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Editorial

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In optimization, the decision maker is frequently not contented with the relatively long solution times resulting from state-of-the-art solvers in linear or mixed-integer linear models. On one hand, input and decision variables may grow fast in order to capture the nature of the underlying problem in its full scale; on the other hand, the time that the decision maker or the beneficiary is willing to wait for an optimal or even a good enough solution to the problem may be extremely limited. Frequently, there are cases in which an out-of-the-box solver is unable to provide a solution to the original problem in its large-scale standard form. Regardless of the cause, and even though optimality may often be a relatively mild requirement, the solution needs to be characterized and accompanied by its distance from the optimum. Exact methods need to be employed to that end.

Decomposition techniques have shown that they can satisfactorily address the above concerns in considerably shorter computation times. In this context, Benders decomposition has attracted the significant interest of many researchers throughout the years and has been

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a popular approach in tackling various types of optimization problems including, most recently, stochastic and bilevel optimization. This special volume is dedicated to the theory and applications of the Benders decomposition method and aims to showcase the benefits acquired when the method is correctly fine-tuned to match the peculiarities and specificities of each problem. In a Benders decomposition framework, a great number of decision parameters can dramatically influence the performance of the algorithms. For instance, the separation of the original problem into *master* and *sub-problem* is not a de facto choice and may be decided in several different ways. The reader will see that the way the authors define these two distinct components often reflects the inherent nature of the problem itself. The settings and communication protocols between the two may also be configured: the incumbent solution that will be communicated by the master to the sub-problem as well as the number or type of cuts that the sub-problem will contribute to the master, both constitute structural decisions in the solution strategy. A warm start given to the master problem may also boost solution time and can be beneficial depending on the nature of the problem addressed.

The guest editors wish to thank the authors of this special volume for having contributed to clarifying some of the above as well as additional important points in their work; we are sure that their individual contributions will provide the readers with further insight into the power and efficiency of Benders decomposition. A brief description of this special volume's contributions follows.

Shim et al. present a branch-and-bound algorithm for discretely constrained mathematical programs with equilibrium constraints. The authors present a dynamic partition scheme to overcome the non-convexity of the Benders sub-problem which ensures convergence to the global optimum. Naoum-Sawaya and Elhedhli present an interior-point branch-and-cut algorithm for structured integer programs based on Benders decomposition and the analytic center cutting plane method (ACCPM). They show that the ACCPM-based Benders cuts are both pareto-optimal and global, making them valid for any node of the branch-and-bound tree. The global cuts are added to a pool of cuts that is used to warm-start the solution of the nodes after branching. The algorithm is tested on two classes of problems: the capacitated facility location problem and the multi-commodity capacitated fixed charge network design problem. Sherali and Lunday also explore certain algorithmic strategies for accelerating the convergence of the Benders decomposition method via the generation of maximal non-dominated cuts. The authors propose an algorithmic strategy that utilizes a preemptively small perturbation of the right-hand side of the Benders sub-problem to generate maximal non-dominated Benders cuts, as well as a complementary strategy that generates an additional cut at each iteration. Lazimy proposes an interactive polyhedral outer approximation method to solve a broad class of multi-objective optimization problems which may include nonlinear and non-differentiable objective and constraint functions, and continuous or discrete decision variables.

The classical implementation of Benders decomposition in some cases results in low density Benders cuts. Covering Cut Bundle (CCB) generation addresses this issue in a novel way, generating a bundle of cuts that could cover more decision variables of the Benders master problem than the classical Benders cut. The motivation to improve further CCB generation led to three cut generation strategies (Saharidis and Ierapetritou, Nader et al., and Tang et al.). These strategies are referred to as the Maximum Density Cut (MDC) generation strategies. MDC are based on the observation that in some cases, when applying CCB generation, it is computationally expensive to cover the maximum number of master decision variables. MDC strategies address this issue by generating the cut that involves the rest of the decision variables of the master problem that are not covered in the Benders cut and/or in the CCB. An extension of MDC strategies was presented by Tang et al. who investigate a logistics facility location problem to determine whether the existing facilities remain open or not, what the expansion size of the open facilities should be, and which potential facilities should be selected. The authors propose three groups of valid inequalities and a high density Pareto cut generation method to accelerate the convergence by lifting Pareto-optimal cuts. MDC strategies and their extensions can be applied as a complementary step to the CCB generation or as a standalone strategy. Finally, You and Grossmann present a multicut version of the Benders decomposition method for solving two-stage stochastic linear programming problems, including stochastic mixed integer programs with only continuous recourse variables. The main idea is to add one cut per realization of the uncertain variable (scenario) to the master problem at each iteration.

Sherali et al. propose a model that integrates the schedule design and fleet assignment processes while considering flexible flight times, schedule balance, and recapture issues, along with optional legs, path/itinerary-based demands, and multiple fare-classes. A polyhedral analysis is conducted to generate several classes of valid inequalities, which are used along with suitable separation routines to tighten the model representation. Solution approaches are designed by applying the Benders decomposition method to the resulting tightened model. Coban and Hooker combine mixed integer programming and constraint programming to solve planning and scheduling problems applying logic-based Benders decomposition. The authors find that a similar technique can be beneficial in solving pure scheduling problems as the problem size scales up. They solve single-facility non-preemptive scheduling problems with time windows and long time horizons. The Benders master problem assigns jobs to predefined segments of the time horizon, and the sub-problem schedules them.

The optimal engineering design problem consists of minimizing the expected total cost of an infrastructure or equipment, including construction and expected repair costs, the latter depending on the failure probabilities of each failure mode. Minguez et al. formulate the optimal engineering design problem as a bilevel optimization problem. The structure of this bilevel problem is advantageously exploited using Benders decomposition. Fakhri and Ghatee deal with preemptive priority-based multi-objective network design problems in which both construction times and travel costs are considered. Cost and time objective functions are ordered lexicographically with respect to manager's strategies in order to decrease total cost and total construction time of the network. The first priority objective function includes both continuous and binary variables, whilst the rest include only binary variables. The problem is solved using a modified Benders decomposition algorithm.

Castillo et al. show how Benders decomposition can be used for estimating the parameters of a fatigue model. The objective function of such a model depends on five different parameters. This makes the parameter estimation problem of the fatigue model suitable for the Benders decomposition, which allows the authors to use well-behaved and robust parameter estimation methods for the different sub-problems. Shen and Smith consider an optimization problem that integrates network design and broadcast domination decisions. The overall problem minimizes the sum of edge construction costs and broadcast domination costs. They then propose a decomposition strategy, which iteratively adds valid inequalities based on optimal broadcast domination solutions corresponding to the first-stage network design solutions. Van Dinter et al. present a unit commitment model which determines generator schedules, associated production and storage quantities, and spinning reserve requirements. Benders decomposition is used to solve the developed model. The model minimizes fixed costs, fuel costs, shortage costs, and emissions costs. A constraint set balances the load, imposes requirements on the way in which generators and storage devices operate, and tracks reserve requirements.

Finally, Benders decomposition is a well-known approach to solve stochastic programming problems. Zheng et al. discuss a stochastic unit commitment model that takes into account various uncertainties affecting thermal energy demand and two types of power generators, i.e., quick-start and non-quick-start generators. This problem corresponds to a stochastic mixed integer program with discrete decision variables in both first and second stages. In order to solve this difficult problem, a method based on Benders decomposition is applied. An additional paper by Kuznia et al. presents a stochastic mixed integer programming model for a comprehensive hybrid power system design problem, including renewable energy generation, storage device, transmission network, and thermal generators for remote areas. Given the complexity of the model, the authors developed a Benders decomposition algorithm with two additional types of cutting planes.

The guest editors would like to extend special thanks to the reviewers of the papers in this special volume. Their dedication and attention to detail has substantially contributed to improving these already high-quality papers. We are most thankful to Prof. Endre Boros, Editor-in-Chief of the *Annals of Operations Research* for giving us the opportunity to arrange this special volume, and to the springer staff for their support. We hope that this special volume has shed light on the advantages and benefits of Benders decomposition.

Guest Editors