ERRATUM TO: ATOMIC DECOMPOSITION OF HARDY SPACES AND CHARACTERIZATION OF BMO VIA BANACH FUNCTION SPACES

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(Received August 27, 2018; accepted February 2, 2019)

Erratum to: *Analysis Mathematica*, **38** (2012), 173–185.
 DOI: 10.1007/s10476-012-0302-5

The main purpose of this short note is to correct a technical error appeared in the proof of [2, Theorem 2.3]. At the end of the proof we use the assumption that for any $f \in \text{BMO}$ and ball B , $(f - f_B)\chi_B \in \text{BMO}$ where χ_B is the characteristic function of B and $f_B = \frac{1}{|B|} \int_B f(x) dx$. This assumption is valid when $f \in L^\infty$ but not necessary true for any $f \in \text{BMO}$.

Even though we have this technical error, the results in [2, Theorem 2.3] are valid. We now present the correction of the proof for it. For brevity, we use the notations given in [2].

In view of the vanishing moment condition satisfied by A_B , we find that

$$\begin{aligned} \int_{\mathbb{R}^n} A_B(x) f(x) dx &= \int_{\mathbb{R}^n} A_B(x) (f(x) - f_B) dx \\ &= \int_B g(x) (f(x) - f_B) dx + \int_{\tilde{B}} \left(-\frac{1}{|B|} \int_B g(y) dy \right) (f(x) - f_B) dx. \end{aligned}$$

By using the Hölder inequality and the fact that $|B| = |\tilde{B}|$, we have

$$\begin{aligned} (1) \quad & \left| \int_B g(x) (f(x) - f_B) dx \right| \\ & \leq \left| \int_{\mathbb{R}^n} A_B(x) f(x) dx \right| + \frac{1}{|B|} \int_B |g(y)| dy \left| \int_{\tilde{B}} (f(x) - f_B) dx \right| \\ & \leq \|A_B\|_{H^1} \|f\|_{\text{BMO}} + \|g\|_{X'} \|\chi_B\|_X |f_{\tilde{B}} - f_B| \\ & \leq C \|\chi_B\|_X \|f\|_{\text{BMO}} + \|\chi_B\|_X |f_{\tilde{B}} - f_B| \end{aligned}$$

because $A_B \in H^1$ with $\|A_B\|_{H^1} \leq C \|\chi_B\|_X$, $\|g\|_{X'} \leq 1$ and $f \in \text{BMO}$.

Notice that there exists a constant $C > 0$ such that for any $B \in \mathbb{B}$, there exists a ball B^* satisfying $B \cup \tilde{B} \subset B^*$ with $|B^*| \leq C|B|$.

We find that

$$\begin{aligned}
 (2) \quad & |f_{\tilde{B}} - f_B| \leq |f_{\tilde{B}} - f_{B^*}| + |f_B - f_{B^*}| \\
 & \leq \frac{1}{|B|} \left(\int_{\tilde{B}} |f(x) - f_{B^*}| dx + \int_B |f(x) - f_{B^*}| dx \right) \\
 & \leq \frac{1}{|B|} \int_{B^*} |f(x) - f_{B^*}| dx \leq C \|f\|_{\text{BMO}}
 \end{aligned}$$

because $|B| = |\tilde{B}|$ and $|B^*| \leq C|B|$.

Therefore, (1) and (2) give

$$\left| \int_B g(x)(f(x) - f_B) dx \right| \leq C \|\chi_B\|_X \|f\|_{\text{BMO}}.$$

Consequently,

$$\begin{aligned}
 \frac{\|(f - f_B)\chi_B\|_X}{\|\chi_B\|_X} & \leq \frac{2}{\|\chi_B\|_X} \left| \int_B g(x)(f(x) - f_B) dx \right| \\
 & \leq C \frac{2}{\|\chi_B\|_X} \|\chi_B\|_X \|f\|_{\text{BMO}} \leq C \|f\|_{\text{BMO}}
 \end{aligned}$$

which shows the embedding $\text{BMO} \rightarrow \text{BMO}_X$.

The above correction also applies to the proof of [1, Theorem 8].

Finally, there is another proof for the [1, Theorem 8] and [2, Theorem 2.3] by using extrapolation theory. By using extrapolation theory, we also have the vector-valued mean oscillations characterization of BMO. We refer the reader to [3] for details.

References

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