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A three-way decision method on multi-scale single-valued neutrosophic decision systems

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Abstract

In this paper, we propose a three-way decision (TWD) method on multi-scale single-valued neutrosophic decision systems (MS-SVNDSs). First, to explore the application of single-valued neutrosophic sets (SVNSs) in multi-scale environment, we establish a rough set model of MS-SVNDSs. Then, aiming at the problem of knowledge acquisition in MS-SVNDSs, we present the corresponding optimal scale selection and reduction methods by using evidence theory, a more direct and simpler algorithm is also discussed. For obtaining decision results that are more in line with human cognition, we further provide a novel three-way decision method. Comparative experiments are subsequently conducted to demonstrate the effectiveness of our approach. The experimental results show that our method not only improves the classification accuracy but also raises decision efficiency.

Keywords Multi-scale · Optimal scales and reducts · Rough sets · Single-valued neutrosophic sets · Three-way decision

1 Introduction

Yao (2009, 2010, 2011) introduced the notion of TWD in 2010s, which is focused on grouping all objects into three distinguishable and relevant sections and then design an appropriate strategy for each section, i.e. non-commitment, rejection, and acceptance. Different from the usual decision-making mode, TWD adds an uncertain part to better deal with the situation that it is hard to make an exact decision of acceptance or rejection directly with insufficient or incomplete information. As it is more in consistent with the characteristics of human mind, three-way decision has been attracted research interest of many researchers.

Up to now, there have been a lot of theoretical and applied achievements on TWD. Some scholars extended the traditional TWD on the basis of different situations. For example,

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Qian et al. (Qian et al. 2022) discussed sequential TWD problem in multi-granularity environment to obtain reliable decisions when there are several different granularity levels. Li et al. (2017a) established a TWD model with generalizations on the basis of subset evaluation. Yao (2018) proposed a model for three-way granular computing. A generalized multi-granularity sequential TWD model with multiple thresholds was developed by Qian et al. (2019). In order to realize the sorting and classification of intuitionistic fuzzy objects, Dai et al. (2023) constructed a TWD model with intuitionistic fuzzy concept consideration of the decision-maker's preference. There has also been significant concern about the TWD procedure improvement. Chen et al. (2012) gave a method to find the threshold of TWD without setting the initial value. According to decision-theoretic rough sets with interval values, Liang and Liu (2014) put forward an optimum method for TWD to go a step further reduce the entire uncertainty. A method of aggregation for combining various TWD spaces was put forth by Hu et al. (2016). A sequential strategy was put up by Xu et al. (2020a) to address the issue of decision conflict by gradually introducing more precise knowledge. Based on criteria importance, Zhu et al. (2021) constructed a novel risk assessment framework to alter traditional failure mode and effect analysis method and aid maintenance supervision. The distinctive preponderance of TWD makes it popular in medical diagnosis (Chu et al. 2020; Yao and Azam 2015), decision support (Liang and Liu 2015; Zhang et al. 2020a), recommender system design (Zhang and Min 2016), classification (Xu et al. 2020b; Yue et al. 2020), uncertainty analysis (Zhang et al. 2020b, 2020c, 2020d), and so forth.

As Smarandache (1999) proposed neutrosophic sets (NSs) in 1999, Dai et al. (2011) developed the idea of SVNSs, which provided a better approach for dealing with uncertain and inaccurate information. Since then, multitudinous scholars have begun to launch intensive study on the theory and application of SVNSs. Özlü (2023a, b) and Özlü and Karaaslan (2022) developed single-valued neutrosophic type-2 hesitant fuzzy sets and gave some basic dice measures. Using the evaluation function, Mohamed et al. (2018) successfully combined TWD with SVNSs, and gave an AHP-QFD framework for resource selection. Ye (2015) optimized the cosine similarity measures of simplified NSs to realize the comparison among simplified NSs. On the basis of NSs, the representation of lattice combining TWD and fuzzy concept was discussed by Singh (2017). Furthermore, Singh (2018) proposed a lattice combining multi-granularity, TWD and n-valued neutrosophic concept. By summarizing various existing sorting methods, Huang et al. (2022) provided an approach for sorting the numbers in SVNSs (SVNNs).

In real-world data processing tasks, participants prefer to data with a hierarchical structure so as to obtain the knowledge of data in different hierarchies, then choose the data in a certain hierarchy that best suits their actual needs to proceed with processing. To address that above problem, Wu and Leung (2011, 2013) introduced the idea of multi-scale, that is, every object may have several values for a single attribute. Aiming at the issue of optimal scale selection (OPS) and reduction (OPR) in multi-scale decision systems (MSDSs), numerous scholars have launched discussions. In order to better identify and process the hyperspectral image, Dao et al. (2021) propose a novel OPS methodology. Li et al. (2017b) initially put forward the concept of the importance of multi-scale attributes and provided a simpler step-by-step OPS method. Wu et al. (2018a, b) considered the problem of global and local granularity selection in generalized decision systems with incomplete multi-granular labels and inconsistent generalized MSDSs respectively. Zhang et al. (2022) provided an updated way for OPS by using TWD and Hasse diagram.

Taking into account the illustrations above, the motives for this study are summarized below:

(1) The existing researches on MSDSs are almost carried out under symbolic data environment, fuzzy data environment, intuitionistic fuzzy data environment and so on. Whereas there are few studies on MS-SVNDSs. For the sake of better solving problems with indeterminacy in real world, we establish the rough set model of MS-SVNDS.
(2) According to the known research, the introduction of Pawlak rough sets, fuzzy rough sets, intuitionistic fuzzy rough sets and so on into MSDSs can effectively simplify data and improve the efficiency of rule extraction, so can the combination of single-valued neutrosophic rough sets and MSDSs perform better? For this problem, we discuss how to obtain optimal scales and optimal scale reducts using multi-scale single-valued neutrosophic rough sets and a pair of functions in evidence theory (DS).
(3) The majority of previous studies focused on obtaining an optimal scale reduct of MSDSs and then make classification directly. While a good deal of judgments cannot be made immediately in reality, so it is necessary to introduce deferred decisions. Therefore, a novel TWD method on MS-SVNDSs is provided in this paper.

Comparison between this paper and our previous studies and the formation of this paper are displayed in Fig. 1. The organization of this paper is as follows. In the next section, we review the notions of TWD, SVNSs, MSDSs and DS. In Sect. 3, we construct a rough set model of MS-SVNDS and present the corresponding OPS and OPR methods. In Sect. 4, we give a novel TWD method. The experimental analyses of our method are carried out in Sect. 5. This paper's work is concluded in Sect. 6.

2 Preliminaries

We review the notions of TWD, SVNSs, MSDSs and DS in this section to facilitate the further analysis.

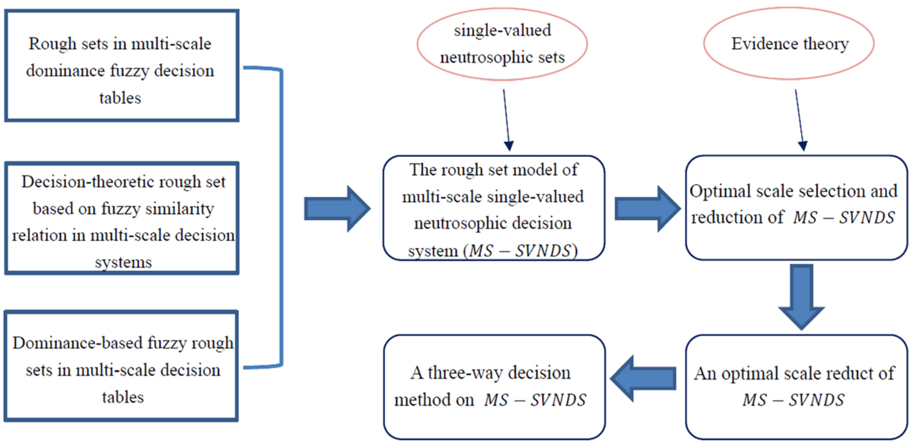

Fig. 1 Comparison and formation of this paper

2.1 TWD

The following defines the TWD model as it was proposed by Yao (2012).

Definition 1 (Yao 2012). Assume $O^{\subseteq}$ is an object set which is non-empty and finite, suppose that $v_a : O^{\subseteq} \to P_a$ and $v_r : O^{\subseteq} \to P_r$ are respectively the acceptance evaluation function and the rejection evaluation function on $O^{\subseteq}$, where $\left(P_a, \leq_a\right)$ and $\left(P_r, \leq_r\right)$ are both partially ordered sets. Let P_a^+ and P_r^- be non-empty subsets of P_a and P_r. Then Three regions induced by $\left(v_a, v_r\right)$ and $\left(P_a^+, P_r^-\right)$ are defined as follows:

$$POS_{(P_a^+,P_r^-)}\left(v_a, v_r\right) = \left\{x \in O^{\subseteq} | v_a(x) \in P_a^+ \bigwedge v_r(x) \notin P_r^-\right\} \tag{1}$$

$$NEG_{(P_a^+,P_r^-)}\left(v_a, v_r\right) = \left\{x \in O^{\subseteq} | v_a(x) \notin P_a^+ \bigwedge v_r(x) \in P_r^-\right\} \tag{2}$$

$$BND_{(P_a^+,P_r^-)}\left(v_a, v_r\right) = \left\{x \in O^{\subseteq} | \begin{array}{l} \left(v_a(x) \in P_a^+ \bigwedge v_r(x) \in P_r^-\right) \vee \\ \left(v_a(x) \notin P_a^+ \bigwedge v_r(x) \notin P_r^-\right) \end{array}\right\} \tag{3}$$

Considering that it is unnecessary to use two evaluation functions in some practical situations, v_a and v_r can be combined into one function for evaluation.

Definition 2 (Yao 2012). Given $O^{\subseteq}$, suppose that $v : O^{\subseteq} \to P$ is an evaluation function, where $(P, \leq)$ is a partially ordered set. Given two disjoint non-empty subsets P^+ and P^- of P, three regions induced by v and $\left(P^+, P^-\right)$ are outlined below:

$$POS_{(P^+,P^-)}(v) = \left\{x \in O^{\subseteq} | v(x) \in P^+\right\} \tag{4}$$

$$NEG_{(P^+,P^-)}(v) = \left\{x \in O^{\subseteq} | v(x) \in P^-\right\} \tag{5}$$

$$BND_{(P^+,P^-)}(v) = \left\{x \in O^{\subseteq} | v(x) \notin P^+ \wedge v(x) \notin P^-\right\} \tag{6}$$

Definition 3 (Yao 2012). Given $O^{\subseteq}$, $(P, \leq)$ is a totally ordered set, suppose that $v : O^{\subseteq} \to P$ is an evaluation function. Given thresholds $\alpha, \beta \in P$ and $\beta < \alpha$, three regions induced by v and (α, β) are outlined below:

$$POS_{(\alpha,\beta)}(v) = \left\{x \in O^{\subseteq} | v(x) \geq \alpha\right\} \tag{7}$$

$$NEG_{(\alpha,\beta)}(v) = \left\{x \in O^{\subseteq} | v(x) \leq \beta\right\} \tag{8}$$

$$BND_{(\alpha,\beta)}(v) = \left\{x \in O^{\subseteq} | \beta < v(x) < \alpha\right\} \tag{9}$$

2.2 SVNSs

To more accurately depict the ambiguity in real-world issues, Wang et al. (2010) provided the following SVNSs definition.

Definition 4 (Wang et al. 2010). Let U be an object set which is non-empty and finite, $O=\{\langle x,T_O(x),I_O(x),F_O(x)\rangle|x\in U\}$ is a SVNS in U, where $T_O(x)\in[0,1]$ represents the degree of truth, $I_O(x)\in[0,1]$ represents the degree of indeterminacy and $F_O(x)\in[0,1]$ represents the degree of falsity. For $\forall x\in U$, the SVNN in SVNS is noted as $\left(T_O(x),I_O(x),F_O(x)\right)$.

Shi and Ye (2017) further gave the definition of single-valued neutrosophic similarity degree.

Definition 5 (Shi and Ye 2017). Let U be an object set which is non-empty and finite, there are two SVNSs $O_1=\{\langle x_i,T_{O_1}(x_i),I_{O_1}(x_i),F_{O_1}(x_i)\rangle|x_i\in U,1\le i\le n\}$ and $O_2=\{\langle x_i,T_{O_2}(x_i),I_{O_2}(x_i),F_{O_2}(x_i)\rangle\,\big|\,x_i\in U,1\le i\le n\}$ in U, then.

$$S(O_1,O_2)=\frac{1}{n}\sum_{i=1}^{n}\frac{T_{O_1}(x_i)T_{O_2}(x_i)+I_{O_1}(x_i)I_{O_2}(x_i)+F_{O_1}(x_i)F_{O_2}(x_i)}{\sqrt{T_{O_1}^2(x_i)+I_{O_1}^2(x_i)+F_{O_1}^2(x_i)}\sqrt{T_{O_2}^2(x_i)+I_{O_2}^2(x_i)+F_{O_2}^2(x_i)}} \tag{10}$$

is defined as the similarity degree between O_1 and O_2.

Property 1 (Shi and Ye 2017). *Given two SVNSs* $O_1=\{\langle x,T_{O_1}(x),I_{O_1}(x),F_{O_1}(x)\rangle|x\in U\}$ *and* $O_2=\{\langle x,T_{O_2}(x),I_{O_2}(x),F_{O_2}(x)\rangle|x\in U\}$, $S(O_1,O_2)$ *is the similarity degree between* O_1 *and* O_2, *then*

(1) (1)$0\le S(O_1,O_2)\le 1$;
(2) (2)$S(O_1,O_2)=S(O_2,O_1)$;
(3) *If* $S(O_1,O_2)=1$, *then* $T_{O_1}(x)=T_{O_2}(x)$, $I_{O_1}(x)$=$I_{O_2}(x)$, $F_{O_1}(x)=F_{O_2}(x)$.

2.3 MSDSs

Definition 6 (Wu and Leung 2011). Tuple $S=(U,AT,V,f)$ represents a multi-scale information system, where $U=\{x_1,x_2,\dots,x_n\}$ is the universe of discourse, $AT=\{a_1,a_2,\dots,a_m\}$ is a non-empty and finite set of conditional attributes, $\forall a_j\in AT(1\le j\le m)$ can assume various values at various scales. V is the domain of conditional attributes, f is a map from $U\times AT$ onto V. $S=\left(U,AT\bigcup\{d\},V,f\right)$ is considered to be a MSDS, where $d\notin AT$ is a decision attribute.

Above is the definition of the original Wu–Leung model (2011, 2013)). Based on it, Li and Hu (2017) provided a novel model definition.

Definition 7 (Li and Hu 2017). Let $S=(U,AT,V,f)=\left(U,\left\{a_j^k|k=1,2,\dots,I_j,j=1,2,\dots,m\right\},V,f\right)$ be a multi-scale information system, where $a_j^k:U\to V_j^k$ is a surjective mapping and V_j^k is the domain of the attribute a_j on the k th scale. For $\forall 1\le k\le I_j-1$, there exists a surjective mapping $g_j^{k,k+1}:V_j^k\to V_j^{k+1}$ such that $a_j^{k+1}=g_j^{k,k+1^\circ}a_j^k$, $a_j^{k+1}(x)=g_j^{k,k+1}\left(a_j^k(x)\right)$, $x\in U$. Based on the above, we consider $S=\left(U,AT\bigcup\{d\},V,f\right)$ a MSDS, where $d\notin AT$ is a decision attribute.

Definition 8 (Li and Hu 2017). Let $S=\left(U,AT\bigcup\{d\},V,f\right)$ be a MSDS, where each attribute a_j has I_j scales $(1\le j\le m)$. There forms a decision system S^K when attributes $a_1,a_2,\dots,a_m$ are respectively restricted on their l_j th scale $(1\le j\le m)$, where $K=\left(l_1,l_2,\dots,l_m\right)$ is the scale combination of S^K in S. $\mathcal{L}=\left\{\left(l_1,l_2,\dots,l_m\right)|1\le l_j\le I_j,j=1,2,\dots,m\right\}$ is the scale collection of S, which contains all the scale combinations in S.

2.4 DS

For better coping with uncertain information and uncertain situations, DS was put forward.

Definition 9 (Niu et al. 2019). Let $S^K=\left(U,AT^K\bigcup\{d\},V,f\right)$, $X\subseteq U$, R_{AT^K} is the equivalent relation on S^K, $\underline{R_{AT^K}}(X)$ and $\overline{R_{AT^K}}(X)$ are respectively the lower approximation and upper approximation of X w.r.t. R_{AT^K}. Then the belief function and plausibility function of U are respectively defined as

$$Bel_{AT^K}(X)=P\left(\underline{R_{AT^K}}(X)\right)=\frac{\left|\underline{R_{AT^K}}(X)\right|}{|U|} \tag{11}$$

$$Pl_{AT^K}(X)=P\left(\overline{R_{AT^K}}(X)\right)=\frac{\left|\overline{R_{AT^K}}(X)\right|}{|U|} \tag{12}$$

where the corresponding mass function is noted as

$$m_{AT^K}(X)=\begin{cases}P(X)=\frac{|X|}{|U|}, & X\in\frac{U}{R_{AT^K}}\\ 0, & X\notin\frac{U}{R_{AT^K}}\end{cases} \tag{13}$$

Hence for $D_i\in U/d$, there exists

$$Bel_{AT^K}(d)=\sum_{D_i\in\frac{U}{d}}Bel_{AT^K}\left(D_i\right)=\sum_{D_i\in\frac{U}{d}}\frac{\left|\underline{R_{AT^K}}\left(D_i\right)\right|}{|U|} \tag{14}$$

$$Pl_{AT^K}(d)=\sum_{D_i\in\frac{U}{d}}Pl_{AT^K}\left(D_i\right)=\sum_{D_i\in\frac{U}{d}}\frac{\left|\overline{R_{AT^K}}\left(D_i\right)\right|}{|U|} \tag{15}$$

3 OPS and OPR of MS-SVNDS

We give the definition of the rough set model of MS-SVNDS in this section. The corresponding OPS and OPR methods are further explored. Hence, we have laid the environmental foundation for the discussion of the following TWD method on MS-SVNDS.

3.1 MS-SVNDS

First, we construct the rough set model of MS-SVNDS and give the related properties.

Definition 10 Let $MS-SVNDS=\left(U,AT\bigcup\{d\},V,f\right)$ be a multi-scale single-valued neutrosophic decision table, where $U=\left\{x_1,x_2,\ldots,x_n\right\}$ is the universe of discourse, $AT=\left\{a_1,a_2,\ldots,a_m\right\}$ denotes a non-empty finite set of conditional attributes and each $a_j\in AT$ has I_j scales $(1\le j\le m)$. When $a_1,a_2,\ldots,a_m$ are respectively restricted on their l_j th $(1\le j\le m)$ scale, there form a single-valued neutrosophic decision table S^K, where $K=\left(l_1,l_2,\ldots,l_m\right)\left(1\le l_j\le I_j,1\le j\le m\right)\in\mathcal{L}$ is denoted as the scale combination of S^K in S and $\mathcal{L}=\left\{\left(l_1,l_2,\ldots,l_m\right)|1\le l_j\le I_j,j=1,2,\ldots,m\right\}$ is the scale collection of S. $d\notin AT$ is a decision attribute, $U/d=\left\{D_1,D_2,\ldots,D_m\right\}$ constitutes an accurate division of U. V is the domain of $AT\bigcup\{d\}$ and $f\colon U\times\left(AT\bigcup\{d\}\right)\rightarrow V$ is a mapping function. For $\forall x\in U,a\in AT,f(x,a)=\left(T_x(a),I_x(a),F_x(a)\right)\left(T_x(a)\in[0,1],I_x(a)\in[0,1],F_x(a)\in[0,1]\right)$ represents the SVNN of object x under the conditional attribute a. The finest scale combination $K_0=\left(l_1=1,l_2=1,\ldots,l_m=1\right)$. For $\forall a_j\in AT(1\le j\le m)$, we have $a_j^1\left(x_i\right)\le a_j^2\left(x_i\right)\le\cdots\le a_j^{I_j}\left(x_i\right)(1\le i\le n,1\le j\le m)$.

Definition 11 Let $MS-SVNDS=\left(U,AT\bigcup\{d\},V,f\right)$, where $AT=\left\{a_1,a_2,\ldots,a_m\right\}$ and each $a_j\in AT$ has I_j scales $(1\le j\le m)$, scale combination $K=\left(l_1,l_2,\ldots,l_m\right)\left(1\le l_j\le I_j,1\le j\le m\right)$, for $\forall x\in U,a\in AT,f(x,a)=\left(T_x(a),I_x(a),F_x(a)\right)\left(T_x(a)\in[0,1],I_x(a)\in[0,1],F_x(a)\in[0,1]\right)$, $D_i\subseteq U(1\le i\le m)$,

$$S^K(x,y)=\frac{1}{m}\sum_{j=1}^{m}\frac{T_x\left(a_j^{l_j}\right)T_y\left(a_j^{l_j}\right)+I_x\left(a_j^{l_j}\right)I_y\left(a_j^{l_j}\right)+F_x\left(a_j^{l_j}\right)F_y\left(a_j^{l_j}\right)}{\sqrt{T_x^2\left(a_j^{l_j}\right)+I_x^2\left(a_j^{l_j}\right)+F_x^2\left(a_j^{l_j}\right)}\sqrt{T_y^2\left(a_j^{l_j}\right)+I_y^2\left(a_j^{l_j}\right)+F_y^2\left(a_j^{l_j}\right)}}\tag{16}$$

$$1\le j\le m,1\le l_j\le I_j$$

is denoted as the single-valued neutrosophic similarity degree between object x and object y in U on K. Then, S^K is the single-valued neutrosophic similarity relation induced by K.

$$[x]_{S^K}=\sum_{y\in U}\frac{S^K(x,y)}{y}\tag{17}$$

is the single-valued neutrosophic similarity set w.r.t. x derived from K.

$$\underline{S^K}\left(D_i\right)(x)=\left\{x\in U\middle|[x]_{S^K}\subseteq D_i\right\}\tag{18}$$

$$\overline{S^K}\left(D_i\right)(x)=\left\{x\in U\middle|[x]_{S^K}\cap D_i\neq\varnothing\right\}\tag{19}$$

We call the pairs $\left(\underline{S^K}\left(D_i\right),\overline{S^K}\left(D_i\right)\right)$ as rough sets of D_i w.r.t. K in $MS-SVNDS$. If $\underline{S^K}\left(D_i\right)=\overline{S^K}\left(D_i\right)$, then D_i is called definable w.r.t. K; if not, D_i is called undefinable w.r.t. K.

For $D_i\in U/d$, the belief function and plausibility function w.r.t. K are defined below:

$$Bel_K(d) = \sum_{D_i \in \frac{U}{d}} Bel_K(D_i) = \sum_{D_i \in \frac{U}{d}} \frac{\left|\underline{S^K}(D_i)\right|}{|U|} \tag{20}$$

$$Pl_K(d) = \sum_{D_i \in \frac{U}{d}} Pl_K(D_i) = \sum_{D_i \in \frac{U}{d}} \frac{\left|\overline{S^K}(D_i)\right|}{|U|} \tag{21}$$

Proposition 1 *The following properties hold based on Definition 9*:

(1) $\forall D_i \subseteq U, \underline{S^K}(D_i) \subseteq \overline{S^K}(D_i)$;
(2) $\underline{S^K}(U) = \overline{S^K}(U) = U$;

3.2 OPS of MS-SVNDS

On the basis of the exposition in the previous section, each object has a set of attributes with several scales and each attribute takes on different SVNNs at different scale levels when in a MS-SVNDS. Obviously, the workload of directly gaining the rules from such a huge amount of data is very difficult and complicated. Therefore, in order to effectively solving the problem of knowledge acquisition, we discuss the OPS method of MS-SVNDS in this section.

For the purpose of elaborating the OPS method of MS-SVNDS, we first review the lattice model proposed by Li and Hu (Li and Hu 2017) as following.

Definition 12 (Li and Hu 2017). Let $K_1 = (l_1^1, l_2^1, \ldots, l_m^1)$, $K_2 = (l_1^2, l_2^2, \ldots, l_m^2) \in \mathcal{L}$.

$$K_1 \le K_2 \iff l_j^1 \le l_j^2, 1 \le j \le m \tag{22}$$

$$K_1 = K_2 \iff l_j^1 = l_j^2, 1 \le j \le m \tag{23}$$

$$K_1 < K_2 \iff K_1 \le K_2 \wedge K_1 \ne K_2 \tag{24}$$

$$K_1 \wedge K_2 = \left(min(l_1^1, l_1^2), min(l_2^1, l_2^2), \cdots, min(l_m^1, l_m^2)\right) \tag{25}$$

$$K_1 \vee K_2 = \left(max(l_1^1, l_1^2), max(l_2^1, l_2^2), \cdots, max(l_m^1, l_m^2)\right) \tag{26}$$

Perceptibly, $(1, 1, \cdots, 1) \le (l_1, l_2, \ldots, l_m) \le (I_1, I_2, \cdots, I_m)$.

$(\mathcal{L}, \le, \wedge, \vee)$ is referred to as a lattice, where $\mathcal{L}$ is the scale collection of MS-SVNDS, $\le$ is a partial relation, $\wedge$ and $\vee$ respectively stand for the operation of taking the maximum and the minimum value.

The definitions of optimal scale combinations of MS-SVNDS are given below.

Definition 13 Let $MS-SVNDS = (U, AT \bigcup \{d\}, V, f)$, where $AT = \{a_1, a_2, \ldots, a_m\}$ and $d \notin AT$ constitutes a set of divisions $U/d = \{D_1, D_2, \ldots, D_m\}$. $K = (l_1, l_2, \ldots, l_m)$ and $K_0 = (1, 1, \ldots, 1)$ are respectively a combination in $\mathcal{L}$ and the finest combination in $\mathcal{L}$, $\mathcal{L} = \{(l_1, l_2, \ldots, l_m) | 1 \le l_j \le I_j, j = 1, 2, \ldots, m\}$.

(1) If there exists $Bel_K(d) = Bel_{K_0}(d)$, K is then considered as a consistent belief scale. Based on above, if there always exists $Bel_{K\prime}(d) \neq Bel_{K_0}(d)$ for $\forall K\prime \in \mathcal{L}$ that satisfies $K\prime > K$, K is then considered as an optimal belief scale.
(2) If there exists $Pl_K(d) = Pl_{K_0}(d)$, K is then considered as a consistent plausibility scale. Based on above, if there always exists $Pl_{K\prime}(d) \neq Pl_{K_0}(d)$ for $\forall K\prime \in \mathcal{L}$ that satisfies $K\prime > K$, K is then considered as an optimal plausibility scale.

In view of the above, we give the OPS algorithm of MS-SVNDS in the following.

Algorithm 1 Optimal belief scale selection algorithm of MS-SVNDS

Input: $MS-SVNDS = (U, AT \cup \{d\}, V, f)$, $U = \{x_1, x_2, \ldots, x_n\}$, $AT = \{a_1, a_2, \ldots, a_m\}$, $I_j (1 \leq j \leq m)$, $D_i \subseteq U (1 \leq i \leq m)$, $K_0 = (1, 1, \cdots, 1)$, $\mathcal{L} = \{(l_1, l_2, \ldots, l_m) | 1 \leq l_j \leq I_j, j = 1,2, \ldots, m\}$.

Output: All the optimal belief scales of MS-SVNDS.

Step1. Let $\varphi = \mathcal{L}$;

Step2. Calculate $Bel_{K_0}(d)$;

Step3. Calculate $Bel_K(d)$ for each K in φ, do $\varphi = \varphi \backslash \{K\}$ if there is $Bel_K(d) \neq Bel_{K_0}(d)$;

Step4. For each K in φ, if there exists K' that satisfies $K' < K$, then do $\varphi = \varphi \backslash \{K'\}$;

Step5. Output φ.

In the above algorithm, we first initialize φ to be equal to the scale collection $\mathcal{L}$. Next, we separately compute the belief functions w.r.t. K_0 and $K (\forall K \in \mathcal{L})$. By comparing the belief functions w.r.t. the above two, we determine whether to delete or retain K, i.e. delete K whose belief function is different from the belief function w.r.t. K_0. Then, we delete all scales finer than the reserved scales and obtain the optimal belief scales of MS-SVNDS.

One can achieve the optimal plausibility scale selection procedure of MS-SVNDS by replacing $Bel_{K_0}(d)$ with $Pl_{K_0}(d)$ and replacing $Bel_K(d)$ with $Pl_K(d)$. Apparently, Algorithm 1's time complexity is $O\left(n^2 \prod_{j=1}^{m} I_j\right)$.

3.3 OPR of MS-SVNDS

After selecting the optimal scales of MS-SVNDS, we acquire the coarsest scales that are coordinated with the finest scale, which effectively simplifies the data. However, not all attributes in MS-SVNDS are necessary and some of them may lead to redundancy. Thus, we discuss the acquisition of optimal scale reducts of MS-SVNDS in this section.

We start by providing the following description of sub-scale. Then, the OPR approach of MS-SVNDS is subsequently presented.

Definition 14 Let$MS-SVNDS = \left(U, AT \bigcup \{d\}, V, f\right)$, $K = \left(l_1, l_2, \ldots, l_m\right) \in \mathcal{L}$, sub-scale $K^{\subseteq} = \left(-, \cdots, -, l_i, -, \ldots, -, l_j, -, \cdots, -\right) \subseteq K$, $\mathcal{L}$ is the scale collection of MS-SVNDS. When there exists$K^{\subseteq} \subset K$, $K^{\subseteq}$ is said to be a proper sub-scale, where "-" denotes the deletion of the relevant attribute.

Definition 15 Let $MS-SVNDS=\left(U,AT\bigcup\{d\},V,f\right)$, $K=\left(l_1,l_2,\ldots,l_m\right)\in\mathcal{L}$, $K^{\subseteq}\subseteq K, U/d=\left\{D_1,D_2,\ldots,D_m\right\}$.

(1) If $Bel_{K^{\subseteq}}(d)=Bel_K(d)$ is met, then $K^{\subseteq}$ is called as a consistent belief sub-scale of K. When $K^{\subseteq}$ is the only one that meets the above conditions, $K^{\subseteq}$ is referred to as a consistent belief scale reduct of K.
(2) If $Pl_{K^{\subseteq}}(d)=Pl_K(d)$ is met, then $K^{\subseteq}$ is called as a consistent plausibility sub-scale of K. When $K^{\subseteq}$ is the only one that meets the above conditions, $K^{\subseteq}$ is referred to as a consistent plausibility scale reduct of K.

Evidently, we can get the reducts of all optimal scales according to the above method. In order to handle the circumstance that we need to quickly obtain a set of the most refined knowledge in real life more effectively, we provide an algorithm to directly obtain an optimal scale reduct of MS-SVNDS.

Algorithm 2 Obtaining an optimal belief scale reduct of MS-SVNDS

Input: $MS-SVNDS=(U,AT\cup\{d\},V,f)$, $U=\{x_1,x_2,\ldots,x_n\}$, $AT=\{a_1,a_2,\ldots,a_m\}$, $I_j(1\leq j\leq m)$, $U/d=\{D_1,D_2,\ldots,D_m\}$, $K_0=(1,1,\cdots,1)$.

Output: An optimal belief scale reduct of MS-SVNDS.

Step1. Let $AT'=AT$, $K'=K_0$;

Step2. Calculate $Bel_{K_0}(d)$;

Step3. Calculate $Bel_{K'\backslash\{a_j\}}(d)$ for each a_j in AT', do $K'=K'\backslash\{a_j\}$ and $AT'=AT'\backslash\{a_j\}$ if there exists $Bel_{K'\backslash\{a_j\}}(d)=Bel_{K'}(d)$, $K'\backslash\{a_j\}=\{l_1',\cdots,l_j',\cdots,l_m'\}$, $l_i'=l_i$, $i\neq j$, $1\leq i\leq m$, $l_j'=-$;

Step4. Otherwise let $l_j=max\left\{k\middle|Bel_{K'(a_j^k)}(d)=Bel_{K'}(d),1\leq k\leq I_j\right\}$, where $K'\left(a_j^k\right)=\{l_1',\cdots,l_j',\cdots,l_m'\}$, $l_i'=l_i$, $i\neq j$, $1\leq i\leq m$, $l_j'=k$;

Step5. Output K'.

In Algorithm 2, we first initialize $AT\prime$ to be equal to the conditional attribute set AT and $K\prime$ to be equal to the finest scale K_0. Next, we separately compute the belief functions w.r.t. $K\prime$ and $K\prime\backslash\left\{a_j\right\}(\forall K\in\mathcal{L})$. We delete a_j when the belief function w.r.t. $K\prime\backslash\left\{a_j\right\}$ are the same as the belief function w.r.t. $K\prime$. Otherwise, we raise the scale level of the reserved a_j to the maximum under the condition of keeping the belief function unchanged. After checking the conditional attributes one by one and outputting the final $K\prime$, we acquire an optimal belief scale reduct of MS-SVNDS.

The algorithm for calculating an optimal plausibility scale reduct of MS-SVNDS can be obtained by replacing all belief functions with plausibility functions. Apparently, Algorithm 2's time complexity is is $O\left(n^2\prod_{j=1}^m I_j\right)$.

4 A TWD method on MS-SVNDS

In the previous section, we construct the rough set model of MS-SVNDS, the selection and reduction of the optimal scale combinations are also investigated. Thus the effective knowledge in the decision table can be obtained and reliable decisions can be made. However, considering that not all decision problems in reality can get two diametrically opposed results, we introduce TWD method to classify objects into three different categories: non-commitment, rejection and acceptance, thereby forming more accurate decision results.

In the contents that follow, we address the TWD issue directly based on the optimal scale reduct obtained by Algorithm 2.

Definition 16 Let $OP-SVNDS=\left(U,AT\bigcup\{d\},V,f\right)$, where $U=\left\{x_1,x_2,\dots,x_n\right\}$ and $AT=\left\{-,\cdots,-,a_i^{l_i},-,\dots,-,a_j^{l_j},-,\cdots,-\right\}\left(1\le i\le j\le m,1\le l_i,l_j\le I_j\right)$, each conditional attribute in AT is restricted on its local optimal scale level and "−" represents that the corresponding conditional attribute is deleted. For $\forall x\in U,a\in AT,f(x,a)=\left(T_x(a),I_x(a),F_x(a)\right)\left(T_x(a)\in[0,1],I_x(a)\in[0,1],F_x(a)\in[0,1]\right)$. $OP=\left\{-,\cdots,-,l_i,-,\dots,-,l_j,-,\cdots,-\right\}$ is the scale combination of $OP-SVNDS$.

$$S^{OP}=\frac{1}{n}\sum_{j=1}^{m}\frac{T_x\left(a_j^{l_j}\right)T_y\left(a_j^{l_j}\right)+I_x\left(a_j^{l_j}\right)I_y\left(a_j^{l_j}\right)+F_x\left(a_j^{l_j}\right)F_y\left(a_j^{l_j}\right)}{\sqrt{T_x^2\left(a_j^{l_j}\right)+I_x^2\left(a_j^{l_j}\right)+F_x^2\left(a_j^{l_j}\right)}\sqrt{T_y^2\left(a_j^{l_j}\right)+I_y^2\left(a_j^{l_j}\right)+F_y^2\left(a_j^{l_j}\right)}} \tag{27}$$

$$a_j\ne -,1\le j\le m,1\le l_j\le I_j$$

is denoted as the single-valued neutrosophic similarity function on OP, where $n\in(0,\ m]$ is the number of reserved conditional attributes in AT.

$$[x]_{S_\omega^{OP}}=\left\{y\in U|S^{OP}(x,y)\ge\omega\right\} \tag{28}$$

is the ω-level ($\omega\in(0,\ 1]$) single-valued neutrosophic similarity class w.r.t. x derived from OP, where S_ω^{OP} is called a ω-level single-valued neutrosophic similarity relation.

Definition 17 Let $OP-SVNDS=\left(U,AT\bigcup\{d\},V,f\right)$ and $\varphi(O_1,O_2)=\frac{|O_1\cap O_2|}{|O_1|}$ be an evaluation function. Given thresholds $\alpha,\beta\in(0,\ 1]$ and $\alpha>\beta$. For any $O\subseteq U$,

$$POS_{(\alpha,\beta)}(O)=\left\{x\in U|\varphi\left([x]_{S_\omega^{OP}},O\right)\ge\alpha\right\} \tag{29}$$

$$NEG_{(\alpha,\beta)}(O)=\left\{x\in U|\varphi\left([x]_{S_\omega^{OP}},O\right)\le\beta\right\} \tag{30}$$

$$BND_{(\alpha,\beta)}(O)=\left\{x\in U|\beta<\varphi\left([x]_{S_\omega^{OP}},O\right)<\alpha\right\} \tag{31}$$

Proposition 2 *Given a three-way decision model* $\left(U,S_\omega^{OP},\varphi,\alpha,\beta\right)$, *for* $O_1,O_2,O_3\subseteq U and O_1\subseteq O_2$, *if there exists* $\varphi\left(O_3,O_1\right)\le\varphi\left(O_3,O_2\right)$, *then*

(1) $POS_{(\alpha,\beta)}\left(O_1\right)\subseteq POS_{(\alpha,\beta)}\left(O_2\right)$;
(2) $NEG_{(\alpha,\beta)}\left(O_2\right)\subseteq NEG_{(\alpha,\beta)}\left(O_1\right)$.

Proof (1) Obviously, for $\forall x \in POS_{(\alpha,\beta)}(O_1)$, there is $\varphi\left([x]_{S_\omega^{OP}}, O_1\right) \geq \alpha$. Considering that $\varphi\left([x]_{S_\omega^{OP}}, O_2\right) \geq \varphi\left([x]_{S_\omega^{OP}}, O_1\right)$, we have $\varphi\left([x]_{S_\omega^{OP}}, O_2\right) \geq \alpha$, which represents that $x \in POS_{(\alpha,\beta)}(O_2)$. Therefore, we obtain that $POS_{(\alpha,\beta)}(O_1) \subseteq POS_{(\alpha,\beta)}(O_2)$.

(2) For $\forall x \in NEG_{(\alpha,\beta)}(O_2)$, there is $\varphi\left([x]_{S_\omega^{OP}}, O_2\right) \leq \beta$. According to $\varphi\left([x]_{S_\omega^{OP}}, O_1\right) \leq \varphi\left([x]_{S_\omega^{OP}}, O_2\right)$, we have $\varphi\left([x]_{S_\omega^{OP}}, O_1\right) \leq \beta$, which represents that $x \in NEG_{(\alpha,\beta)}(O_1)$. Therefore, we obtain that $NEG_{(\alpha,\beta)}(O_2) \subseteq NEG_{(\alpha,\beta)}(O_1)$.

On an optimal scale reduct of MS-SVNDS, we present the specific algorithm for the TWD approach as follows.

Algorithm 3 The algorithm of TWD method

Input: $OP-SVNDS = (U, AT \cup \{d\}, V, f)$, $U = \{x_1, x_2, \ldots, x_n\}$, $O \subseteq U$, $AT = \{-, \cdots, -, a_i^{l_i}, -, \ldots, -, a_j^{l_j}, -, \cdots, -\}$, $OP = \{-, \cdots, -, l_i, -, \ldots, -, l_j, -, \cdots, -\}$, $\varphi(O_1, O_2) = \frac{|O_1 \cap O_2|}{|O_1|}$, $\omega \in (0,1]$, $\alpha, \beta \in (0,1]$, $\alpha > \beta$.

Output: The classification results of TWD method.

Step1. Let $U' = U$;

Step2. For $\forall x, y \in U'$, calculate $S^{OP}(x, y)$;

Step3. For each $x \in U'$, compute $[x]_{S_\omega^{OP}}$.

Step4. For $\forall x \in U'$, if there exists $\varphi([x]_{S_\omega^{OP}}, O) \geq \alpha$, then do $POS_{(\alpha,\beta)}(O) = POS_{(\alpha,\beta)}(O) \cup \{x\}$, $U' = U' \backslash \{x\}$; if there is $\varphi([x]_{S_\omega^{OP}}, O) \leq \beta$, then do $NEG_{(\alpha,\beta)}(O) = NEG_{(\alpha,\beta)}(O) \cup \{x\}$, $U' = U' \backslash \{x\}$; otherwise, do $BND_{(\alpha,\beta)}(O) = BND_{(\alpha,\beta)}(O) \cup \{x\}$ and $U' = U' \backslash \{x\}$;

Step5. Output $POS_{(\alpha,\beta)}(O)$, $NEG_{(\alpha,\beta)}(O)$ and $BND_{(\alpha,\beta)}(O)$.

5 Experimental analysis

In this section, we employ experimental analysis to confirm the accuracy and viability of our method. Figure 2 shows the experimental process. The experiment was conducted with MATLAB R2022a. We utilize seven UCI data sets to assess the efficacy of our method, i.e. Wine, Iris, Algerian Forest Fires, Glass, Thyroid, Bupa and Seeds. Table 1 shows these data sets' specifics.

All data sets are preprocessed before the evaluation experiment. Since the seven data sets are all single-scale, the multi-scale environments of them are firstly constructed. Use the data set Thyroid as an illustration. Figure 3 and 4 present the feature distribution and visualization of Thyroid. Based on the approach in Wu and Leung (2011), we first find out the maximum and minimum values of each feature in Thyroid and do subtraction between the two. Then the difference value obtained from each feature can be divided by 4 to get a 4-value interval $\left[\frac{i-1}{4}\left(A_{max} - A_{min}\right), \frac{i}{4}\left(A_{max} - A_{min}\right)\right]$ as the value on the coarser scale level, and so on to construct multiple scale hierarchies. After the above steps, the scale levels of features in Thyroid are finally set to (1,3,2,2,3).

Table 1 Datasets details

No	Data sets	Instances	Number of features	Number of classes
1	Wine	178	13	3
2	Iris	150	4	3
3	Fire	244	10	2
4	Glass	214	9	6
5	Thyroid	215	5	3
6	Bupa	345	6	2
7	Seeds	199	7	3

After constructing the multi-scale environment, we then convert all the data in data set into SVNNs. We start by calculating the Euclidean distance between each feature value and the maximum, minimum, and average values in the feature column, then use the absolute value that results from deducting the normalized distance from 1 to determine the degree to which the feature in the data set is true, indeterminate or false. Thus, we have established a rough set model of MS-SVNDS in the data set.

In the next step, we employ our Algorithm 2 to acquire an optimal scale reduct based on the established model. First, we calculate the belief function when all the features are at the first scale level, and then eliminate each feature one at a time to determine whether the belief function following the removal of a particular feature is equivalent to the belief function before. If so, confirm the deletion; If not, the belief function on the coarse scale of the feature is examined to see if it equals the original belief function. If they are the same, the scale hierarchy of the feature will be upgraded accordingly; if they are not, the original scale hierarchy of the feature will be kept. The final scale level of the feature is the maximum value that satisfies the above equality condition. Thus, we can get an optimal scale reduct of the data set, which simplifies the data to some extent.

Still take Thyroid as an example. The reserved features of Thyroid after OPR and their corresponding scale levels are (1,2,2,2,0), in which the feature of '0' represents deleted. The feature distribution and visualization of Thyroid after processing are shown in Fig. 5 and 6.

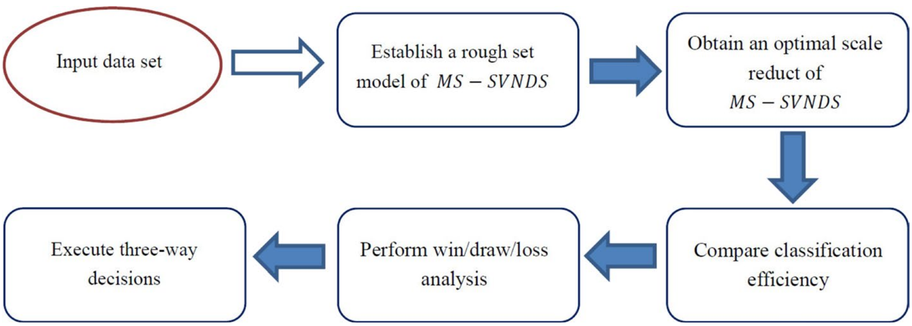

Fig. 2 Experimental flow chart

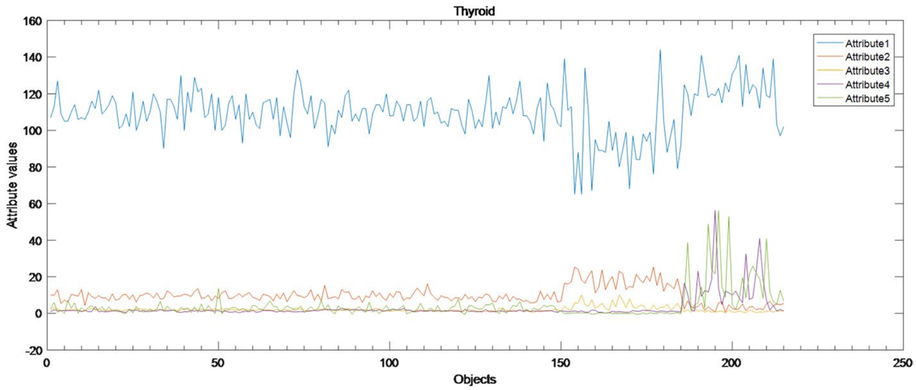

Fig. 3 Feature distribution of Thyroid

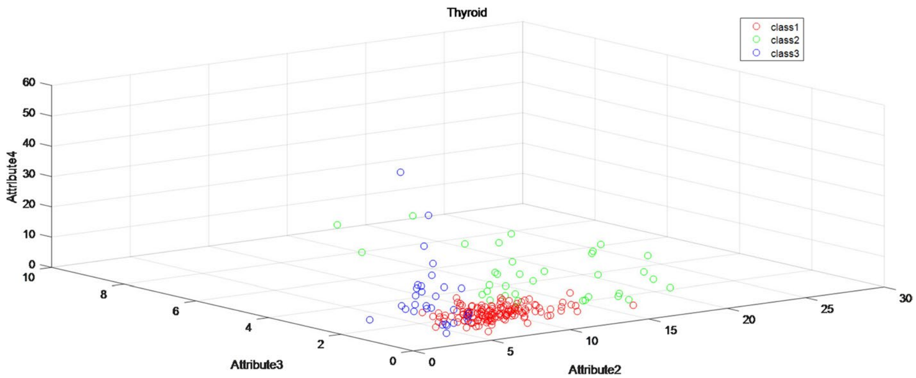

Fig. 4 Visualization of Thyroid

As shown in Table 2, we construct a multi-scale environment in each data set. By using our algorithm to process the multi-scale data sets, several redundant features are deleted and the scale level of some features is improved, thus effectively simplifying the data. The average classification accuracy of the comparison algorithm in Wang et al. (2023) and our algorithm obtained by 5 times of tenfold cross validation with KNN algorithm is shown in Table 3. Apparently, our algorithm can obtain relatively high average classification accuracy with simplified data sets, and the classification performance of our algorithm is improved compared with the comparison algorithm, which verifies the effectiveness and feasibility of our algorithm. The comparison of the experimental results is shown in Fig. 7 and 8.

To carry out further tests, we employ the win/draw/loss analysis presented by Chen et al. (2020) for sign and binomial test of classification accuracy. A total of 50 test samples of classification accuracy are obtained by performing 5 times of tenfold cross validation using the comparison algorithm and our algorithm. The p values here are the two-tail sign and binomial test results. It can be clearly concluded from Table 4 that our algorithm can achieve better classification effects.

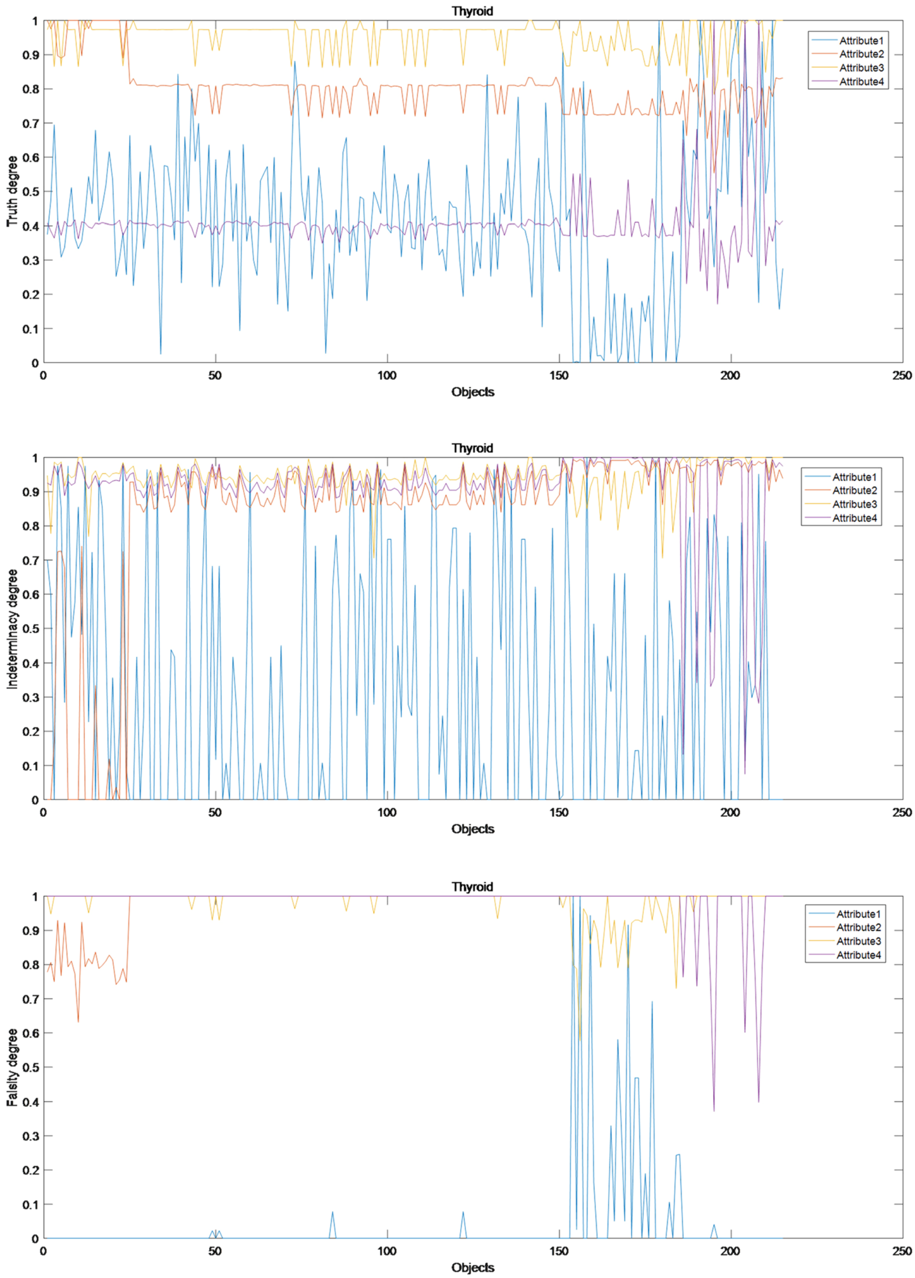

Fig. 5 Feature distribution of Thyroid after reduction

Upon the simplification of the data set, we recalculate the similarity degree between the objects and divide them into ω-level similarity classes by setting the similarity threshold ω. We finally get the partition of objects by comparing the evaluation values of ω-level

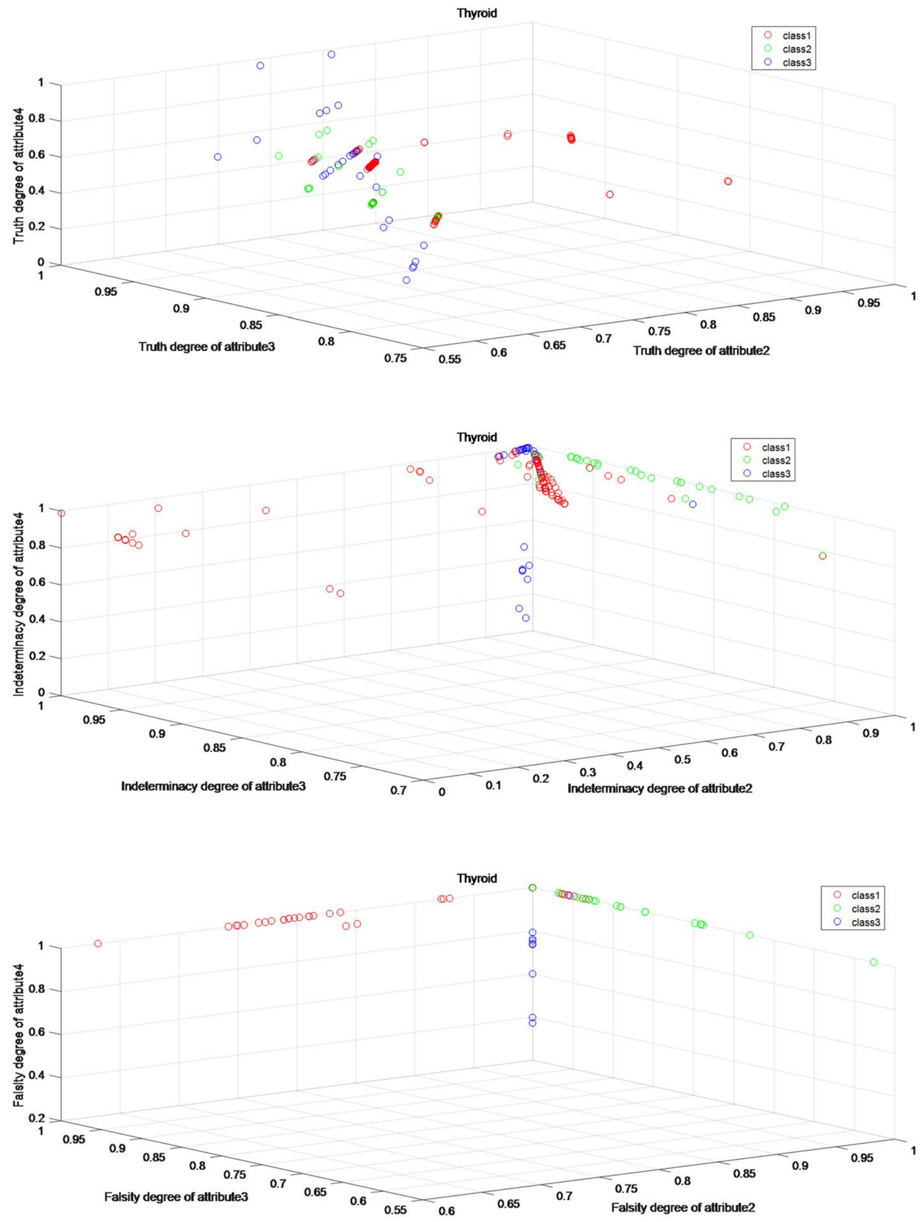

Fig. 6 Visualization of Thyroid after reduction

similarity classes and subsets of objects with the introduced thresholds α and β. The three-way decision results of data set Thyroid are shown below, where ω is set to 0.7259.

The processed data set Thyroid consists of 215 alternatives and 4 features. The scale levels of 4 features are respectively 1, 2, 2, 2. From the classification results of Thyroid in Table 5, we can find that 35 alternatives are considered to belong to $POS(F)$, 34

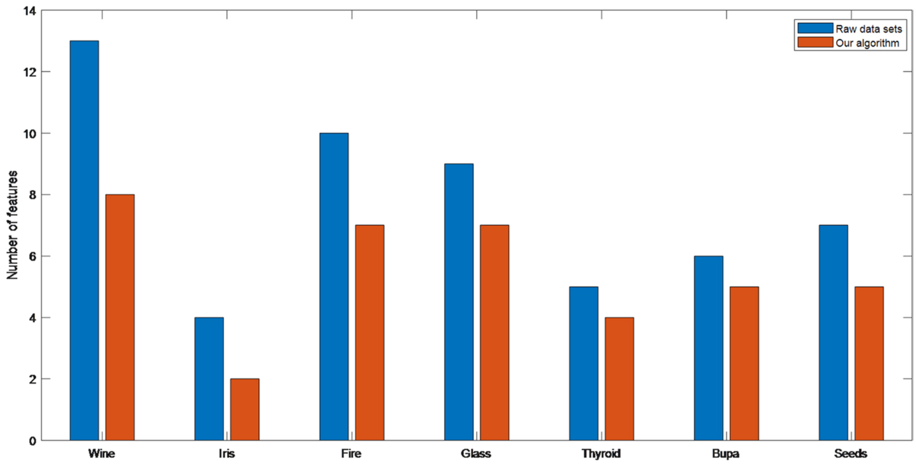

Fig. 7 Comparison of number of features

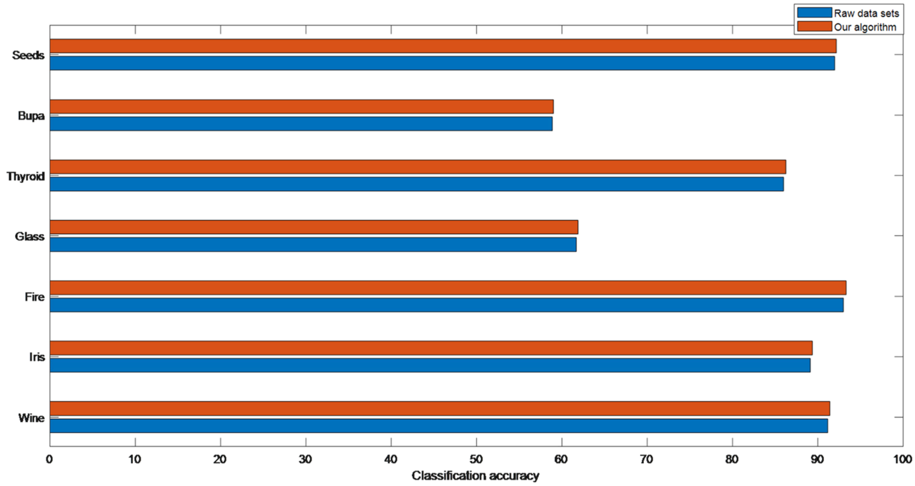

Fig. 8 Comparison of classification accuracy

Table 2 OPS and OPR results

No	Data sets	Set scale	Final scale
1	Wine	(3,2,2,2,2,3,3,2,2,2,3,3,2)	(1,0,2,2,0,0,2,0,2,0,1,1,2)
2	Iris	(3,2,3,2)	(0,0,3,2)
3	Fire	(3,2,2,3,3,2,3,2,2,2)	(1,2,1,0,3,0,0,1,2,1)
4	Glass	(3,2,3,2,2,2,2,3,2)	(0,0,1,1,1,1,1,1,2)
5	Thyroid	(1,3,2,2,3)	(1,2,2,2,0)
6	Bupa	(3,3,2,3,2,2)	(1,1,2,3,0,1)
7	Seeds	(3,2,2,2,3,2,2)	(0,2,0,1,2,2,2)

Table 3 Classification results

No	Data sets	Number of reserved features	Classification accuracy of comparison algorithm (%)	Classification accuracy of our algorithm (%)
1	Wine	8	91.16	91.41
2	Iris	2	89.12	89.36
3	Fire	7	92.99	93.32
4	Glass	7	61.69	61.89
5	Thyroid	4	85.98	86.25
6	Bupa	5	58.88	59.01
7	Seeds	5	91.97	92.17

Table 4 Statistical test results

Wine		Iris		Fire		Glass	
W/D/L	*p*	W/D/L	*p*	W/D/L	*p*	W/D/L	*p*
19/21/10	0.2026	19/19/12	0.3222	24/18/8	0.0328	17/19/14	0.6718
Thyroid		Bupa		Seeds		Total	
W/D/L	*p*	W/D/L	*p*	W/D/L	*p*	W/D/L	*p*
23/20/7	0.0328	15/22/13	0.8877	18/17/15	0.6718	135/136/79	0.0032

Table 5 Classification results of all alternatives in the data set ‘Thyroid’

Category	α	β	Number of alternatives in $POS(F)$	Number of alternatives in $BND(F)$	Number of alternatives in $NEG(F)$
Class 1	3/4	1/2	82	118	15
Class 2	1/7	1/8	35	34	146
Class 3	1/6	1/7	6	90	119

alternatives are considered to belong to $BND(F)$ and 146 alternatives are considered to belong to $NEG(F)$ in view of Class 2. It means that it can be confirmed that Class 2 contains at least 35 alternatives, 34 alternatives need further inspection to conclusively confirm whether they fall into Class 2 and 146 alternatives belong to other classes. The classification of all alternatives is effectively achieved. The effectiveness and feasibility of our approach is therefore validated.

6 Conclusions

We have designed a TWD method on MS-SVNDS in this paper. Considering the inadequacy of existing rough set models in describing uncertainty, we employ the theory of SVNSs, MSDSs and DS so as to construct a rough set model of MS-SVNDS. To address the knowledge acquisition issues involved in MS-SVNDS, we present the corresponding OPS and OPR approaches. Based on an optimal scale reduct of MS-SVNDS, we provide a novel TWD method to implement the classification. The experimental analysis is finally carried out to verify our method. The experimental results show that our method is reliable and effective.

The following list includes this paper's significant contributions and advantages:

(1) A rough set model of MS-SVNDS is established. Compared with other existing models, our model has significantly improved the processing ability of inaccurate and uncertain information. At the same time, the model is closer to the real life needs and data collection way.
(2) The OPS and OPR methods for MS-SVNDS are proposed. Through the above methods, we can obtain the simplified effective knowledge in MS-SVNDS according to the actual needs. And the classification performance based on the knowledge obtained by our method is better.
(3) A TWD method on MS-SVNDS is given. The method differs from other methods in that it is based on simplified effective knowledge and the knowledge is in the form of SVNNs. More reliable classification and decision results can be obtained by using our method.

For our method, there exists two main limitations:

(1) The introduction of SVNSs allows for a more detailed description of the data, but also leads to a much higher volume of data in pre-processing.
(2) Our proposed TWD method is based on an obtained optimal scale reduct of MS-SVNDS. Although it can effectively implement classification, the operation is complicated.

Some of our considerations for future research topics are as follows:

(1) We will further explore the integration of SVNSs and MSDSs.
(2) We will further explore how to apply TWD method to the knowledge acquisition in MSDSs.

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Author contributions XY and BH wrote the main manuscript text and prepared all the figures. All authors reviewed the manuscript.

Declarations

Competing interest The authors declare no competing interests.

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