Optimal selling price, replenishment cycle and payment time among advance, cash, and credit payments from the seller's perspective

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Abstract

Although research on pricing and lot-sizing decisions concerning payment types has been extensive, almost all of it has been done from the buyer's perspective. In this study, we incorporate the following relevant and essential facts. If a seller allows a buyer to pay on credit, it increases sales volume. But if a seller asks a buyer for an advance payment, it decreases sales volume. Sometimes, a seller offers a buyer a price discount for an advance payment to increase sales and profitability. Asking a buyer for an advance payment bears interest earned and has no default risk. If a seller offers a buyer the opportunity to pay on credit, then a longer credit period may mean a higher the sales volume, but the default risk is higher, too. The seller wants to set an optimal selling price, replenishment time, and payment method simultaneously so that the profit per unit time is maximized. To achieve this, we develop and compare the seller's profit per unit time under each of three payment methods—advance, cash and credit. Through numerical analyses, the following managerial insights are highlighted: (1) Under certain conditions, a specific payment type obtains the seller's highest profit among all three payment types. (2) If advance payment is required, then it is critically important for a seller to offer a price discount. (3) An advance payment generates more profit than a credit payment provided that sales volume from a credit payment to an advance payment declines insignificantly and vice versa.

Keywords Pricing · Payment timing · EOQ · Advance payments · Credit payments

1 Introduction

The key target of the retail industry used to be to receive payment immediately after delivering a product, either with a cash-on-delivery or cash payment. However, recent studies indicate that 90% of buyers from the United States prefer a short-term, interest-free credit payment, that is, buy now and pay later. Normally, there are no interest charges if a buyer pays in full within the credit period. However, if the buyer fails to do so, the seller can charge interest on the outstanding balance.

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As opposed to a cash payment, giving buyers a credit payment lowers the buyer's holding cost, and thus increases the seller's sales volume. However, from a seller's perspective, the main problem with credit payment is that the seller may not be able to collect all of the money from some default customers. Furthermore, the seller loses interest incomes during the credit period. To control default risks, interest losses, and cancellations of orders, the seller might prefer to get paid in advance. In that payment method, the seller would demand that the buyer pay for goods and services before the time of delivery.

An advance payment might bring several benefits to a seller. The probability for default risk is zero. There would be no order cancellations. The seller could earn some interest from prepayments. However, if a seller asks a buyer for an advance payment, it increases the buyer's cost, and hence decreases sales volume. On the other hand, from a seller's perspective, offering a credit payment increases not only sales volume but also the default risk and the opportunity cost, meaning interest lost during the credit period.

A seller wants to maximize profit as much as possible, so it is a constant challenge to decide optimal payment type—advance, cash or credit.

Based on economics and marketing theory, if the price of goods or services goes down, then the demand for the product should increase. That makes price one of the most important factors to influence sales and profitability.

When selling goods or services, today's companies can use a variety of pricing strategies to increase sales and profitability. It's up to the seller to determine the optimal selling price. For example, when selling unique goods or services, the seller may use a premium pricing and set its price higher than its competitors do. Another method to attract buyers would be to adopt penetration pricing, putting a lower price on goods and services.

To maximize profitability on new products and services, a company could offer price skimming and set a higher price during the introductory phase and then gradually lower the price when competing goods or services appear on the market. A small company could bundle multiple products, then set a lower price than consumers would pay if they purchased each item individually.

In the inventory literature, almost all models are developed from the buyer's perspective in order to obtain the optimal order quantity under various payment types. Inventory models with various payment terms explored from the seller's perspective to obtain the optimal solution for the seller are rare in the literature.

In addition, the law of demand states that, for nearly all products, the higher the price the lower the demand. However, higher prices can also mean higher profits. Hence, pricing is one of the most important business decisions that management makes.

In this paper, we first study pricing, lot-sizing, and payment-timing decisions in a joint finance, marketing, and operation model. This captures the following relevant facts:

1. Granting a credit payment to buyers, who consider it as a type of price reduction, increases sales volume. Asking buyers for an advance payment can decrease sales volume;
2. Asking buyers for an advance payment can earn interest earned for the seller and has no default risk;
3. Offering buyers, a credit payment can mean that the seller's sales volume will increase, but so will the default risk; and
4. Offering buyers price discounts for an advance payment can increase sales and profitability for the seller.

The goal for the seller is to determine the optimal selling price, replenishment time and payment term simultaneously to maximize the profit per unit time.

The purpose of this study is to answer the following major questions:

- a. Which payment option generates the highest total profit and/or the lowest selling price for the seller, when the demand rate is a multiplicative function of the selling price and the payment time?
- b. Is the total profit significantly improved if the seller offers a price discount when asking for advance payment?
- c. What is the impact of default risk on the seller's optimal selling price and total profit if the seller grants a downstream credit payment?

Hence, the main contribution of the study is to introduce new and important managerial insights to the field of pricing strategies from the seller's perspective under three distinct and well-established payment methods—advance, cash and credit.

The remainder of this paper is organized in the following sections: Sect. 2 reviews the relative literature. Section 3 presents the problem definition and basic assumptions. Section 4 develops a mathematical model for the proposed study. Section 5 finds the optimal solution to the defined problem. Section 6 tests the numerical validity of the model. Section 7 concludes with a discussion of the major findings of the present study.

2 Literature review

There is a vast amount of literature for inventory models with credit payments. The first review of inventory models under trade credits was given by Chang et al. (2008). A second, more recent review was provided by Seifert et al. (2013). In this paper, only those closely related to our proposed model will be mentioned.

For trade credit financing, Hwang and Shinn (1997) investigated the joint determination of price and lot size for an exponentially deteriorating product when the supplier permits delay in payments.

Teng et al. (2005), by assuming that the selling price is necessarily higher than the purchase cost, studied an economic order quantity (EOQ) model to determine the retailer's optimal price and lot size simultaneously when the supplier offers a permissible delay in payments.

Teng et al. (2006) further extended the EOQ model to an economic production quantity (EPQ) model. Ho et al. (2008) developed an integrated supplier-retailer inventory model in which the demand is sensitive to the retail price. Also, the supplier offers a two-part trade credit policy offering cash discount and delayed payment. If the retailer pays off the purchased items within a certain number of days, he will receive a cash discount; otherwise, the full purchasing cost must be paid between a certain number of days to the final payment.

Ouyang et al. (2009) explored an integrated EOQ model to jointly determine the retailer's order quantity and the supplier's production batch in order to maximize the total profit.

Chang et al. (2009) proposed a trade credit linked to the order quantity in which the buyer receives an interest-free credit period if the buyer's order quantity exceeds or equals a predetermined quantity; if it does not, the buyer must pay for the items immediately upon delivery.

Thangam and Uthayakumar (2010) derived optimal pricing and lot-sizing policies for perishable items with two warehouses, when the seller grants buyers a partial trade credit policy. The policy requires that a portion of the purchase amount to be paid on delivery, and the remainder of the purchase amount to be paid in credit.

Jaggi et al. (2014) developed a retailer's optimal ordering and pricing decisions for deteriorating items in a two-warehouse environment, in which shortages are fully backordered.

Chang et al. (2015) investigated optimal pricing and ordering policies for items that don't instantly deteriorate when trade credit is linked to order quantity.

Mahata (2015) proposed a pricing and lot-sizing model for deteriorating and ameliorating items under two levels of trade credit policy. In this model, the retailer receives a full trade credit from the supplier while offering a partial trade credit to customers.

Tsao et al. (2017) explored an inventory model for non-instantaneously deteriorating items for which the selling prices differ in the non-deteriorating and deteriorating periods.

Tiwari et al. (2018) expanded the previous model studied by Mishra et al. (2018) under permissible delay in payments. This allows for partial backlogging and upstream and downstream partial trade credits.

Li and Teng (2018) studied pricing and lot-sizing policies for perishable items when demand depends on selling price, reference price, product freshness, and displayed stocks.

Thereafter, Feng and Chan (2019) derived joint pricing and production decisions for new products, with learning curve effects, under upstream and downstream trade credits.

At the same time, Li et al. (2019a) studied optimal pricing, lot-sizing and backordering decisions when a seller demands an advance-cash-credit payment scheme. The buyer prepays a fraction of the total purchase cost when placing an order, then pays another fraction of the total purchase cost, in cash, upon receiving the order quantity. The buyer then receives a short-term, interest-free credit payment for the remainder of the total purchase cost.

Recently, Taleizadeh et al. (2019) developed an inventory model with probabilistic replenishment intervals and partial backordering under permissible delay in payments.

To obtain the optimal replenishment time and credit period for the seller, Teng and Lou (2012) incorporated the fact that trade credit has a positive impact on demand.

Wang et al. (2014) further generalized the model to allow for deteriorating products that have maximum shelf life.

When the seller offers buyers a credit payment, Chen and Teng (2015) obtained inventory and credit decisions for deteriorating items with upstream and downstream trade credit financing. The longer the credit period, the higher the sales volume, as well as the higher the default risk.

Recently, Li et al. (2019b) explored optimal selling price, credit period, and order quantity simultaneously for perishable goods when demand depends on three things: selling price, expiration date and credit period.

As to advance payments, in order to obtain the ordering quantity and price to maximize the expected profit, Maiti et al. (2009) studied an inventory model with advance payments, stochastic lead-time and price-dependent demand. They used a generalized, reduced-gradient technique and a stochastic search genetic algorithm.

Thangam (2012) established optimal pricing and lot-sizing policies for perishable items. They used an advance-payment scheme in a supply chain in which the retailer receives an upstream trade credit from the supplier, while granting a downstream trade credit to customers.

Taleizadeh (2014) examined the circumstance that a gasoline supplier asks a gas station to prepay a portion of the purchase cost upfront when placing an order and then to pay the remainder in cash on delivery.

Lashgari et al. (2016) built an EOQ model with upstream partial advance payment and downstream partial credit payment with or without shortages.

Teng et al. (2016) explored an economic order quantity model for deteriorating items, with advance payments. In their model, the deterioration rate of a product gradually increased as the expiration date approached.

Li et al. (2017) considered joint pricing and inventory decisions for perishable products under trade credit. They assumed that the demand is influenced by the combined effect of selling price and product freshness when the seller asks the buyer for an advance-cash-credit payment.

Concurrently, Tavakoli and Taleizadeh (2017) examined a buyer's EOQ model for a decaying item with a full prepayment method.

Taleizadeh et al. (2018) further investigate the customer's inventory policy by considering two different scenarios: full prepayment with shortage and partial prepayment–partial delayed payment with shortage.

Li et al. (2018) also proposed an inventory model to determine the jointly optimal ordering quantity and the best payment terms, their choices being advance, cash and credit. This model then determined the optimal payment period to maximize the seller's profit.

Chang et al. (2019) investigated optimal pricing and lot-sizing decisions for perishable products when the supplier demands that the manufacturer use a combination of advance, cash, and credit payments for the total purchase cost.

Currently, Taleizadeh et al. (2020) proposed an EOQ model with mixed sales when the payment scheme is mixed with multiple advance payment and partial credit payment.

All of these papers can be classified into the account of the payment policy; the demand function; the member of the supply chain who is applicable to the model; and the decision variables. This classification is presented in Table 1.

3 Problem description

We use the following notation and assumptions in the model's development. For simplicity, we define the following symbols, then place them in alphabetical order.

C	Purchasing cost per unit, in dollars
d_1	Coefficient of price discount, $d_1 \geq 0$
d_2	Coefficient of default risk on revenue, $d_2 \geq 0$
H	Holding cost per unit per unit time including interest charged, in dollars, $h \geq rP$
L	Payment period, in unit time (a decision variable)
O	Ordering cost per order, in dollars
P	Selling price per unit, in dollars (a decision variable)
R	Interest rate per dollar per unit time
T	Replenishment cycle time, in unit time (a decision variable)
*	Superscript representing an optimal value

The mathematical model with advance, cash, and credit payments is developed under the following assumptions:

Price is an important factor in a consumer's purchasing decision. According to traditional marketing and economic theory, the higher the price, the lower the demand. Hence, demand function is a downward-slope function of the price. By following Robinson and Lakhani (1975), Thompson and Teng (1984), Feng et al. (2017), and Li et al. (2017). The demand rate D is considered as proportional to an exponential function of the price P . Thus, one can have

$$D \approx e^{-\lambda P}, \lambda > 0. \quad (1)$$

Table 1 A brief literature review of related references

References	Payment policy		Demand function	Applicable for	Decision variables
	Advance	Cash			
Chang et al. (2009)	No	No	Price sensitive	Buyers	Lot size and price
Chang et al. (2010)	No	No	Price sensitive	Buyers	Lot size and price
Chang et al. (2015)	No	No	Price sensitive	Buyers	Lot size and price
Chang et al. (2019)	Yes	Yes	Price sensitive	Buyers	Lot size and price
Chen and Teng (2015)	No	No	Credit sensitive	Sellers	Credit period and lot size
Feng and Chan (2019)	No	No	Price sensitive	Buyers	Lot size and price
Ho et al. (2008)	No	No	Price sensitive	Buyers	Lot size and price
Hwang and Shinn (1997)	No	No	Price sensitive	Buyers	Lot size and price
Jaggi et al. (2014)	No	No	Price sensitive	Buyers	Lot size and price
Lashgari et al. (2016)	Yes	No	Price sensitive	Buyers	Lot size and price
Li et al. (2017)	Yes	No	Constant	Buyers	Lot size
Li et al. (2018)	Yes	Yes	Price sensitive	Buyers	Lot size and price
Li et al. (2019a)	Yes	Yes	Payment time sensitive	Sellers	Lot size and payment term
Li et al. (2019b)	No	Yes	Price sensitive	Buyers	Lot size and price
Mahatma (2015)	No	Yes	Price and credit sensitive	Buyers	Credit period, lot size, and Price
Maiti et al. (2009)	Yes	No	Price sensitive	Buyers	Lot size and price
Mishra et al. (2018)	No	No	Price sensitive	Buyers	Lot size and price

Table 1 continued

References	Payment policy		Demand function	Applicable for	Decision variables
	Advance	Cash			
Ouyang et al. (2009)	No	No	Price sensitive	Buyers	Lot size and price
Taleizadeh (2014)	Yes	Yes	Constant	Buyers	Lot size
Taleizadeh et al. (2018)	Yes	No	Constant	Buyers	Lot size
Taleizadeh et al. (2019)	No	No	Stochastic	Buyers	Lot size
Taleizadeh et al. (2020)	Yes	No	Constant	Buyers	Lot size
Tavakoli and Taleizadeh (2017)	Yes	No	Constant	Buyers	Lot size
Teng and Lou (2012)	No	No	Credit sensitive	Sellers	Credit period and lot size
Teng et al. (2005)	No	No	Price sensitive	Buyers	Lot size and price
Teng et al. (2005)	No	No	Price sensitive	Buyers	Lot size and price
Teng et al. (2016)	Yes	No	Constant	Buyers	Lot size
Thangam (2012)	Yes	No	Price sensitive	Buyers	Lot size and price
Thangam and Uthayakumar (2010)	No	Yes	Price sensitive	Buyers	Lot size and price
Tsao et al. (2017)	No	No	Price sensitive	Buyers	Lot size and price
Wang et al. (2014)	No	No	Credit sensitive	Buyers	Credit period and lot size
Wu et al. (2018a)	Yes	Yes	Constant	Buyers	Lot size and
Wu et al. (2018b)	No	No	Time sensitive	Buyers	Lot size
Present paper	Yes	Yes	Payment time and Price sensitive	Sellers	Lot size, payment term, and price

It is observed as in Jaggi et al. (2008) that trade credit offered by the seller to customers has a positive impact on demand. In fact, the longer the trade credit, the higher the demand. Conversely, the earlier the advance payment, the less the demand. As a result, demand rate is an upward-slope function of the payment time L . By following Teng and Lou (2012), and Chen and Teng (2015), the demand rate D is also considered as an exponential function of the payment time L . That is,

$$D \approx e^{\alpha L}, \quad \alpha > 0. \quad (2)$$

Combining (1) and (2), the demand rate D is a multiplicative function of the payment time L and the unit price P can be obtained as:

$$D = D(L, P) = \begin{cases} K e^{\alpha_1 L - \lambda P}, & \text{if } L \leq 0; \\ K e^{\alpha_2 L - \lambda P}, & \text{if } L \geq 0. \end{cases} \quad (3)$$

where K is a positive constant, the payment time $L < 0$ stands for an advance payment with the coefficient $\alpha_1 > 0$, while $L = 0$ is a cash payment on delivery, and $L > 0$ represents a credit payment with the coefficient $\alpha_2 (> d_2)$.

Granting a credit payment to customers, that is, a short-term interest-free loan, stimulates more sales than asking for an advance payment, shown as a short-term, interest-earned prepayment from customers. The longer the prepayment period for an advance payment, the lower the sales volume but the higher the interest earned. To prevent low sales volume, a seller usually offers a price discount (e.g., $(1 + d_1 L)P$) to customers when asking for an advance payment (e.g., $L < 0$). The earlier the prepayment period, the higher the price discount. As a result, the revenue received during the replenishment cycle time T is

$$(1 + d_1 L)PDT = (1 + d_1 L)PK e^{\alpha_1 L - \lambda(1 + d_1 L)P} T = (1 + d_1 L)PTK e^{(\alpha_1 - \lambda d_1 P)L - \lambda P}.$$

For simplicity, we assume that the negative effect of an advance payment on demand α_1 is greater than or equal to the positive effect of price discount on demand $\lambda d_1 P$ (i.e., $\alpha_1 \geq \lambda d_1 P$). Otherwise, the seller would not ask customers for an advance payment.

A 30-year mortgage has a higher default risk than a 15-year mortgage. Likewise, the longer the credit period is, the higher the percentage that the buyer will not be able to pay off the debt. So, the longer the credit period, the higher the default risk. By following Chern et al. (2013, 2014), Wang et al. (2014), and Chen and Teng (2015), we assume that the rate of default risk given credit period L can be taken as follows:

$$R(L) = 1 - e^{-d_2 L}, \quad (4)$$

where d_2 is zero or a positive constant. Notice that the default risk is zero if it is a cash on delivery (i.e., $R(0) = 0$). On the other hand, the default risk is 100% if the payment time $L \rightarrow \infty$ (i.e., $R(\infty) = 1$). We can assume in general that the positive effect of trade credit on demand α_2 is greater than the negative effect of default risk on revenue d_2 (i.e., $\alpha_2 > d_2$). Otherwise, the seller would not provide a credit payment to customers.

Given the above notation and assumptions, the seller's aim is to determine selling price P , payment time L (which corresponds one of three payment terms: advance payment, cash payment, or credit payment), and replenishment cycle time T such that the profit per unit time is maximized.

4 Mathematical model

Given the above notation and assumptions, we can formulate the seller's profit per unit time with respect to advance, cash, and credit payments. The mathematical model is developed for the advance payment first, then for the credit payment and finally for the cash payment.

For the advance payment (i.e., $L < 0$), the seller offers customers a price discount $(1 + d_1)L$ associated with the prepayment period L . The seller's revenue received from sales per cycle time T is $(1 + d_1)L PDT = (1 + d_1)L PK e^{\alpha_1 L - \lambda(1 + d_1)L P} T$, where $\alpha_1 - \lambda d_1 P \geq 0$. Similarly, the purchasing cost per cycle time T is given by $cDT = cK e^{\alpha_1 L - \lambda(1 + d_1)L P} T$. Thus, the inventory holding cost per cycle time T is $\frac{1}{2}hDT^2 = \frac{1}{2}hK e^{\alpha_1 L - \lambda(1 + d_1)L P} T^2$. As the revenue received starts from 0 at time 0 to $(1 + d_1)L PDT = (1 + d_1)L PK e^{\alpha_1 L - \lambda(1 + d_1)L P} T$ at time T , the average revenue is $\frac{1}{2}(1 + d_1)L PK e^{\alpha_1 L - \lambda(1 + d_1)L P} T$. Consequently, the interest earned per cycle T is the average revenue times the interest rate (i.e., r) multiplied by the prepayment period (i.e., $-L$). Therefore, the interest earned per cycle T is obtained as follows:

$$\text{Interest earned} = -\frac{1}{2}rL(1 + d_1)L PK e^{\alpha_1 L - \lambda(1 + d_1)L P} T. \quad (5)$$

In summary, the profit per cycle time T is comprised of the revenue received from sales plus the interest earned minus the total cost including the ordering cost, purchasing cost, holding cost. Hence, the total profit per cycle time T is expressed as follows:

$$\begin{aligned} & (1 + d_1)L PK e^{\alpha_1 L - \lambda(1 + d_1)L P} T - \frac{1}{2}rL(1 + d_1)L PK e^{\alpha_1 L - \lambda(1 + d_1)L P} T \\ & - o - cK e^{\alpha_1 L - \lambda(1 + d_1)L P} T - \frac{1}{2}hK e^{\alpha_1 L - \lambda(1 + d_1)L P} T^2. \end{aligned} \quad (6)$$

Therefore, the profit per unit time for the advance payment is given by $T P_1(L, P, T) = \frac{\text{The total profit per cycle time } T}{\text{The cycle time } T}$

$$\begin{aligned} & = [(1 + d_1)L P - c]K e^{\alpha_1 L - \lambda(1 + d_1)L P} \\ & - \frac{1}{2}rL(1 + d_1)L PK e^{\alpha_1 L - \lambda(1 + d_1)L P} - \frac{o}{T} - \frac{1}{2}hK e^{\alpha_1 L - \lambda(1 + d_1)L P} T \\ & = K e^{\alpha_1 L - \lambda(1 + d_1)L P} \left[(1 - \frac{1}{2}rL)(1 + d_1)L P - c - \frac{1}{2}hT \right] - \frac{o}{T}, \quad L \leq 0. \end{aligned} \quad (7)$$

Next, for the credit payment (i.e., $L \geq 0$), the revenue received per cycle time T after default risk is the revenue from sales multiplied by $1 - R(L)$ as follows:

$$PKT e^{\alpha_2 L - \lambda P} [1 - R(L)] = PKT e^{\alpha_2 L - \lambda P} e^{-d_2 L} = PKT e^{(\alpha_2 - d_2)L - \lambda P}. \quad (8)$$

Note that the positive effect of trade credit on demand α_2 is greater than the negative effect of default risk on revenue d_2 (i.e., $\alpha_2 > d_2$). Otherwise, the seller would not offer a credit payment to customers. On the other hand, the retailer offers customers a credit period of L units of time. Hence, the interest loss per cycle time T is the average revenue (i.e., $\frac{1}{2}PDT = \frac{1}{2}PK e^{\alpha_2 L - \lambda P} T$) times the interest rate (i.e., r) multiplied by the credit period (i.e., L), which is obtained as follows:

$$\text{Interest loss} = \frac{1}{2}rLPK e^{\alpha_2 L - \lambda P} T. \quad (9)$$

Likewise, the total profit after default risk per cycle time T is given as the revenue after default risk minus the total cost, which is consisting of the interest loss, ordering cost,

purchasing cost and holding cost. Thus, the profit after default risk per cycle time T is as follows:

$$PK e^{(\alpha_2 - d_2)L - \lambda P} T - \frac{1}{2} r L P K e^{\alpha_2 L - \lambda P} T - o - c K e^{\alpha_2 L - \lambda P} T - \frac{1}{2} h K e^{\alpha_2 L - \lambda P} T^2. \quad (10)$$

Consequently, the profit per unit time for the credit payment is derived as follows:

$TP_2(L, P, T)$ = The total profit after default risk per cycle time T / The cycle time T

$$\begin{aligned} &= PK e^{(\alpha_2 - d_2)L - \lambda P} - \frac{1}{2} r L P K e^{\alpha_2 L - \lambda P} - \frac{o}{T} - c K e^{\alpha_2 L - \lambda P} - \frac{1}{2} h K e^{\alpha_2 L - \lambda P} T \\ &= K e^{\alpha_2 L - \lambda P} \left[P e^{-d_2 L} - \frac{1}{2} r L P - c - \frac{1}{2} h T \right] - \frac{o}{T}, \quad L \geq 0. \end{aligned} \quad (11)$$

Finally, for the case of cash on delivery, substituting $L = 0$ into (7) or (11), and simplifying terms, we obtain

$$TP_1(0, P, T) = TP_2(0, P, T) = (P - c - \frac{1}{2} h T) K e^{-\lambda P} - \frac{o}{T}. \quad (12)$$

Combining (7), (11), and (12), the total profit per unit time is derived as:

$$TP(L, P, T) = \begin{cases} TP_1(L, P, T), & \text{if } L < 0; \\ TP_1(L, P, T) = TP_2(L, P, T), & \text{if } L = 0; \\ TP_2(L, P, T), & \text{if } L > 0. \end{cases} \quad (13)$$

As a result, we know that $TP(L, P, T)$ is continuous on L, P , and T .

The seller's objective is to obtain the optimal payment time L , selling price P , and replenishment cycle T among one of the three payment terms—advance, cash or credit—such that the profit per unit time $TP(L, P, T)$ is maximized.

In the next section, the optimal solution is characterizing for advance, cash, and credit payments.

5 Optimal solution

In order to find the seller's optimal values for L, P , and T among one of three payment terms, the optimization problem $\Pi = \max_{L, P, T} TP(L, P, T)$ is divided into two sub-problems with respect to payment timing L as follows:

$\Pi_1 = \max_{L, P, T} TP_1(L, P, T)$	$\Pi_2 = \max_{L, P, T} TP_2(L, P, T)$
$L \leq 0$	$L \geq 0$

If $(L_1^*, P_1^*, T_1^*) = \arg \max_{L, P, T} TP_1(L, P, T)$ and $(L_2^*, P_2^*, T_2^*) = \arg \max_{L, P, T} TP_2(L, P, T)$ then the optimal solution of $\Pi = \max_{L, P, T} TP(L, P, T)$ is given by $(L^*, P^*, T^*) = \arg \max \{ \max \{ TP_1(L_1^*, P_1^*, T_1^*), TP_2(L_2^*, P_2^*, T_2^*) \} \}$. Consequently, in order to obtain the optimal solution of $\Pi = \max_{L, P, T} TP(L, P, T)$, two sub-problems $\Pi_1 = \max_{L, P, T} TP_1(L, P, T)$ with $L \leq 0$, and $\Pi_2 = \max_{L, P, T} TP_2(L, P, T)$ with $L \geq 0$, respectively have been solved.

5.1 The optimal solution to Π_1

For any given $L \leq 0$, taking the first-order partial derivatives of $TP_1(L, P, T)$ in (7) with respect to P and T , and simplifying terms, we get

$$\frac{\partial TP_1(L, P, T)}{\partial P} = K(1 + d_1 L)e^{\alpha_1 L - \lambda(1 + d_1 L)P} \left[-\lambda(1 + d_1 L) \left(1 - \frac{1}{2}rL \right) P + \lambda \left(c + \frac{1}{2}hT \right) + 1 - \frac{1}{2}rL \right], \quad (14)$$

and

$$\frac{\partial TP_1(L, P, T)}{\partial T} = \frac{o}{T^2} - \frac{1}{2}hKe^{\alpha_1 L - \lambda(1 + d_1 L)P}. \quad (15)$$

To get the first-order conditions for profit maximization, setting $\frac{\partial TP_1(L, P, T)}{\partial P} = 0$ and $\frac{\partial TP_1(L, P, T)}{\partial T} = 0$, and simplifying terms, the critical point is obtained to maximize $TP_1(L, P, T)$ in (7) as follows:

$$P_1(L) = \frac{\lambda(c + \frac{1}{2}hT) + 1 - \frac{1}{2}rL}{\lambda(1 + d_1 L)(1 - \frac{1}{2}rL)}, \quad (16)$$

and

$$T_1(L) = \sqrt{\frac{2o}{hKe^{\alpha_1 L - \lambda(1 + d_1 L)P}}} = \sqrt{\frac{2o}{hD(L, P)}}. \quad (17)$$

For any given $L \leq 0$, verifying the second-order conditions for a maximum $TP_1(L, P, T)$ in (7), we can demonstrate the optimal values of P and T for profit maximization as shown below.

Theorem 1 For any given $L \leq 0$, if $1 - \frac{1}{2}rL \geq \frac{1}{4}\lambda hT_1(L)$, then the optimal values for P and T to maximize $TP_1(L, P, T)$ in (7) are $P_1(L)$ as shown in (16), and $T_1(L)$ as given in (17), respectively.

Proof See “Appendix 1”.

Generally speaking, the replenishment cycle $T_1(L)$ is less than a year. In addition, both r and λ are relatively small. As a result, we may assume in general that $1 - \frac{1}{2}rL \geq \frac{1}{4}\lambda hT_1(L)$ if the product is not expensive (i.e., the holding cost per unit h is small). By following (16) and (17), we may immediately obtain some analytical properties related to decision variables $P_1(L)$ and $T_1(L)$ as shown in the following Corollary.

Corollary 1 (1) The selling price $P_1(L)$ increases as the purchasing cost c , the coefficient of price discount d_1 , the holding cost h , or the payment timing L increases; while it decreases as the interest rate r or the price coefficient of demand λ increases.
 (2) The replenishment cycle $T_1(L)$ climbs as the ordering cost c , or the selling price $P_1(L)$ increases; but it falls as the holding cost h , or the payment timing L increases.

Proof

$$P_1(L) = \frac{\lambda(c + \frac{1}{2}hT) + 1 - \frac{1}{2}rL}{\lambda(1 + d_1 L)(1 - \frac{1}{2}rL)} = \frac{c + \frac{1}{2}hT}{(1 + d_1 L)(1 - \frac{1}{2}rL)} + \frac{1}{\lambda(1 + d_1 L)}. \quad (18)$$

Hence, $P_1(L)$ is decreasing in both r and λ . This completes the proof. $\square$

On the other hand, for any given $P \geq 0$ and $T \geq 0$, taking the first-order derivatives of $TP_1(L, P, T)$ in (7) with respect to L , and simplifying terms, we obtain

$$\frac{dT P_1(L, P, T)}{dL} = K e^{\alpha_1 L - \lambda(1+d_1)L P} \left\{ (\alpha_1 - \lambda d_1 P) \left[\left(1 - \frac{1}{2} r L\right) (1 + d_1 L) P - c - \frac{1}{2} h T \right] - \frac{1}{2} (1 + d_1 L) r P + \left(1 - \frac{1}{2} r L\right) d_1 P \right\}. \quad (19)$$

For any given $P \geq 0$ and $T \geq 0$, setting $\frac{dT P_1(L, P, T)}{dL} = 0$, and simplifying terms, we obtain the critical point to maximize $TP_1(L, P, T)$ in (7) as follows:

$$\begin{aligned} & \frac{1}{2} (\alpha_1 - \lambda d_1 P) d_1 r P L^2 + \left[(\alpha_1 - \lambda d_1 P) \left(\frac{1}{2} r - d_1 \right) + d_1 r \right] P L \\ & - \left[(\alpha_1 - \lambda d_1 P) \left(P - c - \frac{1}{2} h T \right) + d_1 P - \frac{1}{2} r P \right] \\ & \equiv a_1 L^2 + a_2 L + a_3 = 0, \end{aligned} \quad (20)$$

where

$$a_1 = \frac{1}{2} (\alpha_1 - \lambda d_1 P) d_1 r P \geq 0, \quad (21)$$

$$a_2 = \left[(\alpha_1 - \lambda d_1 P) \left(\frac{1}{2} r - d_1 \right) + d_1 r \right] P, \quad (22)$$

and

$$a_3 = -(\alpha_1 - \lambda d_1 P) \left(P - c - \frac{1}{2} h T \right) - d_1 P + \frac{1}{2} r P < 0. \quad (23)$$

Note that interest rate r in general is significantly less than price discount rate d_1 in today's low inflation environment. Consequently, if $\alpha_1 - \lambda d_1 P = 0$, then (20) is reduced to $d_1 r P L - (d_1 - \frac{r}{2}) P = 0$, which implies there is a unique positive solution $L_1(P, T) = \frac{1}{r} - \frac{d_1}{2}$. Likewise, we can prove that there exists a unique positive solution to (20), if $\alpha_1 - \lambda d_1 P > 0$. Therefore, we conclude in Theorem 2 below that there exists a unique positive solution $L_1(P, T)$ which maximizes $TP_1(L, P, T)$ in (7) for any given $P \geq 0$ and $T \geq 0$.

Theorem 2 For any given $P \geq 0$ and $T \geq 0$, if $(\alpha_1 - \lambda d_1 P) \left[(1 - r L) d_1 - \frac{1}{2} r \right] - d_1 r \leq 0$, then the optimal solution for L to maximize $TP_1(L, P, T)$ in (7) is the unique positive solution to (20).

Proof See “Appendix 2”.

Next, we discuss the case of credit payment.

5.2 The optimal solution to Π_2

Similarly, for any chosen $L \geq 0$, taking the first-order partial derivatives of $TP_2(L, T, P)$ in (11) with respect to P and T , and re-arranging terms, we have

$$\frac{\partial T P_2(L, P, T)}{\partial P} = K e^{\alpha_2 L - \lambda P} \left[(1 - \lambda P) \left(e^{-d_2 L} - \frac{1}{2} r L \right) + \lambda \left(c + \frac{1}{2} h T \right) \right], \quad (24)$$

and

$$\frac{\partial T P_2(L, P, T)}{\partial T} = \frac{o}{T^2} - \frac{1}{2} h K e^{\alpha_2 L - \lambda P}. \quad (25)$$

Likewise, to obtain the first-order conditions for profit maximization, by setting $\frac{\partial T P_2(L, P, T)}{\partial P} = 0$ and $\frac{\partial T P_2(L, P, T)}{\partial T} = 0$, and re-arranging terms, the critical point is obtained to maximize $T P_2(L, T, P)$ in (11) as follows:

$$P_2(L) = \frac{1}{\lambda} + \frac{c + \frac{1}{2}hT}{e^{-d_2L} - \frac{1}{2}rL}, \quad (26)$$

and

$$T_2(L) = \sqrt{\frac{2o}{hK e^{\alpha_2 L - \lambda P}}} = \sqrt{\frac{2o}{hD(L, P)}}. \quad (27)$$

For any chosen $L \geq 0$, verifying the second-order conditions for a maximum $T P_2(L, T, P)$ in (11), we can prove that the optimal values of P and T for profit maximization are given below.

Theorem 3 For any chosen $L \geq 0$, if $\left[\left(e^{-d_2L} - \frac{1}{2}rL \right) \frac{1}{T} - \frac{\lambda h}{4} \right] \geq 0$, then the optimal values for P and T to maximize $T P_2(L, T, P)$ in (11) are $P_2(L)$ as shown in (26), and $T_2(L)$ as given in (27), respectively.

Proof See “Appendix 3”.

Similar to Corollary 1, following from (26) and (27), we immediately get some analytical properties for the decision variables $P_2(L)$ and $T_2(L)$ as given in Corollary 2 below.

Corollary 2 (1) The selling price $P_2(L)$ increases as the purchasing cost c , the coefficient of default risk d_2 , the holding cost h , the payment timing L , or the interest rate r increases; while it decreases as the price coefficient of demand λ increases.
 (2) The replenishment cycle $T_2(L)$ elevates as the ordering cost c , or the selling price $P_1(L)$ climbs; but it falls as the holding cost h , or the payment timing L increases.

Proof It immediately follows from (26) and (27).

For any given $P \geq 0$ and $T \geq 0$, taking the first-order derivatives of $T P_2(L, T, P)$ in (11) with respect to L , and simplifying terms, we derive

$$\frac{dT P_2(L, P, T)}{dL} = K e^{\alpha_2 L - \lambda P} \left[P(\alpha_2 - d_2)e^{-d_2L} - \frac{1}{2}rP(1 + \alpha_2L) - \frac{1}{2}\alpha_2hT - \alpha_2c \right]. \quad (28)$$

By setting $\frac{dT P_2(L, P, T)}{dL} = 0$, and re-arranging terms, the critical point is obtained to maximize $T P_2(L, P, T)$ in (11) as follows:

$$P(\alpha_2 - d_2)e^{-d_2L} - \frac{1}{2}\alpha_2rLP - \left(\frac{1}{2}rP + \frac{1}{2}\alpha_2hT + \alpha_2c \right) = 0. \quad (29)$$

Theorem 4 For any given $P \geq 0$ and $T \geq 0$, if $P(\alpha_2 - d_2) \geq \frac{1}{2}rP + \frac{1}{2}\alpha_2hT + \alpha_2c$, then the optimal solution for L to maximize $T P_2(L, P, T)$ in (11) is the unique positive solution to (29).

Proof See “Appendix 4”.

Table 2 Impacts of α_1 and α_2 on the seller's decisions

α_1 or α_2	L_1^*	P_1^*	T_1^*	TP_1^*	L_2^*	P_2^*	T_2^*	TP_2^*
0.0053	-0.763	33.21	0.76	1259.22	0.000	30.68	0.68	1249.91
0.0153	-0.198	31.28	0.68	1249.92	0.000	30.68	0.68	1249.91
0.1153	0.000	30.68	0.68	1249.91	0.000	30.68	0.68	1249.91
0.2153	0.000	30.68	0.68	1249.91	0.578	31.52	0.65	1253.04
0.3153	0.000	30.68	0.68	1249.91	2.028	35.91	2.03	1412.09

6 Numerical examples and managerial insights

In this coronavirus pandemic, online business transactions have grown exponentially since each party does not need to be physically present in order to make the transaction. To encourage customers to “buy now and pay later”, a store owner for commodity items may grant customers a credit payment (e.g., accepting credit cards). However, these credit payments can present additional risks of fraud and disputed or revoked payments for businesses. To avoid unnecessary risks, a seller for perishable or deteriorating items would prefer an advance payment such as store's gift cards which are prepaid stored-value money cards. Furthermore, the seller for special event tickets or luxury goods could have a strong preference for authenticated or irrevocable cash payments such as debit cards or automated clearing house (ACH) disbursements. Consequently, there are three basic ways for a seller to accept payments online—advance, cash, and credit.

In order to illustrate the results and to highlight the impacts of the financial related parameters to optimal solutions, the following parametric values are used: $c = 10$, $d_1 = 0.1$, $d_2 = 0.125$, $K = 300$, $h = 2$, $o = 30$, $r = 0.02$, and $\lambda = 0.1$.

First of all, the impacts of α_1 and α_2 on the seller's decisions are given in Table 2. For example, if $\alpha_1 = 0.0153$, then $L_1^* = -0.198$, $P_1^* = 31.28$, $T_1^* = 0.682$, and $TP_1^* = TP_1(L_1^*, P_1^*, T_1^*) = 1249.92$. For given $L_1^* = -0.198$, the graphical representation of $TP_1(L_1^* = -0.198, P_1, T_1)$ is given in Fig. 1, which is indeed a concave function in P_1 and T_1 as demonstrated in Theorem 1. Note that the black point in Fig. 1 is the location of the optimal solution. Likewise, for given $P_1^* = 31.28$, and $T_1^* = 0.682$, the graphical representation of $TP_1(L_1, P_1^* = 31.28, T_1^* = 0.682)$ is depicted in Fig. 2 as a strictly concave function in L_1 , which is consistent with the result in Theorem 2.

On the other hand, if $\alpha_2 = 0.2153$, then $L_2^* = 0.578$, $P_2^* = 31.52$, $T_2^* = 0.650$, and $TP_2^* = TP_2(L_2^*, P_2^*, T_2^*) = 1253.04$. For given $L_2^* = 0.578$, the graphical representation of $TP_2(L_2^* = 0.578, P_2, T_2)$ is presented in Fig. 3, which is a concave function in P_2 and T_2 as proven in Theorem 3. Again, the black point in Fig. 3 is the location of the optimal solution. Additionally, for given $P_2^* = 31.52$, and $T_2^* = 0.650$, the graphical representation of $TP_2(L_2, P_2^* = 31.52, T_2^* = 0.65)$ is shown in Fig. 4 clearly as a strictly concave function in L_2 , which is demonstrated in Theorem 4.

Consequently, if $\alpha_1 = 0.0153$ and $\alpha_2 = 0.2153$, then the seller's optimal decision is to offer a credit payment to buyers with a selling price of \$31.52, a downstream credit period of 0.578 years, and a total profit per unit of \$1253.04.

Numerical examples in Table 2 reveal the following important managerial insights:

1. If $\alpha_1 = \alpha_2 \leq 0.0153$, then an advance payment provides the highest profit per unit time for the seller;
2. If $\alpha_1 = \alpha_2 = 0.1153$, then the seller's best decision is a cash payment; and

Fig. 1 Graphical representation of $TP_1(L_1^* = -0.198, P_1, T_1)$

Fig. 2 Graphical representation of $TP_1(L_1, P_1^* = 31.28, T_1^* = 0.682)$

Fig. 3 Graphical representation of $TP_2(L_2^* = 0.578, P_2, T_2)$

3. If $\alpha_1 = \alpha_2 \geq 0.2153$, then the credit payment provides the seller the best profit per unit time among three payments options.

Fig. 4 Graphical representation of $TP_2(L_2, P_2^* = 31.52, T_2^* = 0.65)$

Table 3 The impacts of price discount on the seller's decisions

d_1	L_1^*	P_1^*	T_1^*	TP_1^*	L_2^*	P_2^*	T_2^*	TP_2^*
0.00	0.00	100.00	3.85	166.35	0.58	31.52	0.65	1253.04
0.10	− 0.20	31.28	0.68	1249.92	0.58	31.52	0.65	1253.04
0.15	− 0.20	31.60	0.68	1249.92	0.58	31.52	0.65	1253.04
0.20	− 0.20	31.92	0.68	1249.92	0.58	31.52	0.65	1253.04
0.25	− 0.20	32.26	0.68	1249.92	0.58	31.52	0.65	1253.04

In summary, a high value of α_2 , that is, a high demand rate with a credit payment, not only elevates the selling price but also leads to a credit payment. On the other hand, a low value of α_1 , which is a small demand loss with an advance payment, also elevates the selling price but the best decision is to offer an advance payment.

Second, the impacts of price discount d_1 for an advance payment are investigated on the seller's decisions as shown in Table 3. From Table 3, the following noteworthy managerial insights are obtained:

1. Offering a price discount is essentially necessary for advance payments by comparing the profit of \$166.35 with $d_1 = 0$ and the profit of \$1249.92 with $d_1 = 0.1$; and
2. An increase in price discount rate d_1 causes a small increase in selling price.

Thirdly, the impacts of default risk d_2 for a credit payment are explored on the seller's decisions as obtained in Table 4. The following essentially managerial insights are observed from Table 4 as follows:

1. An increase in default risk reduces selling price, cycle time, payment period, and total profit simultaneously.
2. The credit payment is the best payment optimal for the seller when the default risk is relatively small.

Finally, the impacts of price coefficient of demand λ on the seller's decisions are observed as obtained in Table 5. Table 5 reveals the following managerial results:

1. An advance payment yields more profit for the seller than a credit payment if λ is relatively higher (i.e. the demand is more sensitive to price increase), and vice versa;

Table 4 The impact of default risk on the seller's decisions

d_2	L_1^*	P_1^*	T_1^*	TP_1^*	L_2^*	P_2^*	T_2^*	TP_2^*
0.075	− 0.198	31.28	0.682	1249.92	2.376	35.22	2.376	1385.58
0.100	− 0.198	31.28	0.682	1249.92	1.011	32.32	1.011	1294.87
0.125	− 0.198	31.28	0.682	1249.92	0.578	31.52	0.653	1253.04
0.150	− 0.198	31.28	0.682	1249.92	0.000	30.68	0.681	1249.91

Table 5 The impact of λ on the seller's decisions

λ	L_1^*	P_1^*	T_1^*	TP_1^*	L_2^*	P_2^*	T_2^*	TP_2^*
0.025	0.000	50.59	0.595	3336.87	1.481	54.07	1.481	3466.58
0.050	− 0.198	31.28	0.682	1249.92	0.578	31.52	0.653	1253.04
0.075	− 0.798	26.13	0.798	618.53	0.000	24.11	0.781	617.14
0.100	− 0.927	22.96	0.927	339.58	0.000	20.89	0.899	337.73

2. For credit payment, a higher value of λ drastically reduces selling price, payment period, and total profit; and
3. For advance payment, an increase in λ tremendously increases both payment period and cycle time, while enormously reducing selling price and total profit.

7 Conclusions

Although research on pricing and lot-sizing decisions with respect to a single payment type or a combination of two payment types has been extensive, almost all of it has been done from the buyer's perspective. In this paper, to the best of our knowledge, we have first established a joint operation, marketing, and finance model from the seller's perspective. We have developed the seller's profit under each of the three payment terms: advance, cash and credit.

Moreover, we have obtained explicitly optimal solutions to the problem, and thus theoretically demonstrated the relationship between each parameter and the optimal solution. To obtain managerial insights, we have performed sensitivity analyses to examine the influences of financially related parameters on the seller's decisions and profits. For example, an advance payment generates more profit than a credit payment if the sales volume from a credit payment to an advance payment declines insignificantly. Conversely, if the sales volume from a credit payment to an advance payment falls significantly, then a credit payment creates more profit than an advance payment. Likewise, it is critically important to offer a price discount when the seller asks buyers for an advance payment.

This model can be extended in many directions. As consumers become more health-conscious, the demand for perishable goods, especially organic foods, has increased dramatically in recent years. Additionally, a large pile of seasonal goods such as apples, corns, peaches, etc. in a supermarket often induces consumers to buy more. Hence, the demand could be expanded as a function of selling price, displayed stocks, and time varying as stipulated in Jaggi et al. (2019), Mishra et al. (2017) and Tiwari et al. (2017). Furthermore, in today's supply chain coordination, we may explore the model from a single player's—the

seller's—optimal solution to a two-player—the sellers and the buyers—win-win cooperative Pareto, or integrated solution. Finally, for generality, we could investigate the model by allowing for shortages, partial backordering, and others.

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Appendix 1: Proof of Theorem 1

For any chosen $L \leq 0$, taking the second-order conditions for a maximum $TP_1(L, P, T)$ in (7) at the critical point, we derive the results as follows:

$$\left. \frac{\partial^2 TP_1(L, P, T)}{\partial P^2} \right|_{P=P_1(L)} = -\lambda(1+d_1L)^2 \left(1 - \frac{1}{2}rL\right) K e^{\alpha_1 L - \lambda(1+d_1L)P} \Big|_{P=P_1(L)} < 0, \quad (\text{A1})$$

$$\frac{\partial^2 TP_1(L, P, T)}{\partial T^2} = -\frac{2o}{T^3} < 0, \quad (\text{A2})$$

and

$$\frac{\partial^2 TP_1(L, P, T)}{\partial P \partial T} = \frac{1}{2}h\lambda(1+d_1L)K e^{\alpha_1 L - \lambda(1+d_1L)P} > 0. \quad (\text{A3})$$

For any given $L \leq 0$, to prove the associated Hessian Matrix is strictly negative definite at the critical point, we derive

$$\left[\frac{\partial^2 TP_1(L, P, T)}{\partial P^2} \frac{\partial^2 TP_1(L, P, T)}{\partial T^2} - \left(\frac{\partial^2 TP_1(L, P, T)}{\partial P \partial T} \right)^2 \right] \Big|_{P=P_1(L)} = \lambda(1+d_1L)^2 \left(1 - \frac{1}{2}rL\right) K e^{\alpha_1 L - \lambda(1+d_1L)P} \left(\frac{2o}{T^3} \right) - \left[\frac{1}{2}h\lambda(1+d_1L)K e^{\alpha_1 L - \lambda(1+d_1L)P} \right]^2. \quad (\text{A4})$$

By using (17), and re-arranging terms, we yield

$$\left[\frac{\partial^2 TP_1(L, P, T)}{\partial P^2} \frac{\partial^2 TP_1(L, P, T)}{\partial T^2} - \left(\frac{\partial^2 TP_1(L, P, T)}{\partial P \partial T} \right)^2 \right] \Big|_{P=P_1(L), T=T_1(L)} = \lambda h(1+d_1L)^2 \left[K e^{\alpha_1 L - \lambda(1+d_1L)P} \right]^2 \left[\left(1 - \frac{1}{2}rL\right) \frac{1}{T} - \frac{h\lambda}{4} \right] \geq 0, \quad (\text{A5})$$

if $(1 - \frac{1}{2}rL) \geq \frac{1}{4}\lambda hT$. This completes the proof. $\square$

Appendix 2: Proof of Theorem 2

For any given $P \geq 0$ and $T \geq 0$, taking the second-order condition for a maximum $T P_1(L, P, T)$ in (7) at the critical point, we derive the result as follows:

$$\begin{aligned} & \left. \frac{d^2 T P_1(L, P, T)}{dL^2} \right|_{L=L_1(P, T)} \\ &= K e^{\alpha_1 L - \lambda(1+d_1 L)P} P \left\{ (\alpha_1 - \lambda d_1 P) \left[(1-rL)d_1 - \frac{1}{2}r \right] - d_1 r \right\} \leq 0. \end{aligned} \quad (B1)$$

In addition, if $\alpha_1 - \lambda d_1 P > 0$, then $a_1 > 0$ and $a_3 < 0$. As a result, we can conclude that there exists a unique positive solution $L_1(P, T)$ to (20) such that $T P_1(L, P, T)$ in (7) is maximized in a real-world problem. This completes the proof. $\square$

Appendix 3: Proof of Theorem 3

For any chosen $L \geq 0$, taking the second-order conditions for a maximum $T P_2(L, T, P)$ in (11) at the critical point, we derive the results as follows:

$$\left. \frac{\partial^2 T P_2(L, P, T)}{\partial P^2} \right|_{P=P_2(L)} = -\lambda K e^{\alpha_2 L - \lambda P} \left(e^{-d_2 L} - \frac{1}{2}rL \right) \leq 0, \quad (C1)$$

$$\frac{\partial^2 T P_2(L, P, T)}{\partial T^2} = -\frac{2o}{T^3} < 0, \quad (C2)$$

and

$$\frac{\partial^2 T P_2(L, P, T)}{\partial P \partial T} = \frac{1}{2} \lambda h K e^{\alpha_2 L - \lambda P} > 0. \quad (C3)$$

Again, to verify the associated Hessian Matrix is strictly negative definite at the critical point, we get

$$\begin{aligned} & \left[\frac{\partial^2 T P_2}{\partial P^2} \frac{\partial^2 T P_2}{\partial T^2} - \left(\frac{\partial^2 T P_2}{\partial T \partial P} \right)^2 \right] \Big|_{P=P_2(L)} = \lambda K e^{\alpha_2 L - \lambda P} \left(e^{-d_2 L} - \frac{1}{2}rL \right) \left(\frac{2o}{T^3} \right) \\ & \quad - \left(\frac{1}{2} \lambda h K e^{\alpha_2 L - \lambda P} \right)^2 \end{aligned} \quad ((C4))$$

By applying (27), and re-arranging terms, we yield

$$\begin{aligned} & \left[\frac{\partial^2 T P_2}{\partial P^2} \frac{\partial^2 T P_2}{\partial T^2} - \left(\frac{\partial^2 T P_2}{\partial T \partial P} \right)^2 \right] \Big|_{P=P_2(L), T=T_2(L)} \\ &= (K e^{\alpha_2 L - \lambda P})^2 \lambda h \left[\left(e^{-d_2 L} - \frac{1}{2}rL \right) \frac{1}{T} - \frac{\lambda h}{4} \right] \geq 0, \end{aligned} \quad (C5)$$

if $\left[\left(e^{-d_2 L} - \frac{1}{2}rL \right) \frac{1}{T} - \frac{\lambda h}{4} \right] \geq 0$. This completes the proof. $\square$

Appendix 4: Proof of Theorem 4

For any given $P \geq 0$ and $T \geq 0$, taking the second-order condition for a maximum $T P_2(L, P, T)$ in (11) at the critical point, we get the following result.

$$\left. \frac{d^2 T P_2(L, P, T)}{dL^2} \right|_{L=L_1(P, T)} = -K e^{\alpha_2 L - \lambda P} P \left[d_2(\alpha_2 - d_2) e^{-d_2 L} + \frac{1}{2} \alpha_2 r \right] < 0. \quad (D1)$$

To prove that there exists a unique solution to (29), let us set

$$F(L) = P(\alpha_2 - d_2) e^{-d_2 L} - \frac{1}{2} \alpha_2 r L P - \left(\frac{1}{2} r P + \frac{1}{2} \alpha_2 h T + \alpha_2 c \right). \quad (D2)$$

It is clear that $F(L)$ is continuous in L , $F(0) = P(\alpha_2 - d_2) - \left(\frac{1}{2} r P + \frac{1}{2} \alpha_2 h T + \alpha_2 c \right) > 0$, and $F(\infty) = -\infty$. Applying the Mean Value Theorem and (D1), we know that there exists a unique positive solution $L_2(P, T)$ to (29) such that $T P_2(L, P, T)$ in (11) is maximized. This completes the proof. $\square$

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