Correction to: Involutions of Type G_2 over a Field of Characteristic Two

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We would like to correct two errors in our paper *Involutions of type G_2 over a field of characteristic 2* [2]. The errors relate to the discussions and results of involutions of type I in Sections 3 and 4, and specifically to Lemma 3.3, Lemma 4.8, Proposition 4.10, Theorem 4.11 and Lemma 4.12. The fixed subalgebra C^t of the octonion algebra C by an involution of type I was miscalculated, which led to the mistakes in the listed stated results.

The subalgebra C^t fixed by an involution $t \in \text{Aut}(C)$ of type I is a six dimensional maximal subalgebra referred to by Racine in [3] and Gagola in [1].

Our involution is of the form

$$t(x + yw) = x + ryw. \quad (1.1)$$

If we choose

$$r = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}, \quad (1.2)$$

as in the original paper, the fixed subalgebra C^t has the form

$$\left(\begin{bmatrix} x_0 & x_1 \\ x_2 & x_3 \end{bmatrix}, \begin{bmatrix} y_0 & y_1 \\ 0 & 0 \end{bmatrix} \right),$$

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where $x_i, y_j \in k$ for $0 \leq i \leq 3$ and $0 \leq j \leq 1$. Originally, we stated that the fixed subalgebra was a quaternion algebra D , having elements of the form

$$\left(\begin{bmatrix} x_0 & x_1 \\ x_2 & x_3 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \right).$$

We computed the fixed point group as the subgroup $\text{Aut}(C)^{\mathcal{I}_t}$ of $\text{Aut}(C)$ when t is type I by looking at the subgroup of $\text{Aut}(C)$ that leaves a fixed quaternion subalgebra D invariant. If $g \in \text{Aut}(C)$ and $g(D) = D$, then it can be written in the form

$$g(x + yw) = cxc^{-1} + (pcyc^{-1})w,$$

where $q(c) \neq 0$ and $q(p) = 1$. From this, by using the restraint $pcr = rpc$, we found such a group to be isomorphic to $G_+(k) \times \text{SL}_2(k)$. This is not the correct fixed point group of $\mathcal{I}_t$, but rather the subgroup of $\text{Aut}(C)^{\mathcal{I}_t}$ that leaves D invariant.

The following proposition characterizes the true relationship between an involution t of type I, its fixed subalgebra C^t , and the fixed point group of t in $\text{Aut}(C)$.

Proposition 1.1 *If $t \in \text{Aut}(C)$ such that $t^2 = \text{id}$ then C^t is either a six dimensional subalgebra of a split octonion algebra C that contains a split quaternion subalgebra or C^t is a four dimensional totally singular subalgebra.*

If an involution $t \in \text{Aut}(C)$ fixes a four dimensional totally singular subalgebra we call it an involution of type II. This case is discussed in Section 5 of [2].

Theorem 4.11 in our original paper [2] falsely claims that $\text{Aut}(C)^{\mathcal{I}_t} \cong G_+(k) \times \text{SL}_2(k)$. In Section 3 of [4] Thomas computes the centralizer of such an involution showing

$$\text{Aut}(C)^{\mathcal{I}_t} \cong S \cdot \text{SL}_2(k),$$

where S is a product of root groups. The group S takes the form

$$S = S_\beta S_{\alpha+\beta} S_{2\alpha+\beta} S_{3\alpha+\beta} S_{3\alpha+2\beta},$$

with α and β being simple roots of $\text{Aut}(C)$.

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Nathaniel Schwartz is now unaffiliated. His affiliation and email address is updated accordingly in this correction article.

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