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ERRATUM

Erratum to the Article "On the Left and Right Brylinski-Kostant Filtrations"

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In Section 3.2 of our article [2], the choice of Chevalley basis elements $x_\alpha : \alpha \in \Delta$ together with the corresponding structure constants of $\mathfrak{g}$ is not compatible with the conventions in [3]. We would like to thank Bill Casselman to pointing out to us this issue.

Instead, we refer to [1]. We obtain slightly different formulas but the only significant change is in 3.3, Lemma 3.4 and Proposition 3.5 in which simple coroots must be replaced by simple roots. This is more natural and in particular does not wrongly mislead one into thinking that the Langlands dual is involved.

In the setting of 3.3, choose Chevalley basis elements $x_\alpha : \alpha \in \Delta$. Let $\alpha, \gamma \in \Delta^+$ such that $\alpha + \gamma \in \Delta^+$. Then $\alpha + \gamma + (-\alpha - \gamma) = 0$, and hence

$$\frac{N_{\alpha,-(\alpha+\gamma)}}{(\gamma,\gamma)} = \frac{N_{\gamma,\alpha}}{(\alpha+\gamma,\alpha+\gamma)}$$

by [1, 4.1(ii)]. Since $N_{\gamma,\alpha} = -N_{\alpha,\gamma}$, we conclude that

$$N_{\alpha,-(\alpha+\gamma)} = -\frac{(\gamma,\gamma)}{(\alpha+\gamma,\alpha+\gamma)} N_{\alpha,\gamma}.$$

Thus x_α acts on $S(\mathfrak{b}^-)/I_\eta$ as the differential operator

$$\eta(\alpha^\vee)\partial/\partial x_{-\alpha} - \sum_{\gamma\in\Delta^+ \,|\, \alpha+\gamma\in\Delta^+} \frac{(\gamma,\gamma)}{(\alpha+\gamma,\alpha+\gamma)} N_{\alpha,\gamma} x_{-\gamma}\partial/\partial x_{-(\alpha+\gamma)}.$$

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Consequently, in the proof of Lemma 3.3 we obtain that

$$x_{\alpha_i} \cdot a_{-\gamma} = \Big(n_i \eta(\alpha_i^\vee) - 2 \sum_{j | \alpha_i + \alpha_j \in \Delta^+} c_{i,j} \frac{(\alpha_j, \alpha_j)}{(\alpha_i + \alpha_j, \alpha_i + \alpha_j)} N_{\alpha_i, \alpha_j}\Big) m_i \qquad (1)$$
$$+\text{other terms}$$

for all i. The converse of Lemma 3.3(ii) implies that for all i the coefficient of m_i is zero in (1). By multiplying (1) with (α_i, α_i) and summing up over all i we conclude that $\sum_i (\alpha_i, \alpha_i) n_i \eta(\alpha_i^\vee) = 0$, which is a contradiction to $\eta \in \mathfrak{h}^*_{d\,reg}$ and $\gamma \neq 0$.

In Section 3.4 we identify first $\delta M(0)|_{U(\mathfrak{b})}$ with $S(\mathfrak{b}^-)/I_\eta$ with the left linear action of $\mathfrak{b}$, where $\eta \in \mathfrak{h}^*_{d\,reg}$. The right action of $x_\alpha : \alpha \in \pi$ on $S(\mathfrak{b}^-)/I_\eta$ is a derivation. Using the commutativity of the left and right actions of $\mathfrak{n}$ we obtain that this derivation takes the form

$$\eta(\alpha^\vee)\partial/\partial x_{-\alpha} + \frac{(\alpha, \alpha)}{(\alpha + \alpha', \alpha + \alpha')} N_{\alpha, \alpha'} \frac{\eta(\alpha^\vee)}{\eta(\alpha'^\vee)} x_{-\alpha'} \partial/\partial x_{-(\alpha + \alpha')}$$

up to terms in $S(\mathfrak{n}^-)\partial/\partial x_{-\gamma} : o(\gamma) \geq 2$. Therefore Lemma 3.4 has to be modified:

Lemma 1 *One has $x_{-\gamma} \cdot e^2 = 0$, $\gamma \in \Delta_2^+$, if and only if η takes a fixed positive value on the simple roots.*

Indeed, if $\gamma = x_\alpha + x_{\alpha'}$ with $\alpha, \alpha' \in \pi$, then

$$x_{-\gamma} \cdot e^2 = \frac{N_{\alpha, \alpha'}}{(\gamma, \gamma)} \big((\alpha, \alpha)\eta(\alpha^\vee) - (\alpha', \alpha')\eta(\alpha'^\vee)\big),$$

which implies the claim.

Also, Proposition 3.5 has to be adapted:

Proposition 1 *Assume $\mathfrak{g}$ simple. The filtrations $\mathcal{F}_\eta$, $\mathcal{F}'_e$ coincide on $\delta M(0)$, if and only if $\eta \in \mathfrak{h}^*_{d\,reg}$ takes a fixed positive value on the simple roots.*

The other parts of the paper are not affected by this corrigendum.

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