Artificial Intelligence Review (2024) 57:74
https://doi.org/10.1007/s10462-023-10693-z

An integrated multi-polar fuzzy *N*-soft preference ranking organization method for enrichment of evaluations of the digitization of global economy

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Accepted: 31 December 2023 / Published online: 23 February 2024

Abstract
Digitization represents the ultimate expression of globalization that has revolutionized every facet of global existence, enhancing connectivity, financial terms, trade opportunities, and public services. To further broaden or fortify this digital realm and advance global progress, various strategies for multi-criteria group decision-making (MCGDM) have been developed. This research article extends the literature study on the sensational MCGDM method known as Preference Ranking Organization Method for Enrichment of Evaluations (PROMETHEE), which shares the burden of selecting the best alternative under many terms and conditions. This article aims to make a positive contribution to the development of the digital world. The main principle of the PROMETHEE technique is to calculate positive and negative flows of the selected options according to the divergence of the alternatives' scores. It provides us with partial and full rankings of options by determining score degrees, suitable preference functions, and a multi-criteria preference index. To deal with the multi-polarity of the modern world and assign *N*-ordered levels, this paper proposes the multi-polar fuzzy *N*-soft (*m*F*N*S) PROMETHEE technique. Prior to this, an Analytic Hierarchy Process (AHP) technique is applied to guarantee the credibility of the criterion weights for each criterion. After that, the whole strategy of *m*-polar fuzzy *N*-soft PROMETHEE approach to order the selected options is explained, and all the course of action of this remarkable (MCGDM) technique are synchronized in an extensive flowchart, helping us to learn more about the technology keenly. Moreover, the utility of our discussed technology is illustrated by considering two applications of generating electricity through human movement and choosing the best digital currency. The most suitable choice is extracted with the help of the outranked directed graph. The results obtained by *m*F*N*S PROMETHEE technique benefits us with the selection of most reliable source to digitized the global economy, in addition, It enables us to rank the alternatives from most preferable to least one, that not only saves time but also produces the better outcomes. Subsequently, the eminence of *m*F*N*S PROMETHEE technology is checked by comparability with the prior art. Lastly, the advantages and disadvantages of our proposed technique are supplemented to demonstrate its productivity and shortcomings.

Keywords Multi-polar fuzzy *N*-soft sets · PROMETHEE technique · Outranked directed graph · Digitalization

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1 Introduction

In the rapidly evolving landscape of the 21st century, digitization has emerged as a transformative force, revolutionizing industries, economies, and societies worldwide. Undoubtedly, digitization serves as a significant asset to the global economy, driving growth, innovation, and efficiency across various sectors. Digitization has shrunk the whole world into a single market by the facilitation of communication across borders, sharing of ideas, knowledge and the conduct of multi-million-dollar businesses on a common platform. By this robust tool, more secure, accurate, cheaper transactions can be done within few seconds, that not only enhance a state's economy but also increases the gross domestic product rate (Samoilovych et al. 2022). We can also change the flow of our artificial resources of energy to the natural reservoir, that is also an advance form of digitization. Since human body itself is a full packet of energy, we can utilized this human population's energy in producing electricity to make a state progress day and night. Many advance countries are using different mechanism to make this natural source of energy into maximum use. As energy produced by exercising, walking on the footpaths, human wastage- all these activities can be proved a profound source of energy that not only fulfill an individual's needs but also assists on a massive scale to digitize a state's economy. A novel MCGDM technique named as, *m*F*N*S PROMETHEE, is introduced in this research paper. This technique works efficiently to deal fuzziness of this modern era along with it's multi-polar attributes and can surpass the hurdle of *N*-ordered grading. Concisely, this strategy helps us to make apt decisions and provides us the best alternative selected amid the vagueness of this revolutionizing world. It not only put forward the best alternative but also rank them from best to least one depending upon their multi-polar attributes. The results obtained from practical implication of this proposed technique helps us to navigate the complexity of the digital global economy by which we can utilize more suitable, reliable and apt ways to empower the digital economical growth.

1.1 Literature review

All experts from every field of science are obsessed with taking an active role in digitizing the world and making life better. Like other fields, fuzzy set theory stands out in the complex multipolar world of digitization. Various MCGDM techniques have been developed (Bellman and Zadeh 1970; Hwang and Yoon 1981; Saaty 1986; Alcantud et al. 2023) to apply these amazing techniques in the digitization of the world (Büyüközkan et al. 2021; Kaya and Kahraman 2011; Mahmoudi et al. 2022).

The basic idea of fuzzy sets proposed by Zadeh (1965) is the beginning of all these wonders in fuzzy set theory. By separating the concepts of an attribute's belongingness and non-belongingness, Atanassov (1983) extended the notion of fuzzy sets where the degree of association (μ) and degree of disassociation (ν) of each alternative follows the constraint $\mu + \nu \leq 1$, and the model is known as the intuitionistic fuzzy set. The Pythagorean fuzzy set (a term devised by Yager (2013)) relaxed this constraint, it only requires that the degrees of μ and ν follow the constraint $\mu^2 + \nu^2 \leq 1$. However, these models are unable to decode multi-polar information, so the multi-poles we now observe must be handled by the multi-polar (*m*-polar) fuzzy set (*m*FS) (Chen et al. 2014). They are fabricated to work with *m*-poles, multi-polar agents or multi-polar characteristics.

On the other side of the picture, Molodtsov, in soft set theory (Molodtsov 1999), uses parameterized families of properties. But the concept still exhibits a strong and brittle nature: the required properties are either fulfilled or not. To realistically refine this restricted idea, *N*-ordered ranking (Fatimah et al. 2018) was introduced into a crisp soft set theory, thus pioneering *N*-soft set theory. Models that contain fuzziness for related reasons are enhanced by this generalization of basic soft set theory. In fact, fuzzy soft set theory (Maji et al. 2001) was defined possessing such innovation. The valuation fuzzy soft set Alcantud et al. (2017) processed the idea on real-estate data. Akram et al. (2018) soon combined fuzzy soft set theory with *N*-grading, and put forward fuzzy *N*-soft set theory. It's progress was amplified by the exposure of further hybrid models, mentioning a few names of these: *N*-soft rough sets (Peng et al. 2020), *N*-soft sets approach to rough sets (Alcantud et al. 2020), multi-fuzzy *N*-soft sets (Fatimah and Alcantud 2021).

Let us discuss the numerous MCDM and MCGDM methods developed in crisp set theory and fuzzy set theory (Benayoun et al. 1966; Hwang and Yoon 1981; Saaty 1986). A number of methods have been created in crisp set theory to function in multi-criteria and multi-criteria group decision-making situations. To make this concept realistic, many fuzzy MCDM and MCGDM techniques have been introduced to apply them to practical purposes, and a different version of the fuzzy decision technique was proposed by Bellman and Zadeh (1970) in the first step. Over time, abundant of methodologies and approaches have come forward to tackle with the uncertainty of human decision making power, multi-polarity of criteria or multi-polar agents along with *N*-grading by the use of AHP (Saaty 1986) for reduction of one's personal gains, TOPSIS (Hwang and Yoon 1981) works with ideal solutions, PROMETHEE (Chen 2018) works on incoming and outgoing flow of the alternatives, ELECTRE (Roy 1990), VIKOR (Opricovic and Tzeng 2004) or MULTIMOORA Brauers and Zavadskas (2010) where each methodology has it's own advantages and disadvantages. (Akram et al. 2023) enhanced ELECTRE II method with 2-tuple linguistic *m*-polar fuzzy sets for multi-criteria group decision making, Liu et al. (2023) developed an intuitionistic fuzzy best worst method for deriving weight vector of criteria, an ELECTRE method is being applied for choice of rehabilitation center with *m*-polar fuzzy *N*-soft information (Akram et al. 2023). The acronym PROMETHEE (Chen 2018), which stands for Preference Ranking Organization Method for Enrichment of Evaluation, is based on the idea of comparing options in pairs according to their competing criteria. PROMETHEE I and PROMETHEE II strategics, respectively, are being utilized to determine the incomplete and full ordering of the options. Goumas and Lygerou (2000) extended PROMETHEE method to (DM) in fuzzy scenes. Gul et al. (2018) proposed the PROMETHEE technique, which uses fuzzy numbers and fuzzy logic to solve the MCDM problem. Feng et al. (2020) created a fuzzy soft-set extension of the PROMETHEE method. By utilizing the linguistic terms, Krishankumar et al. (2017) developed the intuitionistic fuzzy PROMETHEE methodology to address the membership and non-membership of data. Akram et al. (2023), Akram et al. (2023) merged Pythagorean fuzzy *N*-soft set with PROMETHEE for group decision analysis. Feng et al. (2019), Feng et al. (2019) discussed general structure of IFSSs and corelated multi-feature decision-making techniques. Fuzzy multi-criteria decision-making for geological storage of carbon dioxide in Turkey (Deveci et al. 2015), using a novel fuzzy LBWA-WASPAS-H decision model (Pamucar et al. 2020) to select the airport ground access mode, interval type 2 fuzzy ARAS method recovery facility location problem (Karagöz et al. 2021), and use the Best-Worst Method and MARCOS (Deveci et al. 2021) based on interval rough numbers to select offshore wind farm sites, a fuzzy perfect agreement method-Dombi-Bonferroni tool for solving transport related problems (Pamucar et al. 2020), a novel PROMETHEE-based method is utilized for

multi-criteria decision making by combining fuzzy distance-based prioritization method based on Pythagorean (Chen 2019), a method for ranking comparison using likelihood-based on the interval type 2 fuzzy PROMETHEE method Chen (2015) and a risk attitude allocation model with Pythagorean fuzzy information and its application to financial decision making are just a few of the models that have been introduced. Many advances have been made by Akram and Shumaiza (2020); Akram et al. (2019) with respect to *m*-pole information and *N*-soft set theory. Jagtap and Karande (2022) applied *m*-polar fuzzy set algorithm for non-traditional machining process selection. In 2023, Akram et al. (2023) developed complex q-rung orthopair fuzzy 2-tuple linguistic group decision-making framework with Muirhead mean operators, Zeng et al. (2023) has developed low-carbon cities comprehensive evaluation method based on Fermatean fuzzy hybrid distance measure, Akram et al. (2023) proposed connectivity indices of *m*-polar fuzzy network model. In conclusion, many MCGDM strategies (Mondal et al. 2023; Roy 1990; Zhang et al. 2023; Xu et al. 2024; Bouraima et al. 2024; Lo et al. 2024; Yüksel et al. 2024) have been introduced to employ concept of fuzziness in our existing-world problems, helping us choose the best option for our situation given the available options and raking the alternatives. Due to its distinctive quality of offering a complete rating of the choices from most preferred to least preferred, the PROMETHEE approach family stands out above the rest.

1.2 Motivation of study

The innovative features of *m*F*N*S PROMETHEE methodology of tackling the multi-polarity of this modern era along with providing *N*-ordered grading to the selected options by parametrization of the attributes motivate us to expand our knowledge on this novel method. It makes itself of best use by providing an opportunity to the policy or decision makers to make the most suitable decision or choice according to their circumstances and provide an *N*-ordering of the selected choices to make a list of most suitable option to least one. The following are some key points that helps us to pursue the study of this amazing method:

- At the stage where crisp set theory connects us with belongingness and non-belongingness of elements to a set, Fuzzy set introduced by Zadeh (1965) provides the notion of the partial belongingness of elements to a set and transforms exactness into fuzzy theory.
- The performance of fuzzy set (Zadeh 1965) is hampered in assigning *N*-ordered grading to alternatives with respect to having a criterion.
- Fuzzy soft sets (Maji et al. 2001) are incorporated with parameterized family of attributes of the alternatives, which lack the ability to arrange the selected options from most suitable to least one, that is, assign *N*-grades. At the same time, many real-life applications are included in this property.
- Fuzzy sets and fuzzy soft sets also fail to comprehend the multi-polarity of the modern world. These set models are discarded while operating with multi-polar agents or multi-polar attributes.
- The *m*F*N*S PROMETHEE technique is proposed to efficiently and accurately execute all multi-tasking agendas at once. Rank the options from most suitable to least one as this proposed technique effectively trades with the fuzziness of human nature, multi-polar information, *N*-ordered grading, multi-polar agents, ranking the alternatives.

– A well-known MCDM technique, for example AHP mechanism (Saaty 1986), is used to calculate normalized weights of the criterion, confirm the uniformness of the criteria, or reduce one's own choice.

The proposed method, *m*F*N*S PROMETHEE, is based on comparing any two alternatives side-by-side and determining their preference by evaluating score degrees, suitable preference functions, computing multi-criteria preference indices, and determining the selected options' outflow and inflow.

1.3 Contribution of research article

This research paper comprises of the following points:

– The procedure of *m*F*N*S PROMETHEE method is explained in detailed among the required operators.
– For a better understanding of the strategy, a detailed flowchart is created, showing each and every step of the process.
– Two numerical examples: production of electricity through human movement and selection of the best digital currency for transactions are being solved and explained by using *m*F*N*S PROMETHEE technique to prove it's practicality and utility in real life scenarios.
– The normalized weights of the criterion are computed by an AHP tool to reduce favoritism of decision-makers' towards the considered alternatives and prohibit personal gains that would affect their final choice or conclusion.
– *m*F*N*S PROMETHEE I technique, in both of the aforementioned examples, provides us incomplete outranking of the choices, however, *m*F*N*S PROMETHEE II gives us the complete outranking of choices which are indicated through directed outranking graphs.
– To underscore the reliability of our proposed *m*F*N*S PROMETHEE strategy, a comparison is being made between *m*F*N*S PROMETHEE technique with *m*F PROMETHEE (Akram and Shumaiza 2020).
– At last, advantages and de merits of *m*F*N*S PROMETHEE strategy are discussed to show its pros and cons.

1.4 Preliminaries

Now, we review the basic concepts underpinning its applicability:

Definition 1 (Fatimah et al. 2018) Let$H_N = \{0, 1, 2, \ldots, N-1\}$ be a set of N-ordered grades, where $N > 1$. An ordered pair (β, X, N) is called a fuzzy N-soft set on $F(Z)$, where $F(Z)$ is a set of all fuzzy sets on Z and $X \subseteq Z$, if $\beta : A \to 2^{F(Z)\times H_N}$ follows condition that for each $x \in X$ and $z \in F(Z)$ there exists a unique $r \in H_N$ such that $(z, r) \in F(x)$.

Definition 2 (Akram et al. 2019) An m-polar fuzzy N-soft set (m, N) is a triple (f, D, m), where $D = (F, C, N)$ is an N-soft set on universal set (U) and f maps any attribute in C with an m-polar fuzzy set (mF) A on $F(c_k)$, which is a convenient subset of $V \times M$ and $c_k \in C$. Therefore, the domain of f is of course C, and it's codomain is $M^{'}(V \times M)$, the family of all sub-mF sets over $V \times M$.

1.5 Structure of research paper

The structure of the complete research paper is in accordance as: Section 2 entitles the procedure of AHP technique (Saaty 1986). Section 3 consists of procedure of *m*F*N*S PROMETHEE technique. Section 4 includes the applications: production of electricity through human movement and selection of the best digital currency for transactions are being explained by using *m*F*N*S PROMETHEE technique. Section 5 and 6 embrace the comparative analysis and discussion between *m*F*N*S PROMETHEE with *m*F PROMETHEE (Akram and Shumaiza 2020). Section 7 consists of merits and de merits of our proposed technique. The acronyms and their description used in the research paper are given in Table 1.

2 Analytic hierarchy process (AHP)

Prior to using the associated F*N*S PROMETHEE method in practical applications, we employ the AHP approach (Saaty 1986) to guarantee eminence of attributes' weights. Using the AHP method, we evaluate normalized weights of the criterion, weight matrix, the consistency index, and the consistency ratio. Whether or not the weights of the criterion are practical will depend on the consistency ratio's value. The AHP technique for the weights of the criteria in *m*F*N*-soft environment comprises of the following steps:

Table 1 Acronyms and their description

Acronyms	Description
FS	Fuzzy set
IFSS	Intuitionistic fuzzy soft sets
*m*FS	*m*-Polar fuzzy set
*m*F*N*SS	*m*-Polar fuzzy *N*-soft set
*m*FNSAO	*m*-Polar fuzzy *N*-soft averaging operator
DM	Decision-making
MCDM	Multi-criteria decision making
MCGDM	Multi-criteria group decision making
AHP	Analytic hierarchy process
TOPSIS	The technique for order of preference by similarity to ideal solution
ELECTRE	ÉLimination et choix traduisant la realité
VIKOR	VlseKriterijumska optimizacija I kompromisno resenje
MULTIMOORA	Multi-objective optimization on the basis of ratio analysis
PROMETHEE	The preference ranking organization method for enrichment of evaluations
LBWA	Level based weight assessment
WASPAS	Weighted aggregated sum product assessment
MARCOS	Measurement of alternative and ranking according to the compromise solution
ARAS	Additive ratio assessment system

(1) Establish all the criteria and compare them pairwise and build a pairwise comparison matrix For m criteria, an $m \times m$ square comparison matrix B is formulated where a criterion b_p is compared with criterion b_q and $b_{qp} = 1$ iff $p = q$ and $b_{qp} = 1/b_{pq}$.

$$B = \begin{pmatrix} b_{11} & b_{12} & \dots & b_{1q} \\ b_{21} & b_{22} & \dots & b_{2q} \\ \vdots & \vdots & \vdots & \vdots \\ b_{p1} & b_{p2} & \dots & b_{pq} \end{pmatrix}_{m\times m}.$$

(2) Construct a normalized comparison matrix B_{norm} and this normalization of weights takes place with the help of Eq. 1 as follows:

$$n_{pq} = \frac{b_{pq}}{\sum\limits_{q=1^m} b_{pq}}. \tag{1}$$

(3) Take the average of all the entries in each row of the normalized matrix by using Eq. 2 as follows:

$$v(p) = \frac{\sum\limits_{q=1^m} n_{pq}}{m}. \tag{2}$$

It will generate a weight vector weight as follows:

$$\varrho = \begin{pmatrix} \gamma(1) \\ \gamma(2) \\ \vdots \\ \gamma(m) \end{pmatrix}^T.$$

(4) Now, multiply the entries of the comparison matrix B with the entries of the weight vector and construct a matrix $B\varrho$.

(5) Calculate the maximum eigenvalue, by using Eq. 3, so the consistency index and consistency ratio can be estimated.

$$\lambda_{max} = \frac{\sum\limits_{p=1^m} (p^{th}\ entry\ in\ B\varrho / p^{th}\ entry\ in\ \varrho)}{m}. \tag{3}$$

(6) Determine the value of consistency index, by using Eq. 4.

$$S = \frac{\lambda_{max} - m}{m-1}. \tag{4}$$

(7) The consistency ratio (Y) is formulated as shown below:

$$Y = \frac{S}{T}, \tag{5}$$

where T is the random index define for different values of n as given in Table 2.

Table 2 Random index for different values of n

n	2	3	4	5	6	7	8	9	10
Y	0	0.58	0.90	1.12	1.24	1.32	1.41	1.45	1.49

If the consistency ratio is less than 0.10 ($S < 0.10$) then the weights are consistent. On the other hand, if ($S \geq 0.10$) then weights are inconsistent and don't produce the meaningful results.

3 Structure of *m*F*N*S PROMETHEE method

The vagueness of fuzzy set theory combined with m-polar attributes and N-soft grading are combined into a *m*F*N*S set, and its utilisation is being explained with the aid of a novel MCGDM technique called the *m*F*N*S PROMETHEE. This technique aids us in making the right decisions in our daily life chores by helping us choose the most appropriate alternative and ranking them in an order from best to worst according to the required terms and conditions. According to the nature and kind of the criteria, the *m*F*N*S PROMETHEE technique chooses the generalized preference function based on the positive and negative flow of the selected options. The following steps summaries *m*F*N*S PROMETHEE technique's methodology:

1. Experts' decision matrices : *m*F*N*S PROMETHEE, an MCGDM technique, embraces a panel of specialists who, after analyzing the selected options on the behalf of their attributes, assigns the membership degrees to the m-polar attributes with ordered grading to the alternatives under *m*F*N*S environment. Therefore, we take a set of $\}z'$ experts, $E = \{e_t\},\ t = 1, 2, \ldots, z$, who examine a set of 'y' alternatives, $A = \{a_l\},\ l = 1, 2, \ldots, y$, according to the 'x' attribute of m-poles under consideration as $C = \{c_s\},\ s = 1, 2, \ldots, x$. By examining the alternatives, each t^{th} expert give the independent membership degrees to the l^{th} alternative according to the s^{th} criterion in the form of m-polar fuzzy N-soft values (*mFNSV*) as $(d_{ls}, \mu_{ls}^1, \mu_{ls}^2, \ldots, \mu_{ls}^m)$. These assessments are shaped in form of *m*F*N*S decision matrix as follows:

$$H^{(t)^i} = \begin{array}{c} \\ a_1 \\ a_2 \\ \vdots \\ a_r \end{array} \begin{array}{c} \begin{array}{cccc} c_1 & c_2 & \cdots & c_p \end{array} \\ \begin{pmatrix} (\langle d_{11}, (b_{11}^{(t)^1}, \ldots, b_{11}^{(t)^m})\rangle & \langle d_{12}, (b_{12}^{(t)^1}, \ldots, b_{12}^{(t)^m})\rangle & \ldots & \langle d_{1p}, (b_{1p}^{(t)^1}, \ldots, b_{1p}^{(t)^m})\rangle) \\ (\langle d_{21}, (b_{21}^{(t)^1}, \ldots, b_{21}^{(t)^m})\rangle & \langle d_{22}, (b_{22}^{(t)^1}, \ldots, b_{22}^{(t)^m})\rangle & \ldots & \langle d_{2p}, (b_{2p}^{(t)^1}, \ldots, b_{2p}^{(t)^m})\rangle) \\ \vdots & \vdots & \ddots & \vdots \\ (\langle d_{r1}, (b_{r1}^{(t)^1}, \ldots, b_{r1}^{(t)^m})\rangle & \langle d_{r2}, (b_{r2}^{(t)^1}, \ldots, b_{r2}^{(t)^m})\rangle & \ldots & \langle d_{rp}, (b_{rp}^{(t)^1}, \ldots, b_{rp}^{(t)^m})\rangle) \end{pmatrix} \end{array}$$

2. Aggregated m **-polar fuzzy** N **-soft decision matrix**: Since each specialist has their own place of importance and value in the decision-making process, they do not all hold equal weight in any realistic scenario. As a result, each expert's distinct decisions, must be added up to create a joint judgement that has an impact on each decision made by each expert. The judgement of all the experts are combined into a single compelling decision using a m-polar fuzzy N-soft averaging operator ((*mFNSAO*)) as follows:

$$Y_{ls} = mFNSA\ ((d_{ls}^{(1)}, \ldots, d_{ls}^{(q)}), (b_{ls}^{(1)}, \ldots, b_{ls}^{(q)})) \tag{6}$$

$$=(max(d_{ls}^{(1)},\ldots,d_{ls}^{(q)}),\frac{\sum_{t=1}^{q}(b_{ls}^{(t)^1}+\ldots+b_{ls}^{(t)^m})}{q}) \tag{7}$$

$$=(d_{ls}^{*},(\mu_{ls}^{*^1},\mu_{ls}^{*^2},\ldots,\mu_{ls}^{*^m})). \tag{8}$$

3. Construct the score matrix: A choice's superiority over the others is always assessed using score degree, that is estimated by utilizing Eq. 9 as follows:

$$\widetilde{s}=\frac{d_{ls}^{*}}{N-1}+\frac{\mu_{ls}^{*^1}+\mu_{ls}^{*^2}+\ldots+\mu_{ls}^{*^m}}{m}. \tag{9}$$

4. Evaluation of divergence of the selected options according to the score degrees: The PROMETHEE method's basic tenet is to assess divergence of the options and the divergence between the options j_l and j_m with respect to the criteria c_t, we use the following Eq. 10:

$$D_k(j_l,j_m)=\widetilde{s}_k(j_l)-\widetilde{s}_k(j_m),\ l,\ m=\ 1,2,\ldots,r. \tag{10}$$

5. Consider an appropriate preference function: Defining the suitable preference function expresses importance of each option under consideration whose values lie between 0 and 1, where 1 shows supremacy and 0 shows no importance of the choices.
6. Evaluation of multiple criteria index: According to the ordering scenario among the choices proposed by preference index, an outranking graph is created to assess how much better or worse one alternative is than the other. The multiple criterion preference index is formulated by considering Eq. 11 as follows:

$$\tau(j_l,j_m)=\frac{\sum\limits_{k=1^p}v(k)\xi_k(j_l,j_m)}{\sum\limits_{k=1^p}v(k)}. \tag{11}$$

By the normalization of criteria weights, Eq. 11 reduces to the Eq. 12 as follows:

$$\tau(j_l,j_m)=\sum_{k=1^p}v(k)\xi_k(j_l,j_m). \tag{12}$$

- $\tau(j_l,j_m)\approx 1$ shows that alternative j_l has a clear advantage over j_m in terms of all attributes. It demonstrates the superiority of j_l over j_m.
- $\tau(j_l,j_m)\approx 0$ shows that, based on all criteria, alternative j_l has a weaker preference than alternative j_m. It demonstrates how j_l is inferior to j_m.

7. Evaluation of incomplete and complete preference ordering: The multiple criterion index is now used to determine the preferred ranking of the alternatives. The preference ranking of the alternatives demonstrates how one alternative dominates and is subjected to all other alternatives. By applying the PROMETHEE I approach for the partial outranking and the PROMETHEE II technique for the full outranking, respectively. The following explanation explains how PROMETHEE I and PROMETHEE II work:

   - The partial outranking flow by (PROMETHEE I): The outgoing flow of the alternatives is estimated by utilizing Eq. 13 as given:

Fig. 1 Outgoing arcs from h_l

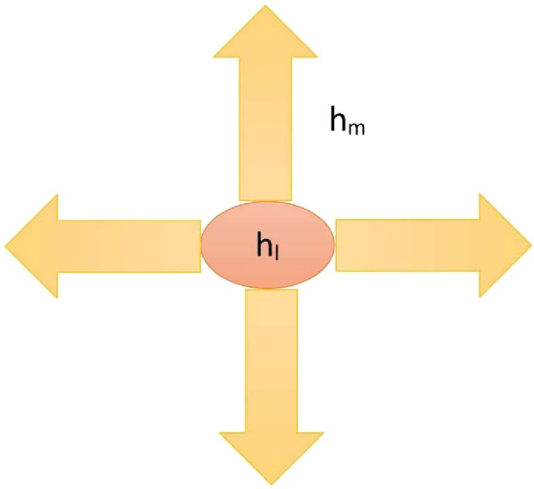
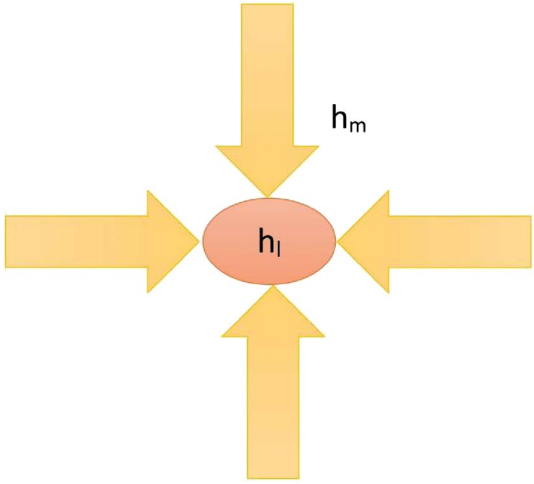


Fig. 2 Incoming arcs towards h_l



$$\Omega^{+}(j_l) = \frac{1}{r-1} \sum_{j_l \in H} \tau(j_l, h_m). \tag{13}$$

The term "outgoing flow," also called as "leaving flow" or "positive flow," refers to the conduct that distinguishes one choice from the others. As can be seen in Fig. 1, it is an average of all arcs drawn from j_1 that are departing.. While, the incoming flow of the selected options is determined by utilizing Eq. 14 as given:

$$\Omega^{-}(j_l) = \frac{1}{r-1} \sum_{j_m \in H} \tau(j_m, j_l). \tag{14}$$

This incoming flow, sometimes referred to as negative flow, gauges an alternative's subdominant behavior relative to all other alternatives. According to Fig. 2, it is actually the average of all incoming arcs to j_1. Over all other alternatives, the one

with a bigger outflow and a smaller inflow is highly appreciable. The selection of the alternatives based on the outgoing and incoming flows is formulated by utilizing the Eqs. 15 and 16:

$$\begin{aligned} &j_l Q^+ j_m, \Leftrightarrow \delta^+(j_l) > \delta^+(j_m) \forall j_l, j_m \in H; \\ &j_l R^+ j_m, \Leftrightarrow \delta^+(j_l) = \delta^+(j_m) \forall j_l, j_m \in H. \end{aligned} \tag{15}$$

$$\begin{aligned} &j_l Q^- j_m, \Leftrightarrow \delta^-(j_l) < \delta^-(j_m) \forall j_l, j_m \in H; \\ &j_l R^- j_m, \Leftrightarrow \delta^-(j_l) = \delta^-(j_m) \forall j_l, j_m \in A. \end{aligned} \tag{16}$$

The intersection of these ordering provides incomplete outranking $(\widetilde{Q}, \widetilde{R}, \widetilde{T})$ of PROMETHEE I method by using the following Eq. 17:

$$\begin{aligned} &j_l \widetilde{Q} j_m, \text{if } j_l Q^+ j_m \text{ and } j_l Q^- j_m, \\ &\qquad \text{or } a_i U^+ a_j \text{ and } j_l R^- j_m, \\ &\qquad \text{or } j_l R^+ j_m \text{ and } j_l Q^- j_m; \\ &j_l \widetilde{R} j_m, \text{iff } j_l R^+ j_m \text{ and } j_l R^- j_m; \\ &j_l \widetilde{K} j_m, \text{otherwise.} \end{aligned} \tag{17}$$

In the Eq. 17, $j_l \widetilde{Q} j_m$ indicates that j_l is supreme over j_m, $j_l \widetilde{R} j_m$ indicates that j_l is similar to j_m, however $j_l \widetilde{K} j_m$ shows that j_l is unable to compare with j_m.

– The complete outranking flow by (PROMETHEE II): The average outranking of the selected options is evaluated by utilizing Eq. 18:

$$\delta(j_l) = \delta^+(j_l) - \delta^-(j_l). \tag{18}$$

Eq. 18 gives the difference between outgoing and incoming flow that provides us the complete outranking (U^*, L^*) in PROMETHEE II by using Eq. 19:

$$\begin{aligned} &j_l Q^* j_m, \text{iff } \delta(j_l) > \delta(j_m); \\ &j_l R^* j_m, \text{iff } \delta(j_l) = \delta(j_m). \end{aligned} \tag{19}$$

In the Eq. 19, $j_l Q^* j_m$ indicates that j_l outranks j_m and $j_l R^* j_m$ indicates that j_l and j_m both are at the same position and indifferent to each other. Hence, the alternative with greatest outranking flow is considered as the most suitable choice. In this way, PROMETHEE II works out with the complete outranking of the alternatives and plays a requisite role in multi-criteria decision making process.

As seen in Fig. 3, the *m*F*N*S PROMETHEE's detailed process is synced up in a flowchart.

4 Applications

This section is indulged with the two prime applications or practical usage of our proposed method as production of electricity through human walking and selection of the best digital currency in this modern era by making use of *m*F*N*S PROMETHEE technique. The proposed technique not only minimizes the human uncertainty but also handles the

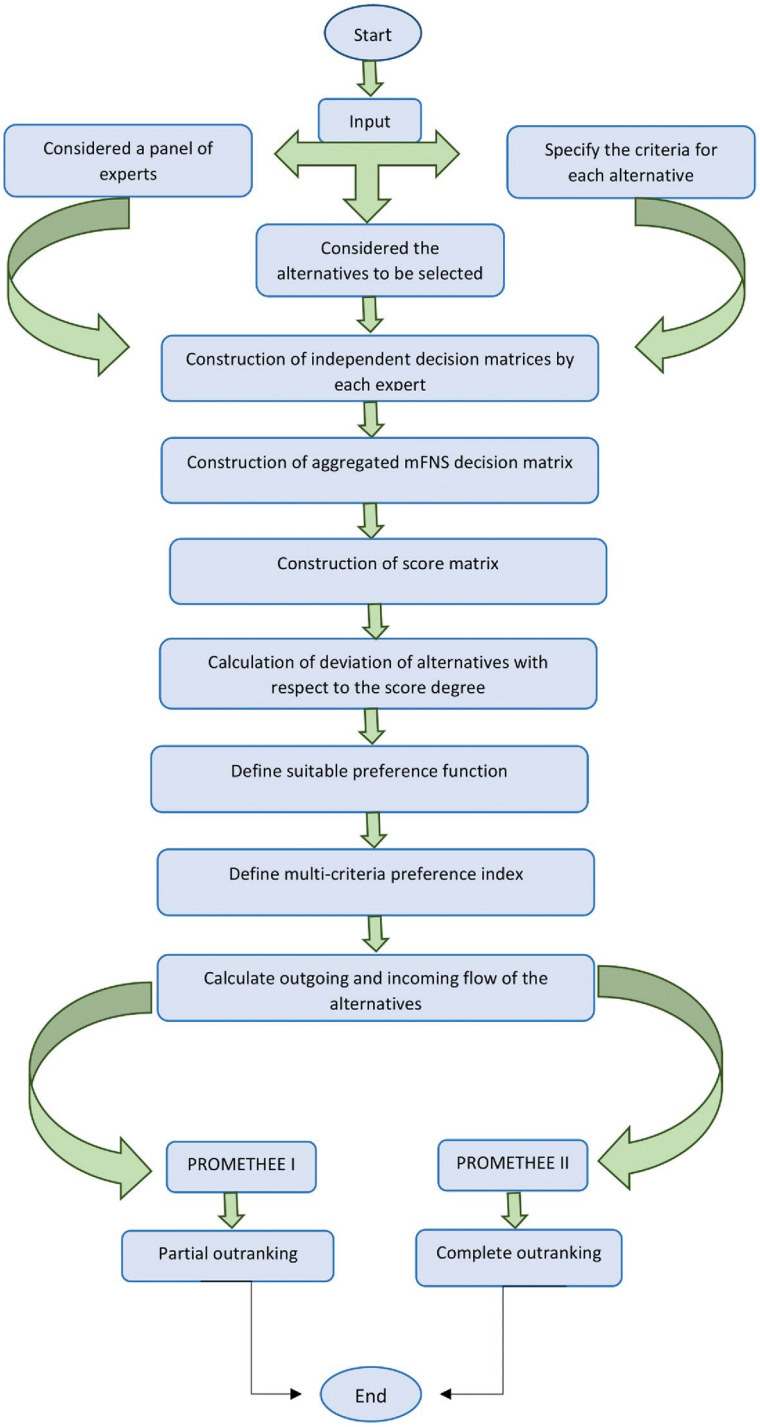


Fig. 3 Flowchart of proposed technique

multi-polarity of the present-day problems along with the ease of assigning the *N*-ordered grading to the selected options regarding the various attributes.

4.1 Production of electricity through human walking

Nature's energy is available all around us in a variety of different forms, it just needs to be properly channelized and used. God has bestowed humans a special quality of critical thinking and inventing various devices according to their needs and requirements. No doubt, electricity is the most dominant invention of humans that revolutionizes the whole mankind. Nowadays, not even a single human being can imagine to live without this marvelous innovation. The need of electricity increases with time as the world evolves or with the rise of the population in the world. To meet up the increasing demand of electricity, scientists started to develop new innovative ways to produce electricity as through utilizing wind energy, solar energy, hydro energy by installing wind mills, solar panels, turbines etc. But from all theses developed techniques, one can imagine that how human body can be a great source of energy that can be utilized to produce enough electricity. Our own self can produce electricity through body movement, work out, human feces etc. In no time, various scientists gather on one platform that how humans can play an active role in producing electricity. Electricity produced through human movement plays a vital role in boosting global economy as by consuming this natural package of energy there would be lesser burden of buying and operating heavy machinery on one state's economy, in addition to perform the daily task all the humans can take their active part in producing necessary electricity even for their own uses (like charging a mobile battery) which enables a country to utilize their valuable resources for well-being of it's people and in result of it there would be a massive increase in it's exports, literacy rate or gross domestic product (gdp).

Consider the body as the most powerful distributed energy source. To meet the crisis and shortfall of electricity in rural as well as urban area, scientists are now moving towards renewable energy sources that are eco-friendly and can be used again and again. Humans can be a major source of electricity production through joint or limb movement, walking or jogging on tracks, by wearing energy harvesting shoes or fabrics, by riding bicycles, motor bikes or cars, through their feces and urine. The details of production of electricity through human movement ia as follows:

– *Exercise* The human body loses energy during exercise that is used to generate electricity. Steppers, elliptical trainers, and stationary bicycles can be found at gyms all throughout the nation. What if each machine generated electricity? Startups like ReRev, Green Revolution, and Human Dynamo are giving the term "man power" a whole new meaning by modifying workout equipment to generate electricity Utilizing micro-inverter technology, the apparatus converts human energy into utility-grade electricity. The apparatus transforms the workout's energy into kilowatts, which may generate more than 160 watt-hours of electricity in a single workout when plugged into an outlet. Along with the electricity produced by a 26 kilowatt rooftop solar array and two 28.9 kilowatt compact wind turbines, the energy produced by the machinery is collected Through an internal micro-inverter, the Verde treadmill can produce up to 200 watts of electricity every hour. In order to transform the captured direct current (DC) power into an alternating current (AC) that can be connected to the grid, these tiny devices are also utilised in solar panels and wind turbines. The use of this equipment has already reduced gyms' dependency on

Fig. 4 Gym producing own electricity

Fig. 5 Treadmill to generate electricity

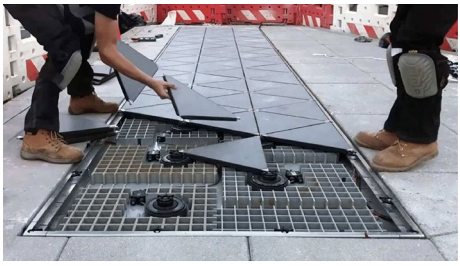


Fig. 6 Smart floor technology

unclean power sources as in Figs. 4 and 5. For instance, the Eco Gym in Rochester converted to using only renewable energy by supplying part of the bikes and ellipticals and augmenting the rest of their power requirements with rooftop solar panels and mini wind turbines.

– *Footsteps* Why not capture the energy produced when people walk and convert it into power? This was the basic concept behind Pavegen, a company that develops footstep-powered tiles in an effort to usher in the future. Depending on how firmly you land, one footstep on the company's tiles might provide one to seven watts of power and it is enough to turn on a street LED for 30 s. The paving slabs or smart floor technologies, Fig. 6, are installed using the Pavegen system, which transforms the energy from people's shoes into minute amounts of electrical power. Various things, such as captivating LED displays, interactive media, gamification, green walls, and more. The patented systems from Pavegen can generate up to 5 watts of power while a person is walking, which is sufficient to operate devices like environmental sensors, LED lighting, and screens as well as to store energy in batteries.

- *Body heat* Researchers are creating wearable fabric that can generate power at a number of renowned institutions, including the Georgia Institute of Technology. One of the researchers is Wake Forest University professor of physics David Carroll. He developed Power Felt, a stretchy material with electrical and thermal insulating properties (https://www.utilitydive.com/news/5-ways-you-can-use-the-human-body-to-generate-electricity/280709/). The new all-fibrous composite nanogenerator (AF-TENG), Fig. 7, made of electrospun polyvinylidene fluoride (PVDF) and nylon, silver nanowires (AgNWs), and polystyrene (PS), is simple to put into garments and can power small devices through regular movements. A blend of fibres developed by scientists at the University of Fukui in Japan can generate a static electrical charge that can be "harvested" through clothing (https://specialtyfabricsreview.com/2022/01/01/smart-fabric-can-generate-electricity/). South Korean researchers have created a bendable, foldable, and wearable fabric that produces electricity as it flexes and bends. To power a small screen or other electronic gadgets, a person wearing a shirt made of the cloth merely needs to move.
- *Urine and feces* One cannot even begin to comprehend how human waste specifically, human pee and excrement, which everyone considers to be waste–can be used to produce electricity. Chinese researchers who created a toilet that helps create fertilizer and electricity claim that human faeces may be processed at a bioreactor to release biogas. A Microbial Fuel Cell Pit Latrine uses composted waste to oxidize it in an anode chamber as opposed to your typical pit toilet. The subsequent release of electrons causes them to flow through a load-bearing circuit, producing electricity. With the help of this microbial fuel, one can browse the web, send and receive SMS messages, and make short phone calls, Fig. 8, (https://www.utilitydive.com/news/5-ways-you-can-use-the-human-body-to-generate-electricity/280709/).
- *Road speed breakers* The quantity of energy that these vehicles transfer across the highways might be enormous and easily captured, particularly over the speed humps where it is considerably more accessible. If configured properly, the speed breaker hump might be equipped with a spring-loaded device that would assist in the required speed braking as well as absorb the energy from the vehicle's motion, providing free recoverable energy right beneath the speed breaker site. The conversion might be accomplished quickly and efficiently using the tried-and-true method of using a motor generator system. The piston is pushed down whenever a vehicle climbs and passes over the speed breaker, rotating the alternator shaft that is linked. This occurs each time a car passes over the speed hump as in Fig. 9. On such a rumble strip, a vehicle carrying 1,000 kg and ascending to a height of 10 cm gener-

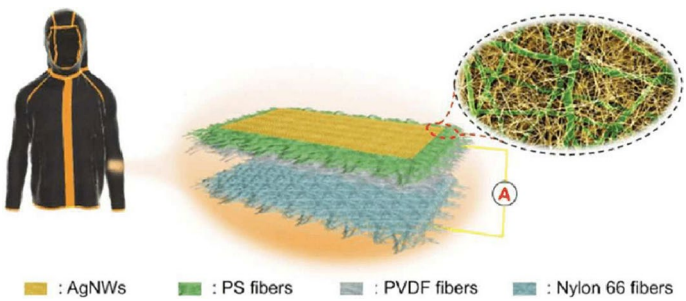


Fig. 7 Fabric producing energy

Fig. 8 Cell charged by human waste energy

Fig. 9 Car producing energy y hitting the speed brakers

ates roughly 0.98 kilowatts of electricity. Therefore, on a busy highway with around 100 vehicles passing by each minute, one such speed-breaker can generate about one kilowatt of electricity every single minute.

To select the most suitable technology for the production of electricity through human movement is not a cup of tea, so here our proposed methodology *m*F*N*S PROMETHEE comes forward that equipping with the features of multi-polarity, ordered *N*-grading and uncertainness of human thinking plays a vital role in selecting the most appropriate technique for renewable energy source. Consider the five techniques or alternatives as $\boldsymbol{\Phi} = \{\varphi_1, \varphi_2, \varphi_3, \varphi_4, \varphi_5\}$ and a team of experts as $\zeta = \{\varsigma_1, \varsigma_2, \varsigma_3, \varsigma_4, \varsigma_5\}$ consisting of the following members:

- Finance expert (ς_1): A budgeting expert will examine the budget of the projects of producing electricity through human movement including all the machines, spare parts, devices being used in this utilization of the nature. He will make sure the availability of the devices and the engineers, technicians, mechanics or workers indulged in the project.
- Technical expert (ς_2): He will manage all the technical work involving repairing of the machines, construction of the devices, maintenance of the spare parts. He will make sure the availability of the second way or chance in case of any emergency by not violating the natural laws or damaging the human health.
- Environmentalist (ς_3): The main task assign to the environmentalist is to take care of the beauty and all healthy aspects of nature that must not be damaged in any case during production of the electricity through human movement, for example the elements

used in electricity generated fabric must not polluted the environment or air quality index.
– Economist (ς_4): After producing electricity through renewable energy sources, the main function is to increase the exports, overcome the crisis or shortfall of electricity, increase the per capita income of the public so that the country or nation becomes successful on behalf of these renewable energy resources.

The aforementioned experts render the decision, $\xi = \{\eta_1, \eta_2, \eta_3, \eta_4\}$, following a careful examination of the factors taken into consideration such as:

– Efficiency (η_1): The efficiency of the implemented devices for production of energy through human movement comprises of 3-poles as *storage of energy*, *management of energy* and *sustainability*. It measures the amount of energy dissipated and stored, managing the energy for future usage and reducing the carbon emission.
– Project cost (η_2): *Maintenance cost*, *making charges* and *cost of implementation* are the three poles on which the whole project cost depend on. It must not be too much expensive that becomes unbearable to afford or not too much cheaper that the quality of the equipment becomes sub-standard so the project does not work out.
– Economic growth (η_3): By implementing the renewable energy sources for production of electricity, the economic growth is some worth while topic for the country. The economic growth is surrounded by 3-poles as *GDP growth*, *increase in exports* and *industrial markup*. To meet up the energy crisis along with the making the country prosper is actually the main goal of using the natural resources of producing energy.
– Health and eco-friendly (η_4): The production of energy through human movement must b safer for human health and also for the purity of the nature. It must be *safer to health*, *healthy for environment* and the energy sources can be *recycled*.

4.2 Weighting criteria using the AHP method

The criteria's weights are evaluated with the help of AHP technique. The comparison table is being displayed in Table 3 based on the Saaty preference scale (Saaty 1986).

The normalized weights are displayed in Table 4.

By utilizing Eq. 2, a weight vector is constructed in Table 5.

Now, multiply the elements of Table 3 with those of in Table 5, $B\varrho$ matrix is shown in Eq. 20:

Table 3 Comparison table for the criteria

B	η_1	η_2	η_3	η_4
η_1	1	0.14	0.20	0.11
η_2	7	1	4	1
η_3	5	0.5	1	0.25
η_4	9	1	2	1

Table 4 Normalized weights

B_{norm}	η_1	η_2	η_3	η_4
η_1	0.045	0.054	0.028	0.047
η_2	0.318	0.378	0.556	0.424
η_3	0.227	0.189	0.139	0.106
η_4	0.409	0.378	0.278	0.424

Table 5 Criteria weights(ϱ)

	ϱ
w_1	0.044
w_2	0.419
w_3	0.165
w_4	0.372

$$BV = \begin{pmatrix} 0.178 \\ 1.759 \\ 0.688 \\ 1.517 \end{pmatrix}. \tag{20}$$

The maximum eigenvalue is calculated as follows:

$$\lambda_{max} = \frac{1}{4} \times [\frac{0.195}{0.049} + \frac{1.803}{0.427} + \frac{0.633}{0.153} + \frac{1.552}{0.372}] \tag{21}$$

$$= 4.123. \tag{22}$$

The consistency index is formulated as

$$S = 0.041$$

by utilizing the Eq. 4. The selected values are compatible as index is near to zero. We determine the consistency ratio by utilizing Eq. 5. Here, we have picked $n = 4$ and $T = 0.90$ from Table 2. The consistency ratio is determined as $Z = 0.046$. As $0.046 < 0.10$, so it is acceptable.

4.3 Steps for the production of electricity through human movement by using *mFNS* PROMETHEE technique

This research paper discussed an MCGDM strategy, *m*F*N*S PROMETHEE, which stands as a leading tool for evaluating the most suitable approach for generating electricity from human movement. At its core, this methodology operates on the fundamental principle of assessing alternative options by comparing their incoming and outgoing flows. Within this framework, we offer both partial and complete rankings of these alternatives, alongside measures of how alternatives deviate from a designated scoring threshold. Moreover, we encompass general preference functions and multi-criteria preference functions to enhance the decision-making process. Following is a detailed description of how to use the *m*F*N*S

Table 6 3*F*6*SDM* of the finance expert (ς_1)

ς_1	η_1	η_2	η_3	η_4
φ_1	(5, 0.96, 0.94, 0.87)	(2, 0.38, 0.27, 0.29)	(4, 0.76, 0.69, 0.72)	(3, 0.48, 0.52, 0.59)
φ_2	(4, 0.72, 0.68, 0.62)	(3, 0.58, 0.49, 0.43)	(2, 0.26, 0.33, 0.39)	(1, 0.16, 0.08, 0.12)
φ_3	(3, 0.57, 0.46, 0.56)	(5, 0.94, 0.87, 0.91)	(3, 0.58, 0.46, 0.49)	(5, 0.83, 0.97, 0.94)
φ_4	(4, 0.62, 0.71, 0.78)	(2, 0.29, 0.31, 0.25)	(2, 0.37, 0.31, 0.25)	(1, 0.05, 0.16, 0.18)
φ_5	(2, 0.38, 0.27, 0.29)	(4, 0.77, 0.73, 0.68)	(1, 0.15, 0.04, 0.11)	(5, 0.86, 0.93, 0.96)

Table 7 3*F*6*SDM* of technical expert (ς_2)

ς_1	η_1	η_2	η_3	η_4
φ_1	(1, 0.17, 0.05, 0.18)	(4, 0.72, 0.65, 0.79)	(3, 0.58, 0.42, 0.55)	(4, 0.78, 0.65, 0.68)
φ_2	(4, 0.63, 0.77, 0.67)	(5, 0.84, 0.93, 0.86)	(2, 0.35, 0.27, 0.38)	(5, 0.93, 0.86, 0.94)
φ_3	(5, 0.85, 0.99, 0.92)	(2, 0.37, 0.28, 0.39)	(4, 0.79, 0.64, 0.72)	(1, 0.07, 0.15, 0.08)
φ_4	(2, 0.38, 0.29, 0.31)	(3, 0.59, 0.42, 0.54)	(5, 0.93, 0.88, 0.92)	(4, 0.74, 0.69, 0.78)
φ_5	(4, 0.75, 0.69, 0.72)	(1, 0.15, 0.16, 0.18)	(3, 0.49, 0.47, 0.52)	(2, 0.27, 0.36, 0.33)

Table 8 3*F*6*SDM* of environmentalist (ς_3)

ς_1	η_1	η_2	η_3	η_4
φ_1	(3, 0.55, 0.44, 0.58)	(1, 0.08, 0.16, 0.05)	(4, 0.69, 0.77, 0.73)	(5, 0.94, 0.87, 0.92)
φ_2	(2, 0.27, 0.36, 0.28)	(4, 0.78, 0.64, 0.72)	(5, 0.98, 0.85, 0.93)	(3, 0.57, 0.48, 0.44)
φ_3	(5, 0.94, 0.97, 0.88)	(3, 0.57, 0.47, 0.42)	(3, 0.35, 0.29, 0.36)	(1, 0.07, 0.04, 0.09)
φ_4	(1, 0.15, 0.18, 0.16)	(2, 0.28, 0.37, 0.33)	(5, 0.94, 0.87, 0.82)	(2, 0.31, 0.38, 0.22)
φ_5	(4, 0.74, 0.69, 0.77)	(3, 0.48, 0.59, 0.54)	(1, 0.06, 0.08, 0.16)	(5, 0.86, 0.94, 0.82)

Table 9 3*F*6*SDM* of economist (ς_4)

ς_1	η_1	η_2	η_3	η_4
φ_1	(4, 0.63, 0.79, 0.75)	(2, 0.27, 0.35, 0.37)	(1, 0.08, 0.15, 0.11)	(5, 0.98, 0.84, 0.95)
φ_2	(5, 0.84, 0.85, 0.97)	(1, 0.06, 0.15, 0.18)	(3, 0.55, 0.48, 0.57)	(2, 0.27, 0.37, 0.29)
φ_3	(2, 0.25, 0.38, 0.36)	(3, 0.53, 0.47, 0.49)	(4, 0.64, 0.78, 0.69)	(4, 0.78, 0.69, 0.72)
φ_4	(4, 0.76, 0.67, 0.77)	(2, 0.25, 0.38, 0.29)	(5, 0.97, 0.88, 0.95)	(3, 0.49, 0.51, 0.42)
φ_5	(3, 0.58, 0.47, 0.56)	(5, 0.98, 0.99, 0.84)	(1, 0.09, 0.16, 0.19)	(4, 0.68, 0.73, 0.65)

PROMETHEE strategy to choose the best way of generating electricity through human movement:

(1) The individual assessments of each of the expert, in form of 3-polar fuzzy 6-soft set, are arranged in Tables 6,7,8 and 9, respectively.

Table 10 Aggregated F6SDM

ς_1	η_1	η_2	η_3	η_4
φ_1	(5, 0.58, 0.56, 0.60)	(4, 0.36, 0.36, 0.38)	(4, 0.53, 0.51, 0.53)	(5, 0.8, 0.72, 0.79)
φ_2	(5, 0.62, 0.67, 0.64)	(5, 0.57, 0.55, 0.55)	(5, 0.54, 0.48, 0.57)	(5, 0.48, 0.45, 0.45)
φ_3	(5, 0.65, 0.7, 0.67)	(5, 0.60, 0.52, 0.55)	(4, 0.59, 0.54, 0.57)	(5, 0.44, 0.46, 0.46)
φ_4	(4, 0.48, 0.46, 0.51)	(3, 0.35, 0.37, 0.35)	(5, 0.80, 0.74, 0.74)	(4, 0.4, 0.44, 0.4)
φ_5	(4, 0.61, 0.53, 0.59)	(5, 0.60, 0.62, 0.56)	(3, 0.2, 0.19, 0.25)	(5, 0.67, 0.72, 0.69)

Table 11 Score matrix

	η_1	η_2	η_3	η_4
φ_1	1.58	1.17	1.32	1.77
φ_2	1.64	1.56	1.53	1.46
φ_3	1.67	1.56	1.37	1.45
φ_4	1.28	0.96	1.76	1.21
φ_5	1.38	1.59	0.81	1.69

(2) By combining the discrete assessments from each decision-maker into a single matrix and using the *PFNSA* operator, which is provided in Eqs. 6 and 8, the expert choice is affected collectively in *AF6SDM* shown in Table 10.

(3) Construct a score matrix displayed in Table 11 of aggregated decision matrix by using Eq. 9.

(4) Evaluate disparity of the given choices with respect to the score degree by using Eq. 10. This divergence of the options is presented in Table 12.

(5) Select an appropriate preference function(Chen 2018) to check the supremacy of an option over another. The generalized preference functions in Table 13 provides various types of attributes.

(6) The generalized criteria preference function by Chen (2018), which is shown in Table 14, is used to compute the level of supremacy of an alternative over another with regard to each criterion.

(7) The multi-criteria preference index, shows favoritism of experts towards the alternatives, derived using Eq. 12. The multi-criteria preference index is displayed in Table 15.

(8) The incomplete ordering of the selected options is estimated with *mFNS* PROMETHEE I method and outgoing and incoming flow of the selected options are estimated by using Eqs. 13 and 14, respectively. The outgoing and incoming flows of an option with respect to other one is deduced by the Eqs. 15 and 16 respectively and displayed in Table 16.

Table 12 Divergence of the alternatives according to assigned criteria

Alternatives	η_1	η_2	η_3	η_4	*Alternatives*	η_1	η_2	η_3	η_4
$\varphi_1\varphi_2$	−0.06	−0.39	−0.21	0.31	$\varphi_1\varphi_3$	−0.09	−0.39	−0.05	0.32
$\varphi_1\varphi_4$	0.3	0.21	−0.44	0.56	$\varphi_1\varphi_5$	0.2	−0.42	0.51	0.08
$\varphi_2\varphi_1$	0.06	0.39	0.21	−0.31	$\varphi_2\varphi_3$	−0.03	0	0.16	0.01
$\varphi_2\varphi_4$	0.36	0.6	−0.23	0.25	$\varphi_2\varphi_5$	0.26	−0.03	0.72	−0.23
$\varphi_3\varphi_1$	0.09	0.39	0.05	−0.32	$\varphi_3\varphi_2$	0.03	0	−0.16	−0.01
$\varphi_3\varphi_4$	0.39	0.6	−0.39	0.24	$\varphi_3\varphi_5$	0.29	−0.03	0.56	−0.24
$\varphi_4\varphi_1$	−0.3	−0.21	0.44	−0.56	$\varphi_4\varphi_2$	−0.36	−0.6	0.23	−0.25
$\varphi_4\varphi_3$	−0.39	−0.6	0.39	0.24	$\varphi_4\varphi_5$	−0.1	−0.63	0.95	−0.48
$\varphi_5\varphi_1$	−0.2	0.42	−0.51	−0.08	$\varphi_5\varphi_2$	−0.26	0.03	−0.72	0.23
$\varphi_5\varphi_3$	−0.29	0.03	−0.56	0.24	$\varphi_5\varphi_4$	0.1	0.63	−0.95	0.48

Table 13 Type of attributes and preference functions

Attributes	*Max or Min*	*Type of attributes*	*Parameters*
φ_1	*Max*	*I*	*Nill*
φ_2	*Max*	*II*	$k = 0.01$
φ_3	*Max*	*II*	$k = 0.01$
φ_4	*Max*	*I*	*Nill*

Table 14 Generalized criteria preference function

Alternatives	η_1	η_2	η_3	η_4	*Alternatives*	η_1	η_2	η_3	η_4
$\varphi_1\varphi_2$	0	0	0	1	$\varphi_1\varphi_3$	0	0	0	1
$\varphi_1\varphi_4$	1	1	0	1	$\varphi_1\varphi_5$	1	0	1	1
$\varphi_2\varphi_1$	1	1	1	0	$\varphi_2\varphi_3$	0	0	1	1
$\varphi_2\varphi_4$	1	1	0	1	$\varphi_2\varphi_5$	1	0	1	0
$\varphi_3\varphi_1$	1	1	1	0	$\varphi_3\varphi_2$	1	0	0	0
$\varphi_3\varphi_4$	1	1	0	1	$\varphi_3\varphi_5$	1	0	1	0
$\varphi_4\varphi_1$	0	0	1	0	$\varphi_4\varphi_2$	0	0	1	0
$\varphi_4\varphi_3$	0	0	1	0	$\varphi_4\varphi_5$	0	0	1	0
$\varphi_5\varphi_1$	0	1	0	0	$\varphi_5\varphi_2$	0	1	0	1
$\varphi_5\varphi_3$	0	1	0	1	$\varphi_5\varphi_4$	1	1	0	1

The intersection of these flows facilitates us with the partial outranking of the alternatives that can be evaluated by the Eq. 17 as follows:

$\varphi_1\hat{P}\varphi_4$, $\varphi_3\hat{P}\varphi_4$, $\varphi_1\hat{P}\varphi_3$, $\varphi_5\hat{P}\varphi_1$,$\varphi_5\hat{P}\varphi_2$.

But φ_1 and φ_2, φ_5 with φ_3 and φ_4 are incomparable, implying partial ranking from PROMETHEE I. The outranking relation among the alternatives by PROMETHEE I is shown in Fig. 10.

(9) *m*F*N*S PROMETHEE II facilitates with the complete ranking relation. The net outranking of the selected options is evaluated by Eq. 18. Now, by using the Eq. 19, we can get the complete ordering of these choices as shown in Table 17.

Table 15 Index of multi-criteria preference

	φ_1	φ_2	φ_3	φ_4	φ_5
φ_1	–	0.372	0.372	0.835	0.581
φ_2	0.628	–	0.537	0.835	0.209
φ_3	0.628	0.044	–	0.835	0.209
φ_4	0.165	0.165	0.165	–	0.165
φ_5	0.419	0.791	0.791	0.835	–

(10) According to the calculations above, alternative φ_5 is determined to be the most effective method for producing electricity through human movement, and the alternatives are listed in the following order: $\varphi_5 > \varphi_2 > \varphi_1 > \varphi_4 > \varphi_3$.

4.4 Selection of the best digital currency

The word digitalization comprises of the long-term technical revolution of the historic world to the modern and digitalized era. Digitalization is the use of digital technology to transform the traditional business to computerized ones, producing more revenue and value-producing opportunities. To mould a country or nation towards digitalization of the modern world, the best way is to convert their traditional business trends into digitized

Table 16 Outgoing and incoming flows by (PROMETHE I)

Alternatives	*Outgoing flow*(Y^+)	*Incoming flow*(Y^-)
φ_1	0.5	0.5
φ_2	0.6	0.3
φ_3	0.4	0.5
φ_4	0.2	0.8
φ_5	0.7	0.3

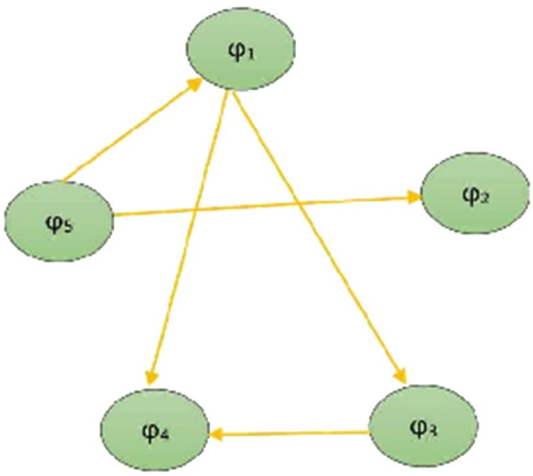


Fig. 10 Outranking of the alternatives by PROMETHEE I

methods. As the business depends on one's country currency, so why not reshape that paper currency into digital one. How much interesting this phenomena is that you can do business in billions and trillions without having cash in bag or accounts in bank. Many developed countries have adopted this astonishing technique for their trade, business, import or export etc and earning the profit from it.

Now-a-days, various forms of digital currencies have been developed as Cryptocurrency, Bitcoin, Ethereum, Tether, USD Coin, Binance Coin, Cardano, Solana or Polkadot etc. First, take a look on, what are these currencies, how these are used, benefits of usage and reason to their popularity.

Cryptocurrency

Cryptography, a method that is distributed over many computers and is nearly impossible to replicate or forge, secures cryptocurrencies, which are digital or virtual forms of money. It is, in essence, a technology that enables the use of virtual tokens as payment methods for safe online transactions. Blockchain technology, an immensely efficient form of recording knowledge, is used to run it on decentralised networks, making it difficult or impossible to manipulate the system or defraud consumers.

Bitcoin

It is a distributed digital money. Bitcoin, Fig. 11, can directly be purchased, trade or bought without any third party like bank. A blockchain is a distributed digital ledger that serves as the foundation of bitcoin. As its name suggests, a blockchain is a network of interconnected data made up of units called blocks that each contain details about a single transaction, such as the date and time, the total amount, the buyer and seller, and a special identification number for each trade.

Ethereum

With Etherum, anyone in the globe will be able to access a decentralized suite of financial products, irrespective of their homeland, race, or religion. Ethereum uses ether, a platform-specific cryptographic token, to power its apps. Although it trails Bitcoin by a wide margin, Ether, which was introduced in 2015, is currently the second-largest digital currency by market capitalization. Ether's market valuation of $147.5 billion as of July 8, 2022, or roughly $1,200 per ETH, is less than half that of Bitcoin. (https://coinmarketcap.com/currencies/ethereum/).

Solana (SOL)

Solana, Fig. 12, is a blockchain platform created in 2017 with the goal of supporting decentralised apps. Solana, often known as a "Ethereum killer," processes far more transactions per second than Ethereum does. In comparison to Ethereum, it also has reduced transaction fees. Solana is the ninth-largest cryptocurrency by market cap with a market worth of $12.8 billion and a price of about $38.

Digital currency have gained success and become quite popular among various developed and developing countries. Most of them have legalized their usage on smaller

Table 17 Net outranking flow of the alternatives (PROMETHE II)

Alternatives	*Net flow*(Y)
φ_1	0
φ_2	0.3
φ_3	−0.1
φ_4	−0.6
φ_5	0.4

Fig. 11 Digital currency-Bitcoin

industrial level as well as on larger business trades. The top cities of the digitalized countries that are hotspot of the digital currency are as follows:

- San Francisco: The technology capital of the United States is home to cryptocurrency trading platforms Coinbase and Kraken (https://www.coinbase.com/about), (https://support.kraken.com/hc/en-us/articles/360001389343-Kraken-s-address).
- New York: 8.4 million residents of New York, a tech hotspot, may use their bitcoins at roughly 30 local businesses, including the Bitcoin Store in lower Manhattan. Additionally, New York serves as a significant hub for media outlets and cryptocurrency startups including CoinDesk, Decrypt, and CoinTelegraph. It also serves as the location of Consensus, one of the biggest yearly events in the cryptocurrency sector. s a tech hub of 8.4 million people can spend their bitcoin at about three dozen merchants in the metro area, including the Bitcoin Store in lower Manhattan. New York is also a major hub for crypto startups and media companies, such as CoinDesk, Decrypt, and CoinTelegraph, as well as home to Consensus, one of the largest annual events in the crypto industry
- London: The capital of the United Kingdom is home to 8.9 million residents, 50 bitcoin ATMs, and about the same number of merchants who will accept bitcoin for payment
- Miami, Florida: A major metropolis has about four dozen merchants who say they accept bitcoin and a surprising 651 bitcoin ATMs and tellers There is also hosted the Miami Bitcoin Conference, one of the biggest and first gatherings in the bitcoin indus-

Fig. 12 Digital currency-Solana

try. Both the European headquarters of BitPay, a company that offers payment services, and Bitfury, a company that makes mining equipment, are located in Amsterdam. There are roughly six bitcoin ATMs available to the city's 840,000 residents

The main reason of it's popularity among developed countries is the astonishing properties that digital currency has maintained. It's level of feasibility, fast services, cyber security system, less transaction charges, decentralization, a user's own ownership, less volatile than cash, acceptable worldwide, transnational digital currency, peer-to-peer transactions, safer for merchants, discrete and confidential make digital currency a worthwhile system to be adopted. The following chart in Fig. 13 highlights some dramatic growth in cryptocurrency over time (https://explodingtopics.com/blog/number-of-cryptocurrencies):

To select the most suitable digital currency to be shifted from the traditional currency in a specified country for the transactions and business, *m*F*N*SS PROMETHEE shares the burden of selecting the best digital currency. Consider the five alternatives as $\alpha = \{\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5\}$ to be selected and a panel of experts as $\beta = \{\beta_1, \beta_2, \beta_3, \beta_4, \beta_5\}$, which consists of the following team:

- Finance expert (β_1): The main responsibility of the finance expert is to analyze the country's own budget and accommodation to shift from the traditional currency to digital one, how much expenses the government has to bear in accordance with public training, advertisements, mass awareness and providing necessary facilitation to it's people.
- Security expert (β_2): To make the transactions transparent and non-repudiable is the foremost duty of a security expert, as e is intended to make the accounts of digital currency holder super secure, decentralized, providing the complete ownership to the account holder and prevent the account from foreign hackers.
- IT expert (β_3): His main expertise must be in the technical advancement of the accounts of digital currency, make sure to provide a simple set up, easy to understand and use, speedy money transactions, fastest operating systems either for a trivial transaction like shopping or dealings of millions and trillions in business, all of these transactions must be utmost speedy, fast, secure and reliable.
- Economist (β_3): Economist will ensure how the country or nation make progress by switching from the paper currency to the digital currency, how the country's economy could be affected by this modern technology, how much will be the revenue generate by this technical advancement and most importantly how the country could stand with the the others in this race of success.

The above mentioned specialists decide on the basis of a survey taken on the considered criteria as $\gamma = \{\gamma_1, \gamma_2, \gamma_3, \gamma_4\}$ that are explained as follows:

- Security (γ_1): Security is the first and foremost concern regarding the digital currency. The accounts and transactions of digital currency holder must be *transparent*, *opaque* and *non-repudiable*. Non-repudiable means that if bitcoin is transacted once, the receiver can't claim that he never received any bitcoin.
- Technical advancement (γ_2): As the paper currency storing in banks or in houses' lockers, it is much prone to be stolen or looted by some one, but as digital currency has no physical existence so it is much safer to use. Technical advancement of digital currency involves 3-poles as it is *easy to set up*, *digital existence* and *value deter-*

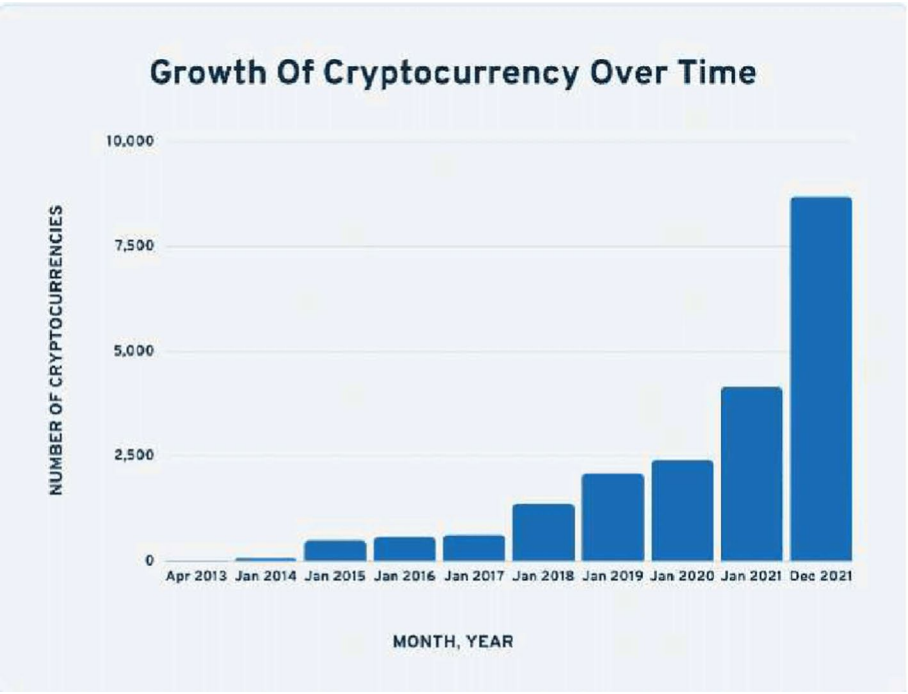


Fig. 13 Growth of cryptocurrency over time

mined by demand. There is no fixed value or price of any digital currency. it's value totally depends on it's demand in market.

- Authority (γ_3): The The main advantage of shifting from traditional to digital currency is that digital currency guarantees the complete ownership to the account holder, ensures the non-interference of any second party in your transactions. Authority of the digital currency comprises of *decentralization*, *commission of own choice* and *unbanked jurisdiction*.
- Global Network (γ_4): Global network means that digital currency enables to globally interact with all the other countries because you can do transactions or business dealings with people in any corner of the world. Globally network makes the transactions *independent of area*, *liberated from converting one currency into other* and *create one person's will*.

4.5 Steps for the selection of the best digital currency through *m*F*N*S PROMETHEE technique

The presented technique, *m*F*N*S PROMETHEE helps us allot in making the apt decisions in every field of life. Here, the numerical example of selection of the best digital currency is considered where by utilizing the multi-polar nature of it's attributes and the property of ranking them, we come to the most satisfied selected alternative. The detailed calculations

Table 18 3*F*6*SDM* of the finance expert (β_1)

β_1	γ_1	γ_2	γ_3	γ_4
α_1	(4, 0.71, 0.69, 0.65)	(4, 0.64, 0.77, 0.69)	(3, 0.57, 0.48, 0.43)	(5, 0.94, 0.87, 0.93)
α_2	(2, 0.38, 0.33, 0.21)	(3, 0.55, 0.44, 0.54)	(2, 0.28, 0.31, 0.27)	(3, 0.58, 0.40, 0.42)
α_3	(5, 0.94, 0.87, 0.97)	(4, 0.73, 0.68, 0.62)	(5, 0.82, 0.95, 0.88)	(5, 0.96, 0.89, 0.87)
α_4	(1, 0.15, 0.08, 0.18)	(2, 0.36, 0.27, 0.29)	(2, 0.34, 0.24, 0.32)	(1, 0.06, 0.09, 0.11)
α_5	(3, 0.49, 0.53, 0.58)	(3, 0.52, 0.46, 0.48)	(4, 0.75, 0.69, 0.72)	(3, 0.58, 0.42, 0.47)

Table 19 3*F*6*SDM* of security specialist (ς_2)

β_2	γ_1	γ_2	γ_3	γ_4
α_1	(3, 0.58, 0.43, 0.57)	(4, 0.75, 0.66, 0.79)	(3, 0.59, 0.48, 0.57)	(3, 0.42, 0.53, 0.46)
α_2	(2, 0.32, 0.22, 0.26)	(2, 0.25, 0.33, 0.27)	(1, 0.14, 0.16, 0.07)	(1, 0.04, 0.06, 0.18)
α_3	(4, 0.78, 0.69, 0.67)	(5, 0.92, 0.85, 0.97)	(5, 0.88, 0.99, 0.97)	(4, 0.79, 0.68, 0.69)
α_4	(1, 0.05, 0.02, 0.11)	(1, 0.01, 0.06, 0.12)	(2, 0.31, 0.26, 0.23)	(1, 0.04, 0.06, 0.03)
α_5	(2, 0.37, 0.29, 0.31)	(1, 0.09, 0.05, 0.13)	(3, 0.53, 0.47, 0.46)	(3, 0.48, 0.52, 0.49)

for selection of the best digital currency through *m*F*N*S PROMETHEE are performed in the following steps:

(1) The individual assessments of each of the expert, in form of 3-polar fuzzy 6-soft set, are arranged in Tables 18, 19, 20 and 21 respectively.

(2) To have a joint effect of these individual assessments, *m*F*N*SA operator is being used given in Eqs. 6 and 8, represented as 3*F*6*SDM* in Table 22.

(3) By using Eq. 9, construct a score matrix displayed in Table 23 which calculates deviation of the alternatives.

(4) Calculate the divergence of the choices according to the score degree by using Eq. 10 and this deviation is displayed in Table 24.

(5) Choose one of the six types of preference functions developed by Chen (2018) based on the preference bounds to check out the superiority of each option. The generalized preference functions is shown in Table 25.

(6) The generalized criteria preference function by Chen (2018), which is shown in Table 26, is used to compute the degree of precedence for each pair of alternatives with respect to each criterion.

(7) The multi-criteria preference index, which represents likelihood of the experts of an alternative over other, is derived using Eq. 12 based on the preference function and the

Table 20 3*F*6*SDM* of IT expert (ς_3)

β_3	γ_1	γ_2	γ_3	γ_4
α_1	(4, 0.68, 0.73, 0.77)	(5, 0.83, 0.84, 0.91)	(3, 0.57, 0.49, 0.42)	(4, 0.75, 0.68, 0.72)
α_2	(3, 0.57, 0.48, 0.41)	(2, 0.39, 0.26, 0.28)	(2, 0.31, 0.26, 0.29)	(3, 0.47, 0.53, 0.48)
α_3	(5, 0.94, 0.87, 0.99)	(5, 0.91, 0.82, 0.89)	(4, 0.78, 0.79, 0.68)	(5, 0.94, 0.86, 0.92)
α_4	(2, 0.24, 0.32, 0.34)	(1, 0.04, 0.03, 0.09)	(1, 0.14, 0.16, 0.08)	(2, 0.36, 0.28, 0.37)
α_5	(4, 0.71, 0.68, 0.64)	(3, 0.45, 0.57, 0.46)	(3, 0.53, 0.48, 0.51)	(3, 0.47, 0.52, 0.58)

Table 21 3*F*6*SDM* of economist (ς_4)

β_4	γ_1	γ_2	γ_3	γ_4
α_1	(4, 0.68, 0.79, 0.63)	(3, 0.54, 0.49, 0.46)	(5, 0.94, 0.88, 0.83)	(4, 0.63, 0.78, 0.74)
α_2	(2, 0.27, 0.33, 0.29)	(2, 0.34, 0.27, 0.36)	(3, 0.47, 0.53, 0.48)	(3, 0.51, 0.47, 0.49)
α_3	(5, 0.95, 0.86, 0.97)	(5, 0.88, 0.99, 0.98)	(4, 0.79, 0.64, 0.78)	(4, 0.73, 0.76, 0.68)
α_4	(1, 0.07, 0.08, 0.13)	(1, 0.14, 0.04, 0.16)	(2, 0.37, 0.26, 0.33)	(2, 0.31, 0.27, 0.37)
α_5	(3, 0.52, 0.44, 0.45)	(4, 0.74, 0.68, 0.72)	(3, 0.58, 0.48, 0.54)	(4, 0.75, 0.69, 0.72)

Table 22 Aggregated 3F6SDM

β^*	γ_1	γ_2	γ_3	γ_4
α_1	(4, 0.63, 0.66, 0.66)	(5, 0.69, 0.69, 0.71)	(5, 0.67, 0.58, 0.56)	(5, 0.69, 0.72, 0.71)
α_2	(3, 0.39, 0.34, 0.29)	(3, 0.38, 0.33, 0.36)	(3, 0.30, 0.32, 0.38)	(3, 0.40, 0.39, 0.39)
α_3	(5, 0.90, 0.82, 0.90)	(5, 0.86, 0.84, 0.87)	(5, 0.82, 0.84, 0.83)	(5, 0.86, 0.80, 0.79)
α_4	(2, 0.13, 0.13, 0.19)	(2, 0.14, 0.10, 0.17)	(2, 0.29, 0.23, 0.24)	(2, 0.19, 0.18, 0.22)
α_5	(4, 0.52, 0.49, 0.50)	(4, 0.45, 0.44, 0.45)	(4, 0.60, 0.53, 0.56)	(4, 0.57, 0.54, 0.57)

Table 23 Score matrix

	γ_1	γ_2	γ_3	γ_4
α_1	1.46	1.70	1.60	1.70
α_2	0.94	0.96	0.90	0.99
α_3	1.88	1.85	1.83	1.81
α_4	0.55	0.53	0.65	0.60
α_5	1.30	1.25	1.36	1.36

criteria weights calculated using the AHP technique 5. In Table 27, the multi-criteria preference index is displayed.

(8) For the deduction of partial outranking relation of the alternatives, *m*F*N*S PROMETHEE I is being used by the formulation of outflow and inflow of the alternatives by using Eqs. 13 and 14, respectively. The outgoing and incoming flows of an alterna-

Table 24 Divergence of the alternatives according to the criteria

Alternatives	γ_1	γ_2	γ_3	γ_4	*Alternatives*	γ_1	γ_2	γ_3	γ_4
$\alpha_1\alpha_2$	0.52	0.74	0.70	0.71	$\alpha_1\alpha_3$	−0.42	−0.15	−0.23	−0.11
$\alpha_1\alpha_4$	0.91	1.17	0.95	1.10	$\alpha_1\alpha_5$	0.16	0.45	0.24	0.34
$\alpha_2\alpha_1$	−0.52	−0.74	−0.70	−0.71	$\alpha_2\alpha_3$	−0.94	−0.89	−0.93	−0.82
$\alpha_2\alpha_4$	0.39	0.43	0.25	0.39	$\alpha_2\alpha_5$	−0.36	−0.29	−0.46	−0.37
$\alpha_3\alpha_1$	0.42	0.15	0.23	0.11	$\alpha_3\alpha_2$	0.94	0.89	0.93	0.82
$\alpha_3\alpha_4$	1.33	1.32	1.18	1.21	$\alpha_3\alpha_5$	0.58	0.60	0.47	0.45
$\alpha_4\alpha_1$	−0.91	−1.17	−0.95	−1.1	$\alpha_4\alpha_2$	−0.39	−0.43	−0.25	−0.39
$\alpha_4\alpha_3$	−1.33	−1.32	−1.18	−1.21	$\alpha_4\alpha_5$	−0.75	−0.72	−0.71	−0.76
$\alpha_5\alpha_1$	−0.16	−0.45	−0.24	−0.34	$\alpha_5\alpha_2$	0.36	0.29	0.46	0.37
$\alpha_5\alpha_3$	−0.58	−0.60	−0.47	−0.45	$\alpha_5\alpha_4$	0.75	0.72	0.71	0.76

Table 25 Type of criteria and preference functions

Criteria	*Max or Min*	*Type of criteria*	*Parameters*
α_1	*Max*	*I*	*Nill*
α_2	*Max*	*II*	$k = 0.01$
α_3	*Max*	*II*	$k = 0.01$
α_4	*Max*	*I*	*Nill*

tive with respect to other alternative is deduced by the equations 15 and 16, respectively and displayed in Table 28.

The intersection of these flows facilitates us with the partial outranking of the alternatives that can be estimated by the Eq. 17 as follows:

$\alpha_1\hat{P}\alpha_2$, $\alpha_1\hat{P}\alpha_4$, $\alpha_1\hat{P}\alpha_5$, $\alpha_2\hat{P}\alpha_4$, $\alpha_3\hat{P}\alpha_1$, $\alpha_3\hat{P}\alpha_2$, $\alpha_3\hat{P}\alpha_4$, $\alpha_3\hat{P}\alpha_5$, $\alpha_5\hat{P}\alpha_2$, $\alpha_5\hat{P}\alpha_4$

Since PROMETHEE I gives the partial outranking relations but this example becomes the special case when PROMETHEE I provides the complete outranking relations among the alternatives. The ordering relation among the selected options by PROMETHEE I method is displayed in Fig. 14.

(9) To deduce the complete outranking of the alternatives with F*N*S PROMETHEE II, the net outranking of the options is evaluated by Eq. 18. Now, by using Eq. 19, complete ranking relation is extracted as shown in Table 29.

(10) According to the results of the computations above, option α_3 is chosen as the best digital money, and the alternatives are listed in the following order: $\varphi_3 > \varphi_1 > \varphi_5 > \varphi_4 > \varphi_2$.

Table 26 Generalized criteria preference function

Alternatives	γ_1	γ_2	γ_3	γ_4	*Alternatives*	γ_1	γ_2	γ_3	γ_4
$\alpha_1\alpha_2$	1	1	1	1	$\alpha_1\alpha_3$	0	0	0	0
$\alpha_1\alpha_4$	1	1	1	1	$\alpha_1\alpha_5$	1	1	1	1
$\alpha_2\alpha_1$	0	0	0	0	$\alpha_2\alpha_3$	0	0	0	0
$\alpha_2\alpha_4$	1	1	1	1	$\alpha_2\alpha_5$	0	0	0	0
$\alpha_3\alpha_1$	1	1	1	1	$\alpha_3\alpha_2$	1	1	1	1
$\alpha_3\alpha_4$	1	1	1	1	$\alpha_3\alpha_5$	1	1	1	1
$\alpha_4\alpha_1$	0	0	0	0	$\alpha_4\alpha_2$	0	0	0	0
$\alpha_4\alpha_3$	0	0	0	0	$\alpha_4\alpha_5$	0	0	0	0
$\alpha_5\alpha_1$	0	0	0	0	$\alpha_5\alpha_2$	1	1	1	1
$\alpha_5\alpha_3$	0	0	0	0	$\alpha_5\alpha_4$	1	1	1	1

Table 27 Index of multi-criteria preference

	α_1	α_2	α_3	α_4	α_5
α_1	–	1	0	1	1
α_2	0	–	0	1	0
α_3	1	1	–	1	1
α_4	0	0	0	–	0
α_5	0	1	0	1	–

Table 28 Alternatives' positive and negative flows (PROMETHE I)

Alternatives	*Outgoing flow*(Y^+)	*Ingoing flow*(Y^-)
φ_1	0.75	0.25
α_2	0.25	0.75
α_3	1	0
α_4	0	1
α_5	0.5	0.5

5 Comparative Analysis

5.1 Comparison of the proposed strategy with existing one (*m*-polar fuzzy PROMETHEE) (Akram and Shumaiza 2020)

This section is entailed with the comparison of our proposed methodology, *m*FNS PROMETHEE, with the existing technique (Akram and Shumaiza 2020) on the application of production of electricity through human movement. We apply *m*-polar fuzzy PROMETHEE technique (Akram and Shumaiza 2020) on this application under *m*-polar environment and will justify the credibility of our proposed technique. Now, consider the example of production of electricity through human movement but under *m*-polar fuzzy environment. The evaluation given by the specialists are given in Tables 30, 31, 32, 33, 34,

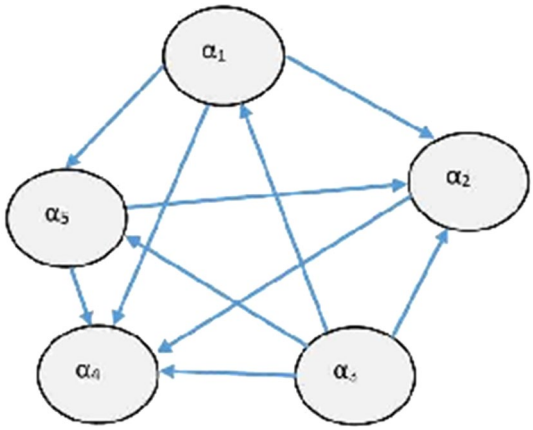


Fig. 14 Outranking of the alternatives by PROMETHEE I

35, 36. Now, the same types of criteria are specified, during application of *m*F*N*S PROMETHEE technique, by the generalized preference functions as given in Table 13. Additionally, Table 37 displays the preference degree for each pair of alternatives in relation to each criterion.

Now, the same types of criteria are specified, during application of *m*F*N*S PROMETHEE technique, by the generalized preference functions as given in Table 13. Additionally, Table 37 displays the preference degree for each pair of alternatives in relation to each criterion.

The criteria weights used for multi-criteria preference index are the same as used under *m*F*N*S PROMETHEE technique in Table 5.The weighted averages of these preference functions make form the multi-criteria preference index of alternatives shown in Table 38.

The outgoing and incoming flows of the alternatives are presented in Table 39.

We can partially outrank the alternatives that can be estimated by Eq. 17 with the help of intersection of these flows:

$\alpha_1 \hat{P} \alpha_4$, $\alpha_2 \hat{P} \alpha_1$, $\alpha_2 \hat{P} \alpha_4$, $\alpha_2 \hat{P} \alpha_3$, $\alpha_3 \hat{P} \alpha_4$, $\alpha_5 \hat{P} \alpha_1$, $\alpha_5 \hat{P} \alpha_2$, $\alpha_5 \hat{P} \alpha_3$, $\alpha_5 \hat{P} \alpha_4$.

But α_1 and α_3 are incomparable. The partial outranking relation among the alternatives by PROMETHEE I is shown in Fig. 15.

Now, Table 40 displays how PROMETHEE II completely outranked the alternatives.

By considering all the computation, we extract that alternative α_5 is selected as the best technique for the production of electricity through human movement, and the ranking of the required options is as follows:

$\varphi_5 > \varphi_2 > \varphi_3 > \varphi_1 > \varphi_4$. Table 41 refers to the comparison between the proposed strategy and the existing one (Akram and Shumaiza 2020). PROMETHEE technique incorporating *m*F*N*S values and *m*F values provides us with the similar best option.

Table 29 Net outranking flow of the alternatives (PROMETHE II)

Alternatives	*Net flow*(*Y*)
α_1	0.5
α_2	−0.5
α_3	1
α_4	−1
α_5	0

Table 30 *PF6SDM* of the finance expert (ς_1)

ς_1	η_1	η_2	η_3	η_4
φ_1	(0.96, 0.94, 0.87)	(0.38, 0.27, 0.29)	(0.76, 0.69, 0.72)	(0.48, 0.52, 0.59)
φ_2	(0.72, 0.68, 0.62)	(0.58, 0.49, 0.43)	(0.26, 0.33, 0.39)	(0.16, 0.08, 0.12)
φ_3	(0.57, 0.46, 0.56)	(0.94, 0.87, 0.91)	(0.58, 0.46, 0.49)	(0.83, 0.97, 0.94)
φ_4	(0.62, 0.71, 0.78)	(0.29, 0.31, 0.25)	(0.37, 0.31, 0.25)	(0.05, 0.16, 0.18)
φ_5	(0.38, 0.27, 0.29)	(0.77, 0.73, 0.68)	(0.15, 0.04, 0.11)	(0.86, 0.93, 0.96)

Table 31 *PF6SDM* of security specialist (ς_2)

ς_1	η_1	η_2	η_3	η_4
φ_1	(0.17, 0.05, 0.18)	(0.72, 0.65, 0.79)	(0.58, 0.42, 0.55)	(0.78, 0.65, 0.68)
φ_2	(0.63, 0.77, 0.67)	(0.84, 0.93, 0.86)	(0.35, 0.27, 0.38)	(0.93, 0.86, 0.94)
φ_3	(0.85, 0.99, 0.92)	(0.37, 0.28, 0.39)	(0.79, 0.64, 0.72)	(0.07, 0.15, 0.08)
φ_4	(0.38, 0.29, 0.31)	(0.59, 0.42, 0.54)	(0.93, 0.88, 0.92)	(0.74, 0.69, 0.78)
φ_5	(0.75, 0.69, 0.72)	(0.15, 0.16, 0.18)	(0.49, 0.47, 0.52)	(0.27, 0.36, 0.33)

Table 32 *PF6SDM* of environmentalist (ς_3)

ς_1	η_1	η_2	η_3	η_4
φ_1	(0.55, 0.44, 0.58)	(0.08, 0.16, 0.05)	(0.69, 0.77, 0.73)	(0.94, 0.87, 0.92)
φ_2	(0.27, 0.36, 0.28)	(0.78, 0.64, 0.72)	(0.98, 0.85, 0.93)	(0.57, 0.48, 0.44)
φ_3	(0.94, 0.97, 0.88)	(0.57, 0.47, 0.42)	(0.35, 0.29, 0.36)	(0.07, 0.04, 0.09)
φ_4	(0.15, 0.18, 0.16)	(0.28, 0.37, 0.33)	(0.94, 0.87, 0.82)	(0.31, 0.38, 0.22)
φ_5	(0.74, 0.69, 0.77)	(0.48, 0.59, 0.54)	(0.06, 0.08, 0.16)	(0.86, 0.94, 0.82)

Table 33 *PF6SDM* of economist (ς_4)

ς_1	η_1	η_2	η_3	η_4
φ_1	(0.63, 0.79, 0.75)	(0.27, 0.35, 0.37)	(0.08, 0.15, 0.11)	(0.98, 0.84, 0.95)
φ_2	(0.84, 0.85, 0.97)	(0.06, 0.15, 0.18)	(0.55, 0.48, 0.57)	(0.27, 0.37, 0.29)
φ_3	(0.25, 0.38, 0.36)	(0.53, 0.47, 0.49)	(0.64, 0.78, 0.69)	(0.78, 0.69, 0.72)
φ_4	(0.76, 0.67, 0.77)	(0.25, 0.38, 0.29)	(0.97, 0.88, 0.95)	(0.49, 0.51, 0.42)
φ_5	(0.58, 0.47, 0.56)	(0.98, 0.99, 0.84)	(0.09, 0.16, 0.19)	(0.68, 0.73, 0.65)

Table 16 shows that the similar best option is selected by considering both of the technique, our proposed strategy and *m*-polar fuzzy PROMETHEE (Akram and Shumaiza 2020), that proves the reliability of our discussed *mFN*S PROMETHEE approach.

Table 34 Aggregated F6SDM

ς_1	η_1	η_2	η_3	η_4
φ_1	(0.58, 0.56, 0.60)	(0.36, 0.36, 0.38)	(0.53, 0.51, 0.53)	(0.8, 0.72, 0.79)
φ_2	(0.62, 0.67, 0.64)	(0.57, 0.55, 0.55)	(0.54, 0.48, 0.57)	(0.48, 0.45, 0.45)
φ_3	(0.65, 0.7, 0.67)	(0.60, 0.52, 0.55)	(0.59, 0.54, 0.57)	(0.44, 0.46, 0.46)
φ_4	(0.48, 0.46, 0.51)	(0.35, 0.37, 0.35)	(0.80, 0.74, 0.74)	(0.4, 0.44, 0.4)
φ_5	(0.61, 0.53, 0.59)	(0.60, 0.62, 0.56)	(0.2, 0.19, 0.25)	(0.67, 0.72, 0.69)

Table 35 Score matrix

	η_1	η_2	η_3	η_4
φ_1	0.58	0.37	0.52	0.77
φ_2	0.64	0.56	0.53	0.46
φ_3	0.67	0.56	0.57	0.45
φ_4	0.48	0.36	0.76	0.41
φ_5	0.58	0.59	0.21	0.69

Table 36 Divergence of the alternatives according to the attributes

Alternatives	η_1	η_2	η_3	η_4	*Alternatives*	η_1	η_2	η_3	η_4
$\varphi_1\varphi_2$	−0.06	−0.19	−0.01	0.31	$\varphi_1\varphi_3$	−0.09	−0.19	−0.05	0.32
$\varphi_1\varphi_4$	0.1	0.01	−0.24	0.36	$\varphi_1\varphi_5$	0	−0.22	0.31	0.08
$\varphi_2\varphi_1$	0.06	0.19	0.01	−0.31	$\varphi_2\varphi_3$	−0.03	0	−0.04	0.01
$\varphi_2\varphi_4$	−0.03	0	−0.04	0.01	$\varphi_2\varphi_5$	0.06	−0.03	0.32	−0.23
$\varphi_3\varphi_1$	0.09	0.19	0.05	−0.32	$\varphi_3\varphi_2$	0.03	0	0.04	−0.01
$\varphi_3\varphi_4$	0.19	0.2	−0.19	0.04	$\varphi_3\varphi_5$	0.09	−0.03	0.36	−0.24
$\varphi_4\varphi_1$	−0.1	−0.01	0.24	−0.36	$\varphi_4\varphi_2$	−0.16	−0.2	0.23	−0.05
$\varphi_4\varphi_3$	−0.19	−0.2	0.19	−0.04	$\varphi_4\varphi_5$	−0.1	−0.23	0.55	−0.28
$\varphi_5\varphi_1$	0	0.22	−0.31	−0.08	$\varphi_5\varphi_2$	−0.06	0.03	−0.32	0.23
$\varphi_5\varphi_3$	−0.09	0.03	−0.36	0.24	$\varphi_5\varphi_4$	0.1	0.23	−0.55	0.28

Table 37 Generalized criteria preference function

Alternatives	γ_1	γ_2	γ_3	γ_4	*Alternatives*	γ_1	γ_2	γ_3	γ_4
$\alpha_1\alpha_2$	0	0	0	1	$\alpha_1\alpha_3$	0	0	0	1
$\alpha_1\alpha_4$	1	0	0	1	$\alpha_1\alpha_5$	0	0	1	1
$\alpha_2\alpha_1$	1	1	0	0	$\alpha_2\alpha_3$	0	0	0	1
$\alpha_2\alpha_4$	1	1	0	1	$\alpha_2\alpha_5$	1	0	1	0
$\alpha_3\alpha_1$	1	1	1	0	$\alpha_3\alpha_2$	1	0	1	0
$\alpha_3\alpha_4$	1	1	0	1	$\alpha_3\alpha_5$	1	0	1	0
$\alpha_4\alpha_1$	0	0	1	0	$\alpha_4\alpha_2$	0	0	1	0
$\alpha_4\alpha_3$	0	0	1	0	$\alpha_4\alpha_5$	0	0	1	0
$\alpha_5\alpha_1$	0	1	0	1	$\alpha_5\alpha_2$	0	1	0	1
$\alpha_5\alpha_3$	0	1	0	1	$\alpha_5\alpha_4$	1	1	0	1

5.2 Comparison of the proposed method with existing (*m*-polar fuzzy ELECTRE-I) technique (Akram et al. 2019)

To underscore the credibility of our proposed technique, we compare this technique with existing *m*-polar fuzzy ELECTRE-I (Akram et al. 2019) by considering the numerical example of selecting the best digital currency that plays a vital role in digitization of the global economy. Now, we consider the data in form of *m*-polar fuzzy (*m*F)set in spite of *m*-polar fuzzy *N*-soft set and perform ELECTRE-I on it. The obtaind results show that the similar option is being selected after applying ELECTRE-I on the numerical values considered in the example but in form of *m*F set which guarantees the credibility of our proposed technique. Consider an example of selection of the best digital currency with the same alternatives, attributes and group of experts. The Aggregated or collective assessment of panel of experts in form of *m*F data is given in Table 42.

The weighted 3-polar fuzzy decision matrix (3FDM) is evaluated in Table 43.

Table 38 Multi-criteria preference index

	α_1	α_2	α_3	α_4	α_5
α_1	–	0.372	0.372	0.416	0.537
α_2	0.463	–	0.372	0.835	0.209
α_3	0.628	0.209	–	0.835	0.209
α_4	0.165	0.165	0.165	–	0.165
α_5	0.419	0.791	0.791	0.835	–

Table 39 Positive and negative flows of the alternatives (PROMETHE I)

Alternatives	*Outgoing flow*(Y^+)	*Ingoing flow*(Y^-)
φ_1	0.42	0.42
α_2	0.47	0.38
α_3	0.47	0.43
α_4	0.17	0.73
α_5	0.71	0.28

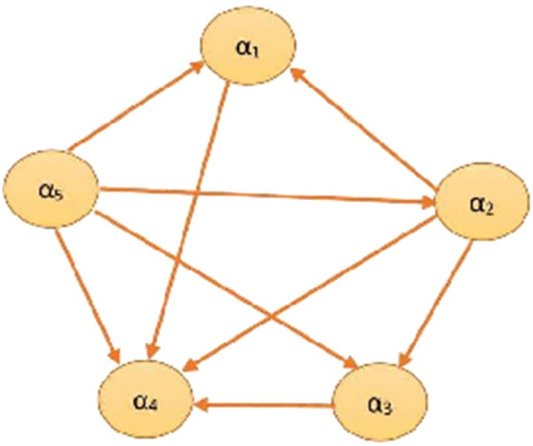


Fig. 15 Outranking of the alternatives by PROMETHEE I

Table 40 Net outranking flow of the alternatives (PROMETHE II)

Alternatives	*Net flow*(Y)
α_1	0
α_2	0.09
α_3	0.04
α_4	−0.56
α_5	0.43

The concordance and discordance set are evaluated in Tables 44 and 45.

The concordance and discordance matrices are represented in Tables 46 and 47.

The concordance and discordance indices are calculated as $\overline{f} = 0.7$ and $\overline{g} = 0.45$, respectively. The 3-polar fuzzy aggregated dominance matrix is given in Table 48.

The outranking relation is shown in the following graph Fig. 16.

The directed graph 16 shows us that all the arrows are emerging from α_3 to all the other alternatives which means that α_3 is the best option that can be selected for digitizing the global economy. This is the same alternative we have selected from *mFNS* PROMETHEE technique and this same outcome from both of the techniques guarantees the validity of our discussed methodology.

6 Sensitivity analysis and discussion

This section comprises of the detailed analysis of the sensitivity of our proposed technique that under which conditions and circumstance this technique can work efficiently or cannot produce useful results. While, on the other hand, we will also take a look on the results being produced in the underlying numerical example of producing electricity through human body and selection of best digital currency for digitization the global economy.

1. Importance of the proposed technique Many advancement have been done in the past few years to tackle the uncertainty of human nature with the parametrization of attributes [?]. But as the world is evolving day by day and researches come up with new problems of making the apt decision in this complex modern life. Only fuzzy *N*-soft sets get insufficient to deal the technicalities of the advanced world. And, on the other side, multi-polar fuzzy set (Akram and Shumaiza 2020) successes in coping up with the multi-

Table 41 Comparative Analysis

Methods	*Outranking relation*	*Best alternative*
mFNS PROMETHEE (*PROPOSED*)	$\varphi_5 > \varphi_2 > \varphi_1 > \varphi_4 > \varphi_3$	φ_5
mF PROMETHEE (Akram and Shumaiza 2020)	$\varphi_5 > \varphi_2 > \varphi_3 > \varphi_1 > \varphi_4$	φ_5

Table 42 Aggregated 3FDM

β^*	γ_1	γ_2	γ_3	γ_4
α_1	(0.63, 0.66, 0.66)	(0.69, 0.69, 0.71)	(0.67, 0.58, 0.56)	(0.69, 0.72, 0.71)
α_2	(0.39, 0.34, 0.29)	(0.38, 0.33, 0.36)	(0.30, 0.32, 0.38)	(0.40, 0.39, 0.39)
α_3	(0.90, 0.82, 0.90)	(0.86, 0.84, 0.87)	(0.82, 0.84, 0.83)	(0.86, 0.80, 0.79)
α_4	(0.13, 0.13, 0.19)	(0.14, 0.10, 0.17)	(0.29, 0.23, 0.24)	(0.19, 0.18, 0.22)
α_5	(0.52, 0.49, 0.50)	(0.45, 0.44, 0.45)	(0.60, 0.53, 0.56)	(0.57, 0.54, 0.57)

Table 43 Weighted 3FDM

β^*	γ_1	γ_2	γ_3	γ_4
α_1	(0.027, 0.029, 0.029)	(0.29, 0.29, 0.297)	(0.111, 0.096, 0.092)	(0.257, 0.268, 0.264)
α_2	(0.017, 0.015, 0.013)	(0.159, 0.138, 0.151)	(0.05, 0.053, 0.063)	(0.149, 0.145, 0.145)
α_3	(0.034, 0.036, 0.034)	(0.36, 0.352, 0.365)	(0.132, 0.139, 0.137)	(0.32, 0.298, 0.294)
α_4	(0.005, 0.005, 0.008)	(0.059, 0.042, 0.071)	(0.048, 0.038, 0.040)	(0.071, 0.067, 0.082)
α_5	(0.023, 0.022, 0.022)	(0.189, 0.184, 0.189)	(0.010, 0.088, 0.092)	(0.212, 0.201, 0.212)

Table 44 3-polar fuzzy concordance set

j	1	2	3	4	5
F_{1j}	–	1, 2, 3, 4		1, 2, 3, 4	1, 2, 3, 4
F_{2j}		–		1, 2, 3, 4	
F_{3j}	1, 2, 3, 4	1, 2, 3, 4	–	1, 2, 3, 4	1, 2, 3, 4
F_{4j}				–	
F_{5j}		1, 2, 3, 4		1, 2, 3, 4	–

Table 45 3-polar fuzzy discordance set

j	1	2	3	4	5
F_{1j}	–		1, 2, 3, 4		
F_{2j}	1, 2, 3, 4	–	1, 2, 3, 4		1, 2, 3, 4
F_{3j}			–		
F_{4j}	1, 2, 3, 4	1, 2, 3, 4	1, 2, 3, 4	–	1, 2, 3, 4
F_{5j}	1, 2, 3, 4		1, 2, 3, 4		–

Table 46 3-polar fuzzy concordance matrix

j	1	2	3	4	5
F_{1j}	–	1	0	1	1
F_{2j}	0	–	0	1	0
F_{3j}	1	1	–	1	1
F_{4j}	0	0	0	–	0
F_{5j}	0	1	0	1	–

Table 47 3-polar fuzzy discordance matrix

j	1	2	3	4	5
G_{1j}	–	0	1	0	0
G_{2j}	1	–	1	0	1
G_{3j}	0	0	–	0	0
G_{4j}	1	1	1	–	1
F_{5j}	1	0	1	0	–

Table 48 Aggregated 3-polar fuzzy dominance matrix

Alternatives	α_1	α_2	α_4	α_4	α_5
α_1	–	1	0	1	1
α_2	0	–	0	1	0
α_3	1	1	–	1	1
α_4	0	0	0	–	0
α_5	0	1	0	1	–

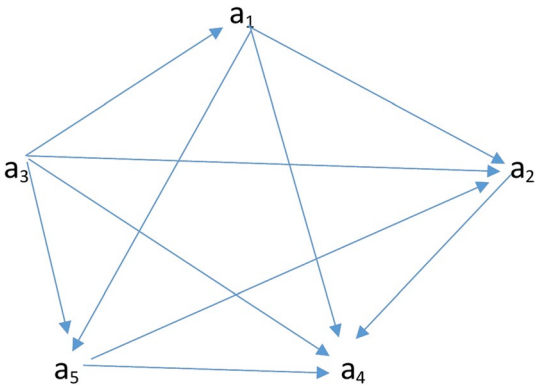


Fig. 16 Directed grah of outranking relation

polar attributes but on the same time fails to make the parametrization of attributes. In order to cover all bounds of this modern world, a novel MCGDM technique, named as, PROMETHEE combined with *m*-polar fuzzy *N*-soft set is proposed in this research paper to cover maximum aspects of having multiple attributes, an *N*-soft grading to the alternatives along with the parametrization of attributes. Hence, *m*F*N*S PROMETHEE technique developed in this paper is very useful for the practical purposes.

2. Qualitative and quantitative analysis The main theme of the proposed technique is to calculate the incoming and outgoing flow of the alternatives based on the deviation

of the alternatives by choosing the suitable preference function. There are variety of preference functions have been developed by Chen (2018) that all have their own characteristics. In the applications of energy producing through human movement and best digital currency, we have used usual or quasi criterion parameter that guarantees more better and authentic result. But by changing the parameters according to the deviation of alternatives, we may have different results. Along with the weights that are assigned to the criteria are also quite essential in the sense that if all the criteria weights are not being normalized then the imprecision rate may be increased.

3. Analysis of the obtained results from the results of existing techniques A sensitive analysis is carried out by performing the proposed strategy and existing ones to the real world problems. The application of generating electricity through human movement is performed by our discussed strategy and the existing one Akram and Shumaiza (2020). Since both of the techniques put forward the same best option but there is difference in their ranking order. The ranking order of the alternatives obtained from both of the techniques is shown in Table 41. This is so because *m*F PROMETHEE technique does not ensure the parametrization of family of attributes, it does not account for the *N*-ordered grading. This technique can work efficiently under multi-polarity of the attributes. While, in the example of best digital currency, the same best alternative is being selected by the proposed technique and existing one, *m*F ELECTRE-I (Akram et al. 2019). This existing technique provides a directed graph showing outranking relations in Fig. 16 but does not compute ordering of the selected options that is not of much reliable in the real world scenario because in this complexity of modern world one requires to have a ranking order from best to worst so that he has a lot more options under consideration according to his terms and conditions.

6.1 Discussion

This subsection is comprised of the discussion and analysis of the research work *m*F*N*S PROMETHEE considering it's working strategy, applications, comparison with other techniques highlighting it's reliability along with the limitations. Prior to this, a notable multi-criteria decision technique, AHP (Saaty 1986) is being utilized to check the consistency of the weights of the criteria and to minimize one's personal interest while making the decision. *m*F*N*S PROMETHEE can work adeptly in the rigorous environment of fuzziness, multi-polar nature of alternatives, group decision scenarios along with the question of ranking the alternatives from best selection to least one. This technique is proved to be of great assistance in decision-making while capturing the fuzziness and multi-polarity of this modern era of digitization. The results obtained from this technique are not only reliable but also strengthens one's ability to select a best way to generate electricity through human movement and digitize the world in an efficient way. To underscore the practicality and applicability of this technique, two numerical examples have been included within the study. The first example explores the generation of electricity through human movement, highlighting the real-world utility of the proposed method. The second example deals with the selection of the best digital currency, showcasing how PROMETHEE under the mFNS framework compares with existing methodologies.

– The most essential breakthrough of this study is to expand the knowledge of a wonderful MCGDM technique, namely, PROMETHEE under *m*-polar fuzzy *N*-soft

(*m*F*N*S) environment that efficiently deals with the multi-polarity of the modern world along with the property of assigning *N*-ordered grades to rank the options by limiting the vagueness of human nature.
- To minimize the vagueness in this revolutionizing era with multiple attributes and make the certain decisions in our daily life challenges, *m*F*N*S PROMETHEE enables us to choose the best alternative depending upon their multi-attributes or features.
- Calculating the inflow and outflow flows of the alternatives in accordance with the divergence of the alternatives' ratings is the primary idea behind the PROMETHEE technique. By calculating score degrees, suitable preference functions, and a multi-criteria preference index, it gives us incomplete and full ordering of the selected options.
- *m*F*N*S PROMETHEE technique provides the best alternative to be selected in multi-polar fuzzy *N*-ordered grading system, in addition, assigns a detailed ordering to the alternatives from the most preferred choice to least one.
- *m*F*N*S PROMETHEE I gives a partial outranking of the alternatives, while *m*F*N*S PROMETHEE II gives us the full ranking of the alternatives. The ranking of both alternatives is shown by the outranking graphs.
- To make the discussed strategy work in a neutral way and produce the results free of one's personal gains, a novel MCDM AHP technique (Saaty 1986) is utilized to calculate the weights of the criteria, normalize them and make the decision free from involvement of any personal influence.
- To demonstrate practical aspects of our proposed technique, two numerical examples are used, such as production of electricity through human movement and selection of the best digital currency used for the transactions, where in the context of *m*F*N*S, the best alternative is selected.
- The outcomes of this strategy are not only trustworthy but also improve one's capacity to choose the optimal method for generating electricity from human activity and effectively digitizing the world.
- To underscore the credibility of our proposed technique, a comparison is being made with *m*-polar fuzzy PROMETHEE (Akram and Shumaiza 2020) and *m*-polar fuzzy ELECTRE-I (Akram et al. 2019) on the similar examples of production of electricity through human movement and choice of best digital currency, respectively. All of the techniques make the similar option the most suitable one that ensures the feasibility of proposed strategy.

7 Merits and demerits of the proposed method

Every technique has it's own pros and cons. Our proposed technique benefits us a lot in variety of ways by empowering us to make a more certain decision in the world of haphazardness, allows us to overcome the vagueness of human nature, makes our decisions or choices more reliable and considerate. This technique suggests a new way to handle the multi-polar nature of this evolving era and in addition of choosing the best option, it also lists the alternatives in *N*-ordered grading strategically. By it's practical implication, it can be proved a great asset in digitization of the global economy. But on the other side of the picture, it also restricts us while facing the complex information in form of complex numbers where phase angles are involved along with the amplitude. This technique can work with multiple membership degrees but only positive ones. It is unable to work efficiently

with non-membership degrees, such as Pythagorean fuzzy sets (Yager 2013). Also when there is a situation where a neutral membership degree (Akram et al. 2023) is involved in making the decisions, this technique fails to impress us.

- *m*F*N*S PROMETHEE is a promising technique to help solve multi-criteria group decision-making problems in our everyday life scenarios to limit the uncertainty of human nature and cope with the multi-polarity of this modern world change.
- In addition to minimizing the vagueness of human nature, the *m*F*N*S PROMETHEE technique also faces multi-valued features or attributes of alternatives.
- In addition to multi-polarity, it is important to rank the alternatives from best to worst, i.e. *N*-ordered grades to the selected alternatives and in this scenario, in this case *m*F*NThe*S PROMETHEE technology proves to be a very coherent tool.
- *m*F*N*S PROMETHEE helped us a lot in choosing the best option among the various options.
- Two numerical examples: production of electricity through human movement and selection of the best digital currency are considered to prove the practicality of our proposed technique.
- The outranking graphs are being made not only shows the partial and complete outranking relations among the alternatives but also put forward the most suitable option.
- For the credibility and validity of our proposed technique, a comparison is being made with the existing technique (Akram and Shumaiza 2020) where the similar outcomes guarantee the applicability of the proposed strategy.

In spite of all the beneficiaries of *m*F*N*S PROMETHEE technique, this technique also have some flaws and pitfalls that are described as follows:

- Besides all the miracles of *m*F*N*S PROMETHEE technique, this technique fails to tackle the complex information. It's performance hampers when the complex data is under the consideration.
- This technique is not useful when discussing the negative aspects as well as the positive aspects of multi-valued features of alternatives.
- *m*F*N*S PROMETHEE technique is not concerned with favor and unfavor of multipolar features as the degree of belongingness and non-belongingness.
- This technique is of no use while considering the neutral membership degree of a decision maker.

8 Conclusion

Digitization serves as an indispensable asset to the global economy, where individuals worldwide converge on a single platform to exchange data, information, knowledge, conduct business transactions, and more. It has extended it's influence far beyond the boundaries of individual industries and nations, revolutionizes the business, transactions, communications and transportation that has been proven extreme beneficial for individuals and organizations worldwide. Undoubtedly, digitization has shrunk the whole world into a single market. Various MCGDM techniques grounded in fuzzy theory have been developed to capture the diverse characteristics and properties essential for enhancing the digital, global, and livable aspects of our world. Among these marvelous strategies for dealing

the uncertainties, PROMETHEE technique has it's own charm in coping with vagueness of human nature in this multi-polar world. The technique proposed in this research paper, *m*F*N*S PROMETHEE assists us in group decision making under multi-attribute fuzzy environment and in addition to select the best option, it also provides the *N*-ordered grading from the most preferred option to least one. Prior to this, an AHP technique is used to minimize one's personal interest in group decision. For practical implication of this technique, two numerical examples are being considered: production of electricity through human movement and selection of the best digital currency. To underscore it's reliability, comparison is being made with the existing *m*F PROMETHEE (Akram and Shumaiza 2020) where the same results guarantee the feasibility of this technique. Finally, a section on merits and de merits is added to demonstrate the productivity and pitfalls of our proposed technique. Because, this technique cannot preserve complex data (amplitude and phase term), negative aspects of an attribute rather than alternatives and the neutral membership degrees. For the future research, we aim to expand the breakthrough features of MCGDM technique PROMETHEE under complex data while showing the modernity and complexity of the world. We would pay special attention to the future implication of the proposed method in field of engineering, medical sciences, information technology etc. We would like to broader this concept under negative membership degrees along with positive ones so that this technique would be able to work in more and more complex environment and makes us enable to opt the best option. Practitioners can use this proposed *m*F*N*S PROMETHEE technique for any kind of practical task where he is faced with multipolar information as well as the artificiality of assigning *N*-grades to alternatives and barriers to limit personal impact of human error.

Author contributions MA, MS and MD: conceptualized and designed the study, analysed the data, and wrote the manuscript.

Funding There is no specific funding for this project.

Data Availibility No data were used to support this study.

Declarations

Conflict of interest The authors declare no conflict of interest.

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