Artificial Intelligence Review (2023) 56:7315–7346
https://doi.org/10.1007/s10462-022-10361-8

An approach to prevent weight manipulation by minimum adjustment and maximum entropy method in social network group decision making

Qi Sun[1,2] · Jian Wu[1,2] · Francisco Chiclana[3,4] · Sha Wang[1,2] · Enrique Herrera-Viedma[4,5] · Ronald R. Yager[6]

Published online: 13 December 2022

Abstract
In social network group decision making (SN-GDM) problem, subgroup weights are mostly unknown, many approaches have been proposed to determine the subgroup weights. However, most of these methods ignore the weight manipulation behavior of subgroups. Some studies indicated that weight manipulation behavior hinders consensus efficiency. To deal with this issue, this paper proposes a theoretical framework to prevent weight manipulation in SN-GDM. Firstly, a community detection based method is used to cluster the large group. The power relations of subgroups are measured by the power index (PI), which depends on the subgroups size and cohesion. Then, a minimum adjustment feedback model with maximum entropy is proposed to prevent subgroups' manipulation behavior. The minimum adjustment rule aims for *'efficiency'* while the maximum entropy rule aims for *'justice'*. The experimental results show that the proposed model can guarantee the rationality of weight distribution to reach consensus efficiently, which is achieved by maintaining a balance between *'efficiency'* and *'justice'* in the mechanism of assigning weights. Finally, the detailed numerical and simulation analyses are carried out to verify the validity of the proposed method.

Keywords Social network group decision making · Weight manipulation · Feedback mechanism · Minimum adjustment · Maximum entropy

1 Introduction

The rapid development of social network makes it convenient for the masses to participate in various group decision-making (GDM) problems, which can be seen as a stepping stone towards the realisation of electronic democracy. Recently, Wu and Chiclana et.al (2014,2015; 2017) proposed the Social network group decision making (SN-GDM), where trust relationship is regarded as a reliable resource to assign expert weighs and generate recommendation advice for the inconsistent expert to reach higher consensus level. It now

✉ Jian Wu
jyajian@163.com

Extended author information available on the last page of the article

becomes a hot issue in the field of science of policy making (Zhang et al. 2017; Wu and Xu 2018; Xu et al. 2019; Tang and Liao 2021; Wan et al. 2021; Li et al. 2022), a differentiation from traditional GDM problems when trust relationship is used to deal with the inconsistency among group (Xu et al. 2019; Zhao et al. 2019; Ding et al. 2020; Chao et al. 2021).

It is well known that differences between people's background and professional opinion may lead to conflicts in GDM. The consensus reaching process (CRP) is an effective method to eliminate conflicts in GDM (Cabrerizo et al. 2014; Amirkhani et al. 2022). Different types of CRP models have been of interest to the research community: CRP in social networks (Zhang et al. 2020; Wu et al. 2022; Zhang et al. 2022), CRP in dynamic context (Pérez et al. 2018), CRP with preferred representative structure (Gong et al. 2018; Wu et al. 2020), adaptive CRP (Rodríguez 2018; Tian et al. 2021), CRP with minimum cost (Zhang et al. 2020; Chen et al. 2021; Gong et al. 2021), and CRP driven by behavior/attitude (Liu et al. 2019b; Sun et al. 2021; Cao et al. 2022). Conflicts in SN-GDM are likely to increase as the size of the group of people involved increases, which subsequently implies that SN-GDM require higher adjustment costs to reach consensus.

A feedback mechanism, as part of a CRP, is an effective method to eliminate conflicts among experts (Dong et al. 2017; Li et al. 2018; Cao et al. 2021; Wu et al. 2021). In the existing literature, the modifications of experts' preferences are usually conducted by two types of feedback mechanisms (Liu et al. 2019; Zhang et al. 2020; Chen et al. 2021; Xing et al. 2022). The first type of feedback mechanisms are designed with the so-called identification rules and direction rules. The first type of rules identify both inconsistent experts and their inconsistent preferences, while the second type of rules provide the direction of preference modification required to increase consensus. The second type of feedback mechanisms are based on minimum adjustment/cost optimization modeling, which performs better than the former type of feedback mechanisms in changing as less as possible the original information of inconsistent experts. The minimum cost improves the consensus efficiency. However, it does not consider the fairness of weight distribution. Thus, in general, the second type of feedback processes have lower adjustment cost and lower computational complexity than the first type of feedback processes. However, they still suffer from the following limitations:

(1) Most of the existing feedback mechanisms for SN-GDM utilize subjective/objective methods to assign weights to subgroups (Zhang et al. 2017; Wu and Xu 2018; Ding et al. 2020; Wan et al. 2021). However, these methods ignore the weight manipulation behavior. Dong et al. (2018) and Liu et al. (2019a) argued that experts may strategically set attribute weights of alternatives to obtain benefits in the decision-making process. Therefore, weight manipulation is also an important issue in GDM since a decision maker may wish to attain a greater importance degree (weight) to increase his/her benefits. For example, a decision maker with lowest status is willing to adopt egalitarianism to assign weight, while a decision maker with highest status expects to use authoritarianism to distribute weight (see Fig. 2). Obviously, manipulation of weights may hinder consensus to a certain extent and affect the cost of reaching consensus, and then, it should be worth studying in SN-GDM.
(2) There are few research studies on how to prevent weight manipulation. Examples of these studies include Yager's study on the use of uninorm operators to prevent manipulation (Yager 2001; 2002), Wu et al.'s (2021a) studied on individual manipulation weights to gain benefit where an optimization model to prevent manipulation behavior with minimum cost was investigated. Although these studies are of interests and

contributed towards manipulation prevention, they are established from an *'efficiency'* policy, which achieves group consensus with minimal adjustment/costs from a group perspective. However, other policy point of view, such as *'justice'*, should also be considered to guarantee individual benefits when assigning weights to DMs. Therefore, a reasonable policy to prevent weight manipulation combining *'justice'* and *'efficiency'* can achieve a balance between individual benefit and group goal.

To deal with the above limitations, a novel approach to prevent weight manipulation in SN-GDM problems is designed in this paper. Thus, the main contribution of this paper are:

(1) A power index is proposed to measure the power relations of subgroups, which combines the subgroups size and cohesion. The power index is used as a reliable resource to determine the importance order relation of subgroups, and as such is embedded in the prevention weight manipulation mechanism.
(2) A maximum entropy mechanism to assign weight to subgroups from the *'justice'* policy point of view is developed. Combining the minimum adjustment feedback and the maximum entropy method, a comprehensive approach to prevent weight manipulation in SN-GDM problems is established, which has the merit of balancing the *'efficiency'* and *'justice'* policy. Specifically, the minimum cost solves the problem of 'efficiency', while the maximum entropy method solves the problem of 'justice' of weight distribution.

The rest of this article is organized as follows: Sect. 2 gives detailed literature review on conflicts problem in SN-GDM for large group, CRP with minimum cost and weight manipulation behavior. Section 3 introduces the concept of 2-tuple linguistic representation. In Sect. 4, a power index (PI) is proposed to determine the importance degree of subgroups. A minimum adjustment and maximum entropy based feedback mechanism to prevent weight manipulation in SN-GDM is proposed in Sect. 5. Section 6 provides a case study to verify the effectiveness of the proposed model. Finally, some conclusions are pointed out in Sect. 7. The main notations used in this paper are summarized in TABLE 1.

2 Related work

This section presents a necessarily short overview of conflicts problem in SN-GDM for large group, CRP with minimum cost and weight manipulation behavior based on their relevance to our proposal.

2.1 Conflicts problem in SN-GDM for large group

As previously point out, there may be conflicts among decision makers due to their differences in knowledge and backgrounds (Del Moral 2018; Xiao et al. 2022). So it is necessary to reach a group consensus before their individual preferences are aggregated. Generally, there are two methods to resolve conflicts of large group in SN-GDM:

Table 1 The main notations in this paper

Notations	Meaning
M	Set of alternatives
C	Set of attributes
E	Set of experts
SG	Set of subgroups
LG	Large-scale group
SM	Social matrix
$A^h = \left(a_{ij}^h\right)_{m\times n}$	Decision matrix of expert e_h
$d\left(a_{ij}^h, a_{ij}^k\right)$	Deviation degree between e_h and e_k
$\vartheta\left(a_{ij}^h, a_{ij}^k\right)$	Similarity degree between e_h and e_k
CD^{hk}	Consensus degree between e_h and e_k
χ_{hk}	Edge between e_h and e_k in network
ε	Network consensus threshold
$N(SG)$	The number of subgroups
$P_{M(r)}$	The number of experts in SG_r
$P_{C(r)}$	Cohesion of subgroup SG_r
PI_{SG_r}	Power index of subgroup SG_r
σ	Permutation
$w = \left(w_1, w_2, \dots, w_r\right)$	A weighting vector for subgroups
α	Attitudinal parameter
$\omega = \left(\omega_1, \omega_2, \dots, \omega_n\right)$	A weighting vector for attributes
ACD^{SG_r}	Consensus degree between SG_r and the rest of subgroups
$\overline{ACD^{SG_r}}$	Consensus degree between SG_r and the rest of subgroups after feedback
$ACA_i^{SG_r}$	Consensus degree between SG_r and the rest of subgroups on the alternative x_i
$ACE_{ij}^{SG_r}$	Consensus degree between SG_r and the rest of subgroups on the alternative x_i with respect to the criterion c_j
γ	Consensus threshold for LG
δ_r	Feedback parameter for inconsistent subgroup SG_r
$\widetilde{a_{ij}^{SG_r}}$	Advice for SG_r
MTC	Minimum total cost of feedback process

(1) The first way is to cluster the large group thereby reducing its dimension, which solves the conflict problem within subgroups. Many commonly used clustering method are utilized to solve SN-DGM problem. For instance, Wu and Xu (2018) identify the subgroups based on the k-means method under the fuzzy preference environment in LSGDM. Mandal et al. (2022) used the grey clustering algorithm to conduct clustering based on the similarity measure among the experts. Li et al. (2022) used the fuzzy cluster analysis method to divide large group into subgroups and integrate heterogeneous information. Wu et al. (2021) proposed a dynamic clustering method, which divides large groups by Louvain algorithm.

(2) The second way is to reach a consensus among the subgroups after clustering, which resolves the conflict problem outside the subgroups. Wu et al. (2018) designed a local

feedback policy with identification rules and direction rules to guide the CRP. Chao et al. (2021) presented a CRP model to address the heterogeneous with non-cooperative behaviors. Mandal et al. (2022) proposed a CRP model to manage non-cooperative behaviors by the cluster consensus index and group consensus index. Wang et al. (2022) proposed a two-stage consensus model with feedback mechanism considering different power structures in SN-GDM.

In general, the basic framework of SN-GDM process consists of the following parts, which are shown in Fig. 1. First, a set of alternatives about a decision problem is presented to a large group of experts. Experts provide their preferences about alternatives are then collected. Then the large group is clustered and the preferences of the clustered subgroups are aggregated. Next, if the consensus degree of subgroup reach a consensus threshold, the resolution process will be executed; otherwise, a feedback mechanism is activated to allow the inconsistent subgroups to modify their opinions, and re-aggregate the preferences until a consensus is reached.

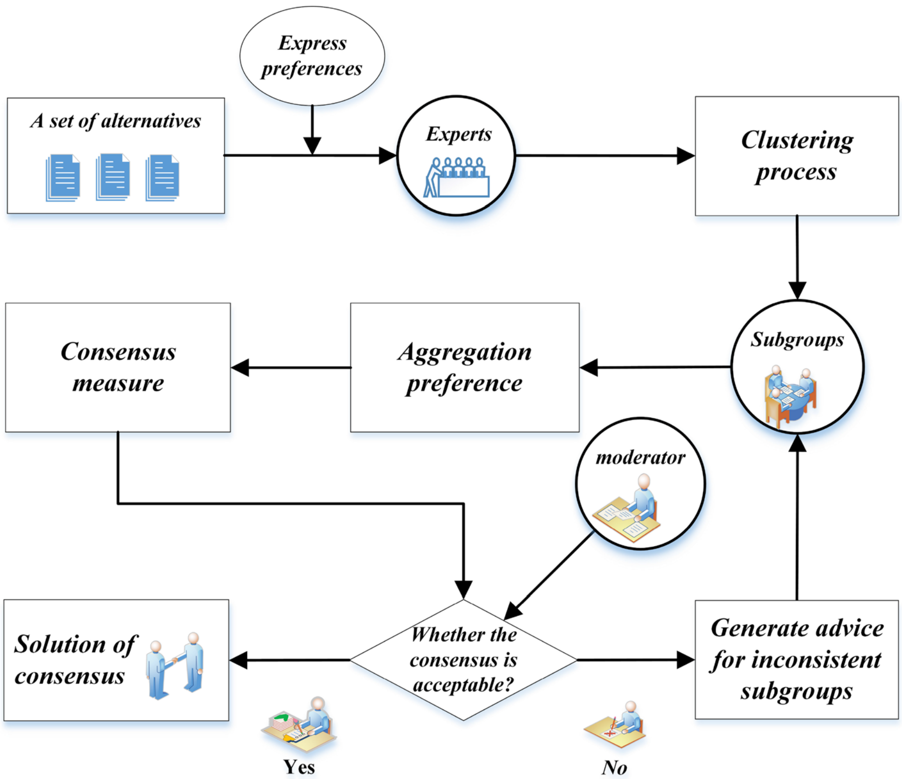


Fig. 1 Basic framework of SN-GDM process

2.2 CRP with minimum cost

In the feedback mechanism of CRP there is usually a moderator responsible for supervising the inconsistent experts and guiding them to modify their preferences to reach a consensus. Generally, the preferences-adjustment are the results of laborious negotiations, which escalate the cost of CRP. Ben-Arieh and Easton (2007) first proposed a consensus minimum cost optimization model for the multi-criteria decision-making consensus problem. In recent years, many minimum cost consensus models based on the method of Ben-Arieh and Easton (2007) have been proposed. For example, Gong et al. (2021) discussed the minimum cost consensus model under uncertain chance-constrained from the perspectives of moderators, individual decision makers, and non-cooperators. Chen et al. (2021) propose an approach to manage the consensus based on minimum adjustments with opinions evolution. Sun et al. (2021) proposed a attitudinal consensus threshold based dynamic minimum adjustment cost feedback model to resolve the GDM problems with different consistency requirements. Xiao et al. (2022) proposed a minimum adjustment element consensus model based on bounded confidences to help the failure mode and effect analysis team reach a consensus. Recently, Zhang et al. (2020) presented a state-of-the-art review of CRP models under minimum cost. By reviewing the research paradigm of minimum cost under classical and complex group decision problems, they pointed out the limitations and some new directions of the minimum cost consensus model.

2.3 Weight manipulation behavior

In the decision-making process, decision-makers may utilize some trick to set their own weights to gain benefits, which is usually called weight manipulation behavior. This behavior can help decision makers, to some extent make the decision result in their desired direction. For instance, Yager (2001,2002) studied the individuals weight manipulation behavior in the process of preference aggregation of group. Besides, considering that weight manipulation may lead to unreasonable decision results, the author proposed the management mechanism to prevent this behavior. Dong et al. (2018) studied the strategic weight manipulation in multiple attribute GDM problem. From an optimization point of view, Liu et al. (2019a) studied the strategic weight manipulation in a group GDM context with interval attribute weight information and proposed a minimum cost strategic weight manipulation model. Dong et al. (2021) investigates the clique-based strategies to manipulate trust relationships to gain the desired decision result. The aforementioned literatures on weigh manipulative behavior focus on the selection process stage in GDM. However, weight manipulation behavior may exist at any stage of GDM. Recently, Wu et al. (2021a) studied the effect of weight manipulation behavior on the efficiency of consensus reaching and proposed an optimization model to prevent weight manipulation to increase the efficiency of CRP. But it is necessary to consider the fairness of weight distribution. In general, the entropy weight method can reduce the uncertainty of weight distribution (Wang et al. 2022). The larger the entropy value, the fairer the weight distribution. Therefore, this paper uses a method based on maximum entropy (O'Hagan 1988) to determine the weight of subgroups.

3 Preliminary

In GDM problems, experts may prefer linguistic terms to numerical values when expressing their preferences (Herrera-Viedma et al. 2021; Li et al. 2021; Yu et al. 2021; Liu et al. 2021). Herrera and Martínez (2000) presented the below 2-tuple linguistic representation model to evaluate decision problems:

Definition 1 (2-tuple linguistic representation) Let $S = \{s_0, \ldots, s_f\}$ and $\beta \in [0,f]$ be a linguistic term set and the result of a symbolic aggregation, respectively. Let $i = round(\beta) \in \{0, \ldots, f\}$. The value $\alpha = \beta - i$ is called a symbolic translation, and (s_i, α) is called the 2-tuple linguistic representation of the symbolic aggregation α.

The 2-tuple linguistic representation of symbolic aggregation can be mathematically formalised as an strictly increasing continuous function:

$$\Delta : [0,f] \rightarrow S \times [-0.5, 0.5), \tag{1}$$

$$\Delta(\beta) = (s_i, \alpha); \begin{cases} i = round(\beta) \\ \alpha = \beta - i \end{cases}, \tag{2}$$

with inverse function $\Delta^{-1} : S \times [-0.5, 0.5) \rightarrow [0,f]$ being $\Delta^{-1}(s_i, \alpha) = i + \alpha = \beta$, and following properties:

1. Let (s_i, α) and (s_j, ψ) be two 2-tuples linguistic.
   (a) If $i < j$, then (s_i, α) is smaller than (s_j, ψ).
   (b) If $i = j$, then
      i. If $\alpha = \psi$, then (s_i, α) and (s_j, ψ) are equivalent.
      ii. If $\alpha < \psi$, then (s_i, α) is smaller than (s_j, ψ).
2. The 2-tuple negation operator is $Neg((s_i, \alpha)) = \Delta(q - \Delta^{-1}(s_i, \alpha))$.
3. Given a set of 2-tuple linguistic labels $\{(s_1, \alpha), (s_2, \alpha), \ldots, (s_f, \alpha)\}$ and let $(\lambda_1, \lambda_2, \ldots, \lambda_f)$ be an associated weighting vector which satisfies $\lambda_i \in [0, 1]$, $\sum_{i=1}^{f} \lambda_i = 1$. The 2-tuple weighted arithmetic average (WAA) is:

$$(\tilde{s}, \tilde{\alpha}) = \Delta\left(\sum_{i=1}^{f} \lambda_i \cdot \Delta^{-1}(s_i, \alpha)\right), \tag{3}$$

   where $\tilde{s} \in S, \tilde{\alpha} \in [-0.5, 0.5)$.

For convenience, (s_i, α) is represented by a. Dong et al. (2010) defined the deviation measure between 2-tuple linguistic labels. Then, the following expert consensus degree functions are proposed:

Definition 2 (Consensus degree (CD)) The consensus degree between experts e_h and e_k with respect to their 2-tuples linguistic preferences on a set of m alternatives with respect a set of n criteria is:

$$CD^{hk} = \frac{1}{mn}\sum_{i=1}^{m}\sum_{j=1}^{n}\vartheta\left(a_{ij}^{h}, a_{ij}^{k}\right), \tag{4}$$

where $\vartheta\left(a_{ij}^{h}, a_{ij}^{k}\right) = 1 - d\left(a_{ij}^{h}, a_{ij}^{k}\right) = 1 - \frac{\left|\Delta^{-1}\left(a_{ij}^{h}\right) - \Delta^{-1}\left(a_{ij}^{k}\right)\right|}{f+1}$.

The consensus degree between expert e_h and the rest of experts in the group with respect to their preferences on a set of m alternatives with respect a set of n criteria is:

$$ACD^{h} = \frac{1}{p-1}\sum_{k=1,h\neq k}^{p} CD^{hk}. \tag{5}$$

4 Clustering analysis process based on community detection in SN-GDM

Clustering analysis process (CAP) is an effective method to simplify SN-GDM, since it classifies individuals with similar preferences or structures into the same subgroup (Wu and Xu 2018; Liu et al. 2019; Xu et al. 2019; Zhao et al. 2019; Rashidi et al. 2019; Ding et al. 2020). One key issues of CAP is the determination of subgroups' weight.

4.1 Community detection based CAP

Community detection is an effective method of CAP to uncover the local characteristics of individual behaviors and the correlation between individuals in the network, i.e. to determine the network structure. Therefore, it can effectively detect the relationship between individuals in SN-GDM and then determine network subgroups (Wu et al. 2021). Raghavan et al. (2007) proposed the following label propagation algorithm (LPA) based community detection.

Definition 3 Assume a large network of experts $LG=\left\{e_1, e_2, \ldots, e_q\right\}$. The social network relation χ_{hk} linking e_h and e_k exists if their consensus degree CD^{hk} is not lower than a network consensus threshold $\varepsilon \in [0, 1]$. Thus, the network edge between e_h and e_k is denoted $\chi_{hk} = CD^{hk}$. Label propagation algorithm (LPA) main idea is as follows.

(1) Initial assignment to each expert a unique label, i.e. , $\forall e_h \in LG(h = 1, \ldots, q) : e_h = l_h$.
(2) Update the labels of all nodes one by one until reaching the convergence requirement. At each iterative round, the rule for updating labels is as follows: For each node, the label shared by most of its neighbors is assigned. If the most shared label is not unique, choose the label with largest sum of the edge weights of nodes connected to destination node:

$$\forall e_h \in LG(h = 1, \ldots, q) : e_h = \arg\max_b \sum_{k \in N_h^b} \chi_{hk}, \tag{6}$$

where N_h^b is the set of all nodes labeled b among the neighbors of node h.

Algorithm 1 for the LPA based CAP in SN-GDM is given as follows.

Algorithm 1 Clustering method based on LPA in SN-GDM

Input:

The preferences of experts: $A^h = \left(a_{ij}^h\right)_{m \times n}, h = 1, ..., q\ (q \geq 20)$;

The initial labels of experts in the large network: $\forall e_h \in LG\ (h = 1, .., q) : e_h = l_h$;

The network consensus threshold: $\varepsilon \in [0, 1]$;

Output:

The labels of all experts after iterative process $L' = \{l'_1, l'_2, ..., l'_q\}$;

The number of communities (subgroups) $N\ (SG)$;

Step 1. Assign each expert a unique label, and apply (4) to compute the consensus degree between pairs of different experts e_h and e_k, $CD^{hk}\ (h, k = 1, ..., q, h \neq k)$.

Step 2. Determine the edges between nodes in the network. If $CD^{hk} \geq \varepsilon$, these exists the edge χ_{hk} between the experts e_h and e_k.

Step 3. Update the label of each node according to (6).

Step 4. Repeat Step 3 until the convergence requirement is achieved.

Since only one node needs to be updated each time, the time complexity of each iteration of the LPA is linear $O(m)$. The higher the value of the network consensus threshold ε, the more sparse the relationship among nodes is and the greater the number of subgroups $N(SG)$ detected.

4.2 Determining the importance order of subgroups

Algorithm 1 divides the large group into t subgroups $SG = \{SG_1, SG_2, \ldots, SG_t\}$ of size $P_{M(r)} = N(SG_r)\ r \in \{1, \ldots, t\}$. As per (5), the cohesion of subgroup SG_r, denoted $P_{C(r)}$, is based on the consensus degree of its experts:

$$P_{C(r)} = \frac{\sum_{h=1}^{P_{M(r)}} ACD_r^h}{P_{M(r)}}, \tag{7}$$

where ACD_r^h is the consensus degree between expert $e_h \in SG_r$ and the rest of experts in subgroup SG_r.

The power index is obtained as a linear combination of the size and cohesion of the subgroups, and it determines their power relations (the permutation of weights) in the community.

Definition 4 (Power index (PI)) The power index of subgroup $SG_r\ (r = 1, \ldots, t)$ is:

$$PI_{SG_r} = \lambda \frac{P_{M(r)}}{\sum_{r=1}^{t} P_{M(r)}} + (1-\lambda) \frac{P_{C(r)}}{\sum_{r=1}^{t} P_{C(r)}}, \tag{8}$$

where $\lambda \in [0,1]$ is a power parameter used to balance the size and cohesion for different LSGDM problems.

The PI values can be used to determine the importance weight of subgroups in LG. Specifically, denoting by σ the permutation such that $PI_{SG_{\sigma(r)}}$ is the $r-th$ highest value of $\{PI_{SG_1}, PI_{SG_2}, \ldots, PI_{SG_t}\}$, it is

$$PI_{SG_{\sigma(r+1)}} \le PI_{SG_{\sigma(r)}} \Rightarrow w_{\sigma(r+1)} \le w_{\sigma(r)}. \tag{9}$$

5 The consensus reaching process against weight manipulation in SN-GDM

5.1 Subgroups consensus measure

At the subgroup level, experts in the same subgroup are regarded as equal important. Meanwhile, each subgroup will be regarded as a new individual with 2-tuples linguistic defined as follows:

Definition 5 (2-tuples linguistic of subgroup and large group) The 2-tuples linguistic of subgroup $SG_r(r = 1, 2, \ldots, t)$ are:

$$a_{ij}^{SG_r} = \Delta\left(\sum_{e_h \in SG_r} \left(\frac{1}{R_{M(r)}} \cdot \Delta^{-1}\left(a_{ij}^{h}\right)\right)\right). \tag{10}$$

The linguistic 2-tuples of the large group are defined as follows:

$$a_{ij}^{LG} = \Delta\left(\sum_{r=1}^{t} \left(w_{\sigma(r)} \cdot \Delta^{-1}\left(a_{ij}^{SG_{\sigma(r)}}\right)\right)\right), \tag{11}$$

where σ is the permutation defined in (9). The subgroup weights, $w_{\sigma(r)} \in [0,1]$, used in (11), have associated orness value $\sum_{r=1}^{t} \frac{t-r}{t-1} w_{\sigma(r)} = \alpha \in [0,1]$, which is herein called the attitudinal parameter.

Applying (5), the consensus degree between any subgroup and large group, $ACD^{SG_r}(r = 1, \ldots, t)$ are computed. A subgroup SG_r will be called *consistent* when its consensus degree is not below the consensus threshold, that is when $ACD^{SG_r} \ge \gamma$; otherwise it is called *inconsistent*. When all subgroups are consistent, the selection process will be implemented. Otherwise the feedback mechanism will be activated to generate recommendations on the inconsistent elements of the members of the set of inconsistent subgroups: $ACD^{SG} = \{r | ACD^{SG_r} < \gamma\}$. Noteworthy, the threshold value can be set according to the consistency requirements of different decision-making problems.

5.2 Identification of inconsistent elements

Let $r \in ACD^{SG}$, in other word, SG_r is identified as an inconsistent subgroup. The value

$$ACE_{ij}^{SG_r} = \frac{1}{t-1} \sum_{s=1, s \neq r}^{t} \vartheta\left(a_{ij}^{SG_r}, a_{ij}^{SG_s}\right) \tag{12}$$

is the consensus degree between the preferences of the subgroup SG_r and the rest of subgroups of the large group at the alternative i and criterion j, which is known as the consensus at the element level. The value

$$ACA_i^{SG_r} = \frac{1}{n} \sum_{j=1}^{n} ACE_{ij}^{SG_r} \tag{13}$$

is the consensus degree between the preferences of the subgroup SG_r and the rest of subgroups of the large group at the alternative i, which is known as the consensus at the alternative level.

The sets of inconsistent alternatives of inconsistent subgroup SG_r is: $ACA^{SG_r} = \{i | r \in ACD^{SG_r} \wedge ACA_i^{SG_r} < \gamma\}$. The set of inconsistent elements of inconsistent subgroup SG_r is: $ACE^{SG_r} = \{j | r \in ACD^{SG_r} \wedge i \in ACA^{SG_r} \wedge ACE_{ij}^{SG_r} < \gamma\}$. Thus, the set of inconsistent elements to consider for feedback recommendation is:

$$APS = \left\{(r, i, j) | r \in ACD^{SG_r} \wedge i \in ACA^{SG_r} \wedge j \in ACE^{SG_r}\right\}. \tag{14}$$

5.3 Advice generation for inconsistent subgroups

For all identified elements $(r, i, j) \in APS$, inconsistent subgroups receive advice based on the following rule:

Value $a_{ij}^{SG_r}$ should be closer to:

$$\widetilde{a_{ij}^{SG_r}} = \left(1 - \delta_r\right) \cdot a_{ij}^{SG_r} + \delta_r \cdot a_{ij}^{LG}, \tag{15}$$

where $\delta_r \in [0, 1]$ is the feedback parameter for inconsistent subgroup SG_r that controls the degree of modification from the original evaluation $a_{ij}^{SG_r}$ to the collective evaluation a_{ij}^{LG}. The feedback parameter is given beforehand by the decision-making moderator.

Notice that ACD^{SG_r} is the consensus degree of subgroup SG_r before the feedback process, while $\overline{ACD^{SG_r}}$ is the consensus degree of subgroup SG_r after the feedback advices to inconsistent groups (15) are implemented. The concept of cost, proposed by Ben-Arieh and Easton (2007), reflects the linear adjustment of the preference of inconsistent individuals required to reach consensus. Thus, the total cost for inconsistent subgroups is expressed as :

$$\begin{aligned} TC &= \sum_{(r,i,j) \in APS} \left| a_{ij}^{SG_r} - \widetilde{a_{ij}^{SG_r}} \right| \\ &= \sum_{(r,i,j) \in APS} \delta_r \cdot \left| a_{ij}^{SG_r} - a_{ij}^{LG} \right|. \end{aligned} \tag{16}$$

5.4 A minimum adjustment and maximum entropy based model to weight prevent manipulation

As aforementioned, experts may strategically set weights for their own benefit in (Dong et al. 2018; Wu et al. 2021a). This article proposes a novel method to prevent weight manipulation in SN-GDM. In detail, the following minimum adjustment feedback mechanism with maximum entropy method (O'Hagan 1988) is established to assign appropriate weights to subgroups.

$$\min\ TC = \sum_{(r,i,j)\in APS} \delta_r \cdot \left| a_{ij}^{SG_r} - a_{ij}^{LG} \right|$$
$$s.t.\ \begin{cases} a_{ij}^{LG} = \sum_{r=1}^{t} w_r \cdot a_{ij}^{SG_r} \\ \widetilde{a_{ij}^{SG_r}} = \left(1-\delta_r\right) \cdot a_{ij}^{SG_r} + \delta_r \cdot a_{ij}^{LG} \\ ACD^{SG_r} < \gamma \wedge \overline{ACD^{SG_r}} \geq \gamma, \left(r \in ACD^{SG}\right) \\ \max\ disp(W) = -\sum_{c=1}^{t} w_c \ln w_c \\ s.t.\ \begin{cases} \sum_{c=1}^{t} \frac{t-c}{t-1} w_c = \alpha \in [0.5, 1) \\ w_c \in [0, 1] \\ \sum_{c=1}^{t} w_c = 1 \end{cases} \end{cases} \tag{17}$$

where $c = \sigma(r)$.

Applying the Lagrange multiplier technique, the maximum entropy method can be transformed into the following geometric OWA operator proposed by Liu and Chen (2004):

$$w_c = aq^{c-1}\ (a > 0,\ q \geq 0,\ c = 1, 2, \ldots, t) \tag{18}$$

The detailed derivation process is as follows. Let

$$L\left(W, \psi_1, \psi_2\right) = -\sum_{c=1}^{t} w_c \ln\left(w_c\right) + \psi_1\left(w_c \frac{t-c}{t-1} - \alpha\right) + \psi_2\left(\sum_{c=1}^{t} w_c - 1\right), \tag{19}$$

The necessary conditions of the solution are

$$\frac{\partial L}{\partial w_c} = -\ln w_c - 1 + \psi_1 \frac{t-c}{t-1} + \psi_2 = 0; \tag{20}$$

$$\frac{\partial L}{\partial \psi_1} = \sum_{i=1}^{t} w_c \frac{t-c}{t-1} - \alpha = 0; \tag{21}$$

$$\frac{\partial L}{\partial \psi_2} = \sum_{c=1}^{t} w_c - 1 = 0. \tag{22}$$

From expression (20), we get that $w_c = e^{\psi_1 \frac{t-c}{t-1} + \psi_2 - 1}$. Let $e^{\psi_1} = 1/\mu$ and $\frac{t-c}{t-1} - \frac{t-(c+1)}{t-1} = \frac{1}{t-1}$ then, $\frac{w_c}{w_{c+1}} = \mu^{-\frac{1}{t-1}}$. Obviously, $\mu^{-\frac{1}{t-1}}$ is a positive number. Therefore, the MEOWA weights is equal to GOWA weights. Since $\sum_{c=1}^{t} w_c = 1$, it is

$$w_c = \frac{q^{c-1}}{\sum_{s=0}^{t-1} q^s}. \tag{23}$$

Since $\sum_{c=1}^{t} \frac{t-c}{t-1} w_c = \alpha$, then q is the solution of the following equation:

$$(t-1)\alpha q^{t-1} + \sum_{c=2}^{t} ((t-1)\alpha - c + 1) q^{t-c} = 0. \tag{24}$$

Thus, optimization model (17) can be rewritten as:

$$\mathbf{min}\ TC = \sum_{(r,i,j)\in APS} \delta_r \cdot d_{ij}$$
$$s.t. \begin{cases} a_{LG}^{ij} = \sum_{r=1}^{t} w_r \cdot a_{SG_r}^{ij} \\ \widetilde{a_{SG_r}^{ij}} = \left(1 - \delta_r\right) \cdot a_{SG_r}^{ij} + \delta_r \cdot a_{LG}^{ij} \\ ACD^{SG_r} < \gamma \wedge \overline{ACD^{SG_r}} \geq \gamma, \left(r \in ACD^{SG}\right) \\ a_{ij}^{SG_r} - a_{ij}^{LG} \leq d_{ij} \\ a_{ij}^{LG} - a_{ij}^{SG_r} \leq d_{ij} \\ w_c = \frac{q^{c-1}}{\sum_{s=0}^{t-1} q^s}, c = 1, 2, \ldots, t \\ (t-1)\alpha q^{t-1} + \sum_{c=2}^{t} ((t-1)\alpha - c + 1) q^{t-c} = 0 \\ \sum_{c=1}^{t} \frac{t-c}{t-1} w_c = \alpha \in [0.5, 1) \\ \sum_{c=1}^{t} w_c = 1 \end{cases} \tag{25}$$

Noteworthy, any feasible solution of model (25) that verifies $d_{ij} > \left| a_{ij}^{SG_r} - a_{ij}^{LG} \right|$ is not a solution of model (17). Thus, only the solutions of (25) that verifies $d_{ij} = \left| a_{ij}^{SG_r} - a_{ij}^{LG} \right|$ are solutions of model (17).

The detailed process of the proposed consensus model is shown in Algorithm 2.

Algorithm 2 Minimum adjustment feedback mechanism based on maximum entropy method for reaching consensus

Input:

The preferences of experts: $A^h = \left(a_{ij}^h\right)_{m\times n}, h = 1, ..., q(q \geq 20)$;

The consensus threshold: γ;

The feedback parameter: δ_r;

Output:

The consensus degree of subgroups after feedback: $\overline{ACD^{SG_r}}$;

The minimum total cost: MTC;

The weight of subgroups: $w_{\sigma(r)}$;

Step 1. Construct social matrix based on expression (4). From the Algorithm 1, the LG is divided into several subgroups SG_r $(r = 1, 2, ..., t)$.

Step 2. Compute the preference of each subgroup based on expression (10), and calculate the consensus degree of subgroups ACD^{SG_r} by expression (5), then sort the weights of the subgroups by the PI of ACD^{SG_r}. If all subgroups' $ACDs \geq \gamma$, then go to step 5; Otherwise, go to step 3.

Step 3. Apply expression (14) to identify the inconsistence subgroups and their alternatives and elements with ACA and ACE values below the γ.

Step 4. Generate the recommended advice to the inconsistent subgroups by expression (15). Then, solve the model (25), the weights of the subgroups $w_{\sigma(r)}$ and the minimum TC are obtained. Go to step 2.

Step 5. The LG moves onto the resolution process.

The detailed execution flow charts of algorithm 1 and algorithm 2 are shown in Fig. 2a, b.

The following properties guarantee that the subgroup consensus degrees are increasing and bounded above.

Proposition 1 *For inconsistent subgroup $SG_r (r \in APS)$, it is*

$$\overline{ACD^{SG_r}} \geq ACD^{SG_r}.$$

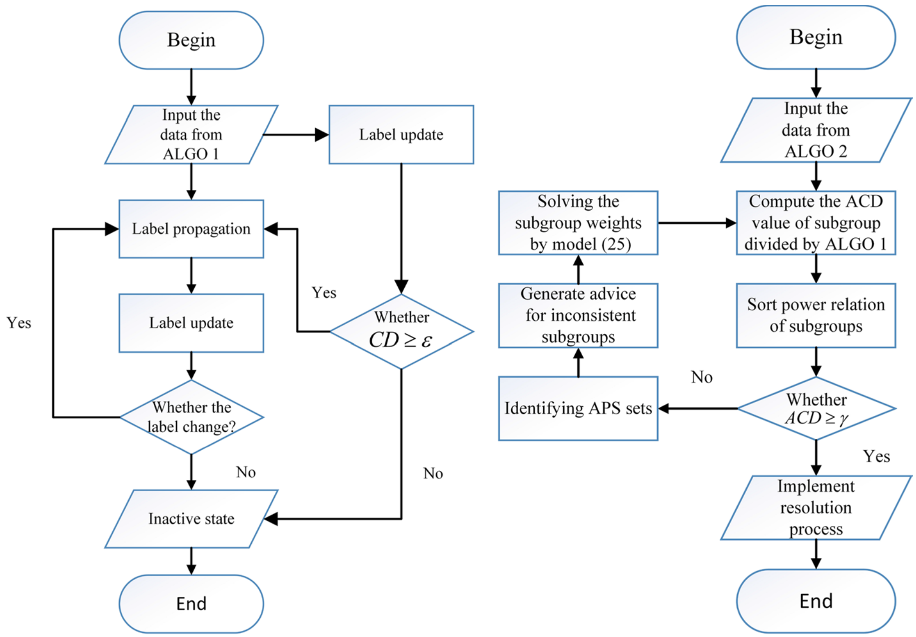
Proof Based on (5), it is

$$ACD^{SG_r} = \frac{1}{(t-1)mn} \sum_{z=1, z\neq r}^{t} \sum_{i=1}^{m} \sum_{j=1}^{n} \left(1 - \left|a_{ij}^r - a_{ij}^z\right|\right). \tag{26}$$

For simplify, $a_{ij}^{SG_r}$ is denoted as a_{ij}^r. From (15), $\overline{ACD^{SG_r}}$ can be split into two values

$$\overline{ACD^{SG_r}} = \overline{ACD_1^{SG_r}} + \overline{ACD_2^{SG_r}},$$

computed using the set of inconsistent elements $\widetilde{a_{ij}^{r1}}((r,i,j) \in APS)$ and consistent elements $a_{ij}^{r2}((r,i,j) \notin APS)$. Noteworthy, $z \in \text{int}[1,t]$ consists of s and r_o. When $s \in \text{int}\left[1,g\right]$ in $\overline{ACD_1^{SG_r}}$, the consensus degree between the inconsistent subgroup SG_r and other consistent subgroups in the large group after feedback process is given in (27) (at the top of the page 13). Notice that $a_{ij}^r = a_{ij}^{r1} \cup a_{ij}^{r2}$ and $a_{ij}^s = a_{ij}^{s1} \cup a_{ij}^{s2}$. While when $r_o \in \text{int}\left[g+1,t\right]$ in $\overline{ACD_2^{SG_r}}$,



(a) The flow chart of algorithm 1 **(b)** The flow chart of algorithm 2

Fig. 2 The flow chart of algorithms 1-2

the consensus degree between the inconsistent subgroup SG_r and other inconsistent subgroups in the large group after feedback process is given in (28) (at the top of the page 13) In this case, $a_{ij}^{r} = a_{ij}^{r^1} \cup a_{ij}^{r^2} \cup a_{ij}^{r^3} \cup a_{ij}^{r^4}$, and $a_{ij}^{r_o} = a_{ij}^{r_o^1} \cup a_{ij}^{r_o^2} \cup a_{ij}^{r_o^3} \cup a_{ij}^{r_o^4}$. Obviously, since δ_r and w_r in the interval [0, 1], so $\left|1 - \delta_r\left(1 - w_r\right)\right| \in [0,1]$ in $\overline{ACD_1^{SG_r}}$. Additionally, since δ_r, δ_{r_o} and w_r are in the interval [0, 1], so $\left|1 - \left(\delta_r\left(1 - w_r\right) + \delta_{r_o} \cdot w_r\right)\right| \in [0,1]$, $\left|1 - \delta_{r_o}\left(1 - w_r\right)\right| \in [0,1]$ and $\left|\delta_r\left(1 - w_r\right) - 1\right| \in [0,1]$ in $\overline{ACD_2^{SG_r}}$. Therefore, we get $ACD^{SG_r} \leq \overline{ACD^{SG_r}}$.

$$
\begin{aligned}
\overline{ACD_1^{SG_r}} &= \frac{1}{(t-1)mn} \sum_{s=1}^{g} \sum_{i=1}^{m} \sum_{j=1}^{n} \left[1 - \left(\left|\widetilde{a_{ij}^{r^1}} - a_{ij}^{s^1}\right| + \left|a_{ij}^{r^2} - a_{ij}^{s^2}\right|\right)\right] \\
&= \frac{1}{(t-1)mn} \sum_{s=1}^{g} \sum_{i=1}^{m} \sum_{j=1}^{n} \left[\left(1 - \left|\left(1 - \delta_r\left(1 - w_r\right)\right)\right| \cdot \left|a_{ij}^{r^1} - a_{ij}^{s^1}\right| + \left|a_{ij}^{r^2} - a_{ij}^{s^2}\right|\right)\right],
\end{aligned}
\tag{27}
$$

$$\overline{ACD_2^{SG_r}} = \frac{1}{(t-1)mn} \sum_{r_o=g+1, r_o \neq r}^{t} \times \sum_{i=1}^{m} \sum_{j=1}^{n} \left[1 - \left(\left| \widetilde{a_{ij}^{r^1}} - \widetilde{a_{ij}^{r_o{}^1}} \right| + \left| a_{ij}^{r^2} - \widetilde{a_{ij}^{r_o{}^2}} \right| + \left| \widetilde{a_{ij}^{r^3}} - a_{ij}^{r_o{}^3} \right| + \left| a_{ij}^{r^4} - a_{ij}^{r_o{}^4} \right| \right) \right]$$
$$= \frac{1}{(t-1)mn} \sum_{r_o=g+1, r_o \neq r}^{t} \sum_{i=1}^{m} \sum_{j=1}^{n} \Big(1 - \Big(\left| \left(1 - \left(\delta_r(1-w_r) + \delta_{r_o} \cdot w_r\right)\right) \right| \cdot \left| \left(a_{ij}^{r^1} - a_{ij}^{r_o{}^1} \right) \right|$$
$$+ \left| \left(1 - \delta_{r_o}(1-w_r)\right) \right| \cdot \left| a_{ij}^{r^2} - a_{ij}^{r_o{}^2} \right| + \left| \left(\delta_r(1-w_r) - 1\right) \right| \cdot \left| a_{ij}^{r^3} - a_{ij}^{r_o{}^3} \right| + \left| a_{ij}^{r^4} - a_{ij}^{r_o{}^4} \right| \Big) \Big). \tag{28}$$

$$ACD^{SG_s} = ACD_1^{SG_s} + ACD_2^{SG_s} = \frac{1}{(t-1)mn} \sum_{z'=1, z' \neq s}^{t} \sum_{i=1}^{m} \sum_{j=1}^{n} \left(1 - \left| a_{ij}^{s} - a_{ij}^{z'} \right| \right). \tag{29}$$

$$\overline{ACD_1^{SG_s}} = \frac{1}{(t-1)mn} \sum_{r=g+1}^{t} \sum_{i=1}^{m} \sum_{j=1}^{n} \left(1 - \left(\left| a_{ij}^{s^1} - \widetilde{a_{ij}^{r^1}} \right| + \left| a_{ij}^{s^2} - a_{ij}^{r^2} \right| \right) \right)$$
$$= \frac{1}{(t-1)mn} \sum_{r=g+1}^{t} \sum_{i=1}^{m} \sum_{j=1}^{n} \left(1 - \left(\left| \left(\delta_r(1-w_r) - 1\right) \right| \cdot \left| a_{ij}^{s^1} - a_{ij}^{r^1} \right| + \left| a_{ij}^{s^2} - a_{ij}^{r^2} \right| \right) \right), \tag{30}$$

$$\overline{ACD_2^{SG_s}} = \frac{1}{(t-1)mn} \sum_{s_o=1, s_o \neq s}^{g} \sum_{i=1}^{m} \sum_{j=1}^{n} \left(1 - \left| a_{ij}^{s} - a_{ij}^{s_o} \right| \right). \tag{31}$$

□

Proposition 2 *For consistent subgroup $SG_s (s \notin APS)$, it is*

$$\overline{ACD^{SG_s}} \geq ACD^{SG_s}.$$

Proof Similar to Proposition 1, based on (5) it is (29) (at the top of the page 13), where $z' \in \text{int}[1, t]$ consists of r and s_o. Then, when $r \in \text{int}\left[g+1, t\right]$ in $\overline{ACD_1^{SG_s}}$ it is (30) (at the top of the page 13), while when $s_o \in \text{int}\left[1, g\right]$ in $\overline{ACD_2^{SG_s}}$ it is (31) (at the top of the page 13). Since δ_r and w_r in the interval [0, 1], so $\left| \delta_r(1-w_r) - 1 \right| \in [0,1]$ in $\overline{ACD_1^{SG_s}}$. Therefore, we get $ACD^{SG_s} \leq \overline{ACD^{SG_s}}$. □

6 Numerical and simulation analysis

This section introduces a numerical and a simulation analysis to verify the effectiveness of the proposed model. With the frequent occurrence of emergencies in the world, large emergency group decision-making (LEGDM) has become a hot research issue. Generally, emergency decision-making problem is time-sensitive. In order to allocate resources efficiently and reduce losses, the location of emergency medical facility is a key issue in LEGDM. In response to emergencies brought about by COVID-19, many module hospitals have been established across China. Assume a modular hospital to be built in Lingang New Area, Shanghai, China. After pre evaluation, four emergency facilities $\{M_1, M_2, M_3, M_4\}$ have remained as alternatives for further evaluation. This paper collects the preference information of twenty experts $e_h(h = 1, \dots, 20)$ from college emergency departments, hospital emergency departments and government departments through a questionnaire survey with respect to three decision criteria: C_1: Geographical factor; C_2: Traffic convenience; C_3: Safety factor. The linguistic term set (LTS) for judging the location with regard to the three criteria is:

$$S = \left\{ \begin{array}{l} s_0 = very\ bad(VB), \quad s_1 = bad(B), \quad s_2 = littlebad(LB), \ \ s_3 = medium(M) \\ s_4 = littlegood(LG), \ s_5 = good(G), \ s_6 = verygood(VG) \end{array} \right\}.$$

6.1 Numerical analysis

1. The twenty experts' preferences are provided as follows.

$$A^1 = \begin{pmatrix} & C_1 & C_2 & C_3 \\ M_1 & (s_4,0) & (s_3,0) & (s_2,0) \\ M_2 & (s_4,0) & (s_4,0) & (s_4,0) \\ M_3 & (s_5,0) & (s_2,0) & (s_2,0) \\ M_4 & (s_3,0) & (s_2,0) & (s_5,0) \end{pmatrix} \quad A^2 = \begin{pmatrix} & C_1 & C_2 & C_3 \\ M_1 & (s_1,0) & (s_1,0) & (s_5,0) \\ M_2 & (s_4,0) & (s_0,0) & (s_6,0) \\ M_3 & (s_2,0) & (s_4,0) & (s_3,0) \\ M_4 & (s_0,0) & (s_5,0) & (s_1,0) \end{pmatrix} \quad A^3 = \begin{pmatrix} & C_1 & C_2 & C_3 \\ M_1 & (s_5,0) & (s_1,0) & (s_3,0) \\ M_2 & (s_2,0) & (s_3,0) & (s_2,0) \\ M_3 & (s_4,0) & (s_4,0) & (s_0,0) \\ M_4 & (s_5,0) & (s_2,0) & (s_4,0) \end{pmatrix}$$

$$A^4 = \begin{pmatrix} & C_1 & C_2 & C_3 \\ M_1 & (s_2,0) & (s_4,0) & (s_1,0) \\ M_2 & (s_2,0) & (s_4,0) & (s_0,0) \\ M_3 & (s_4,0) & (s_2,0) & (s_2,0) \\ M_4 & (s_2,0) & (s_2,0) & (s_6,0) \end{pmatrix} \quad A^5 = \begin{pmatrix} & C_1 & C_2 & C_3 \\ M_1 & (s_5,0) & (s_2,0) & (s_5,0) \\ M_2 & (s_3,0) & (s_3,0) & (s_2,0) \\ M_3 & (s_1,0) & (s_4,0) & (s_4,0) \\ M_4 & (s_3,0) & (s_3,0) & (s_2,0) \end{pmatrix} \quad A^6 = \begin{pmatrix} & C_1 & C_2 & C_3 \\ M_1 & (s_3,0) & (s_4,0) & (s_5,0) \\ M_2 & (s_2,0) & (s_2,0) & (s_4,0) \\ M_3 & (s_4,0) & (s_3,0) & (s_0,0) \\ M_4 & (s_4,0) & (s_0,0) & (s_6,0) \end{pmatrix}$$

$$A^7 = \begin{pmatrix} & C_1 & C_2 & C_3 \\ M_1 & (s_2,0) & (s_4,0) & (s_1,0) \\ M_2 & (s_3,0) & (s_3,0) & (s_0,0) \\ M_3 & (s_5,0) & (s_2,0) & (s_4,0) \\ M_4 & (s_4,0) & (s_0,0) & (s_6,0) \end{pmatrix} \quad A^8 = \begin{pmatrix} & C_1 & C_2 & C_3 \\ M_1 & (s_2,0) & (s_4,0) & (s_1,0) \\ M_2 & (s_5,0) & (s_1,0) & (s_3,0) \\ M_3 & (s_4,0) & (s_3,0) & (s_1,0) \\ M_4 & (s_6,0) & (s_3,0) & (s_4,0) \end{pmatrix} \quad A^9 = \begin{pmatrix} & C_1 & C_2 & C_3 \\ M_1 & (s_1,0) & (s_1,0) & (s_4,0) \\ M_2 & (s_4,0) & (s_2,0) & (s_4,0) \\ M_3 & (s_5,0) & (s_5,0) & (s_3,0) \\ M_4 & (s_1,0) & (s_5,0) & (s_2,0) \end{pmatrix}$$

$$A^{10} = \begin{pmatrix} & C_1 & C_2 & C_3 \\ M_1 & (s_1,0) & (s_4,0) & (s_5,0) \\ M_2 & (s_4,0) & (s_0,0) & (s_4,0) \\ M_3 & (s_3,0) & (s_4,0) & (s_1,0) \\ M_4 & (s_3,0) & (s_2,0) & (s_2,0) \end{pmatrix} \quad A^{11} = \begin{pmatrix} & C_1 & C_2 & C_3 \\ M_1 & (s_3,0) & (s_6,0) & (s_1,0) \\ M_2 & (s_4,0) & (s_2,0) & (s_4,0) \\ M_3 & (s_2,0) & (s_0,0) & (s_3,0) \\ M_4 & (s_5,0) & (s_2,0) & (s_4,0) \end{pmatrix} \quad A^{12} = \begin{pmatrix} & C_1 & C_2 & C_3 \\ M_1 & (s_2,0) & (s_4,0) & (s_3,0) \\ M_2 & (s_2,0) & (s_4,0) & (s_2,0) \\ M_3 & (s_5,0) & (s_5,0) & (s_4,0) \\ M_4 & (s_3,0) & (s_2,0) & (s_1,0) \end{pmatrix}$$

$$A^{13} = \begin{pmatrix} & C_1 & C_2 & C_3 \\ M_1 & (s_3,0) & (s_0,0) & (s_4,0) \\ M_2 & (s_2,0) & (s_3,0) & (s_2,0) \\ M_3 & (s_5,0) & (s_4,0) & (s_2,0) \\ M_4 & (s_5,0) & (s_1,0) & (s_4,0) \end{pmatrix} \quad A^{14} = \begin{pmatrix} & C_1 & C_2 & C_3 \\ M_1 & (s_4,0) & (s_5,0) & (s_3,0) \\ M_2 & (s_2,0) & (s_4,0) & (s_0,0) \\ M_3 & (s_4,0) & (s_2,0) & (s_1,0) \\ M_4 & (s_6,0) & (s_1,0) & (s_4,0) \end{pmatrix} \quad A^{15} = \begin{pmatrix} & C_1 & C_2 & C_3 \\ M_1 & (s_4,0) & (s_2,0) & (s_5,0) \\ M_2 & (s_4,0) & (s_3,0) & (s_2,0) \\ M_3 & (s_5,0) & (s_5,0) & (s_2,0) \\ M_4 & (s_2,0) & (s_1,0) & (s_4,0) \end{pmatrix}$$

$$A^{16} = \begin{pmatrix} & C_1 & C_2 & C_3 \\ M_1 & (s_1,0) & (s_4,0) & (s_3,0) \\ M_2 & (s_4,0) & (s_2,0) & (s_4,0) \\ M_3 & (s_2,0) & (s_6,0) & (s_4,0) \\ M_4 & (s_2,0) & (s_2,0) & (s_5,0) \end{pmatrix} A^{17} = \begin{pmatrix} & C_1 & C_2 & C_3 \\ M_1 & (s_1,0) & (s_1,0) & (s_4,0) \\ M_2 & (s_3,0) & (s_4,0) & (s_5,0) \\ M_3 & (s_1,0) & (s_4,0) & (s_5,0) \\ M_4 & (s_2,0) & (s_4,0) & (s_4,0) \end{pmatrix} A^{18} = \begin{pmatrix} & C_1 & C_2 & C_3 \\ M_1 & (s_5,0) & (s_4,0) & (s_3,0) \\ M_2 & (s_2,0) & (s_2,0) & (s_0,0) \\ M_3 & (s_4,0) & (s_1,0) & (s_2,0) \\ M_4 & (s_3,0) & (s_5,0) & (s_2,0) \end{pmatrix}$$

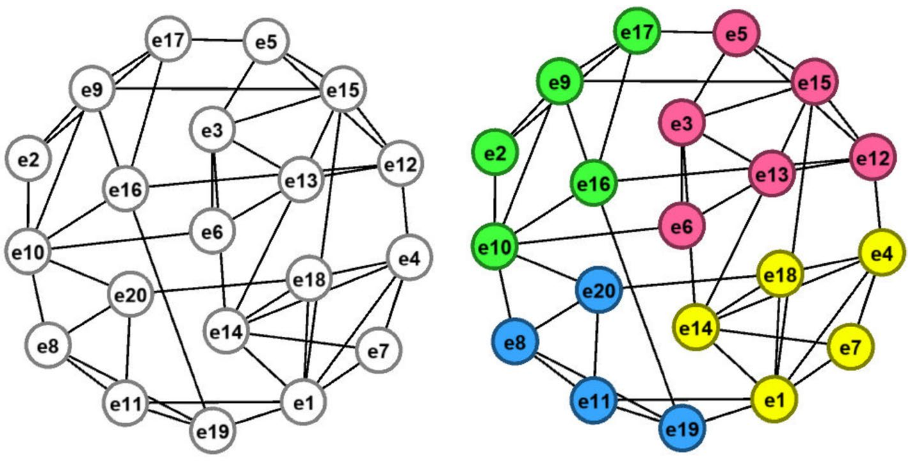
$$A^{19} = \begin{pmatrix} & C_1 & C_2 & C_3 \\ M_1 & (s_6,0) & (s_4,0) & (s_1,0) \\ M_2 & (s_5,0) & (s_4,0) & (s_4,0) \\ M_3 & (s_3,0) & (s_3,0) & (s_4,0) \\ M_4 & (s_3,0) & (s_2,0) & (s_4,0) \end{pmatrix} A^{20} = \begin{pmatrix} & C_1 & C_2 & C_3 \\ M_1 & (s_3,0) & (s_5,0) & (s_2,0) \\ M_2 & (s_4,0) & (s_1,0) & (s_4,0) \\ M_3 & (s_2,0) & (s_2,0) & (s_0,0) \\ M_4 & (s_3,0) & (s_5,0) & (s_2,0) \end{pmatrix}$$

The social matrix $SM = CD^{hk}(h, k = 1, ...q)$ is constructed via (4):

$$SM = \begin{pmatrix} 0 & 0.643 & ... & 0.774 & 0.810 & ... & 0.869 & 0.798 \\ 0.643 & 0 & ... & 0.781 & 0.726 & ... & 0.631 & 0.750 \\ ... & ... & 0 & ... & ... & ... & ... & ... \\ 0.774 & 0.781 & ... & 0 & 0.750 & ... & 0.762 & 0.833 \\ 0.810 & 0.726 & ... & 0.750 & 0 & ... & 0.821 & 0.821 \\ ... & ... & ... & ... & ... & 0 & ... & ... \\ 0.869 & 0.631 & ... & 0.762 & 0.821 & ... & 0 & 0.762 \\ 0.798 & 0.750 & ... & 0.833 & 0.821 & ... & 0.762 & 0 \end{pmatrix}$$

Assuming a network consensus threshold of $\varepsilon = 0.8$, the large network before CAP is established in Fig. 3(a). Applying Algorithm 1, the large network is divided into four subgroups, which are depicted in Fig. 3(b):

$$SG_1 = \{e_1, e_4, e_7, e_{14}, e_{18}\}, SG_2 = \{e_2, e_9, e_{10}, e_{16}, e_{17}\};$$
$$SG_3 = \{e_3, e_5, e_6, e_{12}, e_{13}, e_{15}\}, SG_4 = \{e_8, e_{11}, e_{19}, e_{20}\}.$$



(a) Large network with $\varepsilon = 0.8$ **(b)** LPA Community detection

Fig. 3 The large network before and after CAP

Assuming the power parameter $\lambda = 0.5$ that considers size and the cohesion equally important criteria to measure the power relationship of subgroups, the RI of the four subgroup are computed based on (5) and (7)–(8).

- Subgroup SG_1: $ACD_1^1 = 0.822$; $ACD_1^4 = 0.848$; $ACD_1^7 = 0.81$; $ACD_1^{14} = 0.821$; $ACD_1^{18} = 0.801$; $P_{M(1)} = 5$ and $P_{C(1)} = 0.821$.
- Subgroup SG_2: $ACD_2^2 = 0.813$; $ACD_2^9 = 0.827$; $ACD_2^{10} = 0.804$; $ACD_2^{16} = 0.804$; $ACD_2^{17} = 0.801$; $P_{M(2)} = 5$ and $P_{C(2)} = 0.81$.
- Subgroup SG_3: $ACD_3^3 = 0.829$; $ACD_3^5 = 0.79$; $ACD_3^6 = 0.771$; $ACD_3^{12} = 0.79$; $ACD_3^{13} = 0.838$; $ACD_3^{15} = 0.829$; $P_{M(3)} = 6$ and $P_{C(3)} = 0.808$.
- Subgroup SG_4: $ACD_4^8 = 0.813$; $ACD_4^{11} = 0.821$; $ACD_4^{19} = 0.798$; $ACD_4^{20} = 0.798$; $P_{M(4)} = 4$ and $P_{C(4)} = 0.808$.

The power indices are:

$$PI_{SG_1} = 0.252, PI_{SG_2} = 0.25, PI_{SG_3} = 0.274, PI_{SG_4} = 0.224.$$

Then, the power relations of subgroups are: $w_3 \geq w_1 \geq w_2 \geq w_4$.
where $w_{\sigma(1)} = w_3, w_{\sigma(2)} = w_1, w_{\sigma(3)} = w_2, w_{\sigma(4)} = w_4$ are obtained from expression (9).

2. Applying (10) the preference of four subgroups are:

$$A^{SG_1} = \begin{pmatrix} & C_1 & C_2 & C_3 \\ M_1 & (s_3, 0.4) & (s_4, 0) & (s_2, 0) \\ M_2 & (s_3, -0.4) & (s_3, 0.4) & (s_1, -0.2) \\ M_3 & (s_4, 0.4) & (s_2, -0.2) & (s_2, 0.2) \\ M_4 & (s_4, -0.4) & (s_2, 0) & (s_5, -0.4) \end{pmatrix} \quad A^{SG_2} = \begin{pmatrix} & C_1 & C_2 & C_3 \\ M_1 & (s_1, 0) & (s_2, 0.2) & (s_4, 0.2) \\ M_2 & (s_4, -0.2) & (s_2, -0.4) & (s_5, -0.4) \\ M_3 & (s_3, -0.4) & (s_5, -0.4) & (s_3, 0.2) \\ M_4 & (s_2, -0.4) & (s_4, -0.4) & (s_3, -0.2) \end{pmatrix}$$

$$A^{SG_3} = \begin{pmatrix} & C_1 & C_2 & C_3 \\ M_1 & (s_4, -0.33) & (s_2, 0.17) & (s_4, 0.17) \\ M_2 & (s_3, -0.5) & (s_3, 0) & (s_2, 0.33) \\ M_3 & (s_4, 0) & (s_4, 0.17) & (s_2, 0) \\ M_4 & (s_4, -0.33) & (s_2, -0.5) & (s_4, -0.5) \end{pmatrix} \quad A^{SG_4} = \begin{pmatrix} & C_1 & C_2 & C_3 \\ M_1 & (s_4, -0.5) & (s_5, -0.25) & (s_1, 0.25) \\ M_2 & (s_5, -0.5) & (s_2, 0) & (s_4, -0.25) \\ M_3 & (s_3, -0.25) & (s_2, 0) & (s_2, 0) \\ M_4 & (s_4, 0.25) & (s_3, 0) & (s_4, -0.5) \end{pmatrix}$$

The consensus degrees between each subgroup and the large group are:

$$ACD^{SG_1} = 0.81, ACD^{SG_2} = 0.771, ACD^{SG_3} = 0.833, ACD^{SG_4} = 0.817.$$

3. With $\gamma = 0.8$, the set of inconsistent elements is

$$APS = \{(2,1,1), (2,1,2), (2,1,3), (2,2,3), (2,4,1), (2,4,2)\}.$$

4. Implementing the feedback parameter $\delta_2 = 0.3$, the corresponding model (25) becomes:

$$
\begin{aligned}
&min \sum_{(r,i,j)\in APS} 0.3\times d_{ij}\\
&s.t.\ \begin{cases}
a_{ij}^{LG}=\sum_{r=1}^{4} w_r\cdot a_{ij}^{SG_r}\\
ACD^{SG_2}<0.8\wedge \overline{ACD^{SG_2}}\geq 0.8\\
a_{ij}^{SG_2}-a_{ij}^{LG}\leq d_{ij}\\
a_{ij}^{LG}-a_{ij}^{SG_2}\leq d_{ij}\\
w_{\sigma(r)}=\frac{q^{\sigma(r)-1}}{1+q+q^2+q^3},\sigma(r)=1,2,3,4\\
3\alpha q^3+3\alpha q^2-q^2+3\alpha q-2q+3\alpha-3=0\\
0.5\leq w_{\sigma(1)}+\frac{2}{3}w_{\sigma(2)}+\frac{1}{3}w_{\sigma(3)}=\alpha<1\\
w_{\sigma(1)}+w_{\sigma(2)}+w_{\sigma(3)}+w_{\sigma(4)}=1
\end{cases}
\end{aligned}
\tag{32}
$$

The solution of model (32) results in the following subgroup weights: $w_1=0.277, w_2=0.177, w_3=0.434, w_4=0.113$; with associated attitudinal parameter $\alpha=0.676$, and $MTC=0.391$. And we have the feedback mechanism advices for SG_2:

- Value $a_{11}^{SG_2}$ should be closer to $(s_2,-0.37)$.
- Value $a_{12}^{SG_2}$ should be closer to $(s_2,0.43)$.
- Value $a_{13}^{SG_2}$ should be closer to $(s_4,-0.09)$.
- Value $a_{23}^{SG_2}$ should be closer to $(s_4,-0.04)$.
- Value $a_{41}^{SG_2}$ should be closer to $(s_2,0.12)$.
- Value $a_{42}^{SG_2}$ should be closer to $(s_3,0.17)$.

If SG_2 accepts and implements the advice, the new decision matrix for such subgroup will be:

$$
A_{\delta_2=0.3}^{SG_2}=\begin{pmatrix}
(s_2,-0.37) & (s_2,0.43) & (s_4,-0.09)\\
(s_4,-0.2) & (s_2,-0.4) & (s_4,-0.04)\\
(s_3,-0.4) & (s_5,-0.4) & (s_3,0.2)\\
(s_2,0.12) & (s_3,0.17) & (s_3,-0.2)
\end{pmatrix}
$$

The new ACDs after the feedback process would be:

$$
\overline{ACD^{SG_1}}=0.821,\overline{ACD^{SG_2}}=0.8,\overline{ACD^{SG_3}}=0.84,\overline{ACD^{SG_4}}=0.828.
$$

Subgroup SG_2 reaches the consensus threshold $\gamma=0.8$.

6.2 Analysis of preventing manipulation behavior

Let $PI=\{PI_{SG_1},\dots,PI_{SG_t}\}$ be the set of subgroup PI values. Then the following attitude-OWA (AOWA) operator based on maximum entropy method (Yager 1988; O'Hagan 1988) is used to determine the dynamic subgroups weighting vector $W=(w_1,\dots,w_t)$:

$$\begin{aligned} &\textbf{max}\ \ disp(W) = -\sum_{c=1}^{t} w_c \ln w_c \\ &s.t.\ \ \begin{cases} \sum_{c=1}^{t} \frac{t-c}{t-1} w_c = \alpha \in [0.5, 1] \\ w_c \in [0, 1] \\ \sum_{c=1}^{t} w_c = 1 \end{cases} \end{aligned} \tag{33}$$

If $\alpha = 0.5$, from expression (33), we get weights $w = \left(\frac{1}{n}, \frac{1}{n}, \dots, \frac{1}{n}\right)$, named as '*egalitarianism*' weighting vector. If $\alpha = 1$, then $w = (1, 0, \dots, 0)$, named as '*authoritarianism*' weighting vector. Since the power is too concentrated when α is too large, the value of α considered in this paper is not higher than 0.95. The manipulation behavior of subgroups from '*egalitarianism*' to '*authoritarianism*' is reflected by dynamic weights in Fig. 4.

To verify the prevention manipulation of the proposed model, we simulate the changes in cost with different attitudinal parameter $\alpha \in \{0.6, 0.7, 0.8, 0.9\}$ with $\delta_2 = 0.3$. To do that, we establish model (34), which reflects the weight manipulation of subgroups.

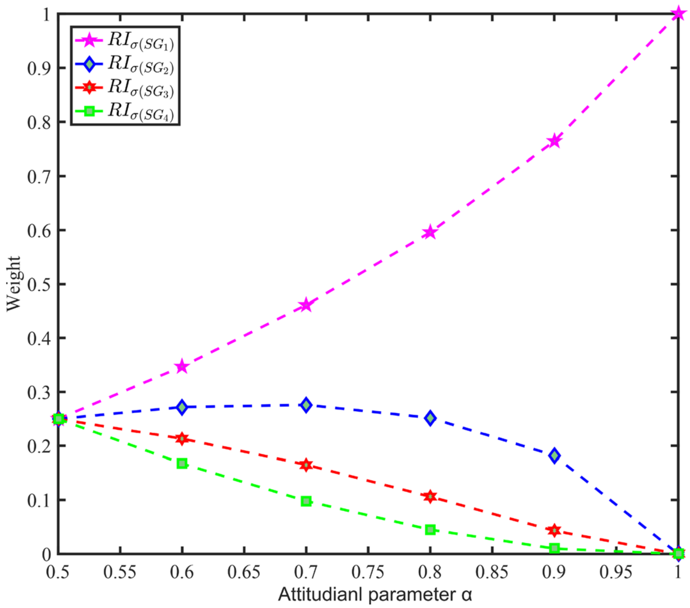


Fig. 4 The manipulation behavior of subgroups

$$\begin{aligned} &\mathbf{max}\ disp(W) = -\sum_{c=1}^{t} w_c \ln w_c \\ &s.t.\ \begin{cases} a_{ij}^{LG} = \sum_{r=1}^{t} w_r a_{ij}^{SG_r} \\ ACD^{SG_r} < \gamma \wedge \overline{ACD^{SG_r}} \geq \gamma, \left(r \in ACD^{SG}\right) \\ \sum_{c=1}^{t} \frac{t-c}{t-1} w_c = \alpha \in [0.5, 1) \\ w_c \in [0,1] (c = \sigma(r)) \\ \sum_{c=1}^{t} w_c = 1 \end{cases} \end{aligned} \tag{34}$$

In model (34), the objective function has only maximum entropy and the attitudinal parameter is a variable. Then $TC = \sum_{(r,i,j)\in APS} \delta_r \cdot \left| a_{ij}^{SG_r} - a_{ij}^{LG} \right|$ can be calculated. In other words, model (34) maximises entropy with known attitudinal parameter. While model (17) obtains the attitudinal parameter by minimum TC. Noteworthy, when $\alpha \in [0.5, 0.676)$, the LG cannot reach consensus after one round of feedback. The model will have an optimal solution only after the inconsistent subgroup accept two rounds of advice, which means the $\overline{ACD^{SG_r}}$ should be changed to $\overline{\overline{ACD^{SG_r}}}$ and TC will be

$$TC(2) = TC_1 + TC_2 = \sum_{r,i,j\in APS} \delta_r \cdot \left(\left| a_{ij}^{SG_r} - a_{ij}^{LG} \right| + \left| \widetilde{a_{ij}^{SG_r}} - \widetilde{a_{ij}^{LG}} \right| \right).$$

Figure 5 illustrates the results of TC in Table 2 obtained with weight manipulation (curve) by model (34) and prevention weight manipulation (blue dot) by model (32), respectively. As previously pointed out, subgroups may have weight manipulative behaviors. The reason for weight manipulation is that each subgroup wants to increase its own weight in LSGDM. When the subgroups with large discourse power (PI) are strong, they hope that the weights are concentrated on themselves, as shown in the curve on the right side of the blue dot in Figure 5. When the subgroups with large discourse power are not very strong, the distribution of weights will be more even, as shown in the curve on the left of the blue dots. But weight manipulative behavior will hinder consensus efficiency to a certain extent (Fig. 6). To this end, we designed an anti-manipulation model (17). Experimental results show that the TC value obtained with our proposed model (32) is the lowest. More specifically, the curve to the left of the blue dot in Fig.5 reflects the situation based on 'justice' policy, where the weight distribution is relatively balanced but requires more cost due to multiple rounds of feedback, so the efficiency is relatively low. While the curve to the right of

Table 2 TC with weight manipulation under different attitudinal parameter α

α	Subgroups' weights	TC	Maximum entropy	$\overline{ACD_{SG_2}}$
0.6	(0.272, 0.213, 0.347, 0.167)	0.677	1.35	0.817
0.7	(0.276, 0.165, 0.461, 0.098)	0.394	1.24	0.801
0.8	(0.252, 0.106, 0.596, 0.045)	0.405	1.03	0.802
0.9	(0.182, 0.043, 0.764, 0.01)	0.411	0.7	0.804
Model (32)	(0.277, 0.177, 0.432, 0.114)	0.391	1.27	0.8

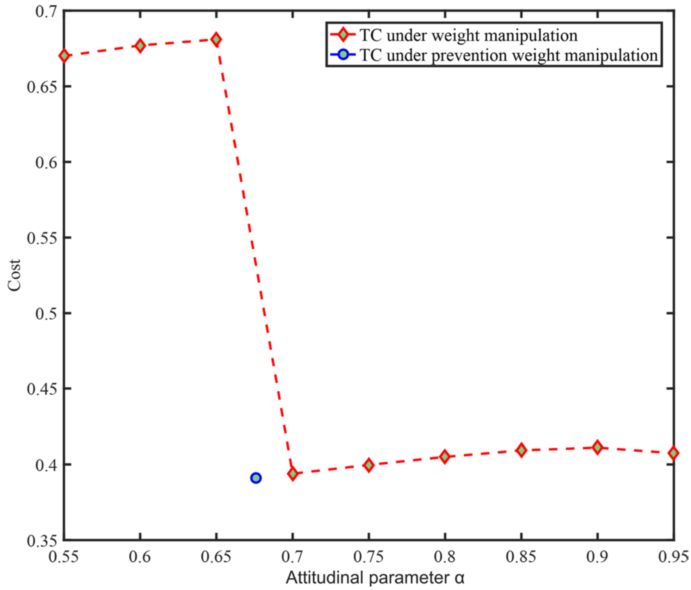


Fig. 5 The TC under weight manipulation and prevention weight manipulation

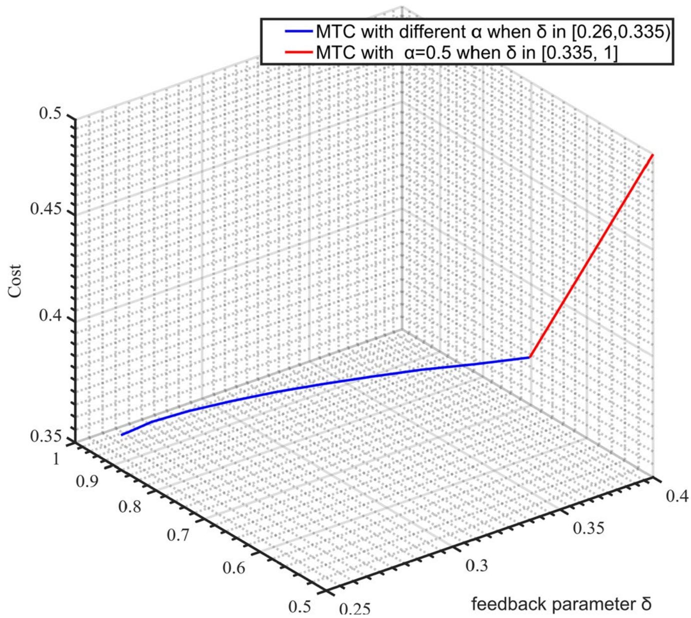


Fig. 6 Simulation of the minimum cost and attitude parameters change with the feedback parameters of inconsistent subgroup SG_2 after feedback

the blue dot reflects the situation based on the efficiency policy, where the cost is relatively low, but the weight distribution is relatively unreasonable. Therefore, our policy is to ensure the justice of the weight distribution as much as possible under the condition of high efficiency.

6.3 Sensitivity analysis

In this subsection, we simulate the change of feedback parameter in the interval $[0, 1]$ to analyze anti-manipulation behavior. Specifically, we randomly generate the feedback parameters of inconsistent experts from 0 to 1, and then substitute these parameters into model (25). By solving the minimum cost and subgroups' weights, the large group can reach consensus by coordinating the 'efficiency' policy and 'justice' policy. To do that, the following three cases are considered:

Case 1. When the value of feedback parameter is small, one round of feedback may not reach consensus, that is, the feedback parameter δ_r for inconsistent subgroup SG_r in the interval $\left[0, \delta_{r_{low}}\right)$ is required more iterations to reach consensus.

Case 2. When the value of feedback parameter δ_r for inconsistent subgroup SG_r in the interval $\left[\delta_{r_{high}}, 1\right]$, the LG can reach consensus after one round of feedback.

Case 3. While, when the feedback parameter is large enough for Case 2, there may be a set of fixed weights with the minimum cost, which is mainly due to the preferences are over-adjusted. Specifically, if the feedback parameter δ_r in the interval $\left[\delta_{r_{high}}, 1\right]$, $\delta_{r_{low}} \le \delta_{r_{high}}$, subgroups' weights could be fixed values.

Noteworthy, due to Case 1 requires multiple rounds of iterations to reach consensus, it will incur a time cost, which is undesirable in the feedback mechanism based on minimum cost. Therefore, the model proposed in this article is mainly to analyze Case 2 and 3. The attitude parameter α is utilized to reflect the distribution of subgroups' weights.

Randomly generate feedback parameters $\delta_2 \in [0, 1]$, from model (25), we get that when $0 \le \delta_2 < 0.26$, the model has no feasible solution, which is applicable to Case 1. Thus, this paper suggests that the value of the feedback parameter should be not lower than 0.26. When $0.26 \le \delta_2 \le 1$, the model has the optimal solution, which is applicable to Case 2. Specifically, when $0.26 \le \delta_2 \le 0.335$, the LG can reach consensus by different attitudinal parameter in the interval $[0.5, 1)$ with minimum cost. while when $0.335 < \delta_2 \le 1$, the LG can can reach consensus by the same attitudinal parameter $\alpha = 0.5$ with minimum cost, which is applicable to Case 3. This can be interpreted as adequate weight should be given when the inconsistent experts are fully willing to modify preferences. A visual simulation of the minimum cost and attitude parameters change with the feedback parameters is shown in Fig. 4.

The blue and red lines in Fig. 4 reflect the conditions after feedback of the feedback parameters in the interval $0.26 \le \delta_2 \le 0.335$ and $0.335 < \delta_2 \le 1$, respectively. The result of the blue line shows that when the weight distribution tends to be justice, the efficiency of CRP will decrease (due to the cost increase). The red line shows when the preferences are over-adjusted (ignore efficiency), the proposed model will give priority to the justice of weight distribution. Therefore, to better retain the original preferences of inconsistent subgroups, the feedback parameter δ_2 should be selected in the interval $[0.26, 0, 335]$ as the alternatives for inconsistent subgroup SG_2, so that the LG can reach consensus by coordinating the 'efficiency' policy and 'justice' policy.

6.4 Ranking order of alternatives

Without loss of generality, it is assumed that the attribute weights of criteria are: $\omega = \left(C_1 = 1/3; C_2 = 1/3; C_3 = 1/3\right)^T$. After the feedback process of the numerical analysis, we get the consensual collective decision matrix with $\alpha = 0.676$:

$$A^{LG} = \begin{pmatrix} & C_1 & C_2 & C_3 \\ M_1 & (s_3, 0.1) & (s_3, -0.03) & (s_3, 0.24) \\ M_2 & (s_3, -0.01) & (s_3, -0.25) & (s_2, 0.47) \\ M_3 & (s_4, -0.28) & (s_3, 0.34) & (s_2, 0.27) \\ M_4 & (s_3, 0.35) & (s_2, 0.18) & (s_4, -0.32) \end{pmatrix}$$

Therefore, the overall consensual preference value of the four alternatives $\{M_1, M_2, M_3, M_4\}$ are:

$$M_1 = (s_3, 0.1), M_2 = (s_3, -0.26), M_3 = (s_3, 0.11), M_4 = (s_3, 0.07).$$

The final consensus ranking of alternatives is:

$$M_3 \succ M_1 \succ M_4 \succ M_2.$$

In addition, we compared the rankings of the alternatives under different policy, as shown in Table 3. The result shows that the final alternative ranking result based on justice policy is different from our model, while the efficiency-based alternative ranking result is consistent with our model. As can be seen from Table 2, the weights obtained by the proposed method in this paper are fairer than the method based on the efficiency policy (the curve to the right of the blue dot in Fig. 5). Therefore, it can be verified that the model proposed in this article not only guarantees efficiency, but also guarantees justice as much as possible.

6.5 Comparative analysis

To clearly differentiate between the proposed method and other methods on weight manipulation behavior, this section compares with other literatures from a theoretical perspective. The characteristic comparisons of our method with other methods are shown in Table 4. To simplify notion, weight manipulation behavior is denoted by *WMB*.

In the aspect of research object: The manipulated objects mainly include attribute weight (Dong et al. 2018; Liu et al. 2019a), individual weight (Yager 2001; 2002; Wu et al.

Table 3 Rankings of the alternatives with different policy

α	Type of policy	Rankings of the alternatives
0.6	Justice	$M_1 \succ M_3 \succ M_4 \succ M_2$.
0.9	Efficiency	$M_3 \succ M_1 \succ M_4 \succ M_2$.
Model (32)	both justice and efficiency	$M_3 \succ M_1 \succ M_4 \succ M_2$.

Table 4 The comparative analysis among different models of weight manipulation behavior

References	Research object	Research perspective of WMB	Prevent WMB
Yager (2001)	Individual weight	Selection process	Yes
Yager (2002)	Individual weight	Selection process	Yes
Dong et al. (2018)	Attribute weight	Selection process	No
Liu et al. (2019a)	Attribute weight	Selection process	No
Dong er al. (2021)	Trust relationship	Selection process	No
Wu et al. (2021a)	Individual weight	Consensus process	Yes
The proposed method	Subgroup weight	Consensus process	Yes

2021a) and trust relationship (Its essence is to get more individual weight) (Dong et al. 2021). Attribute weight manipulation mainly affects the ranking of the alternatives. While the individual weight manipulation will not only affect the final ranking of the alternatives, but also affect the aggregating of individual preference.

In the aspect of research perspective: Weight manipulation behavior mainly exists in the consensus process and selection process in GDM. The purpose of attribute and individual weight manipulation in the selection process is mainly to obtain a desired ranking of alternatives by weight manipulation (Yager 2001; 2002; Dong et al. 2018, 2021; Liu et al. 2019a). But individual weight manipulation can also have an impact on the consensus process. The reason for this effect is that the weight of the individual will affect the aggregation preference of group and feedback mechanisms often suggest discordant individuals to make adjustments based on group preferences. Therefore, individual weight manipulation behavior may affect the efficiency of consensus (Wu et al. 2021a).

In the aspect of preventing manipulation: Although the behavior of weight manipulation can help decision makers to obtain the ideal ranking of alternative to a certain extent, it also has some adverse effects, such as unreasonable decision results or low consensus efficiency. To this end, Yager (2001; 2002) proposed a mechanism for modifying the construction of the group decision function to prevent weight manipulation behavior, but it doesn't take into account consensus issues. Therefore, Wu et al. (2021a) proposed an efficiency policy-based mechanism to prevent individual weight manipulation behavior in CRP.

Here are some advantages of our methods compared with the above literatures:

(1) Compared with Yager (2001; 2002) and Wu et al. (2021a), we extend the research of weight manipulation from individual to subgroup. In general, subgroup behavior is more complex than individual behavior, so we define a power index to determine the importance ranking of subgroups, which provides a new research paradigm for weight manipulation behavior in LSGDM.

(2) Most of articles focuses on the effect of weight manipulation behavior on ranking of alternatives in selection process (Yager 2001; 2002; Dong et al. 2018,2021; Liu et al. 2019a), while this paper mainly focus on the effect of manipulation behavior on consensus process, which is similar to Wu et al. (2021a). In contrast with method of Wu

et al., in the design of the anti-manipulation model, we not only focus on the efficiency of CRP, but also consider the justice of weight distribution.

6.6 Discussion

In GDM, each decision maker wants to obtain greater importance degree (weight) for more benefits. For example, a decision maker with the lowest status is willing to adopt egalitarianism to assign weight. While a decision maker with the highest status expects to use authoritarianism to distribute weight. This behavior will increase the cost (efficiency) of the feedback process to a certain extent and hinder consensus. The existing studies on preventing weight manipulation behaviors have focused on group consensus efficiency: how to minimize interaction costs. However, these mechanisms often ignore the justice of weight distribution. In this article, we define a more reasonable policy to prevent weight manipulation by combining 'justice' and 'efficiency' simultaneously, which can achieve a balance between individual benefit and group goal.

Actually, under situation of weight manipulation, the total cost changes with the behavior (attitude parameters). While only using the maximum entropy method to assign weights cannot know how to assign weights appropriately because 'egalitarianism' or 'authoritarianism' policies are not the optimal choice, which is reflected by dynamic weights in Figs. (4 and 5). Therefore, it is more reasonable to combine the maximum entropy model and the minimum cost model as limiting conditions to assign weights, where (1): Entropy is the largest, which solves the problem of 'justice'. (2): The feedback total cost is minimal, which solves the problem of 'efficiency'. If all two points can be achieved, the weights of subgroups are not manipulated in our proposed model (17). Besides, we demonstrate that subgroup weight manipulation does also affect the final ranking of alternatives. We compare the effect of weight manipulative on the results in Table 3.

7 Conclusion

In SN-GDM, subgroups may exist weight manipulation behavior for certain benefit. This paper focuses on the impact of subgroup weight manipulation behavior on CRP. A method to prevent the weight manipulation in LSGDM problem by combining minimum adjustment and maximum entropy is investigated. The main contributions of the article are as follows.

(i) It develops a new research paradigm to study the subgroup manipulation behavior in the process of preference aggregation by analyzing the network structure relationship of subgroups. To do that, a LPA based community detection method is introduced to cluster the large group into several subgroups. Then, a power index is defined to obtain the power relation of subgroup. An attitude-OWA based on maximum entropy method is introduced to simulate subgroups' manipulation behavior from 'egalitarianism' to 'authoritarianism'.

(ii) Considering that weight manipulation behavior may hinder the efficiency of consensus, it investigates a method to prevent subgroup weight manipulation and facility the convergence of consensus with minimum cost. So, a minimum adjustment feedback mechanism based on maximum entropy method is established to assign reasonable weights for subgroups. The minimum adjustment is used for *'efficiency'* rule while the maximum entropy is used for *'justice'* rule. Compared with the method of Wu et al. (2021a), the proposed method has the advantage of considering the *'justice'* of the weight distribution.

The purposed method also has some problems that need to be further study.

(i) This paper treats the subgroup as a whole during the feedback process, so the differential feedback within the subgroup is not considered. In addition, the selection of parameters such as consensus threshold, network consensus threshold and Power parameter in this paper is random. But in the actual decision-making environment, the choice of these parameters are often related to human behavior (Wu et al 2021, Sun et al 2021). Therefore, it is necessary to conduct further research on them.
(ii) This paper only considers weight manipulation, although there are also other types of manipulation behaviors that deserve more in-depth research. For example, the existing feedback mechanisms usually assume that inconsistent experts are willing to accept feedback suggestions with fixed feedback parameters, which means, more frequent than not, that more adjustment costs are incurred for them than necessary, thereby reducing the independence of inconsistent experts. This type of manipulative behavior is called as group manipulation behavior (Wu et al. 2021a).

It could be an interesting future research topic to discuss the trust relationship of social networks in SN-GDM when studying group manipulation behavior. The other research direction is to apply the proposed method to group recommender systems for social items, such as education recommendation, travel packages and TV shows, which tend to be consumed by groups rather than individuals. A key issue involved in group recommender systems is the consensus reaching process, which inevitably requires consideration of 'justice' and 'efficiency'. Furthermore, a social network-based decision support system can be developed based on the proposed method.

Acknowledgements This work was supported by National Natural Science Foundation of China (NSFC) (Nos. 71971135, 71910107002), Shanghai Foreign Experts Program (No. 22WZ2506100), Innovative Talent Training Project of Graduate Students in Shanghai Maritime University of China (No. 2019YBR017) and was supported in part by the Spanish State Research Agency under Project PID2019-103880RB-I00/AEI/10.13039/501100011033.

References

Amirkhani A, Barshooi AH (2022) Consensus in multi-agent systems: a review. Artif Intell Rev 55:3897–3935. https://doi.org/10.1007/s10462-021-10097-x
Ben-Arieh D, Easton T (2007) Multi-criteria group consensus under linear cost opinion elasticity. Decis Support Syst 43:713–721. https://doi.org/10.1016/j.dss.2006.11.009
Cabrerizo FJ, Ureña R, Pedrycz W, Herrera-Viedma E (2014) Building consensus in group decision making with an allocation of information granularity. Fuzzy Sets Syst 255:115–127. https://doi.org/10.1016/j.fss.2014.03.016
Cao MS, Wu J, Chiclana F et al (2021) A personalized feedback mechanism based on maximum harmony degree for consensus in group decision making. IEEE Trans Syst Man Cybern Syst 51:6134–6146. https://doi.org/10.1109/TSMC.2019.2960052

Cao MS, Liu YJ, Gai TT et al (2022) A comprehensive star rating approach for cruise ships based on interactive group decision making with personalized individual semantics. J Mar Sci Eng 10:638. https://doi.org/10.3390/jmse10050638

Chao XR, Kou G, Peng Y, Herrera-Viedma E (2021) Large-scale group decision-making with non-cooperative behaviors and heterogeneous preferences: An application in financial inclusion. Eur J Oper Res 288:271–293. https://doi.org/10.1016/j.ejor.2020.05.047

Chen X, Ding ZG, Dong YC et al (2021) Managing consensus with minimum adjustments in group decision making with opinions evolution. IEEE Trans Syst Man Cybern Syst 51:2299–2311. https://doi.org/10.1109/TSMC.2019.2912231

Del Moral MJ, Chiclana F, Tapia JM, Herrera-Viedma E (2018) A comparative study on consensus measures in group decision making. Int J Intell Syst 33:1624–1638. https://doi.org/10.1002/int.21954

Ding RX, Palomares I, Wang XQ et al (2020) Large-scale decision-making: characterization, taxonomy, challenges and future directions from an artificial intelligence and applications perspective. Inf Fusion 59:84–102. https://doi.org/10.1016/j.inffus.2020.01.006

Dong QX, Zhü KY, Copper O (2017) Gaining consensus in a moderated group: a model with a twofold feedback mechanism. Expert Syst Appl 71:87–97. https://doi.org/10.1016/j.eswa.2016.11.020

Dong YC, Xu YF, Li HY, Feng B (2010) The OWA-based consensus operator under linguistic representation models using position indexes. Eur J Oper Res 203:455–463. https://doi.org/10.1016/j.ejor.2009.08.013

Dong YC, Liu YT, Liang HM et al (2018) Strategic weight manipulation in multiple attribute decision making. Omega 75:1339–1351. https://doi.org/10.1016/j.omega.2017.02.008

Dong YC, Zha QB, Zhang HJ, Herrera F (2021) Consensus reaching and strategic manipulation in group decision making with trust relationships. IEEE Trans Syst Man Cybern Syst 51:6304–6318. https://doi.org/10.1109/TSMC.2019.2961752

Gong ZW, Zhang N, Chiclana F (2018) The optimization ordering model for intuitionistic fuzzy preference relations with utility functions. Knowl Based Syst 162:174–184. https://doi.org/10.1016/j.knosys.2018.07.012

Gong ZW, Xu XX, Guo WW et al (2021) Minimum cost consensus modelling under various linear uncertain-constrained scenarios. Inf Fusion 66:1–17. https://doi.org/10.1016/j.inffus.2020.08.015

Herrera F, Martínez L (2000) A 2-tuple fuzzy linguistic representation model for computing with words. IEEE Trans Fuzzy Syst 8:746–752. https://doi.org/10.1109/91.890332

Herrera-Viedma E, Palomares I, Li CC et al (2021) Revisiting fuzzy and linguistic decision making: scenarios and challenges for making wiser decisions in a better way. IEEE Trans Syst Man Cybern Syst 51:191–208. https://doi.org/10.1109/TSMC.2020.3043016

Li GX, Kou G, Peng Y (2018) A group decision making model for integrating heterogeneous information. IEEE Trans Syst Man Cybern Syst 48:982–992. https://doi.org/10.1109/TSMC.2016.2627050

Li CC, Gao Y, Dong YC (2021) Managing ignorance elements and personalized individual semantics under incomplete linguistic distribution context in group decision making. Gr Decis Negot 30:97–118. https://doi.org/10.1007/s10726-020-09708-9

Li GX, Kou G, Peng Y (2022) Heterogeneous large-scale group decision making using fuzzy cluster analysis and its application to emergency response plan selection. IEEE Trans Syst Man Cybern Syst 52:3391–3403. https://doi.org/10.1109/TSMC.2021.3068759

Liu XW, Chen LH (2004) On the properties of parametric geometric OWA operator. Int J Approx Reason 35:163–178. https://doi.org/10.1016/j.ijar.2003.09.001

Liu X, Xu YJ, Montes R, Herrera F (2019) Social network group decision making: managing self-confidence-based consensus model with the dynamic importance degree of experts and trust-based feedback mechanism. Inf Sci 505:215–232. https://doi.org/10.1016/j.ins.2019.07.050

Liu YT, Dong YC, Liang HM et al (2019) Multiple attribute strategic weight manipulation with minimum cost in a group decision making context with interval attribute weights information. IEEE Trans Syst Man Cybern Syst 49:1981–1992. https://doi.org/10.1109/TSMC.2018.2874942

Liu YT, Zhang HJ, Wu YZ, Dong YC (2019) Ranking range based approach to madm under incomplete context and its application in venture investment evaluation. Technol Econ Dev Econ 25:877–899. https://doi.org/10.3846/tede.2019.10296

Liu JC, Sheu JB, Li DF, Dai YW (2021) Collaborative profit allocation schemes for logistics enterprise coalitions with incomplete information. Omega 101:102237. https://doi.org/10.1016/j.omega.2020.102237

Mandal P, Samanta S, Pal M, Ranadive AS (2022) Three-way decision model under a large-scale group decision-making environment with detecting and managing non-cooperative behaviors in consensus reaching process. Artif Intell Rev. https://doi.org/10.1007/s10462-021-10133-w

O'Hagan M (1988) Aggregating template or rule antecedents in real-time expert systems with fuzzy set logic. Asilomar Conf Signals IEEE Comput Soc. https://doi.org/10.1109/ACSSC.1988.754637

Pérez IJ, Cabrerizo FJ, Alonso S et al (2018) On dynamic consensus processes in group decision making problems. Inf Sci 459:20–35. https://doi.org/10.1016/j.ins.2018.05.017

Raghavan UN, Albert R, Kumara S (2007) Near linear time algorithm to detect community structures in large-scale networks. Phys Rev E 76:1–11. https://doi.org/10.1103/PhysRevE.76.036106

Rashidi F, Nejatian S, Parvin H, Rezaie V (2019) Diversity based cluster weighting in cluster ensemble: an information theory approach. Artif Intell Rev 52:1341–1368. https://doi.org/10.1007/s10462-019-09701-y

Rodríguez RM, Labella Á, De Tré G, Martnez L (2018) A large scale consensus reaching process managing group hesitation. Knowl Based Syst 159:86–97. https://doi.org/10.1016/j.knosys.2018.06.009

Sun Q, Wu J, Chiclana F et al (2021) A dynamic feedback mechanism with attitudinal consensus threshold for minimum adjustment cost in group decision making. IEEE Trans Fuzzy Syst 30:1287–1301. https://doi.org/10.1109/TFUZZ.2021.3057705

Tang M, Liao HC (2021) From conventional group decision making to large-scale group decision making: what are the challenges and how to meet them in big data era? A state-of-the-art survey. Omega 100:102141. https://doi.org/10.1016/j.omega.2019.102141

Tian ZP, Nie RX, Wang JQ, Long RY (2021) Adaptive consensus-based model for heterogeneous large-scale group decision making: detecting and managing non-cooperative behaviors. IEEE Trans Fuzzy Syst 29:2209–2223. https://doi.org/10.1109/TFUZZ.2020.2995229

Wan QF, Xu XH, Zhuang J, Pan B (2021) A sentiment analysis-based expert weight determination method for large-scale group decision-making driven by social media data. Expert Syst Appl 185:115629. https://doi.org/10.1016/j.eswa.2021.115629

Wang S, Wu J, Chiclana F et al (2022) Two stage feedback mechanism with different power structures for consensus in large-scale group decision-making. IEEE Trans Fuzzy Syst. https://doi.org/10.1109/TFUZZ.2022.3144536

Wang Z, Xiao FY, Cao ZH (2022) Uncertainty measurements for Pythagorean fuzzy set and their applications in multiple-criteria decision making. Soft Comput. https://doi.org/10.1007/s00500-022-07361-9

Wu J, Chiclana F (2014) A social network analysis trust-consensus based approach to group decision-making problems with interval-valued fuzzy reciprocal preference relations. Knowl Based Syst 59:97–107. https://doi.org/10.1016/j.knosys.2014.01.017

Wu ZB, Xu JP (2018) A consensus model for large-scale group decision making with hesitant fuzzy information and changeable clusters. Inf Fusion 41:217–231. https://doi.org/10.1016/j.inffus.2017.09.011

Wu J, Chiclana F, Herrera-Viedma E (2015) Trust based consensus model for social network in an incomplete linguistic information context. Appl Soft Comput J 35:827–839. https://doi.org/10.1016/j.asoc.2015.02.023

Wu J, Chiclana F, Fujita H, Herrera-Viedma E (2017) A visual interaction consensus model for social network group decision making with trust propagation. Knowl Based Syst 122:39–50. https://doi.org/10.1016/j.knosys.2017.01.031

Wu ZB, Jin BM, Fujita H, Xu JP (2020) Consensus analysis for AHP multiplicative preference relations based on consistency control: a heuristic approach. Knowl Based Syst 191:105317. https://doi.org/10.1016/j.knosys.2019.105317

Wu J, Cao MS, Chiclana F et al (2021) An optimal feedback model to prevent manipulation behaviour in consensus under social network group decision making. IEEE Trans Fuzzy Syst 29:1750–1763. https://doi.org/10.1109/TFUZZ.2020.2985331

Wu J, Wang S, Chiclana F, Herrera-Viedma E (2021) Twofold personalized feedback mechanism for social network consensus by uninorm interval trust propagation. IEEE Trans Cybern 1:1–12. https://doi.org/10.1109/TCYB.2021.3076420

Wu T, Liu XW, Qin JD, Herrera F (2021) Balance dynamic clustering analysis and consensus reaching process with consensus evolution networks in large-scale group decision making. IEEE Trans Fuzzy Syst 29:357–371. https://doi.org/10.1109/TFUZZ.2019.2953602

Wu J, Chen J, Liu W et al (2022) A calibrated individual semantic based failure mode and effect analysis and its application in industrial internet platform. Mathematics 10:2492. https://doi.org/10.3390/math10142492

Xiao FY, Wen JH, Pedrycz W (2022) Generalized divergence-based decision making method with an application to pattern classification. IEEE Trans Knowl Data Eng 4347:1. https://doi.org/10.1109/tkde.2022.3177896

Xiao J, Wang XL, Zhang HJ (2022) Exploring the ordinal classifications of failure modes in the reliability management: an optimization-based consensus model with bounded confidences. Gr Decis Negot 31:49–80. https://doi.org/10.1007/s10726-021-09756-9

Xing YM, Cao MS, Liu YJ et al (2022) A Choquet integral based interval Type-2 trapezoidal fuzzy multiple attribute group decision making for sustainable supplier selection. Comput Ind Eng 165:107935. https://doi.org/10.1016/j.cie.2022.107935

Xu Xh, Du ZJ, Chen XH, Cai CG (2019) Confidence consensus-based model for large-scale group decision making: a novel approach to managing non-cooperative behaviors. Inf Sci 477:410–427. https://doi.org/10.1002/int.22122

Xu YJ, Gao PQ, Martínez L (2019) An interindividual iterative consensus model for fuzzy preference relations. Int J Intell Syst 34:1864–1888. https://doi.org/10.1016/j.ins.2019.07.050

Xu YJ, Zhu SN, Liu X et al (2022) Additive consistency exploration of linguistic preference relations with self-confidence. Artif Intell Rev. https://doi.org/10.1007/s10462-022-10172-x

Yager RR (1988) On ordered weighted averaging aggregation operators in multicriteria decisionmaking. IEEE Trans Syst Man Cybern 18:183–190. https://doi.org/10.1109/21.87068

Yager RR (2001) Penalizing strategic preference manipulation in multi-agent decision making. IEEE Trans Fuzzy Syst 9:393–403. https://doi.org/10.1109/91.928736

Yager RR (2002) Defending against strategic manipulation in uninorm-based multi-agent decision making. Eur J Oper Res 141:217–232. https://doi.org/10.1016/S0377-2217(01)00267-3

Yu GF, Li DF, Liang DC, Li GX (2021) An intuitionistic fuzzy multi-objective goal programming approach to portfolio selection. Int J Inf Technol Decis Making 20:1477–1497. https://doi.org/10.1142/S0219622021500395

Zhang Z, Guo CH, Martínez L (2017) Managing multigranular linguistic distribution assessments in large-scale multiattribute group decision making. IEEE Trans Syst Man Cybern Syst 47:3063–3076. https://doi.org/10.1109/TSMC.2016.2560521

Zhang HJ, Zhao SH, Kou G et al (2020) An overview on feedback mechanisms with minimum adjustment or cost in consensus reaching in group decision making: Research paradigms and challenges. Inf Fusion 60:65–79. https://doi.org/10.1016/j.inffus.2020.03.001

Zhang Z, Gao Y, Li ZL (2020) Consensus reaching for social network group decision making by considering leadership and bounded confidence. Knowl Based Syst 204:106240. https://doi.org/10.1016/j.knosys.2020.106240

Zhang YJ, Chen X, Gao L et al (2022) Consensus reaching with trust evolution in social network group decision making. Expert Syst Appl 188:116022. https://doi.org/10.1016/j.eswa.2021.116022

Zhao ZY, Li C, Zhang XJ et al (2019) An incremental method to detect communities in dynamic evolving social networks. Knowl Based Syst 163:404–415. https://doi.org/10.1016/j.knosys.2018.09.002

Authors and Affiliations

Qi Sun[1,2] · Jian Wu[1,2] · Francisco Chiclana[3,4] · Sha Wang[1,2] · Enrique Herrera-Viedma[4,5] · Ronald R. Yager[6]

Qi Sun
qisun1207@126.com

Francisco Chiclana
chiclana@dmu.ac.uk

Sha Wang
wangshaws@126.com

Enrique Herrera-Viedma
viedma@decsai.ugr.es

Ronald R. Yager
yager@panix.com

[1] School of Economics and Management, Shanghai Maritime University, Shanghai 201306, China

[2] Center for Artificial Intelligence and Decision Sciences, Shanghai Maritime University, Shanghai 201306, China

[3] Faculty of Computing, Engineering and Media, Institute of Artificial Intelligence, De Montfort University, Leicester, UK

[4] Department of Computer Science and AI, Andalusian Research Institute in Data Science and Computational Intelligence, University of Granada, 18071 Granada, Spain

[5] Department of Electrical and Computer Engineering, Faculty of Engineering, King Abdulaziz University, Jeddah 21589, Saudi Arabia

[6] Machine Intelligence Institute, Iona College, New Rochelle, NY 10801, USA