, Data and Digital Marketing Practice, 17*(2), 130–149.

Chellappa, R. K., & Mukherjee, R. (2021). Platform Preannouncement Strategies: The Strategic Role of Information in Two-Sided Markets Competition. *Management science, 67*(3), 1527–1545.

Chernonog, T. (2020). Strategic information sharing in online retailing under a consignment contract with revenue sharing. *Annals of Operations Research*. https://doi.org/10.1007/s10479020038071.

Carpenter, J., Robbett, A., & Akbar, P. A. (2018). Profit sharing and peer reporting. *Management science, 64*(9), 3971–4470.

Dou, G., He, P., & Xu, X. (2016). One-side value-added service investment and pricing strategies for a two-sided platform. *International Journal of Production Research, 54*(13), 3808–3821.

Dou, G., Lin, X., & Xu, X. (2018). Value-added service investment strategy of a two-sided platform with the negative intra-group network externality. *Kybernetes, 47*(5), 937–956.
Giri, R. N., Mondal, S. K., & Maiti, M. (2019). Government intervention on a competing supply chain with two green manufacturers and a retailer. *Computers & Industrial Engineering, 128,* 104–121.
Gruner, R., Vomberg, A., Homburg, C., & Lukas, B. A. (2019). Supporting new product launches with social media communication and online advertising: sales volume and profit implications. *Journal of Product Innovation Management, 36*(2), 172–195.
Gruner, R., & Power, D. (2018). To integrate or not to integrate? Understanding B2B social media communications. *Online Information Review, 42*(1), 73–92.
Hagiu, A., & Halaburda, H. (2014). Information and two-sided platform profits. *International Journal of Industrial Organization, 34*(1), 25–35.
Hagiu, A., & Spulber, D. F. (2013). First-Party Content and Coordination in Two-Sided Markets. *Management Science, 59*(4), 933–949.
Hong, Z., Li, M., Han, X., & He, X. (2020). Innovative green product diffusion through word of mouth. *Transportation Research Part E-logistics and Transportation Review, 134,* 101833.
Hu, X., Caldentey, R., & Vulcano, G. (2013). Revenue Sharing in Airline Alliances. *Management Science, 59*(5), 1177–1195.
Huang, N., Sun, T., Chen, P., & Golden, J. (2019). Word-of-mouth system implementation and customer conversion: A randomized field experiment. *Information Systems Research, 30*(3), 805–818.
Jung, D., Kim, B., Park, M., & Straub, D. (2019). Innovation and policy support for two-sided market platforms: Can government policy makers and executives optimize both societal value and profits? *Information Systems Research, 30*(3), 1037–1050.
Kung, L. C., & Zhong, G. Y. (2017). The optimal pricing strategy for two-sided platform delivery in the sharing economy. *Transportation Research Part E: Logistics and Transportation Review, 101,* 1–12.
Li, H., Shen, Q. W., & Bart, Y. (2020). Dynamic resource allocation on multi-category two-sided platforms. *Management Science*. https://doi.org/10.1287/mnsc.2020.3586.
Li, X., & Wu, L. (2014). Herding and social media word-of-mouth: evidence from groupon. *Management Information Systems Quarterly, 42*(4), 1331–1351.
Martínez-López, F. J., Li, Y., Liu, H., & Feng, C. (2020). Do safe buy buttons and integrated path-to-purchase on social platforms improve users' shopping-related responses? *Electronic Commerce Research and Applications, 39,* 100913.
Myerson, R. B. (1997). Game theoretic models of bargaining: an introduction for economists studying the transnational commons.?*The economics of transnational commons*, 17-34.
Nadeem, W., Andreini, D., Salo, J., & Laukkanen, T. (2015). Engaging consumers online through websites and social media: a gender study of italian generation y clothing consumers. *International Journal of Information Management, 35*(4), 432–442.
Pei, J., Yan, P., Kumar, S., & Liu, X. (2021). How to React to Internal and External Sharing in B2C and C2C. *Production and Operations Management, 30*(1), 145–170.
Qin, X., Lin, L., Lysecky, S., Roveda, J., Son, Y., & Sprinkle, J. (2016). A modular framework to enable rapid evaluation and exploration of energy management methods in smart home platforms. *Energy Systems, 7,* 215–235.
Rochet, J., & Tirole, J. (2003). Platform competition in two-sided markets. *Journal of the European Economic Association, 1*(4), 990–1029.
Rysman, M. (2009). The economics of two-sided markets. *The Journal of Economic Perspectives, 23*(3), 125–143.
Sarmah, B., Kamboj, S., & Kandampully, J. (2018). Social media and co-creative service innovation: an empirical study. *Online Information Review, 42*(7), 1146–1179.
Schiff, A. (2007). Basic pricing principles in two-sided Markets: Some simple models. *SSRN Electronic Journal*. https://doi.org/10.2139/ssrn.1010553.
Tan, B., Anderson, E. G., & Parker, G. G. (2020). Platform pricing and investment to drive third-party value creation in two-sided networks. *Information Systems Research, 31*(1), 217–239.
Tian, L., & Jiang, B. (2018). Effects of consumer-to-consumer product sharing on distribution channel. *Production and Operations Management, 27*(2), 350–367.
Uratnik, M. (2016). Interactional Service Innovation with Social Media Users. *Management Science, 8*(3), 300–319.
Villeda, M. (2019). Social media recruitment: How to successfully integrate social media into recruitment strategy. *Social media and society, 8*(1), 285–287.
Wang, L., Cai, G. G., Tsay, A. A., & Vakharia, A. J. (2017b). Design of the Reverse Channel for Remanufacturing: Must Profit-Maximization Harm the Environment? *Production and Operations Management, 26*(8), 1585–1603.

Wang, S., Chen, H., & Wu, D. (2017a). Regulating platform competition in two-sided markets under the O2O era. *International Journal of Production Economics, 215,* 131–143.
Zhao, D., & Chen, M. (2019). Ex-ante versus ex-post destination information model for on-demand service ride-sharing platform. *Annals of Operations Research, 279*(1), 301–341.
Zhu, W., & He, Y. (2017). Green product design in supply chains under competition. *European Journal of Operational Research, 258*(1), 165–180.