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Fire Hawk Optimizer: a novel metaheuristic algorithm

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Abstract

This study proposes the Fire Hawk Optimizer (FHO) as a novel metaheuristic algorithm based on the foraging behavior of whistling kites, black kites and brown falcons. These birds are termed Fire Hawks considering the specific actions they perform to catch prey in nature, specifically by means of setting fire. Utilizing the proposed algorithm, a numerical investigation was conducted on 233 mathematical test functions with dimensions of 2–100, and 150,000 function evaluations were performed for optimization purposes. For comparison, a total of ten different classical and new metaheuristic algorithms were utilized as alternative approaches. The statistical measurements include the best, mean, median, and standard deviation of 100 independent optimization runs, while well-known statistical analyses, such as Kolmogorov–Smirnov, Wilcoxon, Mann–Whitney, Kruskal–Wallis, and Post-Hoc analysis, were also conducted. The obtained results prove that the FHO algorithm exhibits better performance than the compared algorithms from literature. In addition, two of the latest Competitions on Evolutionary Computation (CEC), such as CEC 2020 on bound constraint problems and CEC 2020 on real-world optimization problems including the well-known mechanical engineering design problems, were considered for performance evaluation of the FHO algorithm, which further demonstrated the superior capability of the optimizer over other metaheuristic algorithms in literature. The capability of the FHO is also evaluated in dealing with two of the real-size structural frames with 15 and 24 stories in which the new method outperforms the previously developed metaheuristics.

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1 Introduction

Optimization is the process of decision-making between multiple approaches to achieve the best performance for dealing with a specific system problem. In recent decades, the importance of optimization in performance improvements of different engineering and economic design problems has gained increasing awareness. Specifically, the best decision or solution for a predefined design problem is identified by evaluating different alternative approaches. The predefined measure for the quality of a decision is considered by determining an objective function which is addressed in most of the cases as a performance evaluation index. In other words, optimization concerns the process of selecting the best decision among multiple alternative choices by considering the satisfaction of an objective function.

Regarding the rapid progression of various software programs and high-speed parallel processors in the computer science and technology fields, optimization has received heightened attention especially by engineering and economic experts. However, most calculus-based optimization algorithms are incapable of finding the global optimum solutions, which is considered the main deficiency of these algorithms. For instance, gradient-based algorithms require differentiable objective functions that are not achievable in dealing with complex optimization problems. In this regard, the metaheuristic algorithms have been proposed as successful practical methods that provide acceptable accuracy for different optimization purposes. The history of metaheuristic algorithms (Sörensen et al. 2018) can be broken down into five time-periods: (1) "Pre-Theoretical Period" (before 1940) had limited formal presentation of metaheuristics; (2) "Early Period" (1940–1980) witnessed the formal presentation of the mathematical formulation of metaheuristics; (3) "Method-Centric Period" (1980–2000) saw the introduction and application of many metaheuristic algorithms in different fields; (4) "Framework-Centric Period" (2000-present) observed the utilization of metaheuristics as strong frameworks in different optimization fields; and (5) "Scientific or Future Period" where the metaheuristics are assumed to turn into matter of science instead of art.

Based on the developments of different metaheuristic algorithms in recent decades, four categories can be determined by considering the main concept of these algorithms. The first category, "Evolutionary Algorithms," represents the algorithms that are developed based on biological reproduction and evolution, such as the Genetic Algorithm (GA) (Holland 1984), Differential Evolution (DE) (Storn and Price 1997), and Biogeography-Based Optimizer (BBO) (Simon 2008). The second category includes the algorithms that are developed based on "Swarm Intelligence," such as the Particle Swarm Optimization (PSO) (Eberhart and Kennedy 1995), Ant Colony Optimization (ACO) (Dorigo et al. 1996), and Firefly Algorithm (FA) (Yang 2012). In the third category, the "Physics-Inspired Algorithms" are the Harmony Search (HS) (Geem et al. 2001), Gravitational Search Algorithm (GSA) (Rashedi et al. 2009), Big-Bang Big-Crunch (BBBC) (Erol and Eksin 2006), Charged System Search (CSS) (Kaveh and Talatahari 2010a, b, c, d), Wind Driven Optimization (WDO) (Bayraktar et al. 2010), Multi-verse Algorithm (MVO) (Mirjalili et al. 2016), Rain Fall Optimization (RFO) algorithm (Aghay Kaboli et al. 2017), Chaos Game Optimization (CGO) algorithm (Talatahari and Azizi 2020b, 2021a), Crystal Structure Algorithm (Talatahari et al. 2021a, b, c, d, e), Material Generation Algorithm (Talatahari et al. 2021a), and Atomic Orbital Search (Azizi 2021). In the last category, the algorithms are developed based on the lifestyle of humans and animals and include the Bees Algorithm (BA) (Pham et al. 2006), Imperialistic Competitive Algorithm (ICA) (Atashpaz-Gargari

and Lucas 2007), Bat Inspired Algorithm (BIA) (Yang 2010a), Sine Cosine Algorithm (SCA) (Mirjalili 2016), Jaya Algorithm (JA) (Rao 2016), Whale Optimization Algorithm (WOA) (Mirjalili and Lewis 2016), Grey Wolf Optimizer (GWO) (Mirjalili et al. 2014), Harris Hawks Optimization (HHO) (Heidari et al. 2019), Butterfly Optimization Algorithm (BOA) (Arora and Singh 2019), Pity Beetle Algorithm (PBA) (Kallioras et al. 2018), Arithmetic Optimization Algorithm (AOA) (Abualigah et al. 2021a), Aquila Optimizer (AO) (Abualigah et al. 2021b), Interior search algorithm (ISA) (Gandomi 2014), and Drone Squadron Optimization (DSO) (de Melo and Banzhaf 2018). It should be noted that some of the standard algorithms have been improved or hybridized for specific applications (Azizi et al. 2019a, b, 2020a, b; Sadollah et al. 2018; Talatahari and Azizi 2020a, c, 2021b; Talatahari et al. 2021b, d, e).

In recent years, some newer metaheuristics have not yet been specifically categorized into the mentioned classifications. For example, Hayyolalam and Pourhaji Kazem (2020) proposed the Black Widow Optimization (BWO) algorithm, which mimics the unique mating behavior of black widow spiders in nature. Nematollahi et al. (2020) developed the Golden Ratio Optimization Method (GROM) as a novel metaheuristic algorithm inspired by the golden ratio of plant and animal growth in nature. Zhang and Jin (2020) presented the Group Teaching Optimization Algorithm (GTOA) that mimics the group teaching mechanism of humans. Li and Tam (2020) developed the Virus Spread Optimization (VSO) as a novel metaheuristic algorithm inspired by the spread of viruses among hosts. Alsattar et al. (2020) proposed the Bald Eagle Search (BES) algorithm for optimum design purposes in which the hunting intelligence and strategy of bald eagles in searching fishes is followed. Feng et al. (2021) presented the Cooperation Search Algorithm (CSA), which is motivated by the team cooperation behaviors in modern enterprises. Ghasemian et al. (2020) developed the Human Urbanization Algorithm (HUA) as a new metaheuristic algorithm that mimics the human behaviors and actions for improving life situations and urbanization. Kaveh et al. (2020) proposed the Black Hole Mechanics Optimization (BHMO) for optimization purposes, which is based on the mechanics of black holes in space. Braik et al. (2021) presented Capuchin Search (CSA) as a novel metaheuristic algorithm for optimum design purposes based on the dynamic behavior of capuchin monkeys. Ahmia and Aider (2019) developed Monarchy Metaheuristic (MN) for optimization purposes that is inspired by the monarchy government system. Brammya et al. (2019) proposed the Deer Hunting Optimization Algorithm (DHOA) as an optimization algorithm, which considers the behavior humans exhibit when hunting deer. Besides, the most important application of these algorithms can be in dealing with engineering design problems including the stress-based topology optimization by Xia et al. (2018), material and shape optimization of structures by Wang et al. (2020), structural engineering design by Zhao et al. (2018), optimization of phase change material and insulation layer thickness (Daqiqnia et al. 2021), performance-based structural design optimization by Gholizadeh et al. (2020), and Optimal design of structures by Gholizadeh and Salajegheh (2009).

In this paper, the Fire Hawk Optimizer (FHO) is proposed as a novel metaheuristic algorithm inspired by the foraging behavior of whistling kites, black kites, and brown falcons. These birds catch prey in nature by means of setting fire and, thus, are aptly called Fire Hawks. A numerical investigation was conducted to evaluate the performance of the FHO algorithm by considering 233 mathematical test functions with dimensions of 2–100 and completing 150,000 function evaluations for optimization purposes. For comparative purposes, a total number of 10 different classical and new metaheuristic algorithms were analyzed as alternative approaches. The best, mean, median and standard deviation results of 100 independent optimization runs were

obtained for comparison, and Kolmogorov–Smirnov (KS) test, Wilcoxon (W) sign rank test, Mann Whitney (MW) test, Kruskal Wallis (KW) test, and Post Hoc (PH) analyses were conducted accordingly. Moreover, two of the latest Competitions on Evolutionary Computation (CEC), including CEC 2020 on bound constraint problems and CEC 2020 on real-world optimization problems, were utilized for further performance evaluation of the FHO algorithm and comparison to other metaheuristic algorithms in the literature. The capability of the FHO is also evaluated in dealing with two of the real-size structural frames with 15 and 24 stories in which the new method outperforms the previously developed metaheuristics.

This research's novelty can be seen from inspirational and computational points of view. The foraging behavior of fire hawks is utilized for the first time in this paper for developing a novel metaheuristic algorithm. Besides, the complexity level of the utilized test functions is also a different novelty aspect of this paper. It should be noted that the overall performance of numerous algorithms must be evaluated under the same conditions and with similar problems, and under diverse cases, the superiority of each algorithm cannot be proved or disproved by means of different examples and datasets. In this regard, the benchmark test functions of well-known competitions on evolutionary computation should be utilized for having a fair judgment so the capability of FHO as a novel algorithm has been evaluated by utilization of different sets of CEC test problems. This level of complexity in choosing test factions have been utilized for the first time in evaluating novel algorithms. However, the FHO algorithm's advantages include being parameter-free, having quick convergence behaviour, and having the lowest possible objective function evaluations in dealing with different design examples. On the other hand, it cannot produce accurate answers; in other words, the FHO algorithm is an approximation algorithm like other metaheuristic algorithms. Nonetheless, a plethora of metaheuristic algorithms has been proposed through miscellaneous inspirational concepts from nature, which the mathematical models and the specific aspects of the algorithms in their searching groups should be distinct and novel so as to prepare and backup the research from the stable point. PSO, for example, is one of the pioneer algorithms in the metaheuristic area, in which the position updating process by the solution candidates is conducted using the global best and local best of each particle. In stark contrast, in the FHO algorithm, the position updating process is carried out by utilizing the better solution's not the global best, and the mean of the solution candidates, which makes the searching process avoid entrapping in local optimum points. Furthermore, in the Genetic Algorithm (GA), a new solution candidate is created by combining two populations, so there is a possibility of reaching a solution that can be entrapped in the local optimum point. However, in the FHO algorithm, the mean of solution candidates in the specific territory is utilized to avoid entrapment in the local optimum and provide solutions that can finally reach the global optimum.

The main contribution of this paper is as follows:

- Fire Hawks bizarre behaviour spread fire intentionally by carrying burning sticks in their beaks, and talons are examined and analysed to develop a mathematical model.
- A unique nature-inspired FHO algorithm is developed using this model.
- The FHO algorithm's solution updating depends on the preys' new position and safe places under/outside the fire.
- FHO's performance is extensively evaluated against a set of 233 benchmark functions and well-known CEC design examples. It is compared to a plethora of state-of-the-art metaheuristic algorithms.

The rest of the paper is divided into the following sections. In Sect. 2, the inspiration and mathematical model of the proposed FHO algorithm are presented. The numerical investigations, including the mathematical test functions, are indicated in Sect. 3. Alternative Metaheuristic algorithms used in this paper are described in Sect. 4. Furthermore, numerical results of the mathematical functions, statistical analysis, bound constraint benchmark problems of CEC 2020, and computational complexity and cost analysis are demonstrated in Sects. 5, 6, 7, and 8, respectively. Sections 9 and 10 indicate real-world constrained optimization problems of CEC 2020 and structural optimization, respectively. Finally, in Sect. 11, the core findings of this study are presented as concluding remarks.

2 Fire Hawk optimizer (FHO)

In this section, the inspiration concept of the FHO alongside the mathematical model of the proposed metaheuristic algorithm are illustrated.

2.1 Inspiration

Native Australians utilize fire as an effective tool to control and maintain balance of the local ecosystem and landscape, which has been a part of cultural and ethnical traditions for many years. Most of the time, the fires that are started on purpose or may naturally occur due to lightning can be spread by people and other factors, increasing the vulnerability of the native landscape and wildlife. Moreover, whistling kites, black kites, and brown falcons are also responsible for spreading fires across the country—this alternative cause has only been realized recently. These birds, known as Fire Hawks, try to spread fire intentionally by carrying burning sticks in their beaks and talons, which is reported as a destructive phenomenon in nature. Figure 1 provides images showing the behavior of these birds around fires.

As a mechanism to control and capture their prey, the birds pick up burning sticks and drop them in other unburned places in order to set small fires. These small fires scare the prey, including rodents, snakes, and other animals, and force them flee in a most hasty and nervous way that makes it much easier for the hawks to catch.

2.2 Mathematical model

The FHO metaheuristic algorithm mimics the foraging behavior of fire hawks, considering the process of setting and spreading fires and catching prey. At first, a number of solution candidates (X) are determined as the position vectors of the fire hawks and prey. A random initialization process is utilized to identify the initial positions of these vectors in the search space.

$$X = \begin{bmatrix} X_1 \\ X_2 \\ \vdots \\ X_i \\ \vdots \\ X_N \end{bmatrix} = \begin{bmatrix} x_1^1 x_1^2 \cdots x_1^j \cdots x_1^d \\ x_2^1 x_2^2 \cdots x_2^j \cdots x_2^d \\ \vdots \vdots \vdots \ddots \vdots \\ x_i^1 x_i^2 \cdots x_i^j \cdots x_i^d \\ \vdots \vdots \vdots \ddots \vdots \\ x_N^1 x_N^2 \cdots x_N^j \cdots x_N^d \end{bmatrix}, \begin{cases} i = 1, 2, \dots, N. \\ j = 1, 2, \dots, d. \end{cases} \tag{1}$$
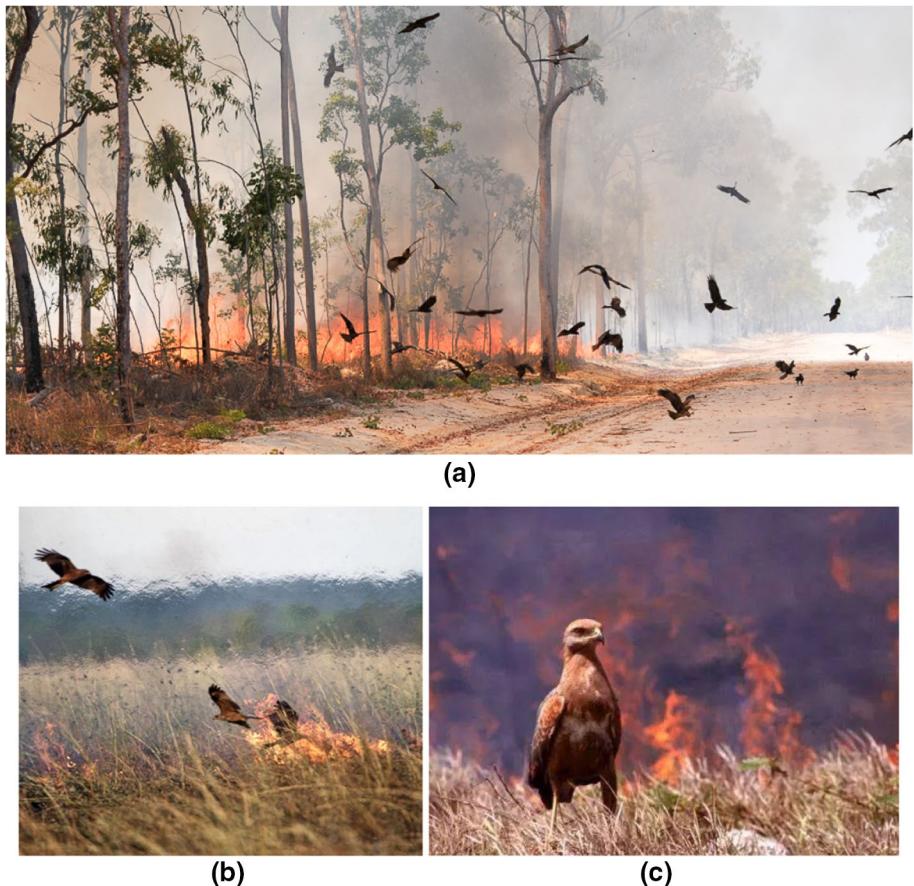
Fig. 1 Photographs of Fire Hawks' behavior around fires: **a** www.theaggie.org. **b** www.dailymail.co.uk. **c** www.twitter.com

$$x_i^j(0) = x_{i,min}^j + \text{rand.}\left(x_{i,max}^j - x_{i,min}^j\right), \begin{cases} i = 1, 2, \ldots, N. \\ j = 1, 2, \ldots, d. \end{cases} \qquad (2)$$

where X_i represents the *ith* solution candidate in the search space; d represents the dimension of the considered problem; N is the total number of solution candidates in the search space; x_i^j is the *jth* decision variable of the *ith* solution candidate; $x_i^j(0)$ represents the initial position of the solution candidates; $x_{i,min}^j$ and $x_{i,max}^j$ are the minimum and maximum bounds of the *jth* decision variable for the *ith* solution candidate; and rand is a uniformly distributed random number in the range of [0,1].

In order to determine the locations the Fire Hawks in the search space, the objective function evaluation for the solution candidates considers the selected optimization problem. Some of the solution candidates with better objective function values are represented as Fire Hawks, while the rest of the solution candidates are the prey. The selected Fire Hawks are utilized for spreading fires around the prey in the search space to make the

hunting easier. Besides, the global best solution is assumed to be the main fire that is first utilized by the Fire Hawks to spread fires through the search space (nature). In Fig. 2a, b, the schematic presentation of these aspects is provided, which are mathematically presented as follows:

$$PR = \begin{bmatrix} PR_1 \\ PR_2 \\ \vdots \\ PR_k \\ \vdots \\ PR_m \end{bmatrix}, k = 1, 2, \dots, m, \tag{3}$$

(a)

(b)
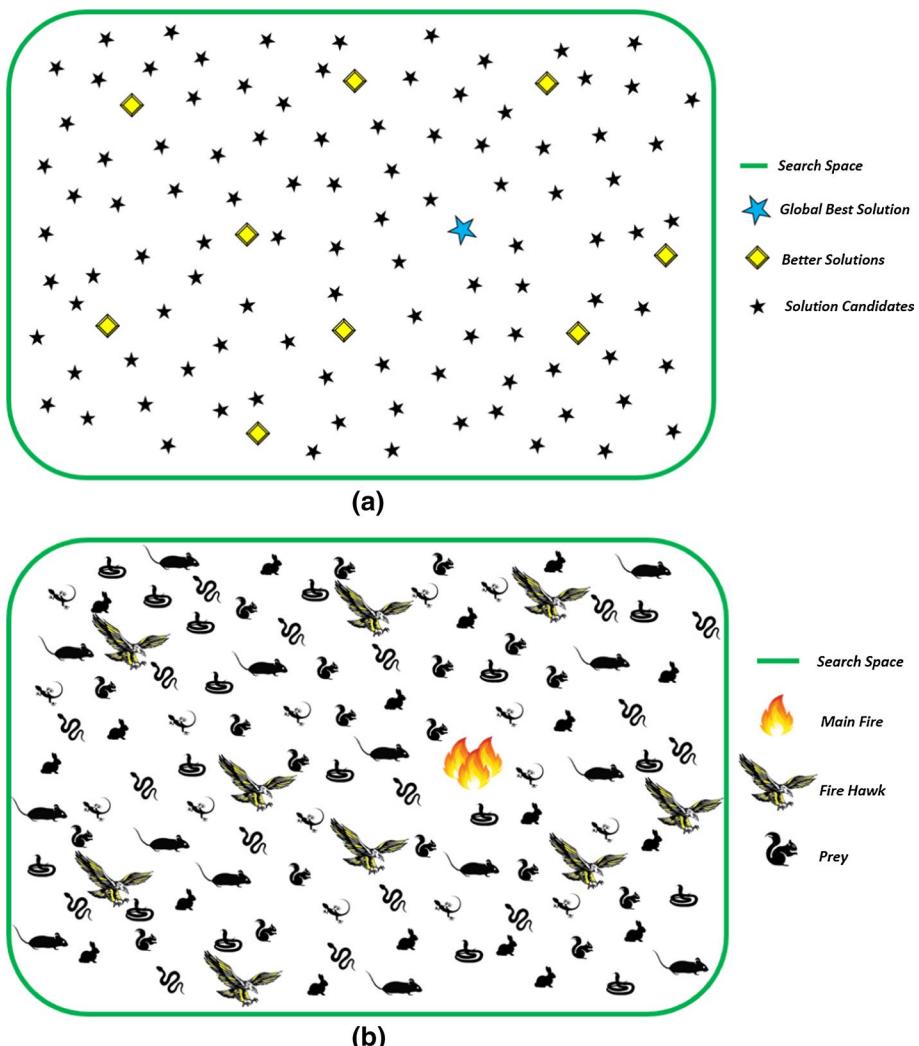
Fig. 2 Schematic presentation of determining fire hawks and prey in the search space

$$FH = \begin{bmatrix} FH_1 \\ FH_2 \\ \vdots \\ FH_l \\ \vdots \\ FH_n \end{bmatrix}, l = 1, 2, \ldots n, \tag{4}$$

where PR_k is the *kth* prey in the search space regarding the total number of *m* preys; and FH_l is the *lth* fire hawk considering a total number of *n* fire hawks in the search space.

In the next phase of the algorithm, the total distance between the Fire Hawks and the prey is calculated. As a result, the nearest prey to each bird is determined so that the effective territory of these birds is distinguished. It should be noted that the nearest prey to the first Fire Hawk with the best objective function value is determined, while the territory of the other birds are considered by means of the remaining prey. Figure 3 provides an illustration of this perspective, where D_k^l is determined by means of the following equation:

$$D_k^l = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}, \begin{cases} l = 1, 2, \ldots, n. \\ k = 1, 2, \ldots, m. \end{cases} \tag{5}$$

where D_k^l is the total distance between the *lth* fire hawk and the *kth* prey; m is the total number of prey in the search space; n is the total number of fire hawks in the search space; and (x_1, y_1) and (x_2, y_2) represent the coordinates of the Fire Hawks and prey in the search space.
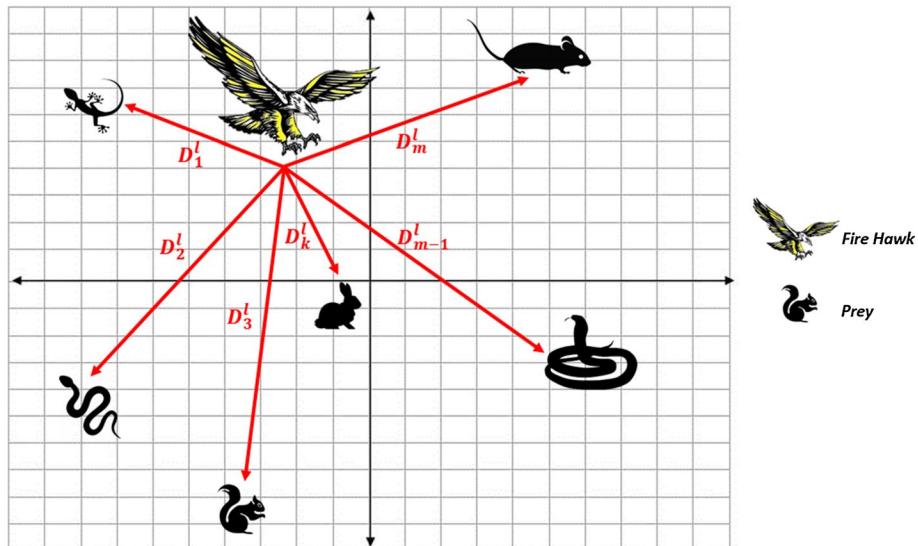
Fig. 3 Schematic presentation for measuring the total distance between the Fire Hawks and the prey

After conducting the mentioned procedure for measuring the total distance between the Fire Hawks and prey, the territory of these birds is distinguished by means of the nearest prey around them. By classifying the Fire Hawks and prey, the searching process of the algorithm is configured. It should be noted that the Fire Hawk with the better objective function value selects the best nearest prey in the search space for its specific territory. Then, the other Fire Hawks accomplish the next nearest prey in the search space, which supports that the strongest Fire Hawks accomplish perform more successful hunting than the weaker birds. In Fig. 4, the schematic presentation of determining Fire Hawks' territory in the search space is provided.

In the next phase of the algorithm, the Fire Hawks collect burning sticks from the main fire in order to set fire in the selected area. In this stage, each bird picks up a burning stick then drops it in its specific territory to force the prey to hastily flee. Meanwhile, some birds are eager to use the burning sticks from other Fire Hawks' territories; therefore, these two behaviors can be utilized as position updating procedures in the main search loop of FHO, as indicated in the following equation:

$$FH_l^{new} = FH_l + \left(r_1 \times GB - r_2 \times FH_{Near} \right), \ l = 1, 2, \ldots, n, \tag{6}$$

where FH_l^{new} is the new position vector of the *lth* Fire Hawk (FH_l); GB is the global best solution in the search space considered as the main fire; FH_{Near} is one of the other Fire Hawks in the search space; and r_1 and r_2 are uniformly distributed random numbers in the range of (0, 1) for determining the movements of Fire Hawks toward the main fire and the other Fire Hawks' territories (see Fig. 5).
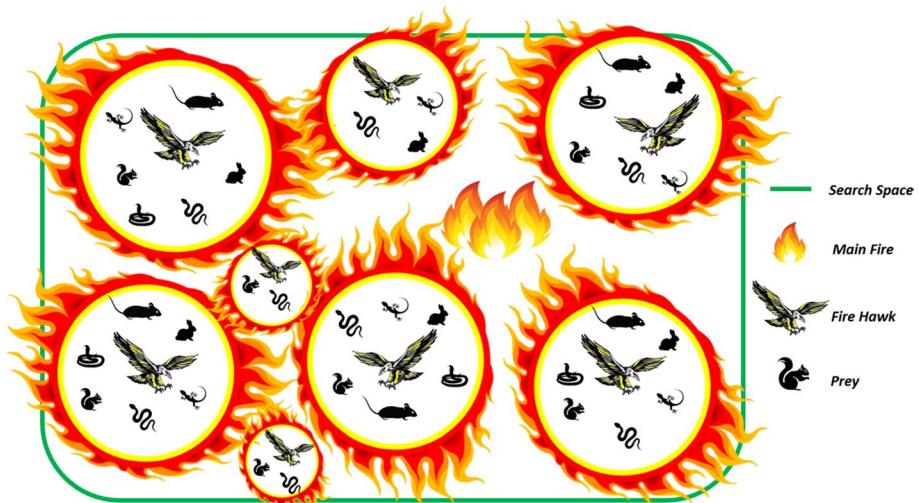
Fig. 4 Schematic presentation of determining Fire Hawks' territory in the search space
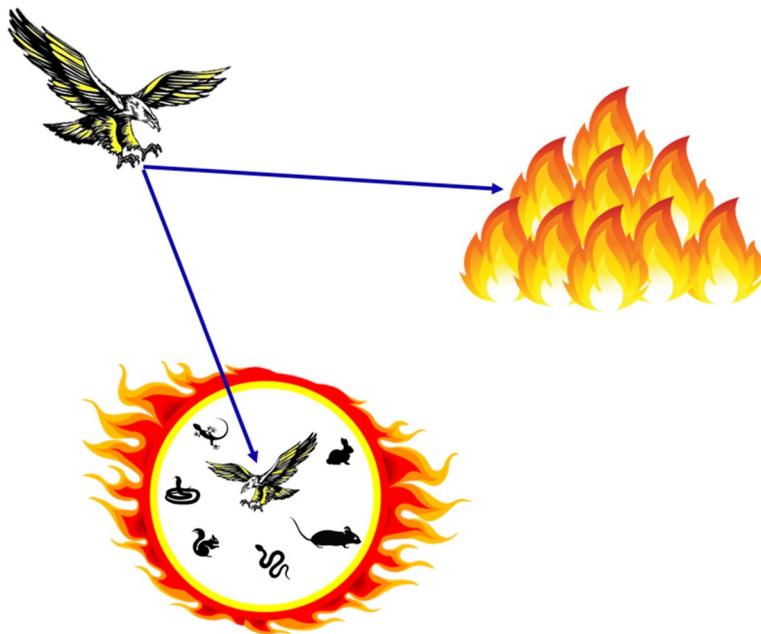
Fig. 5 Schematic presentation of the Fire Hawks' position updating process in the search space

In the next phase of the algorithm, the movement of prey inside the territory of each Fire Hawk is considered a key aspect of animal behavior for the position updating process. When a burning stick is dropped by a Fire Hawk, the prey decide to hide, run away, or will run towards the Fire Hawk by mistake. These actions can be considered in the position updating process by using the following equation:

$$PR_q^{new} = PR_q + \left(r_3 \times FH_l - r_4 \times SP_l\right), \begin{cases} l = 1, 2, \dots, n. \\ q = 1, 2, \dots, r. \end{cases} \tag{7}$$

where PR_q^{new} is the new position vector of the *qth* prey (PR_q) surrounded by the *lth* Fire Hawk (FH_l); GB is the global best solution in the search space considered as the main fire; SP_l is a safe place under the *lth* Fire Hawk territory; and r_3 and r_4 are uniformly distributed random numbers in the range of (0, 1) for determining the movements of prey toward the Fire Hawks and the safe place (see Fig. 6).

Besides, the prey may have movements toward the other Fire Hawks' territory while there is a possibility in which the preys may get closer to the Fire Hawks in the near ambushes or even try to hide in a safer place outside the Fire Hawk's territory in which they are entrapped. These actions can be considered in the position updating process by using the following equation (Fig. 7):

$$PR_q^{new} = PR_q + \left(r_5 \times FH_{Alter} - r_6 \times SP\right), \begin{cases} l = 1, 2, \dots, n. \\ q = 1, 2, \dots, r. \end{cases} \tag{8}$$

where PR_q^{new} is the new position vector of the *qth* prey (PR_q) surrounded by the *lth* fire hawk (FH_l); FH_{Alter} is one of the other fire hawks in the search space; SP is a safe place outside the *lth* Fire Hawk's territory; r_5 and r_6 are uniformly distributed random numbers

Fig. 6 Schematic presentation of the preys position updating process inside the fire hawks' territory
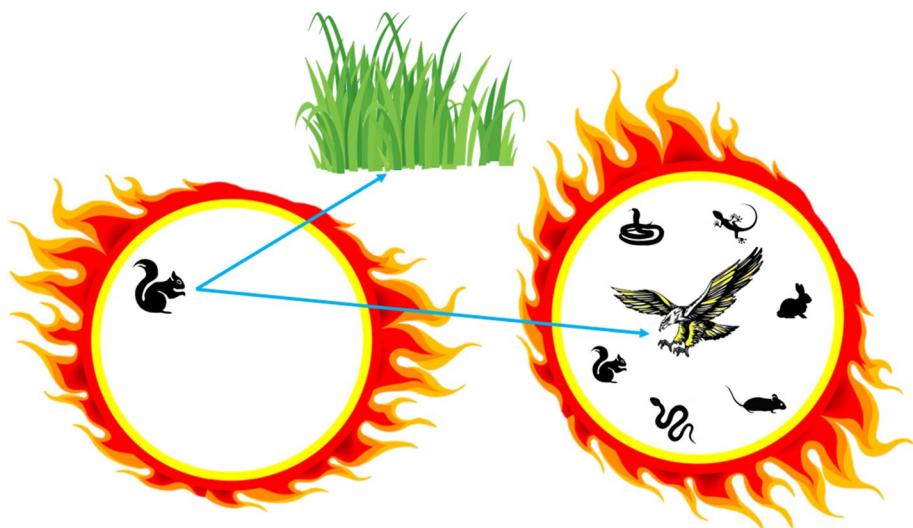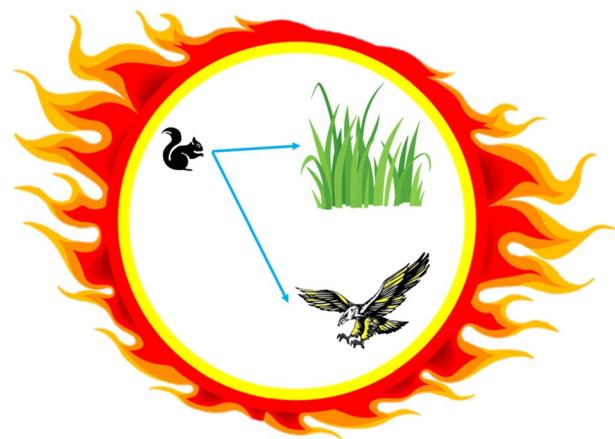
Fig. 7 Schematic presentation of the preys position updating process outside the fire hawks' territory

in the range of (0, 1) for determining the movements of preys toward the other Fire Hawks and the safe place outside the territory.

Based on the fact that the safe place in nature is a place that most of the animals gather to gather in order to remain safe and sound during a hazard, the mathematical presentation of SP_l and SP are formulated as follows:

$$SP_l = \frac{\sum_{q=1}^{r} PR_q}{r}, \begin{cases} q = 1, 2, \ldots, r. \\ l = 1, 2, \ldots, n. \end{cases} \qquad (9)$$

$$SP = \frac{\sum_{k=1}^{m} PR_k}{m}, k = 1, 2, \ldots, m. \tag{10}$$

where PR_q is the *qth* prey surrounded by the *lth* fire hawk (FH_l); PR_k is the *kth* prey in the search space.

It should be noted that the territory of each fire hawk is assumed as a circular area for schematic presentation purposes, so the exact definition of the territory is dependent on the overall distances of the prey and the considered fire hawk. In other words, when prey is positioned in a specific fire hawk's territory, it is assumed to be affected by the considered fire hawk and not the other ones, so the number of the preys and their distances to the considered fire hawk determine the limits of the territory of this fire hawk. Meanwhile, the possibility of the preys being outside their own territory is also considered in the position updating process regarding the fact that the preys should be affected by the fire hawks from other territories. The number of preys in each search loop is the total number of solution candidates minus the number of fire hawks determined randomly through the Brownian motion with a Gaussian distribution as one of the well-known distributions utilized in randomization procedures.

The general aspects of FHO including the boundary violation of solution candidates alongside the termination criterion are also considered in the mathematical model of this algorithm. In this regard, a mathematical flag is implemented in the FHO in which a boundary control for violating decision variables is determined while a predefined number of objective function evaluations or iterations can be utilized as termination criteria. In Fig. 8, the pseudo-code of the FHO algorithm is provided, and Fig. 9 presents the flowchart of this algorithm.

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procedure *Fire Hawk Optimizer (FHO)*
    Determine initial positions of solution candidates (X_i*) in the search space with N candidates*
    Evaluate fitness values for initial solution candidates
    Determine the Global Best (GB) solution as the main fire
    while *Iteration < Maximum number of iterations*
        Generate n *as a random integer number for determining the number of Fire Hawks*
        *Determine Fire Hawks (*FH*) and Preys (*PR*) in the search space*
        Calculate the total distance between the Fire Hawks and the preys
        Determine the territory of the Fire Hawks by dispersing the preys
        for *l=1:* n
            Determine the new position of the Fire Hawks by Eq. 6.
            for *q=1:*r
                Calculate the safe place under lth Fire Hawk territory by Eq. 9.
                Determine the new position of the preys by Eq. 7.
                Calculate the safe place outside the lth Fire Hawk territory by Eq. 10.
                Determine the new position of the preys by Eq. 8.
            end
        end
        Evaluate fitness values for the newly created Fire Hawks and preys
        Determine the Global Best (GB) solution as the main fire
    end while
    return GB
end procedure

Fig. 8 Pseudo-code of FHO
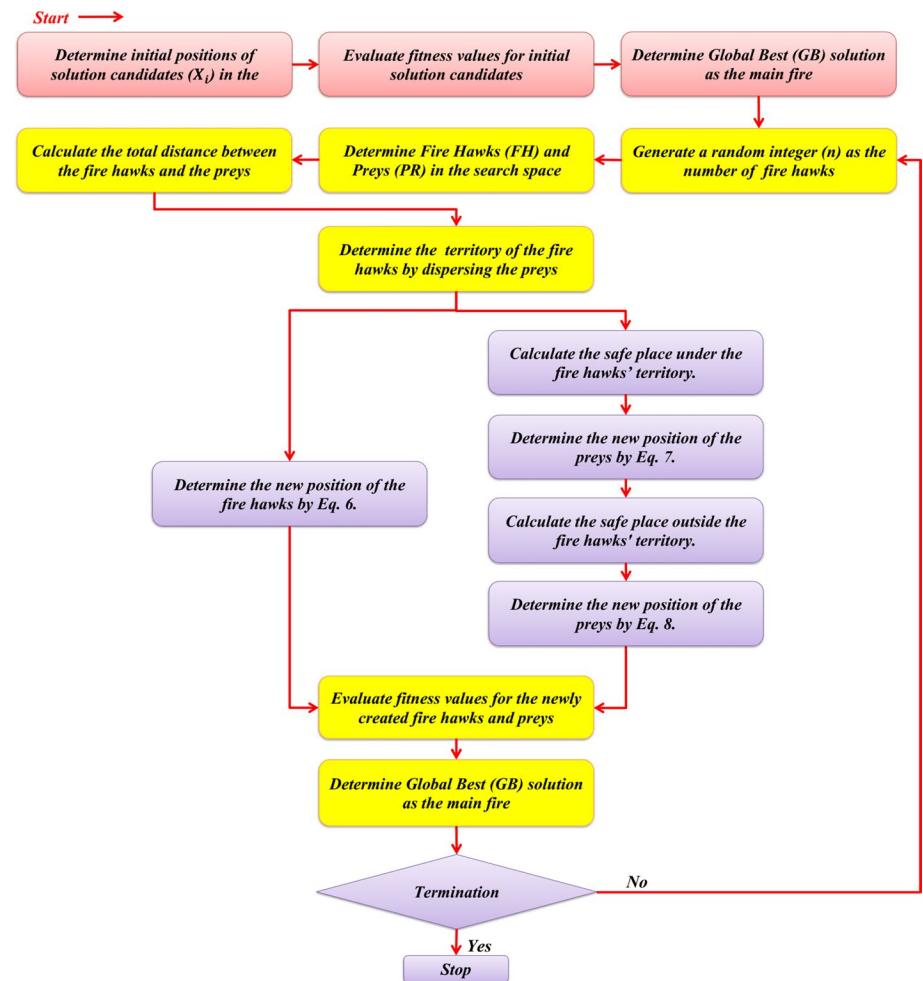
Fig. 9 Flowchart of FHO

3 Mathematical test functions

In order to conduction a comprehensive investigation of FHO, a total of 233 mathematical test functions were collected, which are briefly described in this section. Meanwhile, all tests to evaluate the performance of FHO algorithm were conducted using a PC with the detailed parameters shown in Table 1. These functions have different dimensions varying from 2 to 100 and are among the most well-known unconstrained mathematical test functions in the field of global optimization. In this selection, 117 test functions were selected with minimum dimension of 2 and maximum dimension of 4 (F1–F117), which are denoted "Small Scale (SS) functions" in this paper. In addition, 58 selected functions with a dimension of 50 (F118–F175) were named "50D functions," and 58 functions (F176–F233) with a dimension of 100 were termed "100D functions." The general description of these test functions are provided in Table 2, and the 3D plots are provided in Figs. 10, 11, 12, 13. The fully-detailed mathematical presentations of these

Table 1 Details of the utilized system in optimization process in the current study

Name	Detailed settings
Hardware	
CPU	CORE i7
Frequency	2.8 GHz
RAM	8 GB
Hard drive	2 TB
Software	
Operating system	Windows 10
Language	MATLAB R2020

test functions were obtained from (Jamil and Yang 2013), (Jamil et al. 2013), (Yang 2010b), and (Liang et al. 2005).

4 Alternative metaheuristics

Regarding the fact that the overall performance of FHO should be evaluated by comparative means, a total of 16 metaheuristic algorithms, particularly latest and most important algorithms in the fields of optimization and metaheuristics, were chosen for this purpose. These include the Bat-Inspired Algorithm (BIA), Big-Bang Big-Crunch (BBBC) algorithm, Butterfly Optimization Algorithm (BOA), Cyclical Parthenogenesis Algorithm (CPA), Charged System Search (CSS) algorithm, Galactic Swarm Optimization (GSO), Jaya Algorithm (JA), Rain Fall Optimization (RFO) algorithm, Tug of War Optimization (TWO), Wind Driven Optimization (WDO) algorithm, Whale Optimization Algorithm (WOA) algorithm, Grey Wolf Optimizer (GWO), and the Ray Optimization Algorithm (ROA). It should be noted that some of these algorithms require a number of parameters to be tuned to improve their performance. Therefore, a parameter investigation process was conducted for this purpose, in which the most successful sets of parameters were derived from the literature. Table 3 specifies the parameters of these algorithms.

5 Numerical results of the mathematical functions

The results of FHO alongside the other 10 metaheuristic algorithms in dealing with the considered 233 mathematical test functions are described in this section. A total number of 150,000 objective function evaluations were considered while a tolerance of 1×10^{-12} is considered as the stopping criterion for the FHO and the selected alternatives. A total number of 100 optimization runs were also considered for statistical purposes in order to calculate the mean and standard deviation of the optimization results. Besides, a fixed random state was also considered in dealing with the FHO and the other alternatives in order to have a comparative investigation under equal condition. Since a total of 233 test functions were utilized for comparative purposes, a tolerance of 1×10^{-12} is achievable as stopping criterion of the optimization process due to the fact that all of the alternative algorithms have been implemented individually in this study and none of the previously published results have been utilized in this purpose which increase the complexity levels of the study.

Table 2 Basic properties of the collected mathematical test functions

No.	Name	Type	R	D	Min.
F_1	Ackley 2 function	C, D, NS, Sc,M	[− 35, 35]	2	− 200
F_2	Ackley 3 function	C, D, NS, NSc,U	[− 32, 32]	2	− 195.629
F_3	Adjiman function	C, D, NS, NSc, M	[− 1, 2] & [− 1, 1]	2	− 2.02181
F_4	Bartels conn function	C, ND, NS, NSc, M	[− 500, 500]	2	1
F_5	Beale function	C, D, NS, NSc, U	[− 4.5, 4.5]	2	0
F_6	Becker-lago function	S	[− 10, 10]	2	0
F_7	Biggs EXP2 function	C, D, NS, NSc, M	[0, 20]	2	0
F_8	Bird function	C, D, NS, NSc, M	[− 2π, π]	2	− 106.765
F_9	Bohachevsky 1 function	C, D, S, NSc, M	[− 100, 100]	2	0
F_{10}	Bohachevsky 2 function	C, D, NS, NSc, M	[− 100, 100]	2	0
F_{11}	Bohachevsky 3 function	C, D, NS, NSc, M	[− 100, 100]	2	0
F_{12}	Booth function	C, D, NS, NSc, U	[− 10, 10]	2	0
F_{13}	Branin RCOS function	C, D, NS, NSc M	[− 5, 10] & [0, 15]	2	0.397887
F_{14}	Branin RCOS 2 function	C, D, NS, NSc, M	[− 5, 15]	2	5.559037
F_{15}	Brent function	C, D, NS, NSc, U	[− 10, 10]	2	0
F_{16}	Bukin 4 function	C, ND, S, NSc, M	[− 15, − 5] & [− 3, 3]	2	0
F_{17}	Bukin 6 function	C, ND, NS, NSc, M	[− 15, − 5] & [− 3, 3]	2	0
F_{18}	Camel function—three hump	C, D, NS, NSc, M	[− 5, 5]	2	0
F_{19}	Camel function—six hump	C, D, NS, NSc, M	[− 5, 5]	2	− 1.0316
F_{20}	Carrom table function	NS	[− 10, 10]	2	− 24.1568
F_{21}	Chen bird function	C, D, NS, NSc, M	[− 500, 500]	2	− 2000
F_{22}	Chen V function	C, D, NS, NSc, M	[− 500, 500]	2	− 2000
F_{23}	Chichinadze function	C, D, S, NSc, M	[− 30, 30]	2	− 42.9444
F_{24}	Cross-in-tray function	C, NS, NSc, M	[− 10, 10]	2	− 2.06261
F_{25}	Cube function	C, D, NS, NSc, U	[− 10, 10]	2	0
F_{26}	Damavandi function	C, D, NS, NSc, M	[0, 14]	2	0
F_{27}	Deckkers-aarts function	C, D, NS, NSc, M	[− 20, 20]	2	− 24771.1
F_{28}	Easom function	C, D, S, NSc, M	[− 100, 100]	2	− 1
F_{29}	El-Attar-Vidyasagar-Dutta function	C, D, NS, NSc, M	[− 500, 500]	2	1.7128
F_{30}	Egg crate function	C, D, NS, Sc, M	[− 5, 5]	2	0
F_{31}	Exp 2 function	S	[0, 20]	2	0
F_{32}	Freudenstein Roth function	C, D, NS, NSc, M	[− 10, 10]	2	0
F_{33}	Giunta function	C, D, S, Sc, M	[− 1, 1]	2	0.060447
F_{34}	Goldstein price function	C, D, NS, NSc, M	[− 2, 2]	2	3
F_{35}	Hansen function	C, D, S, NSc, M	[− 10, 10]	2	− 165.953
F_{36}	Himmelblau function	C, D, NS, NSc, M	[− 5, 5]	2	0
F_{37}	Hosaki function	C, D, NS, NSc, M	[0, 5] & [0, 6]	2	− 2.3458
F_{38}	Jennrich-Sampson function	C, D, NS, NSc, M	[− 1, 1]	2	124.3612
F_{39}	Keane function	C, D, NS, NSc, M	[0, 10]	2	− 0.67367
F_{40}	Leon function	C, D, NS, NSc, U	[− 1.2, 1.2]	2	0

Table 2 (continued)

No.	Name	Type	R	D	Min.
F_{41}	Levy 3 function	S	[− 10, 10]	2	− 176.542
F_{42}	Levy 5 function	NS	[− 10, 10]	2	− 176.138
F_{43}	Matyas function	C, D, NS, NSc, U	[− 10, 10]	2	0
F_{44}	McCormick function	C, D, NS, NSc, M	[− 1.5, 4] & [− 3, 3]	2	− 1.9133
F_{45}	Mexican hat function	NS	[− 10, 10]	2	− 19.6683
F_{46}	Michaelewicz 2 function	S	[0, π]	2	− 1.8013
F_{47}	Mishra 3 function	C, D, NS, NSc, M	[− 10, 10]	2	− 0.18465
F_{48}	Mishra 4 function	C, D, NS, NSc, M	[− 10, 10]	2	− 0.19941
F_{49}	Mishra 5 function	C, D, NS, NSc, M	[− 10, 10]	2	− 1.01983
F_{50}	Mishra 6 function	C, D, NS, NSc, M	[− 10, 10]	2	− 2.28395
F_{51}	Mishra 8 function	C, D, NS, NSc, M	[− 10, 10]	2	0
F_{52}	Mishra 10 function	C, D, NS, NSc, M	[− 10, 10]	2	0
F_{53}	Parsopoulos function	C, D, S, Sc, M	[− 5, 5]	2	0
F_{54}	Pen Holder function	C, D, NS, NSc, M	[− 11, 11]	2	− 0.96354
F_{55}	Periodic function	S	[− 10, 10]	2	0.9
F_{56}	Price 1 function	C, ND, S, NSc, M	[− 500, 500]	2	0
F_{57}	Price 2 function	C, D, NS, NSc, M	[− 10, 10]	2	0.9
F_{58}	Price 3 function	C, D, NS, NSc, M	[− 500, 500]	2	0
F_{59}	Price 4 function	C, D, NS, NSc, M	[− 500, 500]	2	0
F_{60}	Quadratic function	C, D, NS, NSc	[− 10, 10]	2	− 3873.72
F_{61}	Ripple 1 function	NS	[0, 1]	2	− 2.2
F_{62}	Ripple 25 function	NS	[0, 1]	2	− 2
F_{63}	Rosenbrock modified function	C, D, NS, NSc, M	[− 2, 2]	2	34.3712
F_{64}	Rotated ellipse function	C, D, NS, NSc, U	[− 500, 500]	2	0
F_{65}	Rotated ellipse 2 function	C, D, NS, NSc, U	[− 500, 500]	2	0
F_{66}	Rump function	C, D, NS, NSc, U	[− 500, 500]	2	0
F_{67}	Scahffer 1 function	C, D, NS, NSc, U	[− 100, 100]	2	0
F_{68}	Scahffer 2 function	C, D, NS, NSc, U	[− 100, 100]	2	0
F_{69}	Scahffer 3 function	C, D, NS, NSc, U	[− 100, 100]	2	0.001567
F_{70}	Scahffer 4 function	C, D, NS, NSc, U	[− 100, 100]	2	0.292579
F_{71}	Schwefel 2.6 function	C, D, NS, NSc, U	[− 100, 100]	2	0
F_{72}	Schwefel 2.36 function	C, D, S, Sc, M	[0, 500]	2	− 3456
F_{73}	Table 1/holder Table 1 function	C, D, S, NSc, M	[− 10, 10]	2	− 26.9203
F_{74}	Table 2/holder Table 2 function	C, D, S, NSc, M	[− 10, 10]	2	− 19.2085
F_{75}	Table 3/carrom table function	C, D, NS, NSc, M	[− 10, 10]	2	− 24.1568
F_{76}	Testtube holder function	C, D, S, NSc, M	[− 10, 10]	2	− 10.8723
F_{77}	Trecanni function	C, D, S, NSc, U	[− 5, 5]	2	0
F_{78}	Trefethen function	C, D, NS, NSc, M	[− 10, 10]	2	− 3.30687
F_{79}	Tripod function	C, D, NS, NSc, M	[− 100, 100]	2	0
F_{80}	Ursem 1 function	S	[− 2.5, 3] & [− 2, 2]	2	− 4.81681
F_{81}	Ursem 3 function	NS	[− 2, 2] & [− 1.5, 1.5]	2	− 2.5

Table 2 (continued)

No.	Name	Type	R	D	Min.
F_{82}	Ursem 4 function	NS	[− 2, 2]	2	− 1.5
F_{83}	Ursem waves function	NS	[− 0.9, 1.2] & [− 1.2, 1.2]	2	− 7.307
F_{84}	Venter Sobiezcczanski-Sobieski function	C, D, S, NSc	[− 50, 50]	2	− 400
F_{85}	Wayburn Seader 1 function	C, D, NS, Sc, U	[− 500, 500]	2	0
F_{86}	Wayburn Seader 2 function	C, D, NS, Sc, U	[− 500, 500]	2	0
F_{87}	Wayburn Seader 3 function	C, D, NS, Sc, U	[− 500, 500]	2	21.35
F_{88}	Zettl function	C, D, NS, NSc, U	[− 5, 10]	2	− 0.00379
F_{89}	Zirilli or Aluffi-Pentini's function	C, D, S, NSc, U	[− 10, 10]	2	− 0.3523
F_{90}	Zirilli function 2	C, D, S, S, M	[− 500, 500]	2	0
F_{91}	Biggs EXP3 function	C, D, NS, NSc, M	[0, 20]	3	0
F_{92}	Gulf research problem	C, D, NS, NSc, M	[0.1, 100] & [0, 25.6] & [0, 6.5]	3	0
F_{93}	Hartman 3 function	C, D, NS, NSc, M	[0, 1]	3	− 3.86278
F_{94}	Helical valley	C, D, NS, Sc, M	[− 10, 10]	3	0
F_{95}	Meyer-Roth function	NS	[0, 1]	3	4.00E−05
F_{96}	Mishra 9 function	C, D, NS, NSc, M	[− 10, 10]	3	0
F_{97}	Wolfe function	C, D, S, Sc, M	[0, 2]	3	0
F_{98}	Biggs EXP4 function	C, D, NS, NSc, M	[0, 20]	4	0
F_{99}	Colville function	C, D, NS, NSc, M	[− 10, 10]	4	0
F_{100}	Corana function	DC, ND, S, Sc, M	[− 500, 500]	4	0
F_{101}	DeVilliers Glasser 1 function	C, D, NS, NSc, M	[1, 100]	4	0
F_{102}	Gear function	NS	[12, 60]	4	2.70E−12
F_{103}	Kowalik function	NS	[− 5, 5]	4	0.000308
F_{104}	Miele Cantrell function	C, D, NS, NSc, M	[− 1, 1]	4	0
F_{105}	Shekel 5	C, D, NS, Sc, M	[0, 10]	4	− 10.1532
F_{106}	Shekel 7	C, D, NS, Sc, M	[0, 10]	4	− 10.4029
F_{107}	Shekel 10	C, D, NS, Sc, M	[0, 10]	4	− 10.5364
F_{108}	Biggs EXP5 function	C, D, NS, NSc, M	[0, 20]	5	0
F_{109}	DeVilliers Glasser 2 function	C, D, NS, NSc, M	[1, 60]	5	0
F_{110}	Dolan function	C, D, NS, NSc, M	[− 100, 100]	5	− 529.871
F_{111}	Langerman-5 function	C, D, NS, Sc, M	[0, 10]	5	− 0.965
F_{112}	Biggs EXP6 function	C, D, NS, NSc, M	[− 20, 20]	6	0
F_{113}	Hartman 6 function	C, D, NS, NSc, M	[0, 1]	6	− 3.32236
F_{114}	Trid 6 function	C, D, NS, NSc, M	[− 36, 36]	6	− 50
F_{115}	Ann-XOR function	NS	[− 1, 1]	9	0.95979
F_{116}	Paviani function	C, D, NS, Sc, M	[2.0001, 10]	10	− 45.778
F_{117}	Trid 10 function	C, D, NS, NSc, M	[− 100,100]	10	− 210
F_{118}	Ackley 1 function	C, D, NS, Sc,M	[− 35, 35]	50	0

Table 2 (continued)

No.	Name	Type	R	D	Min.
F_{119}	Alpine 1 function	C, ND, S, NSc,U	[− 10, 10]	50	0
F_{120}	Brown function	C, D, NS, Sc, U	[− 1, 4]	50	0
F_{121}	Chung Reynolds function	C, D, PS, Sc, U	[− 100, 100]	50	0
F_{122}	Csendes function	C, D, S, Sc, M	[− 1, 1]	50	0
F_{123}	Deb 1 function	C, D, S, Sc, M	[− 1, 1]	50	− 1
F_{124}	Deb 3 function	C, D, S, Sc, M	[0, 1]	50	− 1
F_{125}	Dixon & Price function	C, D, NS, Sc, U	[− 10, 10]	50	0
F_{126}	Extended Easom function	C, D, S, NSc, M	[− 2π, 2π]	50	− 1
F_{127}	Exponential function	C, D, NS, Sc, M	[− 1, 1]	50	− 1
F_{128}	Griewank function	C, D, NS, Sc, M	[− 100,100]	50	0
F_{129}	Holzman 2 function	S	[− 10, 10]	50	0
F_{130}	Hyper-ellipsoid function	C, U	[− 500, 500]	50	0
F_{131}	Inverted cosine wave function	NS	[− 10, 10]	50	− 49
F_{132}	Levy 8 function	NS	[− 10, 10]	50	0
F_{133}	Mishra 1 function	C, D, NS, Sc, M	[0, 1]	50	2
F_{134}	Mishra 2 function	C, D, NS, Sc, M	[0, 1]	50	2
F_{135}	Mishra 7 function	C, D, NS, NSc, M	[− 10, 10]	50	0
F_{136}	Mishra 11 function	C, D, NS, NSc, M	[− 10, 10]	50	0
F_{137}	Pathological function	C, D, NS, NSc, M	[− 100, 100]	50	0
F_{138}	Pint´er function	C, D, NS, Sc, M	[− 10, 10]	50	0
F_{139}	Powell singular function	C, D, NS, Sc, U	[− 4, 5]	50	0
F_{140}	Powell singular 2 function	C, D, NS, Sc, U	[− 4, 5]	50	0
F_{141}	Powell sum function	C, D, S, Sc, U	[− 1, 1]	50	0
F_{142}	Rastrigin function	C, D, S, M	[− 5.12, 5.12]	50	0
F_{143}	Qing function	C, D, S, Sc, M	[− 500, 500]	50	0
F_{144}	Quintic function	C, D, S, NSc, M	[− 10, 10]	50	0
F_{145}	Rosenbrock function	C, D, NS, Sc, U	[− 30, 30]	50	0
F_{146}	Salomon function	C, D, NS, Sc, M	[− 100, 100]	50	0
F_{147}	Schumer Steiglitz function	C, D, S, Sc, U	[− 100, 100]	50	0
F_{148}	Schwefel function	C, D, PS, Sc, U	[− 100, 100]	50	0
F_{149}	Schwefel 1.2 function	C, D, NS, Sc, U	[− 100, 100]	50	0
F_{150}	Schwefel 2.4 function	C, D, S, NSc, M	[0, 10]	50	0
F_{151}	Schwefel 2.20 function	C, ND, S, Sc, U	[− 100, 100]	50	0
F_{152}	Schwefel 2.21 function	C, ND, S, Sc, U	[− 100, 100]	50	0
F_{153}	Schwefel 2.22 function	C, D, NS, Sc, U	[− 100, 100]	50	0
F_{154}	Schwefel 2.23 function	C, D, NS, Sc, U	[− 10, 10]	50	0
F_{155}	Schwefel 2.25 function	C, D, S, NSc, M	[0, 10]	50	0
F_{156}	Schwefel 2.26 function	C, D, S, Sc, M	[− 500, 500]	50	− 418.983
F_{157}	Sphere function	C, D, S, Sc, M	[0, 10]	50	0
F_{158}	Step function	DC, ND, S, Sc, U	[− 100, 100]	50	0
F_{159}	Step 2 function	DC, ND, S, Sc, U	[− 100, 100]	50	0
F_{160}	Step 3 function	DC, ND, S, Sc, U	[− 100, 100]	50	0
F_{161}	Stepint function	DC, ND, S, Sc, U	[− 5.12, 5.12]	50	− 275

Table 2 (continued)

No.	Name	Type	R	D	Min.
F_{162}	Streched V sine wave function	C, D, NS, Sc, U	[− 10, 10]	50	0
F_{163}	Sum squares function	C, D, S, Sc, U	[− 10, 10]	50	0
F_{164}	Styblinski-Tang function	C, D, NS, NSc, M	[− 5, 5]	50	− 1958.3
F_{165}	Trid function	C, D, NS, NSc, U	[− D^2, D^2]	50	− 22050
F_{166}	Trigonometric 1 function	C, D, NS, Sc, M	[0, π]	50	0
F_{167}	Trigonometric 2 function	C, D, NS, Sc, M	[− 500, 500]	50	1
F_{168}	W/Wavy function	C, D, S, Sc, M	[− π, π]	50	0
F_{169}	Xin-She Yang (function 1)	DC, ND, NS, Sc, M	[− 20, 20]	50	− 1
F_{170}	Xin-She Yang (function 2)	DC, ND, NS, Sc, M	[− 10, 10]	50	0
F_{171}	Xin-She Yang (function 3)	DC, ND, NS, Sc, M	[− 2π, 2π]	50	0
F_{172}	Xin-She Yang (function 4)	DC, ND, NS, Sc, M	[− 5, 5]	50	0
F_{173}	Xin-She Yang (function 5)	DC, ND, NS, Sc, M	[− 10, 10]	50	− 1
F_{174}	Xin-She Yang (function 6)	DC, ND, NS, Sc, M	[− 5, 5]	50	0
F_{175}	Zakharov function	C, D, NS, Sc, M	[− 5, 10]	50	0
F_{176}	Ackley 1 function	C, D, NS, Sc,M	[− 35, 35]	100	0
F_{177}	Alpine 1 function	C, ND, S, NSc,U	[− 10, 10]	100	0
F_{178}	Brown function	C, D, NS, Sc, U	[− 1, 4]	100	0
F_{179}	Chung Reynolds function	C, D, PS, Sc, U	[− 100, 100]	100	0
F_{180}	Csendes function	C, D, S, Sc, M	[− 1, 1]	100	0
F_{181}	Deb 1 function	C, D, S, Sc, M	[− 1, 1]	100	− 1
F_{182}	Deb 3 function	C, D, S, Sc, M	[0, 1]	100	− 1
F_{183}	Dixon & Price function	C, D, NS, Sc, U	[− 10, 10]	100	0
F_{184}	Extended Easom function	C, D, S, NSc, M	[− 2π, 2π]	100	− 1
F_{185}	Exponential function	C, D, NS, Sc, M	[− 1, 1]	100	− 1
F_{186}	Griewank function	C, D, NS, Sc, M	[− 100,100]	100	0
F_{187}	Holzman 2 function	S	[− 10, 10]	100	0
F_{188}	Hyper-ellipsoid function	C, U	[− 500, 500]	100	0
F_{189}	Inverted cosine wave function	NS	[− 10, 10]	100	− 99
F_{190}	Levy 8 function	NS	[− 10, 10]	100	0
F_{191}	Mishra 1 function	C, D, NS, Sc, M	[0, 1]	100	2
F_{192}	Mishra 2 function	C, D, NS, Sc, M	[0, 1]	100	2
F_{193}	Mishra 7 function	C, D, NS, NSc, M	[− 10, 10]	100	0
F_{194}	Mishra 11 function	C, D, NS, NSc, M	[− 10, 10]	100	0
F_{195}	Pathological function	C, D, NS, NSc, M	[− 100, 100]	100	0
F_{196}	Pint´er function	C, D, NS, Sc, M	[− 10, 10]	100	0
F_{197}	Powell singular function	C, D, NS, Sc, U	[− 4, 5]	100	0
F_{198}	Powell singular 2 function	C, D, NS, Sc, U	[− 4, 5]	100	0
F_{199}	Powell Sum function	C, D, S, Sc, U	[− 1, 1]	100	0
F_{200}	Rastrigin function	C, D, S, M	[− 5.12, 5.12]	100	0
F_{201}	Qing function	C, D, S, Sc, M	[− 500, 500]	100	0
F_{202}	Quintic function	C, D, S, NSc, M	[− 10, 10]	100	0
F_{203}	Rosenbrock function	C, D, NS, Sc, U	[− 30, 30]	100	0
F_{204}	Salomon function	C, D, NS, Sc, M	[− 100, 100]	100	0

Table 2 (continued)

No.	Name	Type	R	D	Min.
F_{205}	Schumer Steiglitz function	C, D, S, Sc, U	[− 100, 100]	100	0
F_{206}	Schwefel function	C, D, PS, Sc, U	[− 100, 100]	100	0
F_{207}	Schwefel 1.2 function	C, D, NS, Sc, U	[− 100, 100]	100	0
F_{208}	Schwefel 2.4 function	C, D, S, NSc, M	[0, 10]	100	0
F_{209}	Schwefel 2.20 function	C, ND, S, Sc, U	[− 100, 100]	100	0
F_{210}	Schwefel 2.21 function	C, ND, S, Sc, U	[− 100, 100]	100	0
F_{211}	Schwefel 2.22 function	C, D, NS, Sc, U	[− 100, 100]	100	0
F_{212}	Schwefel 2.23 function	C, D, NS, Sc, U	[− 10, 10]	100	0
F_{213}	Schwefel 2.25 function	C, D, S, NSc, M	[0, 10]	100	0
F_{214}	Schwefel 2.26 function	C, D, S, Sc, M	[− 500, 500]	100	− 418.983
F_{215}	Sphere function	C, D, S, Sc, M	[0, 10]	100	0
F_{216}	Step function	DC, ND, S, Sc, U	[− 100, 100]	100	0
F_{217}	Step 2 function	DC, ND, S, Sc, U	[− 100, 100]	100	0
F_{218}	Step 3 function	DC, ND, S, Sc, U	[− 100, 100]	100	0
F_{219}	Stepint function	DC, ND, S, Sc, U	[− 5.12, 5.12]	100	− 575
F_{220}	Streched V sine wave function	C, D, NS, Sc, U	[− 10, 10]	100	0
F_{221}	Sum Squares function	C, D, S, Sc, U	[− 10, 10]	100	0
F_{222}	Styblinski-Tang function	C, D, NS, NSc, M	[− 5, 5]	100	− 3916.6
F_{223}	Trid function	C, D, NS, NSc, U	[− D^2, D^2]	100	− 171600
F_{224}	Trigonometric 1 function	C, D, NS, Sc, M	[0, π]	100	0
F_{225}	Trigonometric 2 function	C, D, NS, Sc, M	[− 500, 500]	100	1
F_{226}	W/Wavy function	C, D, S, Sc, M	[− π, π]	100	0
F_{227}	Xin-She Yang (function 1)	DC, ND, NS, Sc, M	[− 20, 20]	100	− 1
F_{228}	Xin-She Yang (function 2)	DC, ND, NS, Sc, M	[− 10, 10]	100	0
F_{229}	Xin-She Yang (function 3)	DC, ND, NS, Sc, M	[− 2π, 2π]	100	0
F_{230}	Xin-She Yang (function 4)	DC, ND, NS, Sc, M	[− 5, 5]	100	0
F_{231}	Xin-She Yang (function 5)	DC, ND, NS, Sc, M	[− 10, 10]	100	− 1
F_{232}	Xin-She Yang (function 6)	DC, ND, NS, Sc, M	[− 5, 5]	100	0
F_{233}	Zakharov function	C, D, NS, Sc, M	[− 5, 10]	100	0

C Continuous, *NC* Non-Continuous, *D* Differentiable, *ND* Non-Differentiable, *S* Separable, *NS* Non-Separable, *Sc* Scalable, *NSc* Non-Scalable, *U* Unimodal and *M* Multi-modal, *R* Variables Range, *D* Variables Dimension, *Min* global minimum of the functions
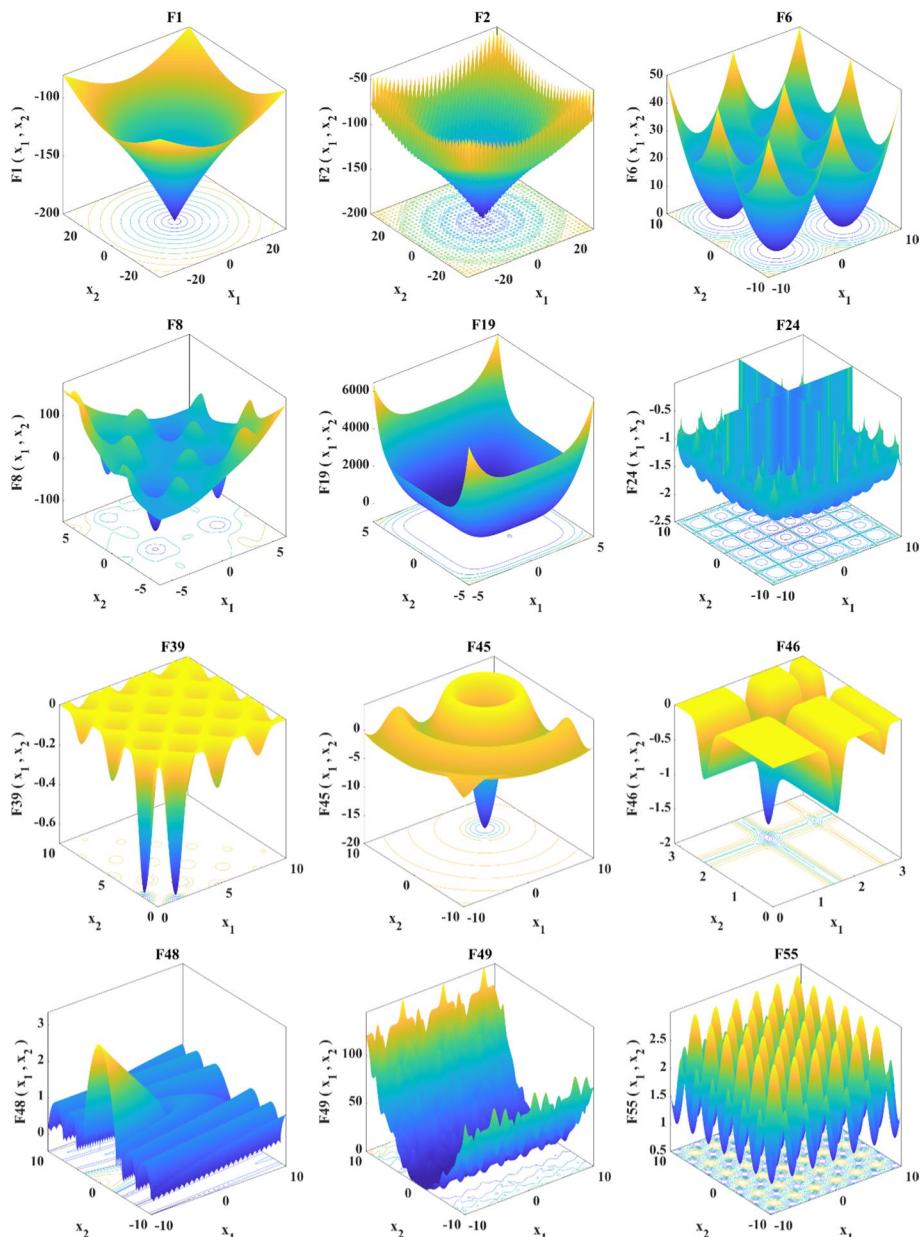
Fig. 10 The 3D plots of the 2D mathematical functions (first set)

It also should be noted that this number of test functions are utilized for the first time for testing a metaheuristic algorithm; thus for a smaller number of test functions, a smaller tolerance as 10^{-30} or 10^{-50} can be utilized.

The best values of the considered 100 optimization runs are all presented in Table 4 for the SS, 50D and 100D mathematical test functions. Based on the results, FHO
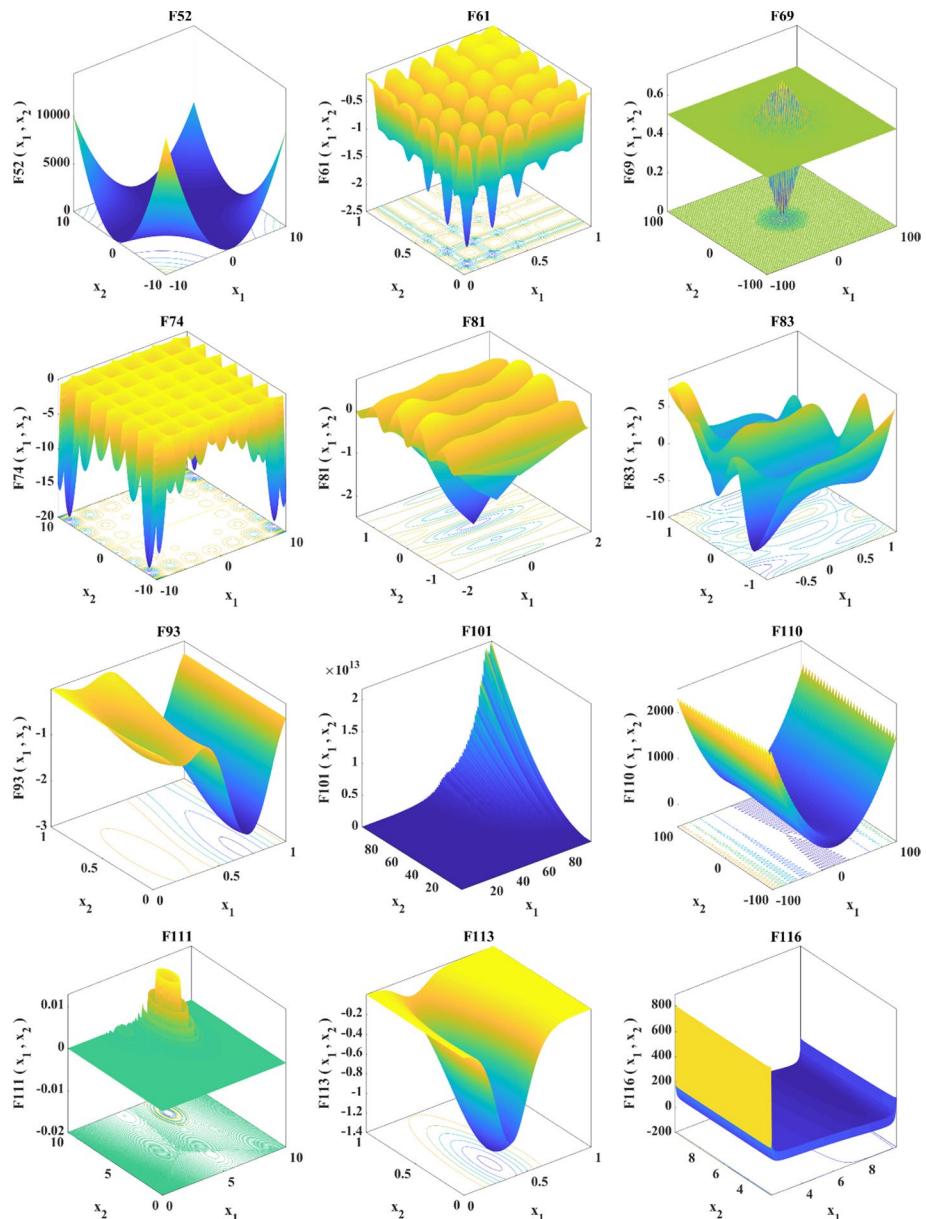
Fig. 11 The 3D plots of the 2D mathematical functions (second set)

outranks the other metaheuristic algorithms in most of the cases. The mean values of the Function Evaluations (Fun. Evl.) in each of the conducted optimization runs regarding the alternative metaheuristics are presented in Table 27 in the Appendix. It is obvious that FHO is able to converge to better results than the other metaheuristics with a
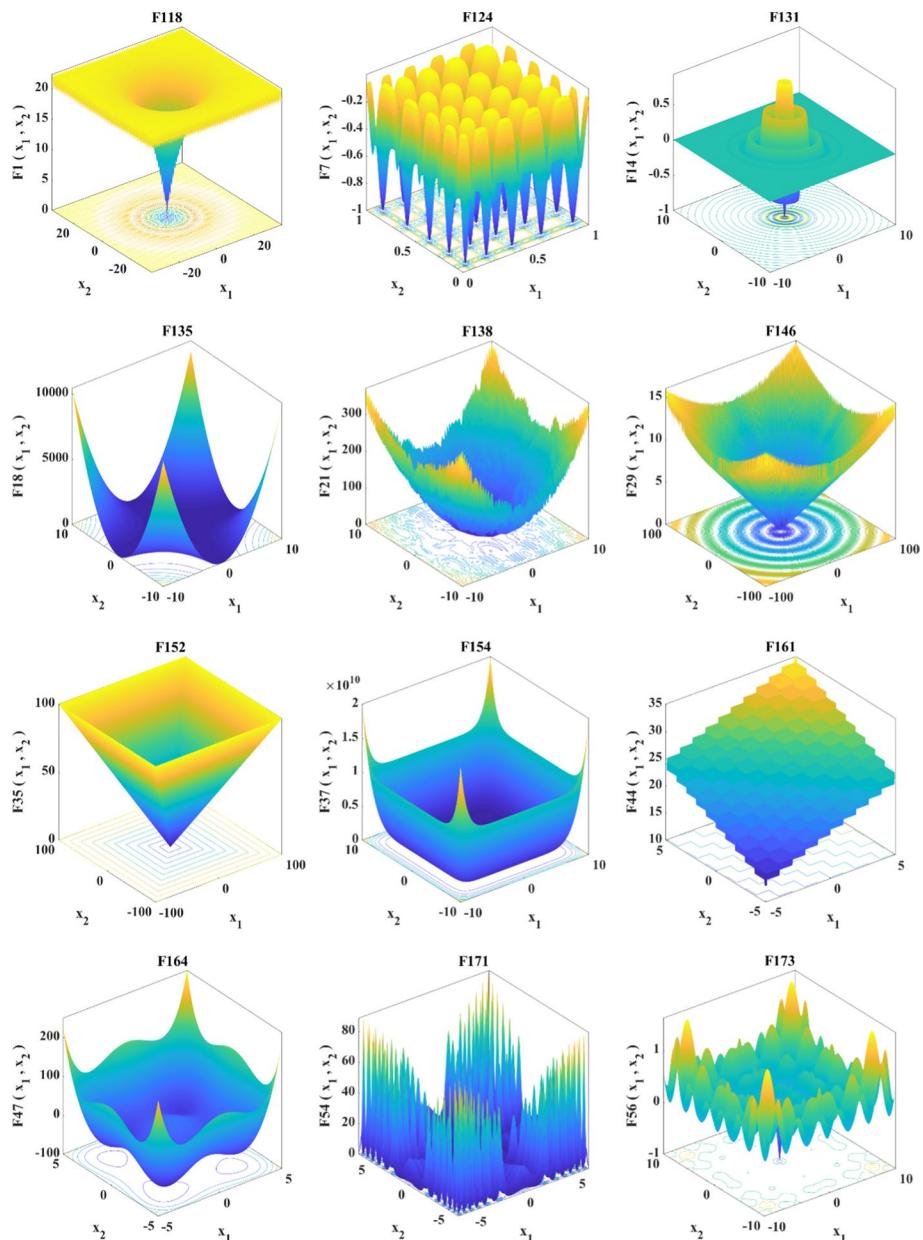
Fig. 12 The 3D plots of the 50D mathematical functions

minimum number of required function evaluations which represents the ability of this algorithm to deal with computational complexity issues.

The convergence curves of the 100 optimization runs for FHO in Fig. 14 help to clarify the algorithm's ability to deal with the mathematical test functions, in which the
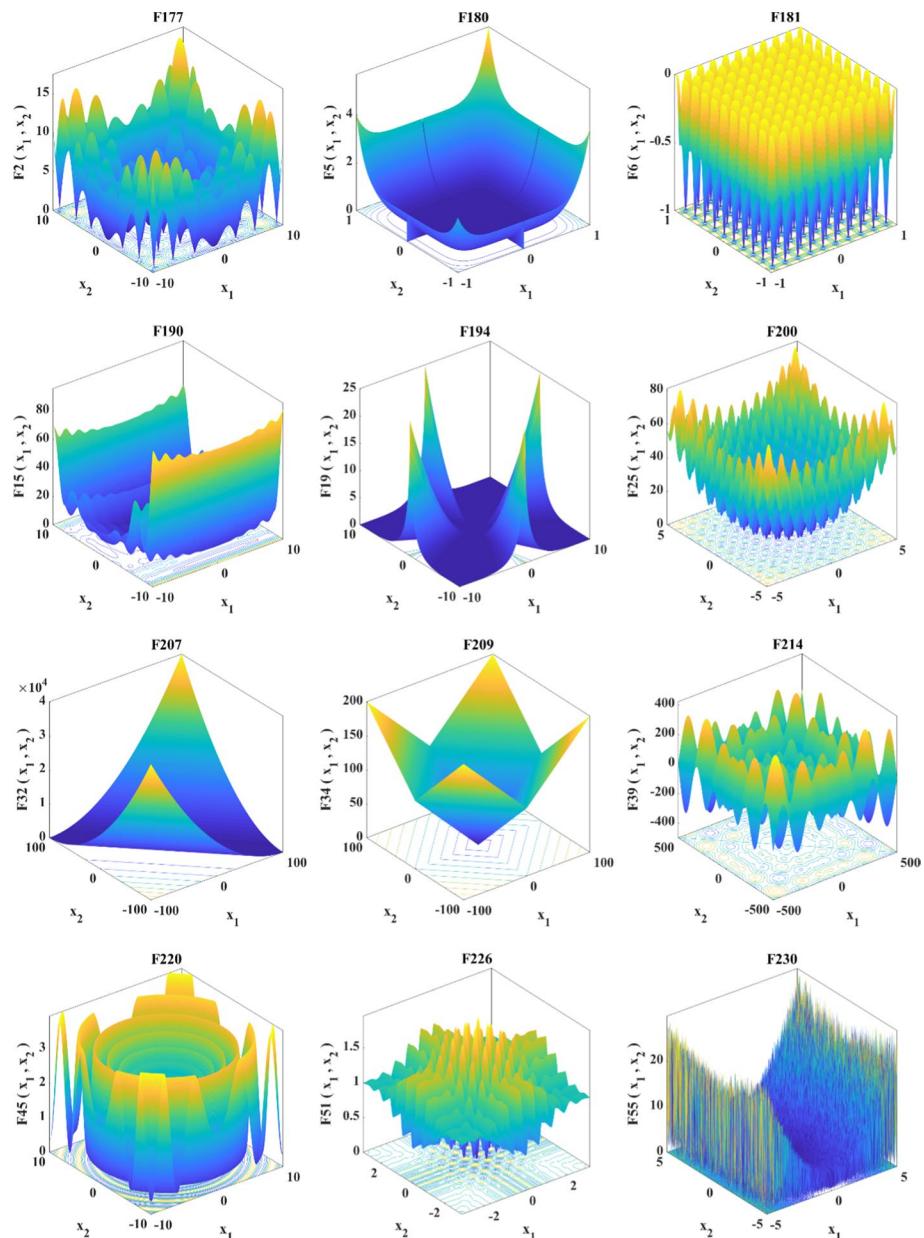
Fig. 13 The 3D plots of the 100D mathematical functions

best, median and worst runs are highlighted for clarification. It can be concluded that FHO performs a fast optimization procedures in most of the cases, while completing the predefined 150,000 objective function evaluations is not required to achieve the tolerance of 1×10^{-12}.

Table 3 Parameter presentation of the alternative metaheuristic algorithms

Metaheuristic	Parameter	Description	Value
CPA	N_{pop}	Number of population	50
	N_A	Number of aphids	60
	N_C	Number of colonies	10
	F_r	Ratio of aphids	0.4
	α_1	Search step size parameter 1	1
	α_2	Search step size parameter 2	2
TWO	N_{pop}	Number of population	50
	Meus	Static coefficients of friction	1
	Meuk	Kinematic coefficients of friction	1
	δ	Time step in the movement equation	1
	α	Controlling parameter	0.99
	β	Scaling factor of team's movement	0.1
BIA	N_{pop}	Number of bats	50
	A	Loudness	0.5
	r	Pulse rate	0.5
	Q_{Min}	Frequency minimum	0
	Q_{Max}	Frequency maximum	2
BBBC	N_{pop}	Number of population	50
	β	Parameter for the center of mass	0.2
	α	Parameter for size of the initial search space	1
BOA	N_{pop}	Number of butterflies	50
	P_S	Probability switch	0.8
	P_E	Power exponent	0.1
	S_M	Sensory modality	0.01
CSS	N_{pop}	Number of charged particles	50
	a	Radius of charged sphere	0.1
	HMCR	Harmony memory consideration rate	0.85
	PAR	Pitch adjustment rate	0.15
	kt	Attract-repel coefficient	0.9
	N_{cm}	Charged memory size	12
	k_a	Acceleration coefficient	0.5
	k_v	Velocity coefficient	0.5
RFO	N_{pop}	Population size	50
	α	Constant for movement update	360
	G	Gravity	10
	t	Time constant	1
	P	Power	1
	N	Norm	1
WDO	N_{pop}	Population size	50
	RT	RT coefficient	3
	g	Gravitational constant	0.2
	α	Constant for updating process	0.4
	c	Coriolis effect	0.4
	V	Maximum allowed speed	0.3

Table 4 The best results of metaheuristic algorithms considering mathematical test functions

No.	Alternative metaheuristic algorithms										
	BIA	BBBC	BOA	CPA	CSS	GSO	JA	RFO	TWO	WDO	FHO
F_1	9.75E−07	6.27E−05	0.00E+00	0.00E+00	8.21E−10	0.00E+00	2.57E−02	1.18E−01	0.00E+00	0.00E+00	0.00E+00
F_2	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	4.08E−01	9.40E−05	3.01E−03	0.00E+00	0.00E+00	0.00E+00
F_3	0.00E+00	3.22E−06	0.00E+00	3.22E−06	3.22E−06	3.22E−06	3.22E−06	7.00E−03	3.22E−06	3.22E−06	3.22E−06
F_4	3.13E−09	4.72E−06	0.00E+00	0.00E+00	1.87E+01	0.00E+00	4.47E−02	4.30E−02	0.00E+00	0.00E+00	0.00E+00
F_5	7.01E−12	1.03E−11	2.40E−06	0.00E+00	0.00E+00	9.71E+00	7.05E−08	4.71E−04	0.00E+00	7.05E−09	0.00E+00
F_6	0.00E+00	5.03E−11	3.49E−08	0.00E+00	0.00E+00	0.00E+00	4.32E−07	8.21E−05	0.00E+00	3.77E−10	0.00E+00
F_7	0.00E+00	6.92E−12	3.20E−07	0.00E+00	0.00E+00	2.08E+00	5.14E−07	2.37E−04	0.00E+00	9.01E−08	0.00E+00
F_8	2.51E−07	2.56E−07	0.00E+00	2.51E−07	2.51E−07	8.07E+01	5.75E−05	2.02E−02	2.51E−07	2.94E−05	2.51E−07
F_9	2.83E−10	8.58E−09	0.00E+00	0.00E+00	0.00E+00	0.00E+00	6.77E−03	8.50E−03	0.00E+00	0.00E+00	0.00E+00
F_{10}	2.18E−01	9.07E−09	0.00E+00	0.00E+00	0.00E+00	0.00E+00	6.71E−03	1.91E−01	0.00E+00	0.00E+00	0.00E+00
F_{11}	1.49E−11	7.94E−09	0.00E+00	0.00E+00	0.00E+00	0.00E+00	3.33E−03	1.95E−01	0.00E+00	0.00E+00	0.00E+00
F_{12}	0.00E+00	1.16E−10	3.14E−06	0.00E+00	0.00E+00	2.00E+00	2.38E−05	4.15E−03	0.00E+00	8.66E−07	0.00E+00
F_{13}	5.77E−08	5.78E−08	5.39E−06	5.77E−08	5.77E−08	7.81E−05	3.55E−06	7.21E−04	5.77E−08	6.36E−08	5.77E−08
F_{14}	0.00E+00	0.00E+00	6.04E−04	0.00E+00	0.00E+00	9.93E+00	0.00E+00	1.04E−02	0.00E+00	0.00E+00	0.00E+00
F_{15}	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
F_{16}	5.71E−10	1.84E−09	1.71E−06	0.00E+00	0.00E+00	2.00E−02	1.13E−05	4.18E−05	0.00E+00	0.00E+00	0.00E+00
F_{17}	9.70E−03	4.32E−03	7.39E−02	2.55E−05	2.21E−04	5.00E−02	1.92E−02	7.86E−01	8.91E−03	2.75E−02	1.02E−04
F_{18}	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	8.13E−07	6.25E−04	0.00E+00	0.00E+00	0.00E+00
F_{19}	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	1.03E+00	0.00E+00	6.82E−04	0.00E+00	0.00E+00	0.00E+00
F_{20}	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	2.78E−05	1.75E−02	0.00E+00	0.00E+00	0.00E+00
F_{21}	1.00E+03	1.00E+03	0.00E+00	1.00E+03	2.00E+03	0.00E+00	1.00E+03	1.00E+03	1.00E+03	1.10E−05	0.00E+00
F_{22}	1.00E+03	1.00E+03	0.00E+00	1.00E+03	1.07E+03	1.00E+03	1.01E+03	1.00E+03	0.00E+00	1.00E+03	0.00E+00
F_{23}	1.30E−05	1.30E−05	0.00E+00	1.30E−05	1.30E−05	4.47E−01	9.47E−04	4.80E−03	1.30E−05	1.66E−05	1.30E−05
F_{24}	3.09E−07	3.09E−07	0.00E+00	3.09E−07	3.09E−07	3.09E−07	3.36E−07	6.78E−06	3.09E−07	3.09E−07	3.09E−07
F_{25}	5.04E−12	2.34E−10	5.21E−07	0.00E+00	0.00E+00	0.00E+00	7.23E−05	1.31E−02	0.00E+00	4.23E−10	0.00E+00

Table 4 (continued)

No.	Alternative metaheuristic algorithms										
	BIA	BBBC	BOA	CPA	CSS	GSO	JA	RFO	TWO	WDO	FHO
F_{26}	2.35E−08	2.00E+00	3.37E−03	0.00E+00	0.00E+00	2.55E−02	1.92E−03	4.66E−01	0.00E+00	2.00E+00	2.00E+00
F_{27}	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	2.48E+04	0.00E+00	3.95E+00	0.00E+00	0.00E+00	0.00E+00
F_{28}	0.00E+00	7.43E−01	0.00E+00	0.00E+00	3.96E−11	0.00E+00	7.26E−04	1.06E−02	0.00E+00	4.46E−11	0.00E+00
F_{29}	0.00E+00	0.00E+00	6.80E−05	0.00E+00	7.53E+01	6.90E+01	1.04E+00	7.54E−01	0.00E+00	0.00E+00	0.00E+00
F_{30}	6.84E−12	3.25E−10	0.00E+00	0.00E+00	0.00E+00	0.00E+00	3.02E−05	2.46E−03	0.00E+00	0.00E+00	0.00E+00
F_{31}	0.00E+00	0.00E+00	2.71E−08	0.00E+00	0.00E+00	1.95E+00	4.75E−07	7.79E−04	0.00E+00	2.43E−08	0.00E+00
F_{32}	5.77E−12	5.05E−10	5.08E−04	0.00E+00	0.00E+00	1.40E+00	4.11E−04	4.59E−03	0.00E+00	4.86E−07	0.00E+00
F_{33}	4.02E−03	4.02E−03	4.02E−03	4.02E−03	4.02E−03	4.02E−03	4.02E−03	4.03E−03	4.02E−03	4.02E−03	4.02E−03
F_{34}	1.52E−10	1.73E−10	6.58E−05	0.00E+00	0.00E+00	2.97E+01	8.46E−05	1.94E−02	0.00E+00	9.54E−07	0.00E+00
F_{35}	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	7.10E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
F_{36}	5.86E−12	1.86E−10	1.45E−05	0.00E+00	0.00E+00	8.00E+00	3.03E−05	1.11E−03	0.00E+00	4.28E−08	0.00E+00
F_{37}	0.00E+00	2.15E−03	0.00E+00	0.00E+00	0.00E+00	9.45E−01	0.00E+00	6.35E−05	0.00E+00	0.00E+00	0.00E+00
F_{38}	9.82E−04	9.82E−04	1.18E−03	9.82E−04	9.82E−04	9.82E−04	1.72E−03	2.04E−01	9.82E−04	9.82E−04	9.82E−04
F_{39}	4.79E−07	2.33E−05	0.00E+00	4.79E−07	4.79E−07	6.74E−01	4.79E−07	2.67E−06	4.79E−07	4.79E−07	4.79E−07
F_{40}	1.68E−12	3.21E−12	8.19E−07	0.00E+00	0.00E+00	0.00E+00	9.16E−08	1.31E−03	0.00E+00	1.36E−09	0.00E+00
F_{41}	2.07E−04	2.07E−04	0.00E+00	2.07E−04	2.07E−04	8.15E+00	6.98E−03	8.61E−03	2.07E−04	2.10E−04	2.07E−04
F_{42}	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	8.10E+00	9.41E−03	2.53E−01	0.00E+00	0.00E+00	0.00E+00
F_{43}	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	9.35E−07	8.21E−05	0.00E+00	0.00E+00	0.00E+00
F_{44}	0.00E+00	7.70E−05	0.00E+00	7.70E−05	7.70E−05	7.70E−05	7.73E−05	1.20E−04	7.70E−05	7.71E−05	7.70E−05
F_{45}	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
F_{46}	0.00E+00	3.59E−02	0.00E+00	0.00E+00	0.00E+00	8.00E−01	0.00E+00	1.25E−03	0.00E+00	0.00E+00	0.00E+00
F_{47}	0.00E+00	4.43E−04	1.21E−02	0.00E+00	3.03E−05	1.73E−02	8.47E−03	3.61E−03	0.00E+00	4.79E−03	0.00E+00
F_{48}	0.00E+00	6.45E−05	2.50E−02	0.00E+00	0.00E+00	6.46E−04	5.10E−03	5.32E−03	0.00E+00	5.01E−03	0.00E+00
F_{49}	0.00E+00	4.80E−07	0.00E+00	4.80E−07	4.80E−07	8.89E−01	4.80E−07	4.80E−07	4.80E−07	4.80E−07	4.80E−07

Table 4 (continued)

No.	Alternative metaheuristic algorithms										
	BIA	BBBC	BOA	CPA	CSS	GSO	JA	RFO	TWO	WDO	FHO
F_{50}	1.62E−07	1.62E−07	0.00E+00	1.62E−07	1.62E−07	4.48E−01	7.75E−06	1.10E−03	1.62E−07	1.84E−07	1.62E−07
F_{51}	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
F_{52}	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	1.70E−09	0.00E+00	0.00E+00	0.00E+00
F_{53}	0.00E+00	1.25E−11	3.92E−08	0.00E+00	0.00E+00	1.00E+00	7.84E−08	5.77E−05	0.00E+00	7.56E−09	0.00E+00
F_{54}	0.00E+00	3.20E−05	0.00E+00	5.17E−06	5.17E−06	0.00E+00	5.17E−06	1.23E−05	5.17E−06	5.17E−06	5.17E−06
F_{55}	1.67E−12	8.00E−12	0.00E+00	0.00E+00	0.00E+00	0.00E+00	2.49E−06	1.76E−03	0.00E+00	0.00E+00	0.00E+00
F_{56}	0.00E+00	1.26E−07	1.10E−06	0.00E+00	3.17E+00	0.00E+00	6.27E−03	5.38E−03	2.96E−06	7.03E−11	0.00E+00
F_{57}	1.67E−12	8.00E−12	0.00E+00	0.00E+00	0.00E+00	0.00E+00	2.49E−06	1.76E−03	0.00E+00	0.00E+00	0.00E+00
F_{58}	6.31E−11	2.14E−06	2.52E−07	0.00E+00	3.94E+03	0.00E+00	3.07E−01	1.59E+00	0.00E+00	1.67E−09	0.00E+00
F_{59}	2.79E−12	0.00E+00	0.00E+00	0.00E+00	2.15E+03	0.00E+00	2.21E−05	7.71E−02	0.00E+00	0.00E+00	0.00E+00
F_{60}	1.18E−04	1.18E−04	0.00E+00	1.18E−04	1.18E−04	2.93E+00	1.36E−03	9.93E−02	1.18E−04	1.18E−04	1.18E−04
F_{61}	9.71E−08	7.61E−09	0.00E+00	0.00E+00	0.00E+00	5.95E−03	1.40E−03	4.05E−02	0.00E+00	9.17E−04	0.00E+00
F_{62}	1.47E−10	7.42E−03	0.00E+00	0.00E+00	0.00E+00	1.14E−04	4.22E−06	3.01E−03	0.00E+00	9.13E−09	0.00E+00
F_{63}	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	2.85E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
F_{64}	7.14E−12	6.07E−07	0.00E+00	0.00E+00	1.06E+02	0.00E+00	6.76E−02	1.20E+00	0.00E+00	0.00E+00	0.00E+00
F_{65}	0.00E+00	2.41E−08	0.00E+00	0.00E+00	6.54E+00	0.00E+00	1.02E−02	2.64E−02	0.00E+00	0.00E+00	0.00E+00
F_{66}	4.66E−07	4.93E−08	0.00E+00	0.00E+00	1.07E+04	0.00E+00	2.53E−03	1.72E−02	1.53E−08	3.51E−12	0.00E+00
F_{67}	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	1.08E−10	9.38E−10	0.00E+00	0.00E+00	0.00E+00
F_{68}	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	3.15E−08	5.98E−06	0.00E+00	0.00E+00	0.00E+00
F_{69}	4.53E−09	4.84E−09	1.26E−06	4.53E−09	4.84E−09	4.98E−01	3.08E−05	1.31E−04	1.33E−08	5.46E−09	4.53E−09
F_{70}	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	2.07E−01	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
F_{71}	7.38E−07	7.51E−06	1.15E−03	0.00E+00	3.00E−07	1.00E+00	3.34E−02	1.38E−01	0.00E+00	3.11E−04	0.00E+00
F_{72}	0.00E+00	2.52E+02	0.00E+00	0.00E+00	8.22E+00	4.99E−04	3.70E−01	9.91E+00	0.00E+00	2.37E−04	0.00E+00
F_{73}	0.00E+00	4.44E−07	0.00E+00	4.44E−07	4.44E−07	0.00E+00	1.60E−05	1.10E−02	4.44E−07	4.67E−07	4.44E−07

Table 4 (continued)

No.	Alternative metaheuristic algorithms										
	BIA	BBBC	BOA	CPA	CSS	GSO	JA	RFO	TWO	WDO	FHO
F_{74}	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	6.44E−06	2.41E−03	0.00E+00	0.00E+00	0.00E+00
F_{75}	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	2.78E−05	1.75E−02	0.00E+00	0.00E+00	0.00E+00
F_{76}	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	5.44E+00	7.78E−05	1.48E−03	0.00E+00	0.00E+00	0.00E+00
F_{77}	1.92E−12	6.41E−12	0.00E+00	0.00E+00	0.00E+00	0.00E+00	4.33E−09	8.56E−05	0.00E+00	0.00E+00	0.00E+00
F_{78}	3.03E−01	5.43E−03	0.00E+00	2.52E−09	2.52E−09	1.16E+00	4.62E−02	5.35E−01	2.52E−09	3.29E−06	2.52E−09
F_{79}	1.11E−06	1.29E−04	4.28E−03	0.00E+00	5.41E−05	2.00E+00	5.09E−02	3.05E−02	0.00E+00	1.97E−03	0.00E+00
F_{80}	0.00E+00	2.35E−12	0.00E+00	0.00E+00	0.00E+00	0.00E+00	2.84E−07	8.18E−05	0.00E+00	2.64E−09	0.00E+00
F_{81}	0.00E+00	3.35E−06	0.00E+00	0.00E+00	5.31E−11	0.00E+00	4.84E−04	8.88E−03	0.00E+00	0.00E+00	0.00E+00
F_{82}	0.00E+00	2.03E−07	0.00E+00	0.00E+00	1.09E−10	0.00E+00	3.24E−04	6.24E−03	0.00E+00	0.00E+00	0.00E+00
F_{83}	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	3.86E−06	9.22E−05	0.00E+00	3.58E−07	0.00E+00
F_{84}	0.00E+00	2.16E−10	0.00E+00	0.00E+00	0.00E+00	0.00E+00	1.17E−04	3.29E−03	0.00E+00	0.00E+00	0.00E+00
F_{85}	9.24E−11	4.31E−06	1.31E−03	0.00E+00	6.83E+03	2.77E−01	3.62E−01	1.92E−01	0.00E+00	1.65E−07	0.00E+00
F_{86}	0.00E+00	2.62E−08	2.03E−08	0.00E+00	2.01E+00	3.36E+00	6.52E−02	1.08E−01	1.84E−11	1.63E−09	0.00E+00
F_{87}	0.00E+00	0.00E+00	0.00E+00	0.00E+00	1.49E+02	2.15E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
F_{88}	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	1.03E−04	4.08E−06	1.58E−04	0.00E+00	0.00E+00	0.00E+00
F_{89}	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	3.17E−01	0.00E+00	1.40E−03	0.00E+00	0.00E+00	0.00E+00
F_{90}	0.00E+00	9.56E−09	0.00E+00	0.00E+00	2.28E+01	0.00E+00	1.12E−02	3.55E−02	0.00E+00	0.00E+00	0.00E+00
F_{91}	1.23E−11	1.26E−10	9.19E−06	0.00E+00	0.00E+00	1.96E+00	3.70E−05	3.19E−03	0.00E+00	2.38E−06	0.00E+00
F_{92}	0.00E+00	3.33E−10	2.91E−04	0.00E+00	8.17E−09	0.00E+00	1.21E−05	1.40E−03	0.00E+00	6.32E−07	0.00E+00
F_{93}	2.21E−06	2.21E−06	0.00E+00	2.21E−06	2.21E−06	1.96E+00	1.14E−04	9.08E−03	2.21E−06	2.38E−06	2.21E−06
F_{94}	8.92E−12	2.97E−12	7.53E−10	0.00E+00	0.00E+00	7.46E+00	5.33E−04	1.17E−02	0.00E+00	5.00E−10	0.00E+00
F_{95}	3.55E−06	2.45E−02	2.46E−02	2.45E−02	2.45E−02	2.89E−02	2.45E−02	2.48E−02	2.45E−02	2.45E−02	2.45E−02
F_{96}	0.00E+00	0.00E+00	2.64E−05	0.00E+00	0.00E+00	0.00E+00	2.92E−05	3.94E−02	0.00E+00	6.15E−12	0.00E+00
F_{97}	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	3.07E−01	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00

Table 4 (continued)

No.	Alternative metaheuristic algorithms										
	BIA	BBBC	BOA	CPA	CSS	GSO	JA	RFO	TWO	WDO	FHO
F_{98}	1.92E−10	1.00E−08	2.03E−03	0.00E+00	0.00E+00	2.83E+00	5.67E−03	4.98E−02	0.00E+00	2.12E−05	0.00E+00
F_{99}	5.21E−08	2.52E−06	1.03E−02	0.00E+00	0.00E+00	0.00E+00	1.48E+00	1.13E+01	0.00E+00	3.21E−06	0.00E+00
F_{100}	2.95E+04	0.00E+00	0.00E+00	0.00E+00	3.86E+05	0.00E+00	3.19E+04	2.41E+04	0.00E+00	0.00E+00	0.00E+00
F_{101}	0.00E+00	2.05E−01	1.67E+03	0.00E+00	2.67E+01	7.30E+06	1.12E+03	4.48E+03	4.57E−02	8.14E+01	0.00E+00
F_{102}	0.00E+00	2.04E−11	9.89E−10	0.00E+00	0.00E+00	7.32E−01	0.00E+00	8.86E−10	0.00E+00	0.00E+00	0.00E+00
F_{103}	0.00E+00	0.00E+00	8.81E−07	0.00E+00	0.00E+00	1.35E−03	3.42E−04	8.65E−04	1.57E−05	6.05E−09	0.00E+00
F_{104}	0.00E+00	0.00E+00	3.72E−05	0.00E+00	0.00E+00	2.86E−03	0.00E+00	7.51E−04	0.00E+00	0.00E+00	0.00E+00
F_{105}	4.91E−07	3.34E−06	0.00E+00	3.21E−07	3.21E−07	2.52E−01	1.74E+00	5.19E+00	3.21E−07	1.25E−06	3.21E−07
F_{106}	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	2.52E−01	1.65E+00	5.24E+00	0.00E+00	0.00E+00	0.00E+00
F_{107}	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	2.51E−01	2.08E+00	1.35E+00	0.00E+00	0.00E+00	0.00E+00
F_{108}	7.47E−06	1.93E−07	6.07E−04	4.42E−07	1.44E−08	6.11E+00	3.40E−03	1.76E−02	1.09E−03	1.33E−04	0.00E+00
F_{109}	0.00E+00	1.51E+02	5.52E+02	1.40E−05	9.34E+01	1.89E+06	2.43E+02	3.61E+02	8.68E+01	1.57E+02	1.40E+00
F_{110}	0.00E+00	1.79E−04	0.00E+00	0.00E+00	1.63E+01	4.96E+02	6.35E+00	4.82E+01	0.00E+00	3.26E−02	0.00E+00
F_{111}	1.10E−06	0.00E+00	0.00E+00	0.00E+00	0.00E+00	9.56E−01	0.00E+00	1.67E−01	0.00E+00	0.00E+00	0.00E+00
F_{112}	2.21E−06	1.81E−06	2.35E−03	2.13E−05	6.95E−07	6.55E−01	1.68E−02	1.45E−01	3.37E−03	5.12E−06	3.62E−09
F_{113}	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	2.74E+00	3.33E−02	3.10E−01	0.00E+00	1.96E−04	0.00E+00
F_{114}	2.80E−08	1.40E−05	7.65E+00	0.00E+00	6.30E−05	2.00E+01	6.96E+00	3.49E−01	0.00E+00	7.28E−04	0.00E+00
F_{115}	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	4.02E−02	2.48E−03	7.37E−03	0.00E+00	0.00E+00	0.00E+00
F_{116}	0.00E+00	0.00E+00	1.05E+01	0.00E+00	0.00E+00	0.00E+00	6.43E+00	1.09E+01	0.00E+00	0.00E+00	0.00E+00
F_{117}	2.67E+00	1.18E−03	1.41E+02	4.26E−09	8.23E−01	1.20E+02	2.07E+02	1.56E+02	4.13E−08	6.30E−02	0.00E+00
F_{118}	1.36E+01	1.21E−01	0.00E+00	1.86E+01	6.83E+00	0.00E+00	1.93E+01	1.33E+01	7.82E−12	0.00E+00	0.00E+00
F_{119}	3.10E+00	6.06E−01	0.00E+00	5.66E−06	2.08E−01	0.00E+00	6.66E+01	4.00E+01	8.36E−03	0.00E+00	0.00E+00
F_{120}	6.05E−05	2.74E−04	0.00E+00	0.00E+00	1.01E−03	0.00E+00	3.90E+02	7.04E+01	0.00E+00	2.04E−02	0.00E+00
F_{121}	2.58E+07	3.89E−02	0.00E+00	0.00E+00	2.64E+06	0.00E+00	3.22E+09	3.07E+07	0.00E+00	0.00E+00	0.00E+00

Table 4 (continued)

No.	Alternative metaheuristic algorithms										
	BIA	BBBC	BOA	CPA	CSS	GSO	JA	RFO	TWO	WDO	FHO
F_{122}	9.67E−07	0.00E+00	1.01E−11	0.00E+00	5.34E−10	0.00E+00	6.47E−01	5.21E−02	0.00E+00	0.00E+00	0.00E+00
F_{123}	4.22E−04	3.06E−04	0.00E+00	0.00E+00	4.15E−02	0.00E+00	4.13E−01	4.88E−01	6.86E−02	5.68E−02	0.00E+00
F_{124}	0.00E+00	6.34E−05	0.00E+00	0.00E+00	3.36E−02	1.61E−07	3.80E−01	4.67E−01	2.00E−02	3.24E−02	1.54E−03
F_{125}	6.68E−01	7.20E−01	9.83E−01	6.67E−01	5.11E+00	2.50E−01	1.25E+06	4.44E+04	6.77E−01	6.71E−01	2.50E−01
F_{126}	1.00E+00	1.00E+00	1.00E+00	1.00E+00	2.33E−03	0.00E+00	1.00E+00	1.00E+00	1.00E+00	1.26E−01	1.00E+00
F_{127}	1.33E−05	9.13E−06	0.00E+00	0.00E+00	1.39E−05	0.00E+00	9.41E−01	5.10E−01	0.00E+00	0.00E+00	0.00E+00
F_{128}	2.27E+00	8.22E−03	0.00E+00	1.93E−11	1.41E+00	0.00E+00	1.52E+01	2.63E+00	2.96E−06	0.00E+00	0.00E+00
F_{129}	1.09E−09	4.91E−06	0.00E+00	0.00E+00	1.20E−02	0.00E+00	3.30E+05	1.47E+04	0.00E+00	0.00E+00	0.00E+00
F_{130}	3.89E+17	2.82E+14	0.00E+00	1.96E+07	3.32E+17	0.00E+00	9.41E+17	2.22E+17	7.25E+14	0.00E+00	0.00E+00
F_{131}	2.99E+01	4.12E+01	0.00E+00	3.56E+01	2.19E+01	0.00E+00	4.20E+01	4.05E+01	1.41E+01	1.52E+01	0.00E+00
F_{132}	4.79E+00	1.47E+01	2.96E+00	4.05E+01	3.04E−01	0.00E+00	1.41E+02	4.12E+01	1.29E−10	9.04E−03	2.05E−04
F_{133}	0.00E+00	1.81E−02	2.92E+16	0.00E+00	7.60E−03	0.00E+00	1.05E+13	8.00E+23	1.27E+00	0.00E+00	0.00E+00
F_{134}	0.00E+00	1.85E−02	2.92E+16	0.00E+00	1.05E−02	0.00E+00	1.99E+13	1.99E+24	2.43E+00	0.00E+00	0.00E+00
F_{135}	1.51E+109	9.25E+128	9.25E+128	9.25E+128	9.25E+128	9.25E+128	9.25E+128	9.25E+128	9.25E+128	9.25E+128	9.25E+128
F_{136}	0.00E+00	5.50E−12	0.00E+00	0.00E+00	1.60E−06	0.00E+00	7.40E−02	0.00E+00	0.00E+00	0.00E+00	0.00E+00
F_{137}	2.00E+01	1.54E+01	1.62E+01	1.02E+01	1.86E+01	0.00E+00	1.85E+01	4.78E+00	8.87E+00	2.96E+00	0.00E+00
F_{138}	7.76E+03	2.79E+03	0.00E+00	1.30E+04	8.25E+02	0.00E+00	2.75E+04	1.26E+04	1.07E+02	0.00E+00	0.00E+00
F_{139}	2.76E−02	9.91E−03	0.00E+00	5.64E−02	8.54E−01	0.00E+00	3.44E+03	9.00E+02	5.67E−01	7.04E−01	0.00E+00
F_{140}	5.35E−03	4.26E−02	0.00E+00	2.74E−09	4.20E−01	0.00E+00	4.16E+04	6.00E+03	7.96E−03	3.35E−01	0.00E+00
F_{141}	9.22E−10	9.35E−10	0.00E+00	0.00E+00	0.00E+00	0.00E+00	2.41E−02	2.36E−03	1.40E−11	0.00E+00	0.00E+00
F_{142}	4.88E+01	1.28E+02	0.00E+00	2.12E+02	5.82E+01	0.00E+00	5.28E+02	4.05E+02	6.96E+00	2.03E+01	0.00E+00
F_{143}	4.45E+09	5.13E+02	9.44E+03	5.30E−07	3.85E+08	1.04E+04	1.06E+11	3.55E+08	2.45E+02	1.11E+02	9.59E+03
F_{144}	3.12E+01	4.15E+01	1.22E+02	9.16E−06	2.52E+01	0.00E+00	7.87E+04	1.46E+03	2.18E−10	3.29E+00	2.27E−04
F_{145}	7.71E+03	3.96E+01	4.88E+01	3.59E+01	1.44E+04	0.00E+00	1.34E+08	2.59E+06	4.69E+01	4.74E+01	6.66E−07

Table 4 (continued)

No.	Alternative metaheuristic algorithms										
	BIA	BBBC	BOA	CPA	CSS	GSO	JA	RFO	TWO	WDO	FHO
F_{146}	1.24E+01	5.00E−01	2.15E−01	7.30E+00	4.50E+00	0.00E+00	2.36E+01	8.28E+00	4.54E−01	9.99E−02	0.00E+00
F_{147}	7.63E+05	1.94E−03	0.00E+00	0.00E+00	1.07E+05	0.00E+00	1.69E+08	1.86E+06	0.00E+00	0.00E+00	0.00E+00
F_{148}	1.72E+22	3.66E−05	5.41E+21	0.00E+00	6.44E+18	0.00E+00	3.34E+28	6.16E+22	0.00E+00	0.00E+00	0.00E+00
F_{149}	0.00E+00	0.00E+00	0.00E+00	0.00E+00	1.18E−12	0.00E+00	1.85E−09	4.30E−08	0.00E+00	0.00E+00	0.00E+00
F_{150}	2.30E−04	2.63E−03	9.10E+00	1.06E−04	9.93E−03	4.09E+05	1.50E+04	4.66E+01	4.91E−01	6.49E−02	1.97E−10
F_{151}	4.65E+02	2.65E+00	0.00E+00	8.13E−06	2.19E+02	0.00E+00	1.27E+03	4.49E+02	2.29E−10	0.00E+00	0.00E+00
F_{152}	2.87E+01	1.73E−01	0.00E+00	3.07E+01	1.15E+01	0.00E+00	5.44E+01	2.34E+01	3.20E−09	0.00E+00	0.00E+00
F_{153}	1.21E+39	1.78E+03	1.15E+62	1.92E−02	2.37E−09	0.00E+00	7.47E+53	7.32E+36	2.27E+01	0.00E+00	0.00E+00
F_{154}	0.00E+00	0.00E+00	0.00E+00	0.00E+00	1.37E−06	0.00E+00	3.95E+08	6.09E+05	0.00E+00	0.00E+00	0.00E+00
F_{155}	2.71E−04	2.93E−03	9.05E+00	1.94E−04	1.10E−02	4.09E+05	1.50E+04	4.45E+01	4.55E−01	4.45E−02	1.08E−08
F_{156}	0.00E+00	9.14E+01	2.73E+02	1.08E+02	3.21E+02	0.00E+00	2.72E+02	2.17E+02	8.59E+01	1.71E+02	4.96E−03
F_{157}	4.03E−05	2.52E−04	0.00E+00	0.00E+00	7.63E−06	5.00E+03	5.64E+02	0.00E+00	5.07E−03	0.00E+00	0.00E+00
F_{158}	6.23E+02	0.00E+00	0.00E+00	0.00E+00	1.91E+02	0.00E+00	1.25E+03	4.27E+02	0.00E+00	0.00E+00	0.00E+00
F_{159}	1.39E+04	0.00E+00	0.00E+00	0.00E+00	1.53E+03	0.00E+00	5.66E+04	7.60E+03	0.00E+00	0.00E+00	0.00E+00
F_{160}	1.40E+04	0.00E+00	1.00E+00	1.00E+00	1.65E+03	0.00E+00	5.55E+04	6.14E+03	0.00E+00	0.00E+00	0.00E+00
F_{161}	0.00E+00	2.10E+01	1.56E+02	1.70E+01	0.00E+00	0.00E+00	1.23E+02	0.00E+00	4.90E+01	0.00E+00	0.00E+00
F_{162}	8.66E+00	1.13E+01	7.59E−02	1.36E+01	7.10E+00	0.00E+00	4.12E+01	2.01E+01	2.68E+00	2.63E−01	0.00E+00
F_{163}	1.28E−03	5.13E−02	0.00E+00	2.19E−10	1.94E+00	0.00E+00	1.30E+04	3.17E+03	9.97E−02	0.00E+00	0.00E+00
F_{164}	1.98E+02	1.84E+02	0.00E+00	1.84E+02	1.55E−01	0.00E+00	8.27E+02	7.94E+02	1.83E+02	3.04E+02	0.00E+00
F_{165}	7.08E+06	1.52E+02	2.20E+04	9.17E+02	4.28E+07	1.96E+04	2.45E+07	1.72E+06	3.17E+01	5.53E+03	1.94E+04
F_{166}	5.67E−01	1.26E+00	0.00E+00	2.17E+00	6.74E−03	8.06E−04	1.90E+04	0.00E+00	8.68E+01	0.00E+00	0.00E+00
F_{167}	3.60E+05	1.63E+02	1.37E+02	9.74E+01	7.45E+04	0.00E+00	1.41E+06	1.21E+05	3.38E+00	5.79E−02	0.00E+00
F_{168}	3.10E−01	4.53E−01	6.37E−01	4.38E−01	9.19E−02	0.00E+00	6.83E−01	7.12E−01	1.04E−01	0.00E+00	0.00E+00
F_{169}	1.00E+00	1.00E+00	1.00E+00	1.00E+00	1.00E+00	0.00E+00	1.00E+00	1.00E+00	1.00E+00	1.00E+00	0.00E+00

Table 4 (continued)

No.	Alternative metaheuristic algorithms										
	BIA	BBBC	BOA	CPA	CSS	GSO	JA	RFO	TWO	WDO	FHO
F_{170}	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
F_{171}	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
F_{172}	2.19E+03	3.05E−01	7.42E−07	2.34E+07	2.67E−10	0.00E+00	1.60E+13	3.81E+07	4.24E−10	1.16E−11	0.00E+00
F_{173}	1.00E+00	1.00E+00	1.00E+00	1.00E+00	1.00E+00	0.00E+00	1.00E+00	1.00E+00	1.00E+00	1.00E+00	0.00E+00
F_{174}	1.02E−04	7.87E−06	2.58E−05	6.99E−08	1.52E−06	1.53E−06	2.87E−06	2.11E−06	4.82E−06	1.30E−06	3.30E−07
F_{175}	1.17E+02	6.87E−03	0.00E+00	1.86E+01	7.49E+00	0.00E+00	5.88E+02	5.05E+02	5.94E+01	7.21E+01	0.00E+00
F_{176}	1.47E+01	2.32E−01	0.00E+00	2.00E+01	7.56E+00	0.00E+00	2.01E+01	1.41E+01	1.07E−11	0.00E+00	0.00E+00
F_{177}	9.90E+00	2.63E+00	0.00E+00	5.19E−02	4.26E+00	0.00E+00	1.72E+02	8.24E+01	3.36E−02	0.00E+00	0.00E+00
F_{178}	3.04E−04	1.25E−03	0.00E+00	5.12E−05	6.37E−02	0.00E+00	3.07E+04	1.66E+02	1.56E+01	2.04E−01	0.00E+00
F_{179}	4.80E+07	1.04E+00	0.00E+00	3.61E−06	2.34E+07	0.00E+00	2.12E+10	1.74E+08	0.00E+00	0.00E+00	0.00E+00
F_{180}	1.97E−06	0.00E+00	0.00E+00	5.76E−07	3.07E−07	0.00E+00	4.41E+00	1.44E−01	0.00E+00	0.00E+00	0.00E+00
F_{181}	1.30E−03	7.33E−04	0.00E+00	1.48E−06	1.51E−01	0.00E+00	4.88E−01	5.24E−01	1.45E−01	1.48E−01	0.00E+00
F_{182}	0.00E+00	1.78E−04	0.00E+00	7.11E−07	1.49E−01	3.45E−08	4.73E−01	5.22E−01	1.07E−01	1.13E−01	6.24E−03
F_{183}	6.95E−01	1.16E+00	9.97E−01	6.76E−01	1.61E+02	2.50E−01	8.93E+06	3.40E+05	5.59E+00	6.85E−01	2.50E−01
F_{184}	1.00E+00	1.00E+00	1.00E+00	1.00E+00	1.00E+00	0.00E+00	1.00E+00	1.00E+00	1.00E+00	4.81E−02	1.00E+00
F_{185}	6.33E−05	5.31E−05	0.00E+00	9.51E−08	4.65E−04	0.00E+00	9.99E−01	7.72E−01	0.00E+00	0.00E+00	0.00E+00
F_{186}	2.73E+00	2.30E−02	0.00E+00	5.47E−05	2.19E+00	0.00E+00	3.74E+01	4.88E+00	8.45E−03	0.00E+00	0.00E+00
F_{187}	6.18E−08	1.52E−04	0.00E+00	1.65E−06	1.61E+01	0.00E+00	2.27E+06	8.42E+04	1.81E−03	0.00E+00	0.00E+00
F_{188}	4.78E+32	4.74E+33	3.81E+33	3.26E+22	6.42E+32	0.00E+00	1.12E+33	2.83E+32	1.13E+30	0.00E+00	0.00E+00
F_{189}	6.86E+01	8.39E+01	0.00E+00	7.98E+01	5.83E+01	0.00E+00	8.98E+01	8.77E+01	4.05E+01	3.22E+01	0.00E+00
F_{190}	1.68E+01	7.53E+01	8.02E+00	1.08E+02	3.84E+00	0.00E+00	4.10E+02	1.25E+02	1.23E−05	4.04E−02	1.26E−04
F_{191}	0.00E+00	7.52E−02	3.46E+50	0.00E+00	1.51E−01	0.00E+00	2.66E+46	2.53E+66	2.30E+18	1.41E+06	0.00E+00
F_{192}	0.00E+00	8.27E−02	3.29E+51	0.00E+00	1.77E−01	0.00E+00	2.47E+46	2.43E+66	5.47E+18	4.34E+02	0.00E+00
F_{193}	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00

Table 4 (continued)

No.	Alternative metaheuristic algorithms										
	BIA	BBBC	BOA	CPA	CSS	GSO	JA	RFO	TWO	WDO	FHO
F_{194}	0.00E+00	5.18E−11	2.53E−09	0.00E+00	3.93E−04	0.00E+00	3.03E−01	0.00E+00	0.00E+00	0.00E+00	0.00E+00
F_{195}	4.25E+01	3.43E+01	3.75E+01	2.53E+01	3.97E+01	0.00E+00	4.14E+01	9.84E+00	2.52E+01	1.82E+01	0.00E+00
F_{196}	3.32E+04	2.87E+04	0.00E+00	7.52E+04	1.37E+04	0.00E+00	1.36E+05	6.62E+04	2.87E+03	0.00E+00	0.00E+00
F_{197}	1.86E−01	1.13E−01	0.00E+00	1.49E+00	8.29E+00	0.00E+00	1.42E+04	2.72E+03	7.45E+00	9.82E−01	0.00E+00
F_{198}	3.62E−02	2.35E−01	0.00E+00	1.29E−03	2.01E+01	0.00E+00	1.54E+05	1.61E+04	5.63E−02	1.92E+00	0.00E+00
F_{199}	1.08E−09	1.30E−09	0.00E+00	0.00E+00	0.00E+00	0.00E+00	8.54E−02	2.36E−03	0.00E+00	0.00E+00	0.00E+00
F_{200}	1.14E+02	4.05E+02	0.00E+00	5.47E+02	2.86E+02	0.00E+00	1.18E+03	9.30E+02	0.00E+00	0.00E+00	0.00E+00
F_{201}	1.09E+10	4.90E+03	1.16E+05	9.78E+00	6.18E+08	8.33E+04	3.34E+11	1.99E+09	4.11E+03	3.36E+03	8.25E+04
F_{202}	7.70E+01	1.59E+01	2.97E+02	2.93E−01	2.42E+02	0.00E+00	3.37E+05	7.13E+03	1.04E+02	8.94E+00	2.21E−04
F_{203}	4.55E+05	1.00E+02	9.88E+01	9.00E+01	1.34E+05	0.00E+00	4.27E+08	8.28E+06	9.91E+01	9.79E+01	2.34E−04
F_{204}	1.83E+01	1.30E+00	2.61E−01	2.09E+01	7.21E+00	0.00E+00	3.95E+01	1.27E+01	1.60E+00	9.99E−02	0.00E+00
F_{205}	8.81E+06	3.52E−02	0.00E+00	1.85E−04	5.81E+05	0.00E+00	5.35E+08	5.60E+06	5.43E−09	0.00E+00	0.00E+00
F_{206}	1.11E+23	9.28E−01	7.45E+29	0.00E+00	5.84E+21	0.00E+00	9.50E+30	1.06E+25	0.00E+00	0.00E+00	0.00E+00
F_{207}	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	4.49E−09	1.96E−07	0.00E+00	0.00E+00	0.00E+00
F_{208}	1.37E+01	1.35E−02	1.97E+01	4.81E−02	5.66E−01	8.18E+05	5.43E+04	9.41E+01	1.44E+02	2.91E−01	7.51E−06
F_{209}	1.24E+03	8.09E+01	0.00E+00	2.25E−01	5.22E+02	0.00E+00	3.03E+03	8.66E+02	3.65E−02	0.00E+00	0.00E+00
F_{210}	3.45E+01	7.02E+00	0.00E+00	5.68E+01	1.40E+01	0.00E+00	7.77E+01	2.58E+01	3.02E+00	0.00E+00	0.00E+00
F_{211}	5.07E+93	2.38E+53	2.41E+138	2.96E+03	2.21E+40	0.00E+00	2.67E+123	4.73E+74	4.45E+02	0.00E+00	0.00E+00
F_{212}	0.00E+00	0.00E+00	0.00E+00	0.00E+00	3.86E−02	0.00E+00	7.21E+09	2.15E+06	1.06E−07	0.00E+00	0.00E+00
F_{213}	7.56E+00	1.42E−02	1.85E+01	6.09E−02	6.49E−01	8.18E+05	5.42E+04	9.40E+01	1.59E+02	2.78E−01	2.21E−05
F_{214}	0.00E+00	9.41E+01	3.09E+02	1.27E+02	3.49E+02	0.00E+00	3.18E+02	2.43E+02	1.68E+02	2.05E+02	1.57E−03
F_{215}	8.81E+01	2.03E−03	0.00E+00	0.00E+00	3.36E−03	1.00E+04	1.68E+03	0.00E+00	5.40E+01	0.00E+00	0.00E+00
F_{216}	1.45E+03	0.00E+00	0.00E+00	4.00E+01	4.86E+02	0.00E+00	2.98E+03	9.16E+02	0.00E+00	0.00E+00	0.00E+00
F_{217}	3.15E+04	0.00E+00	0.00E+00	2.60E+01	5.12E+03	0.00E+00	1.46E+05	1.23E+04	0.00E+00	0.00E+00	0.00E+00

Table 4 (continued)

| No.	Alternative metaheuristic algorithms										
	BIA	BBBC	BOA	CPA	CSS	GSO	JA	RFO	TWO	WDO	FHO
F_{218}	2.97E+04	0.00E+00	1.40E+01	1.20E+01	4.60E+03	0.00E+00	1.48E+05	1.38E+04	0.00E+00	0.00E+00	0.00E+00
F_{219}	0.00E+00	1.10E+02	3.57E+02	9.60E+01	0.00E+00	0.00E+00	3.34E+02	0.00E+00	1.60E+02	0.00E+00	0.00E+00
F_{220}	1.83E+01	2.54E+01	1.68E−01	3.09E+01	2.44E+01	0.00E+00	9.57E+01	4.12E+01	1.09E+01	1.45E−01	0.00E+00
F_{221}	4.44E−02	4.66E−01	0.00E+00	3.92E−03	1.32E+02	0.00E+00	6.46E+04	1.24E+04	1.55E+01	0.00E+00	0.00E+00
F_{222}	5.09E+02	4.10E+02	0.00E+00	4.38E+02	6.38E+01	0.00E+00	2.14E+03	1.81E+03	7.97E+02	7.35E+02	0.00E+00
F_{223}	2.88E+08	5.23E+03	1.72E+05	2.21E+05	1.99E+09	1.62E+05	1.09E+09	5.77E+07	5.73E+05	1.24E+05	1.60E+05
F_{224}	2.13E+01	8.02E+00	0.00E+00	7.63E+01	6.44E−01	2.16E−03	3.94E+05	0.00E+00	2.19E+03	0.00E+00	0.00E+00
F_{225}	8.08E+05	4.77E+02	3.28E+02	3.93E+02	1.53E+05	0.00E+00	3.64E+06	2.58E+05	1.10E+02	2.29E−01	0.00E+00
F_{226}	3.46E−01	4.97E−01	7.52E−01	4.84E−01	2.21E−01	0.00E+00	7.97E−01	8.08E−01	1.27E−01	1.20E−01	0.00E+00
F_{227}	1.00E+00	1.00E+00	1.00E+00	1.00E+00	1.00E+00	0.00E+00	1.00E+00	1.00E+00	1.00E+00	1.00E+00	0.00E+00
F_{228}	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
F_{229}	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
F_{230}	1.29E+13	3.81E+11	1.80E+50	2.02E+28	3.10E−10	0.00E+00	1.26E+38	9.23E+19	7.18E−07	1.89E−10	0.00E+00
F_{231}	1.00E+00	1.00E+00	1.00E+00	1.00E+00	1.00E+00	0.00E+00	1.00E+00	1.00E+00	1.00E+00	1.00E+00	0.00E+00
F_{232}	4.43E−05	1.29E−05	2.81E−07	1.19E−06	2.74E−06	1.42E−06	6.87E−06	8.97E−05	1.27E−05	8.69E−06	1.87E−07
F_{233}	2.29E+03	2.46E−01	0.00E+00	7.84E+02	1.10E+02	0.00E+00	1.75E+03	1.53E+03	1.52E+02	5.94E+02	0.00E+00
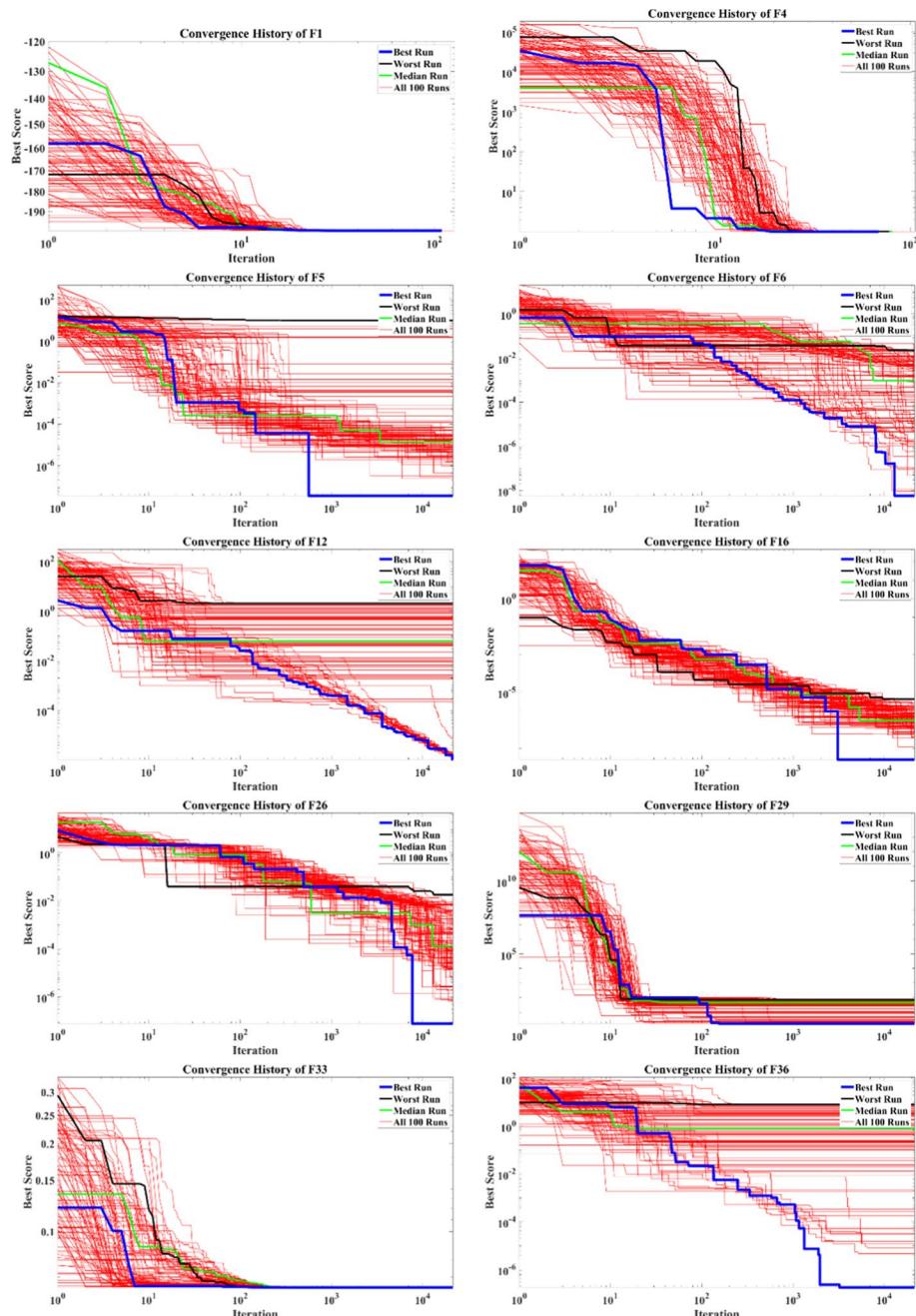
**Fig. 14** Convergence curves of the FHO for the mathematical functions
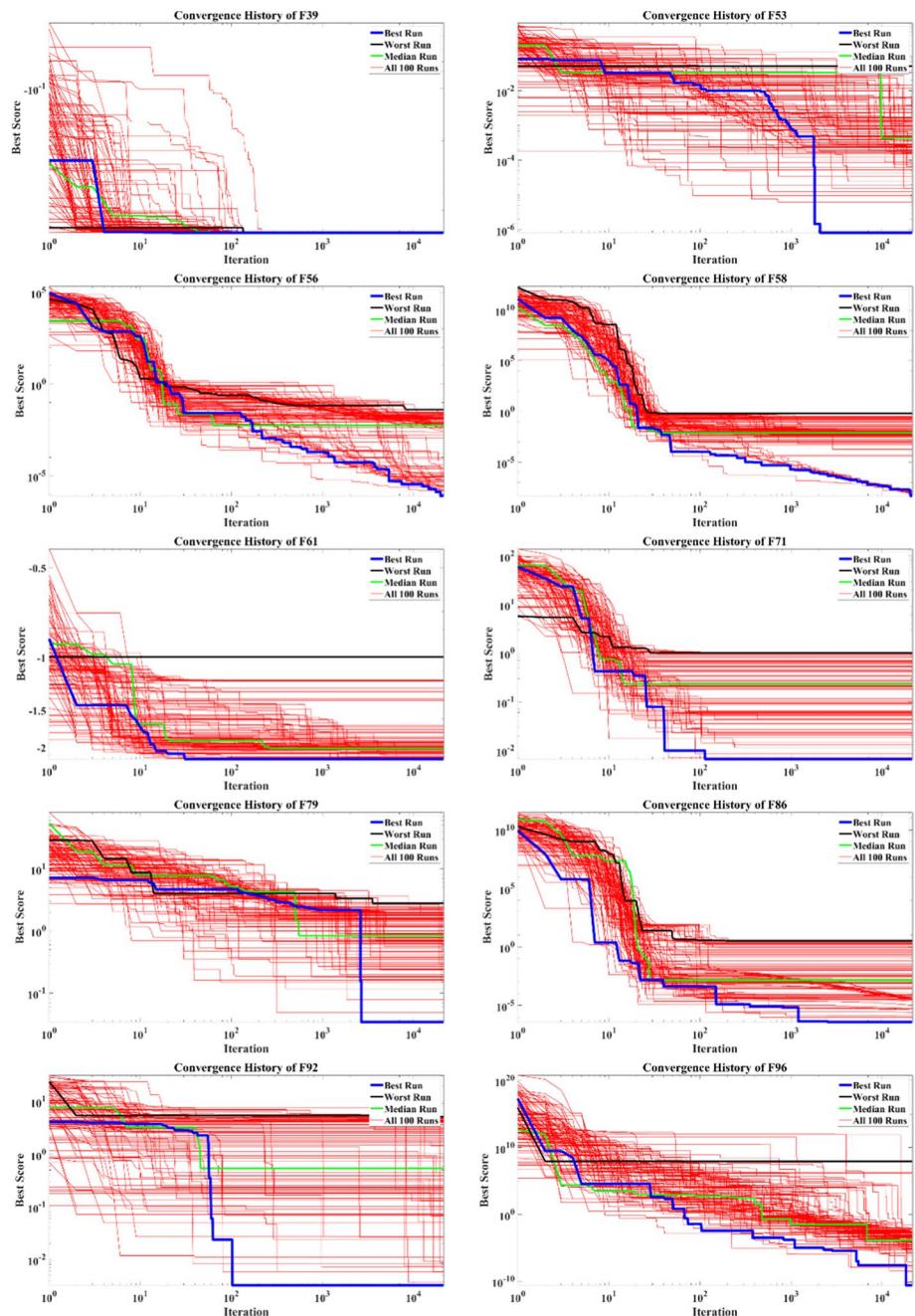
**Fig. 14** (continued)
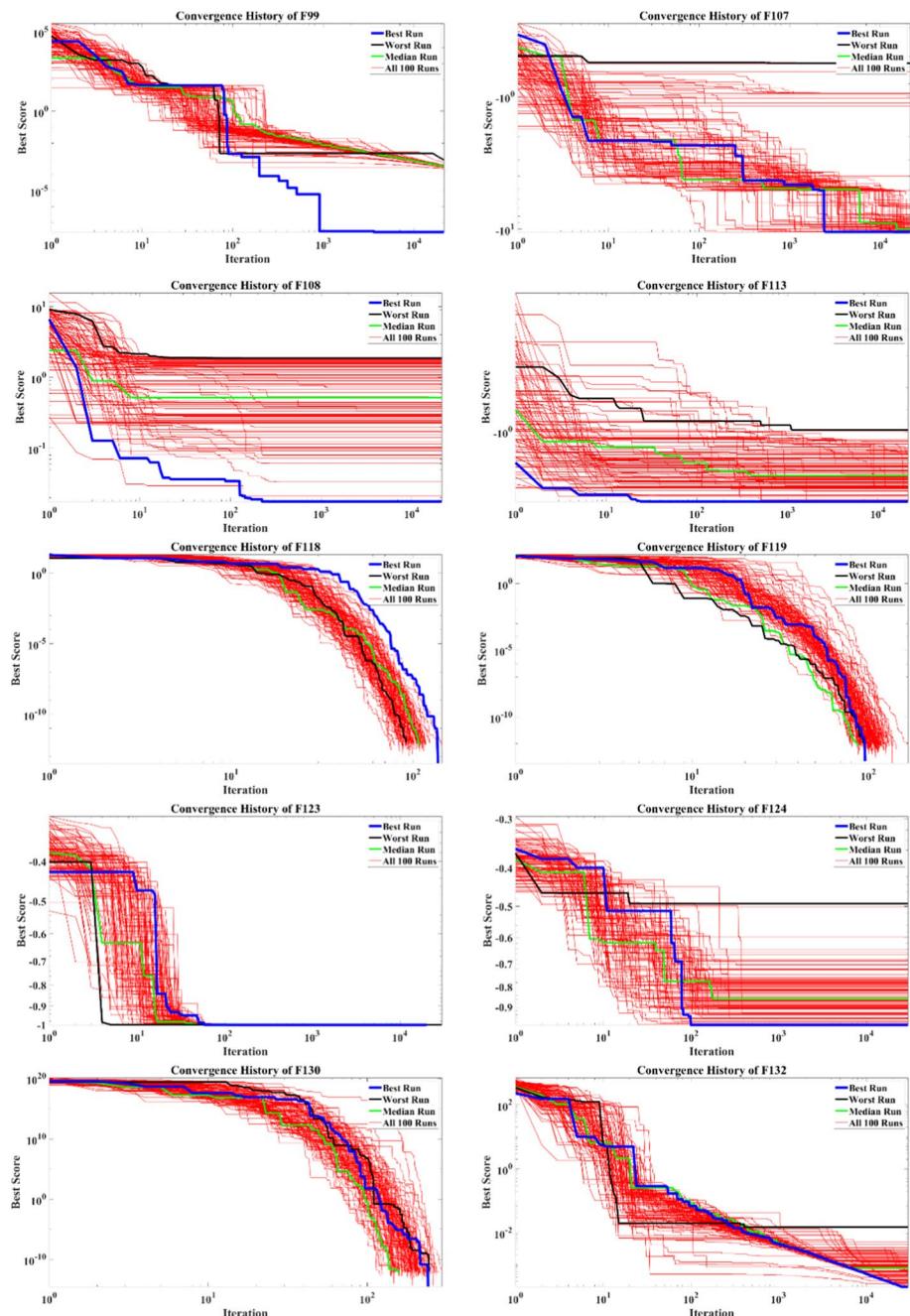
**Fig. 14** (continued)
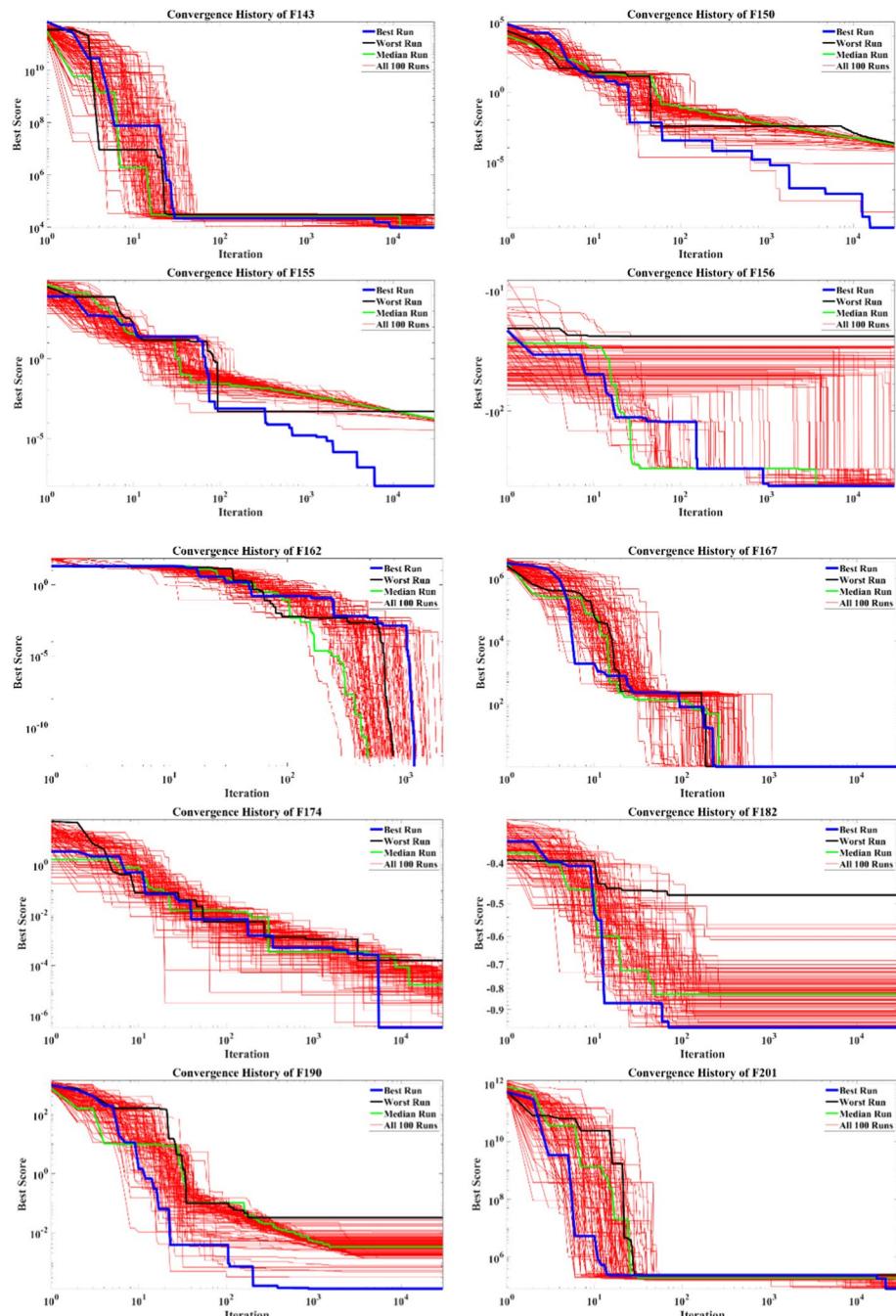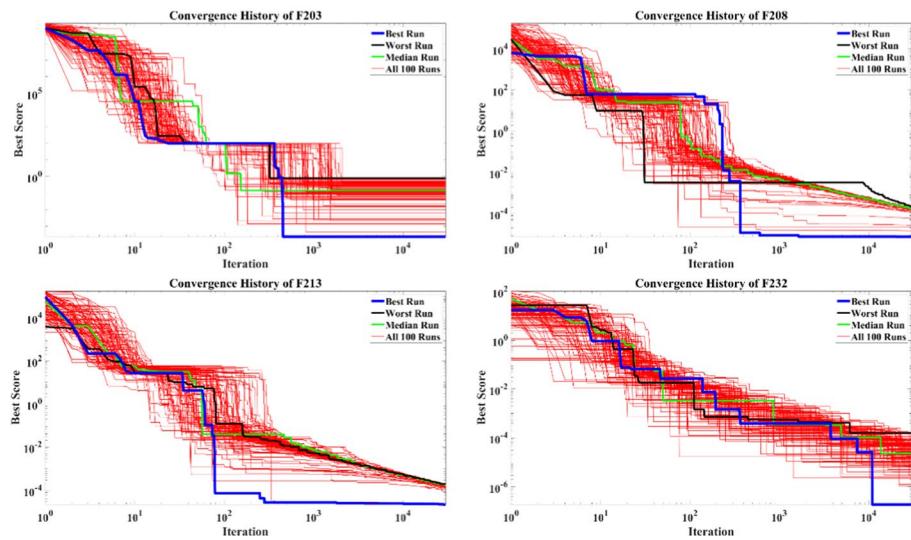
Fig. 14 (continued)

Fig. 14 (continued)

6 Statistical analysis

As the mean and standard deviation of the results do not effectively represent the overall performance of the algorithms in dealing with test functions, a complete statistical analysis was performed using the most conclusive statistical tests. Specifically, the Kolmogorov Smirnov (KS) test was used for evaluating the normality of results, the Mann Whitney (MW) test for two-by-two comparison of the summation of different metaheuristics' ranks, Wilcoxon (W) signed ranks test for two-by-two comparison of the summation of different algorithms' ranks, and the Kruskal Wallis (KW) test for comparing the mean of the metaheuristic algorithms' ranks for representing the overall rankings of these algorithms. A Post Hoc (PH) analysis was also conducted for comprehensive comparative purposes.

6.1 Kolmogorov Smirnov (KS) test

The KS test was utilized to determine if the parametric or non-parametric statistical tests should be conducted on the collected dataset, considering the mathematical test functions and alternative metaheuristic algorithms. Specifically, if the p value of this statistical test is less than 0.05, the dataset is not distributed normally and the non-parametric test should be considered. If p value > 0.05, then parametric test should be conducted. Table 5 provides the results of this test for the collected dataset in the previous section regarding the SS, 50D, and 100D test functions. Based on the results, it is obvious that the non-parametric statistical tests, including the W, MW, and KW tests, should be determined for further investigation.

Table 5 The KS test results (p values) for different metaheuristic algorithms

Main algorithm	Function type	Data type	Alternative metaheuristic algorithms									
			BIA	BBBC	BOA	CPA	CSS	GSO	JA	RFO	TWO	WDO
FHO	SS	Min	6.95E−04	4.73E−12	1.22E−02	1.00E+0	1.94E−03	3.36E−15	1.26E−22	2.19E−34	9.40E−01	6.92E−07
		Mean	3.20E−11	9.99E−15	6.95E−04	9.40E−01	3.92E−05	4.73E−12	1.79E−21	1.62E−29	3.16E−03	3.30E−08
		Std	1.15E−16	4.80E−22	2.85E−06	9.40E−01	7.24E−08	1.18E−09	7.49E−29	3.44E−30	3.92E−05	9.99E−15
		Fun. Evl	2.01E−10	1.00E−18	6.92E−07	1.17E−03	4.73E−12	6.59E−21	1.21E−25	7.49E−29	9.99E−15	3.20E−11
	50D	Min	3.23E−11	2.04E−12	2.85E−03	2.37E−05	2.48E−14	1.00E+0	2.36E−19	1.09E−15	1.61E−07	6.67E−04
		Mean	1.54E−09	4.40E−10	2.85E−03	1.61E−07	4.87E−13	4.87E−13	7.70E−18	5.30E−15	1.61E−07	2.37E−05
		Std	1.69E−08	5.31E−08	5.70E−02	1.32E−06	8.27E−12	8.27E−12	5.30E−15	4.87E−13	4.69E−07	5.76E−05
		Fun. Evl	4.40E−10	2.04E−12	8.27E−12	8.27E−12	1.12E−13	1.69E−08	5.30E−15	2.04E−12	8.27E−12	1.21E−10
	100D	Min	1.21E−10	3.23E−11	6.67E−04	4.40E−10	5.30E−15	9.99E−01	2.36E−19	1.09E−15	1.54E−09	1.35E−04
		Mean	1.21E−10	3.23E−11	6.67E−04	1.21E−10	1.12E−13	4.87E−13	1.37E−18	1.09E−15	3.23E−11	2.37E−05
		Std	1.54E−09	1.54E−09	3.37E−02	1.54E−09	2.04E−12	8.27E−12	2.48E−14	4.87E−13	1.21E−10	3.06E−04
		Fun. Evl	4.40E−10	8.27E−12	8.27E−12	8.27E−12	2.48E−14	5.31E−08	5.30E−15	4.87E−13	8.27E−12	1.21E−10

Table 6 The MW test results (summation of ranks) for different metaheuristic algorithms

Main algorithm	Function type	Data type	Alternative metaheuristic algorithms									
			BIA	BBBC	BOA	CPA	CSS	GSO	JA	RFO	TWO	WDO
FHO	SS	Min.	15271.50	16788.50	15120.00	13925.50	15554.50	17338.50	18647.00	19583.00	14177.00	16105.50
			12223.50	10706.50	12375.00	13569.50	11940.50	10156.50	8848.00	7912.00	13318.00	11389.50
		Mean	17372.50	17231.00	15449.50	14053.50	15991.50	17000.00	18435.00	19433.00	15117.00	16556.50
			10122.50	10264.00	12045.50	13441.50	11503.50	10495.00	9060.00	8062.00	12378.00	10938.50
		Std.	17694.00	17766.00	15827.00	14136.50	16391.00	16318.00	18524.00	19309.00	15519.00	16944.00
			9801.00	9729.00	11668.00	13358.50	11104.00	11177.00	8971.00	8186.00	11976.00	10551.00
		Fun. Evl.	15778.00	17902.00	15882.00	14760.00	16565.00	16281.00	18557.00	18877.00	16794.00	16186.00
			11717.00	9593.00	11613.00	12735.00	10930.00	11214.00	8938.00	8618.00	10701.00	11309.00
	50D	Min.	4523.50	4549.50	3876.00	4178.00	4700.00	3331.50	4857.00	4713.50	4320.00	3890.00
			2262.50	2236.50	2910.00	2608.00	2086.00	3454.50	1929.00	2072.50	2466.00	2896.00
		Mean	4464.50	4458.50	3809.00	4358.50	4673.00	4561.50	4850.00	4678.50	4354.50	4085.00
			2321.50	2327.50	2977.00	2427.50	2113.00	2224.50	1936.00	2107.50	2431.50	2701.00
		Std.	4368.50	4351.50	3708.00	4278.00	4625.50	4470.00	4750.00	4584.00	4290.50	4023.00
			2417.50	2434.50	3078.00	2508.00	2160.50	2316.00	2036.00	2202.00	2495.50	2763.00
		Fun. Evl.	4380.00	4547.00	4347.50	4444.00	4617.00	4171.00	4644.00	4586.00	4483.50	4320.00
			2406.00	2239.00	2438.50	2342.00	2169.00	2615.00	2142.00	2200.00	2302.50	2466.00
	100D	Min.	4555.50	4585.50	3925.50	4498.50	4719.50	3325.00	4870.50	4730.50	4464.50	3964.00
			2230.50	2200.50	2860.50	2287.50	2066.50	3461.00	1915.50	2055.50	2321.50	2822.00
		Mean	4559.00	4581.50	3886.00	4574.00	4739.00	4596.00	4845.00	4721.50	4608.50	4076.50
			2227.00	2204.50	2900.00	2212.00	2047.00	2190.00	1941.00	2064.50	2177.50	2709.50
		Std.	4464.00	4474.00	3780.00	4486.00	4637.00	4505.00	4726.00	4611.00	4526.00	4012.00
			2322.00	2312.00	3006.00	2300.00	2149.00	2281.00	2060.00	2175.00	2260.00	2774.00
		Fun. Evl.	4380.00	4549.00	4339.00	4534.00	4586.50	4128.00	4613.00	4590.00	4511.00	4311.00
			2406.00	2237.00	2447.00	2252.00	2199.50	2658.00	2173.00	2196.00	2275.00	2475.00

6.2 Mann Whitney (MW) test

In this statistical test, the null hypothesis denotes the difference between two randomly selected variables from different datasets by considering the summation of the variables' ranks. In other words, the statistical dataset with smaller summation of ranks has better statistical behavior. In Table 6, the results of the MW test, including the summation of ranks for different metaheuristics in dealing with the SS, 50D and 100D functions, are presented in a two-by-two manner. In this table, if the summations of ranks regarding MW test in the second rows are lower than the values in the first rows, FHO is superior to the other metaheuristics. It should be noted that the proposed FHO has lower summation of ranks in most of the cases, which verifies its superiority over the other algorithms.

6.3 Wilcoxon (W) signed ranks test

In this section, the results of the W test are presented, in which the means of ranks are considered in a two-by-two manner for comparative purposes. The null hypothesis of this test denotes the difference between two randomly selected variables from two datasets, considering the mean of the variables' ranks. In this test, the dataset with the lower mean of ranks represents better statistical behavior. The results of this test in dealing with the SS, 50D and 100D functions, including the summation of ranks for the considered alternative metaheuristic algorithms, are provided in Table 7 in a two-by-two manner. In this table, if the summation of ranks regarding W test in the second rows are lower than the values in the first rows, FHO is superior to the other metaheuristics. It is obvious that FHO provides a lower mean of ranks in most of the cases and, thus, outranks the other algorithms.

6.4 Kruskal Wallis (KW) test

The KW test is a well-known statistical test in which the overall rankings of multiple variables in different datasets are considered. Since the previously conducted MW and W tests are applied in a two-by-two manner based on the summation and mean of ranks, the KW test is utilized for comparing the mean of ranks between different datasets simultaneously. In this test, the dataset with the lower mean of ranks has better statistical behavior. The result of the KW test for different metaheuristics in dealing with the considered SS, 50D and 100D mathematical test functions are presented in Figs. 15, 16, and 17, respectively, in which the results of the algorithms with lowest mean of ranks (superior to others) are bolded. Again, it is obvious that FHO provides a lower mean of ranks in all of the cases and outranks the other algorithms accordingly.

6.5 Post Hoc (PH) analysis

In most statistical investigations, additional analyses are conducted for further evaluation and comparison of the statistical data from a different point-of-view. In this work, the results of the KW test were considered as the base data for PH analysis, in which the overall rankings of different metaheuristics were determined by considering the results of 233

Table 7 The W test results (mean of ranks) for different metaheuristic algorithms

| Main algorithm	Function type	Data type	Alternative metaheuristic algorithms									
			BIA	BBBC	BOA	CPA	CSS	GSO	JA	RFO	TWO	WDO
FHO	SS	Min.	1442	3003	1451	26	799	2440	4861	5900	192	2080
			388	0	379	19	21	45	89	95	18	0
		Mean	4902	5386	2668	691	2625	3570	6013	6638	1745	3784
			558	179	987	485	778	258	428	32	811	132
		Std.	4563	5044	2589	618	2604	2838	5600	6264	1678	3711
			588	521	651	558	799	817	841	406	950	205
		Fun. Evl.	3956	4340	4262	3430	4152	5300	4358	4360	4290	4002
			994	31	1624	1035	313	1603	13	11	175	369
	50D	Min.	1151	1122	327	650	1461	71	1540	1275	807	338
			74	103	24	91	24	34	0	0	96	68
		Mean	1135	1162	422	1010	1563	1241	1596	1326	987	618
			90	113	13	118	33	34	0	0	94	85
		Std.	1158	1061	314	958	1462	1183	1493	1283	938	563
			67	164	121	123	78	42	47	43	97	103
		Fun. Evl.	1035	1031	992	1072	1031	1480	1035	1030	1031	1035
			93	4	89	9	4	231	0	5	4	0
	100D	Min.	1159	1078	375	1025	1326	68	1485	1275	944	366
			17	98	3	56	0	37	0	0	46	69
		Mean	1238	1180	459	1279	1485	1289	1485	1326	1279	560
			37	146	37	99	0	37	0	0	47	70
		Std.	1219	1196	372	1239	1415	1229	1441	1281	1213	533
			56	130	124	139	70	46	44	45	113	97
		Fun. Evl.	1041	1035	997	1035	1035	1471	1035	1035	1035	1035
			87	0	84	0	0	240	0	0	0	0
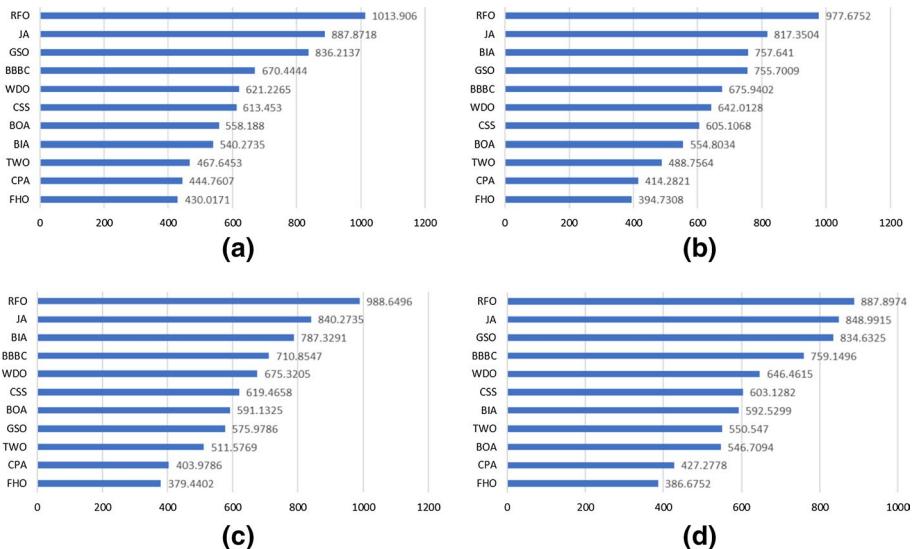
Fig. 15 The KW test results including mean of the ranks considering SS mathematical test functions: **a** min, **b** mean, **c** Std., **d** Fun. Evl
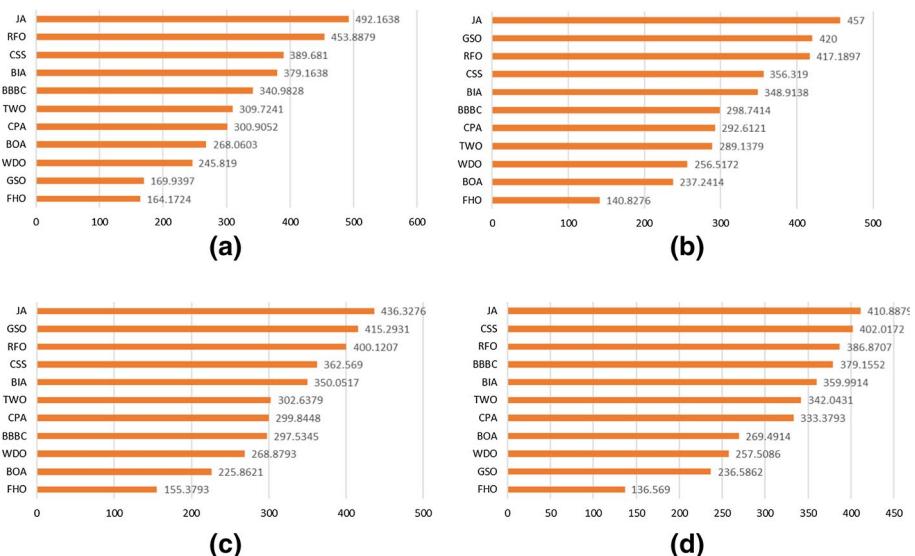

Fig. 16 The KW test results including mean of the ranks considering 50D mathematical test functions: **a** min, **b** mean, **c** Std., **d** Fun. Evl
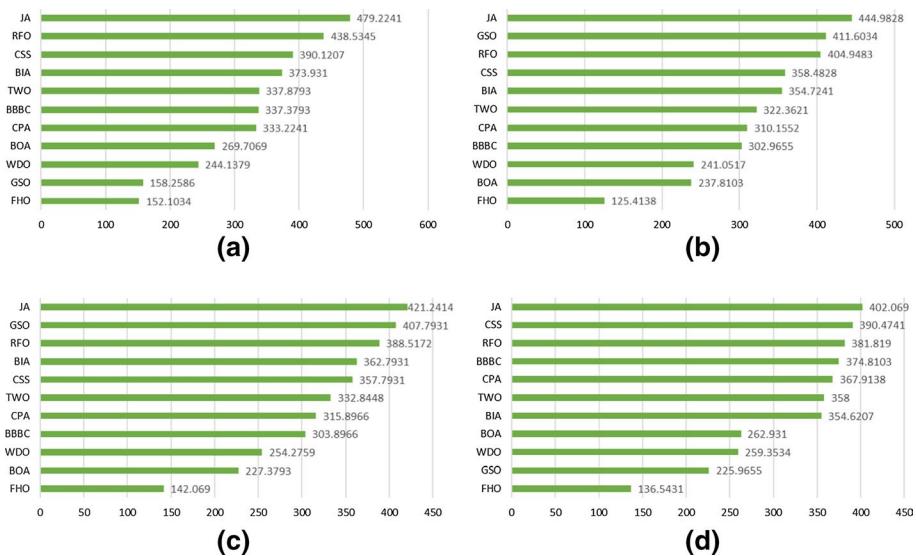
Fig. 17 The KW test results including mean of the ranks considering 100D mathematical test functions: **a** min, **b** mean, **c** Std., **d** Fun. Evl
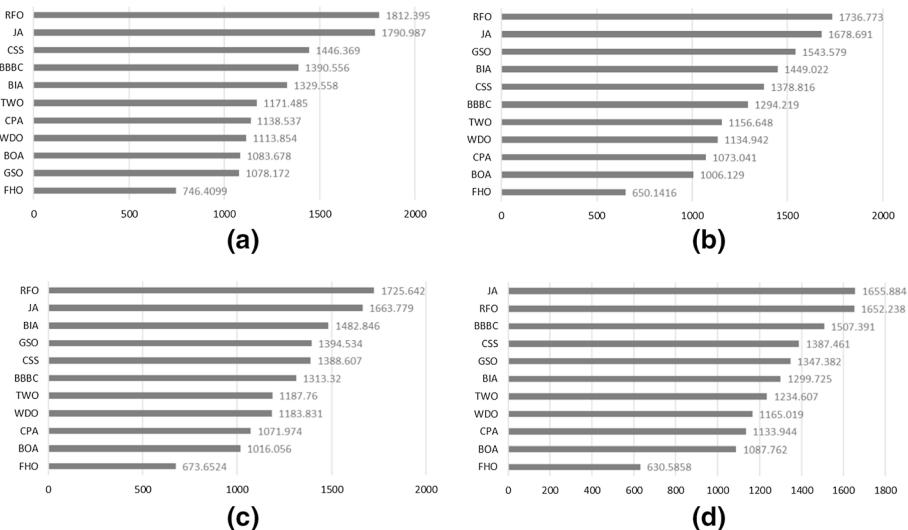

Fig. 18 The PH analysis results including mean of ranks for all of the considered mathematical test functions: **a** min, **b** mean, **c** Std., **d** Fun. Evl

Table 8 Summary of the CEC 2020 bound constraint benchmark problems (Yue et al. 2020)

	Fun. No.	Function detail	D	Fun. Min.
Unimodal function	G_1	Shifted and rotated bent cigar function (CEC 2017[4] F1)	5	100
Basic functions	G_2	Shifted and rotated Schwefel's function (CEC 2014[3] F11)	5	1100
	G_3	Shifted and rotated Lunacek bi-Rastrigin function (CEC 2017[4] F7)	5	700
	G_4	Expanded Rosenbrock's plus Griewangk's function (CEC2017[4] f19)	5	1900
Hybrid functions	G_5	Hybrid function 1 (N = 3) (CEC 2014[3] F17)	5	1700
	G_6	Hybrid function 2 (N = 4) (CEC 2017[4] F16)	5	1600
	G_7	Hybrid function 3 (N = 5) (CEC 2014[3] F21)	5	2100
Composition functions	G_8	Composition function 1 (N = 3) (CEC 2017[4] F22)	5	2200
	G_9	Composition function 2 (N = 4) (CEC 2017[4] F24)	5	2400
	G_{10}	Composition function 3 (N = 5) (CEC 2017[4] F25)	5	2500

D Dimension, *Fun. No.* Function Number, *Fun. Min*. Function Minimum, *Search range* $[-100,100]^D$

mathematical functions simultaneously. According to the overall rankings of metaheuristics in Fig. 18, it is apparent that FHO has a higher ranking (bolded values) in all cases.

7 Bound constraint benchmark problems of CEC 2020

Regarding the fact that the proposed metaheuristic algorithms should be evaluated by means of difficult optimization problems which have been benchmarked in the competitions, one of the recent Competitions on Evolutionary Computation (CEC) about the single objective bound constrained numerical optimization as CEC 2020 which is proposed by Yue et al. (2020) including 10 of the well-established benchmark mathematical test problems is utilized in this paper to thoroughly investigate the overall behavior of the FHO in order to demonstrate its capability in dealing with the complex problems by comparing to the state-of-the-art algorithms. A brief description of these problems are presented in Table 13 while the complete mathematical formulation of these problems are presented in (Sallam et al. 2020; Yue et al. 2020).

Since the proposed metaheuristic algorithms should be evaluated using difficult optimization problems that have been benchmarked in competition, such as the Competitions on Evolutionary Computation (CEC). Recently, CEC 2020 on the single objective bound constrained numerical optimization was proposed by Yue et al. (2020). Herein, ten of the well-established benchmark mathematical test problems from CEC 2020 were utilized to thoroughly investigate the overall behavior of FHO and demonstrate its capability in dealing with the complex problems via comparison to other state-of-the-art algorithms. A brief description of these problems are presented in Table 8, while the complete mathematical formulation of these problems are given in (Yue et al. 2020).

The comparative results of FHO and other alternative metaheuristic algorithm, obtained from CEC 2020, are presented in Tables 9, 10, 11, regarding the 30 optimization runs that were conducted for statistical purposes. By considering the best, mean, and standard deviation of these runs for FHO and the alternative methods, it can be concluded that FHO

Table 9 The best results of multiple optimization runs regarding CEC 2020 for different algorithms

Approaches	G_1	G_2	G_3	G_4	G_5	G_6	G_7	G_8	G_9	G_{10}
IMODE	0.00E+00	0.00E+00	5.15E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
EBO with CMAR	0.00E+00	0.00E+00	5.15E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	1.00E+02
HSES	0.00E+00	0.00E+00	5.15E+00	1.09E-01	9.95E-01	0.00E+00	0.00E+00	0.00E+00	1.00E+02	3.47E+02
LSHADE-cnEpSin	0.00E+00	1.25E-01	1.03E-01	3.53E-07	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	3.00E+02
LSHADE-SPACMA	0.00E+00	6.90E-03	5.15E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	3.47E+02
j2020	0.00E+00	2.00E-04	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
GSK	0.00E+00	6.14E-01	4.38E-07	1.67E-03	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
FHO (Present study)	0.00E+00	2.33E+00	5.38E+00	1.05E-01	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00

IMODE Improved Multi-Operator Differential Evolution (Sallam et al. 2020), *EBOwithCMAR* Effective Butterfly Optimizer with Covariance Matrix Adapted Retreat Phase (Sallam et al. 2020), *HSES* Hybrid Sampling Evolution Strategy (Sallam et al. 2020), *LSHADE-cnEpSin* LSHADE with an Ensemble Sinsoidal Parameter Adaptation (Sallam et al. 2020), *LSHADE-SPACMA* LSHADE with Semi-Parameter Adaptation and Covariance Matrix Adaptation (Sallam et al. 2020), *j 2020* Developed Differential Evolution (Brest et al. 2020), *GSK* Gaining Sharing Knowledge based algorithm (Mohamed et al. 2020)

Table 10 The mean results of multiple optimization runs regarding CEC 2020 for different algorithms

Approaches	G_1	G_2	G_3	G_4	G_5	G_6	G_7	G_8	G_9	G_{10}
IMODE	0.00E+00	8.33E−02	5.15E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	2.44E+02
EBO with CMAR	0.00E+00	1.54E−01	5.19E+00	6.07E−02	1.41E−01	0.00E+00	0.00E+00	1.21E+01	9.67E+01	3.38E+02
HSES	0.00E+00	4.76E+01	5.41E+00	2.57E−01	3.32E+00	2.50E−01	4.16E+00	4.76E+01	1.00E+02	3.47E+02
LSHADE-cnEpSin	0.00E+00	2.98E+00	1.30E+00	6.88E−02	1.40E−01	0.00E+00	0.00E+00	3.14E−01	3.33E+00	3.02E+02
LSHADE-SPACMA	0.00E+00	4.42E−01	5.25E+00	9.81E−06	2.08E−02	0.00E+00	0.00E+00	0.00E+00	9.67E+01	3.47E+02
j2020	0.00E+00	3.23E+00	3.42E+00	7.68E−02	1.37E−01	0.00E+00	0.00E+00	6.28E−01	2.05E+01	1.26E+02
GSK	0.00E+00	1.64E+01	2.87E+00	1.11E−01	0.00E+00	0.00E+00	0.00E+00	0.00E+00	3.33E+01	2.25E+02
FHO (Present study)	0.00E+00	1.00E+01	6.34E+00	2.86E−01	0.00E+00	0.00E+00	0.00E+00	0.00E+00	2.02E+02	3.47E+02

Table 11 The standard deviation of the results regarding multiple optimization runs for different algorithms considering CEC 2020

Approaches	G_1	G_2	G_3	G_4	G_5	G_6	G_7	G_8	G_9	G_{10}
IMODE	0.00E+00	8.89E−02	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	1.36E+02
EBO with CMAR	0.00E+00	1.02E−01	8.35E−02	2.54E−02	3.89E−01	0.00E+00	0.00E+00	3.08E+01	1.83E+01	4.57E+01
HSES	0.00E+00	5.91E+01	1.97E−01	1.17E−01	2.12E+00	3.11E−01	2.16E+01	3.84E+01	2.52E−11	1.17E−02
LSHADE-cnEpSin	0.00E+00	3.27E+00	6.96E−01	3.91E−02	3.66E−01	0.00E+00	0.00E+00	1.72E+00	1.83E+01	8.65E+00
LSHADE-SPACMA	0.00E+00	1.24E+00	1.43E−01	5.37E−05	1.14E−01	0.00E+00	0.00E+00	0.00E+00	1.83E+01	5.23E−04
j2020	0.00E+00	3.74E+00	2.33E+00	6.40E−02	2.86E−01	0.00E+00	0.00E+00	2.39E+00	3.75E+01	9.03E+01
GSK	0.00E+00	2.58E+01	2.05E+00	6.05E−02	0.00E+00	0.00E+00	0.00E+00	0.00E+00	4.79E+01	1.32E+02
FHO (Present study)	0.00E+00	8.71E+00	8.01E−01	1.06E−01	0.00E+00	0.00E+00	0.00E+00	0.00E+00	1.12E+02	1.17E−02

exhibits the ability to equally compete with these algorithms. The values in Table 9 are the error values between the global optimum solutions and the optimum values calculated by each algorithm while the values in Tables 10 and 11 are the related mean and standard deviation of these error values.

Four prominent statistical analyses, including Kolmogorov–Smirnov (KS), Wilcoxon (W), Mann–Whitney (MW), and Kruskal–Wallis (KW), were used to make a better judgment about the performance of the FHO in dealing with the CEC 2020 bound-constrained test functions. In Table 12, the p values of the KS, MW, and W statistical tests are presented for comparative purposes, while in Table 13, the maximum difference between different approaches are presented considering the KS, MW, and W statistical tests in which the capability of the FHO is in perspective. It is obvious that the summation (MW test) and mean (W test) of ranks in FHO is lower than the other approaches in most of the cases, which represent the ability of the FHO in dealing with these kinds of difficult problems. In Table 14, the overall rankings of the algorithms in dealing with CEC 2020 benchmark test functions are presented in which the mean of ranks are in perspective. It should also be noted that FHO has the third ranking, which is somehow acceptable.

In order to obtain better judgment about the performance of FHO in dealing with the CEC 2020 bound constrained test functions, four well-known statistical analyses, including Kolmogorov–Smirnov (KS), Wilcoxon (W), Mann–Whitney (MW) and Kruskal–Wallis (KW), were utilized. In Table 12, the p values of the KS, MW, and W statistical tests are presented for comparative purposes. Table 13 displays the maximum difference between different approaches, i.e. KS, MW and W statistical tests, in which the capability of FHO is in perspective. It is obvious that the summation (MW test) and mean (W test) of ranks in FHO is lower than the other approaches in most of the cases, which represents the proposed algorithm's ability to handle these kinds of difficult problems. In Table 13, the overall rankings of the algorithms in dealing with CEC 2020 benchmark test functions are presented, including the mean of ranks. It should also be noted that FHO has the third rank, which is somehow acceptable regarding the fact that the first two are amongst the most powerful and well-formulated algorithms which can be a great challenge for the improved versions of FHO to compete with in the future.

8 Computational complexity and cost analysis

In recent decades, complexity issues have been a major concern of new algorithms due to their importance in conducting time- and complexity-efficient optimization procedures. For this purpose, the computational complexity procedures of the CEC 2020 benchmark suit on bound constrained were utilized. In these functions, T0 denotes the run time of a specific mathematical procedure as presented in detail by Yue et al. (2020) as follows which should be repeated for 1,000,000 times:

$$x = 0.55; x = x + x; x = \frac{x}{2}; x = x * x; x = \sqrt{x}; x = \log(x); x = \exp(x); x = \frac{x}{x+2} \quad (11)$$

TheT1 refers to the computational time for 200,000 function evaluations of G1 function; T2 presents the computational time of the considered algorithms (FHO in this paper) considering 200,000 function evaluations for G1 function; and $\hat{T}_2$ is the mean value of five calculations of T2. Table 15 presents the computational time complexity results of FHO

Table 12 The p-values of different statistical tests considering CEC 2020 test functions

Main algorithm	Statistical test	Alternative metaheuristic algorithms						
		IMODE	EBOwithCMAR	HSES	LSHADE-cnEpSin	LSHADE-SPACMA	j2020	GSK
FHO	KS test	2.00E−01	9.36E−01	2.59E−02	7.60E−01	9.97E−01	9.97E−01	9.97E−01
	MW test	5.64E−02	5.40E−01	2.10E−02	4.46E−01	9.23E−01	8.10E−01	9.74E−01
	W test	1.71E−02	4.46E−01	7.65E−02	2.15E−01	3.40E−02	1.36E−01	4.14E−01

Table 13 The maximum difference of metaheuristics considering CEC 2020 test functions

Main algorithm	Statistical test	Alternative metaheuristic algorithms						
		IMODE	EBOwithC-MAR	HSES	LSHADE-cnEpSin	LSHADE-SPACMA	j2020	GSK
FHO	KS test	0.27	0.13	0.37	0.17	0.10	0.10	0.10
	MW test	808	954	1066.5	964	921.5	930.5	912.5
		1022	876	763.5	866	908.5	899.5	917.5
	W test	12	68	212	57	27	45	33
		79	103	88	114	109	108	58

Table 14 The KW test results (mean of the ranks) considering CEC 2020 test functions

Rankings	Algorithms	Mean of Ranks
1	IMODE	86.43
2	GSK	113.43
3	**FHO**	**115.43**
4	LSHADE-SPACMA	116.52
5	j2020	119.60
6	EBOwithCMAR	126.42
7	LSHADE-cnEpSin	128.43
8	HSES	157.73
Chi-sq	19.540088	
Prob > Chi-sq	0.006653	

We bold our method; you can change it if necessary

Table 15 Computational time complexity of different algorithms regarding CEC 2020 complexity procedure

Metaheuristics	Properties	Results
IMODE	T0	0.01117 s
	T1	0.2235 s
	$\hat{T}_2$	0.3330 s
	$(\hat{T}_2 - T1)/T0$	0.9780 (unitless)
j2020	T0	0^* s
	T1	0.0465 s
	$\hat{T}_2$	0.1818s
	$(\hat{T}_2 - T1)/T0$	Inf (unitless)
GSK	T0	0.0411 s
	T1	1.12E-05 s
	$\hat{T}_2$	1.76E-05 s
	$(\hat{T}_2 - T1)/T0$	1.58E-04(unitless)
FHO (Present study)	T0	0.0195 s
	T1	0.0108 s
	$\hat{T}_2$	2.9279 s
	$(\hat{T}_2 - T1)/T0$	149.5949(unitless)

*It is a rounded value

and other algorithms by means of the mentioned procedures, in which the superiority of FHO is demonstrated.

In order to conduct the computational complexity analysis for a metaheuristic algorithm, the "Big O notation" can be utilized as a well-known mathematical notation in computer science. Herein, the required run time and memory usage of the algorithms were determined for comparative purposes. It should be noted that it is common to set numerical values for the complexity of an algorithm for testing; however, finding a solution for evaluating the run time issues in such algorithms is another concern that should be largely considered. In this regard, other complexity procedures should be utilized, while the complexity of an algorithm can be described regardless of the type of computer or hardware restrictions. In computer science, "Big O notation" refers to the required run time and memory usage of the algorithms, which are determined for comparative purposes. First, NP is determined as the total number of initial solution candidates, and D is the dimension of the considered problem. In the initialization phase of FHO, the computational complexity is determined as $O(NP \times D)$, then the computational complexity of the objective function evaluation in this phase is calculated as $O(NP) \times O(F(x))$, where $F(x)$ is the objective function of the considered problem. Each line in the main search loop of FHO has a computational complexity equal to the total number of iterations (MxIter). The position updating process for each of the solution candidates in the search space has a computational complexity of $O(M \times Iter \times NP \times D \times 3)$ regarding the movements of Fire Hawks and prey. Finally, the objective function evaluation in the main search loop of the FHO has a computational complexity of $O(M \times Iter \times NP \times D \times 3) \times O(F(x))$.

To further investigate the complexity issues of FHO, the balance between exploration and exploitation was considered as a key factor that lends insight about the convergence speed, which is one critical property of metaheuristic algorithms. Therefore, the diversity graphs for the FHO algorithm in dealing with F2, F8, F24, F47, F50, F61, F66, and F78 mathematical test functions were obtained and are depicted in Figs. 19, 20, 21, 22, 23, 24,
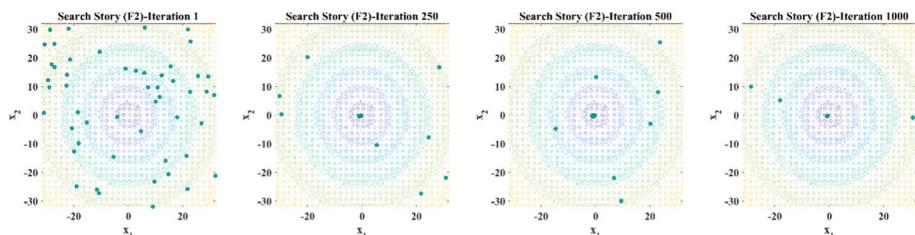

**Fig. 19** Diversity plots of F2 function considering FHO
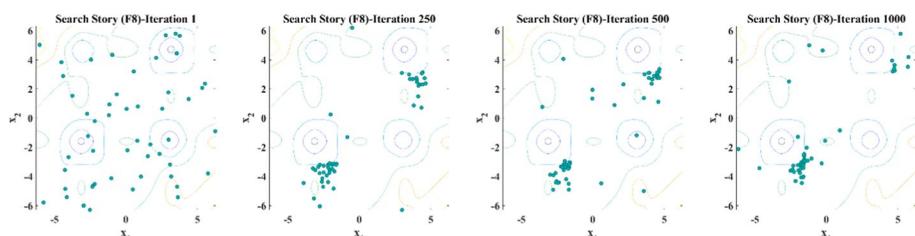
**Fig. 20** Diversity plots of F8 function considering FHO
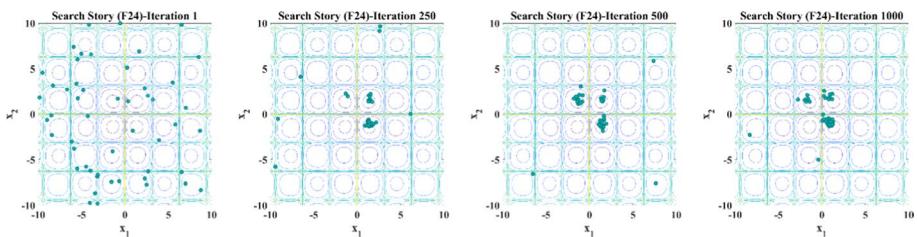
Fig. 21 Diversity plots of F24 function considering FHO
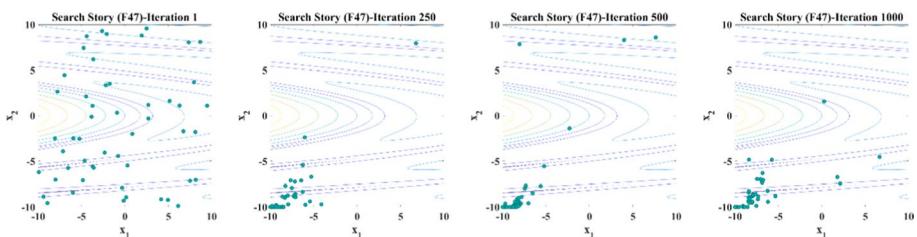

**Fig. 22** Diversity plots of F47 function considering FHO
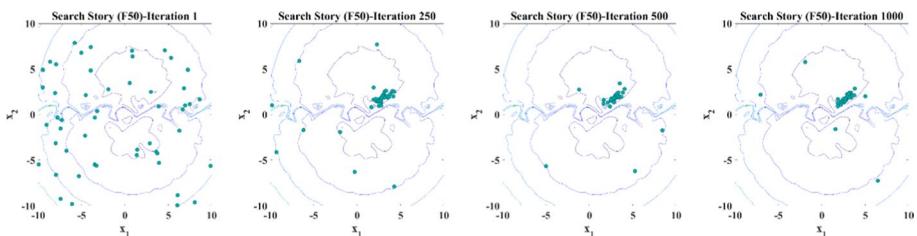
**Fig. 23** Diversity plots of F50 function considering FHO
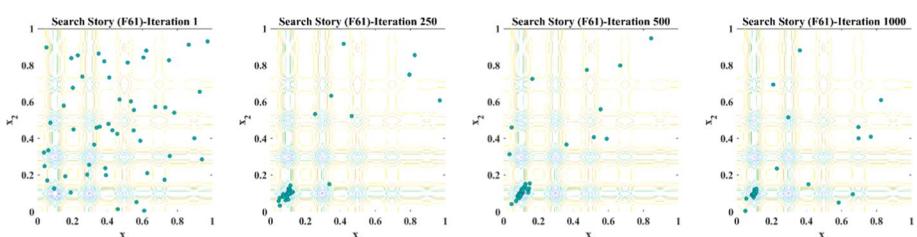
Fig. 24 Diversity plots of F61 function considering FHO

25, 26, respectively. It is obvious that the current population of FHO tends to localize the search during the optimization process for providing optimum results.
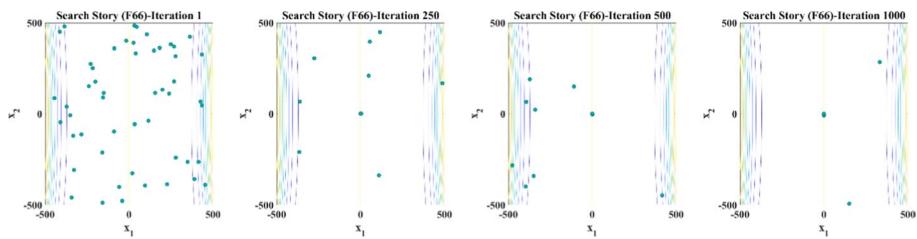
**Fig. 25** Diversity plots of F66 function considering FHO
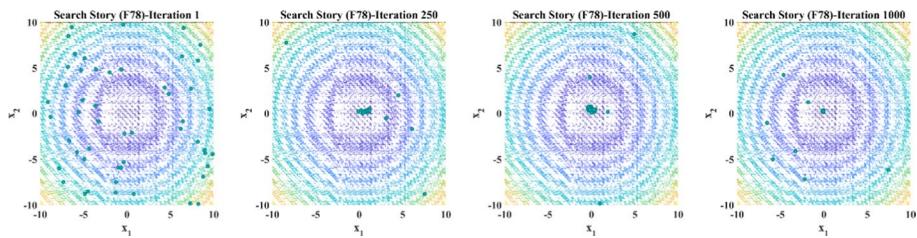
Fig. 26 Diversity plots of F78 function considering FHO

9 Real-world constrained optimization problems of CEC 2020

Often, the capability of a metaheuristic algorithm can be determined using real-world optimization problems, in which some sort of design constraints and bound constraints should be considered for obtaining proper or feasible solutions. In this regard, the capability of FHO was evaluated in dealing with the mentioned problems to make a valid judgment about its overall performance. To do so, the real-world constraint optimization problems from CEC 2020 (Kumar et al. 2020) and some other structural engineering problems from the literature were selected for numerical investigations. A brief description of these problems is presented in Table 16, while the complete mathematical formulation of these problems can be found in the literature. It is pertinent to mention that 25 independent optimization runs were conducted by using 20,000 function evaluations, and a simple penalty approach was utilized for constraint handling purposes.

In Tables 17 and 18, the comparative results of FHO and other algorithms are presented, including the design variables related to the best optimization run and the statistical results

Table 16 Basic characteristics of the considered real-world constrained problems

No.	Name	D	g	h
H_1	Tension/compression spring	3	4	0
H_2	Speed reducer	7	11	0
H_3	Step-cone pulley	5	8	3
H_4	Multiple disk clutch brake	5	8	0

D Dimensions, *g* Number of inequality constraints, *h* Number of equality constraints

Table 17 Best results of different approaches for the tension or compression spring

	Coello (2000); Coello Coello (2000)	Ray and Liew (2003)	Han et al. (2018)	Gandomi et al. (2013b)	Present study (FHO)
Best	0.0127047800	0.0126692000	0.0126653400	0.0126652200	0.0126652328
d	0.0514800000	0.0521600000	0.0516800000	0.0516900000	0.0516880695
D	0.3516610000	0.3681590000	0.3565001000	0.3567300000	0.3566938859
N	11.6322010000	10.6484420000	11.3018335000	11.2885000000	11.2903643694
$g_1(x)$	− 0.0033370000	− 0.0000000075	− 0.0000062180	0.0000000000	0.0000000000
$g_2(x)$	− 0.0001100000	− 0.0000000037	− 0.0000016910	0.0000000000	0.0000000000
$g_3(x)$	− 4.0263180000	− 4.0758050000	− 4.0533150000	− 4.0538000000	− 4.0537385161
$g_4(x)$	− 0.7312390000	− 0.7197870000	− 0.7278799000	− 0.7277000000	− 0.7277453631

D diameter of the coil, *d* diameter of the wire, *N* number of active coils

Table 18 Statistical results for the tension or compression spring considering different approaches

Approaches	Best	Mean	Worst	Std-dev
Coello (2000)	0.0127047800	0.0127692000	0.0128220800	0.0000393900
Ray and Liew (2003)	0.0126692000	0.0129227000	0.0167172000	0.0000519850
Han et al. (2018)	0.0126653400	0.0126859200	0.0127296800	0.0000216720
Gandomi et al. (2013b)	0.0126652200	0.0135005200	0.0168954000	0.0014202720
Present study (FHO)	0.0126652328	0.0126768632	0.0127196031	0.0000153798

of 25 optimization runs. It is obvious that FHO provides very competitive result in this case.

In Tables 19 and 20, the best and statistical results of the conducted 25 optimization runs by FHO are compared with those of other algorithms reported in the literature. It can be seen that best result of multiple runs was achieved by FHO (2994.44), which can be compared to 2994.47 of another algorithm, thus verifying the acceptable performance of FHO. Meanwhile, the FHO is capable of providing better statistical results, including the mean of multiple runs which is 2994.46.

Tables 21 and 22 present the competitive and statistical results of multiple metaheuristics in dealing with the step-cone pulley constraint problem. It is apparent that the results of FHO are superior, achieving the best optimum value of 16.07 for the objective function, while the other algorithms could only provide 16.63.

In Tables 23 and 24, the competitive and statistical results of multiple metaheuristics in dealing with the multiple disk clutch brake constraint problem are presented. It can be seen that FHO achieved the best optimum value of 0.26 for the objective function, which is superior the other algorithms could only provide 0.31.

Comparison of the results of FHO and other metaheuristics from literature demonstrates that this algorithm is capable of yielding acceptable results in dealing with the complex optimization problems of CEC 2020 and other challenges. Besides, the statistical results of FHO, including the mean, worst, and standard deviation, of 25 independent optimization runs were also outstanding compared to the other approaches.

Table 19 Best results of different approaches for the speed reducer problem

	Mezura-Montes et al. (2003)	Akhtar et al. (2002)	Gandomi et al. (2013a)	Zhang et al. (2008)	Present study (FHO)
Best	3025.00500000	3008.08000000	3000.98100000	2994.47106600	2994.44536810
b	3.50616300	3.50612200	3.50150000	3.50000000	3.50001119
m	0.70083100	0.70000600	0.70000000	0.70000000	0.70000014
z	17.00000000	17.00000000	17.00000000	17.00000000	17.00000087
l_1	7.46018100	7.54912600	7.60500000	7.30000000	7.30070577
l_2	7.96214300	7.85933000	7.81810000	7.71531991	7.71553368
d_1	3.36290000	3.36557600	3.35200000	3.35021467	3.35054862
d_2	5.30900000	5.28977300	5.28750000	5.28665447	5.28665896
$g_1(x)$	− 0.07770000	− 0.07550000	− 0.07430000	− 0.0739152	− 2.15510628
$g_2(x)$	− 0.20130000	− 0.19940000	− 0.19830000	− 0.1979985	− 98.13683213
$g_3(x)$	− 0.47410000	− 0.45620000	− 0.43490000	− 0.9999967	− 1.92404012
$g_4(x)$	− 0.89710000	− 0.89940000	− 0.90080000	− 0.9999995	− 18.30831434
$g_5(x)$	− 0.01100000	− 0.01320000	− 0.00110000	− 0.66685260	− 0.00626454
$g_6(x)$	− 0.01250000	− 0.00170000	− 0.00040000	− 0.0000000	− 0.00213570
$g_7(x)$	− 0.70220000	− 0.70250000	− 0.70250000	− 0.70250000	− 28.09999703
$g_8(x)$	− 0.00060000	− 0.00170000	− 0.00040000	− 0.0000000	− 0.00001500
$g_9(x)$	− 0.58310000	− 0.58260000	− 0.58320000	− 0.5833333	− 6.99998500
$g_{10}(x)$	− 0.06910000	− 0.07960000	− 0.08900000	− 0.0513257	− 0.37488283
$g_{11}(x)$	− 0.02790000	− 0.01790000	− 0.01300000	− 0.0000000	− 0.00020882

m teeth module, b face width, $l1$ length of the first shaft between bearings, $d1$ diameter of the first shaft, z number of teeth on pinion, $l2$ length of the second shaft between bearings, $d2$ diameter of the second shaft

Table 20 Statistical results for the speed reducer problem considering different approaches

Approaches	Best	Mean	Worst	Std-dev
Mezura-Montes et al. (2003)	3025.00500000	3088.77780000	3078.59180000	NA
Akhtar et al. (2002)	3008.08000000	3012.12000000	3028.28000000	NA
Gandomi et al. (2013a)	3000.98100000	3007.19970000	3.00900000	4.96340000
Zhang et al. (2008)	2994.47106600	2994.47106600	2994.47106600	0.00000000
Present study (FHO)	2994.44536810	2994.46114878	2994.49374909	0.02816046

Table 21 Best results of different approaches for the step-cone pulley problem

	(TLBO) Rao et al. (2011)	(WOA) Yildiz et al. (2020)	(WCA) Yildiz et al. (2020)	(MBA) Yildiz et al. (2020)	Present study (FHO)
Best	16.63451000	16.63452130	16.63450849	16.63450780	16.07152201
d_1	40.00000000	40.00000000	40.00000000	40.00000000	38.39584030
d_2	54.76430000	54.76432600	54.76430000	54.76430000	52.80599130
d_3	73.01318000	54.76432600	54.76430000	54.76430000	70.40315003
d_4	73.01318000	54.76432600	54.76430000	88.42841900	84.49831395
w	73.01318000	85.98629700	54.76430000	85.98624200	89.99992563

TLBO Teaching–Learning Based Optimization, *WOA* Whale Optimization Algorithm, *WCA* Water Cycle Algorithm, *MBA* Mine Blast Algorithm, *w* width of the pulley, *d1, d2, d3, d4* diameters of the steps in the pulley

Table 22 Statistical results for the step-cone pulley problem considering different approaches

Approaches	Best	Mean	Worst	Std-dev
(TLBO) Rao et al. (2011)	16.63451000	24.01135770	74.02295100	0.34000000
(WOA) Yildiz et al. (2020)	16.63452130	20.93829477	24.84882590	3.34980000
(WCA) Yildiz et al. (2020)	16.63450849	17.53037682	18.83302997	0.92290000
(MBA) Yildiz et al. (2020)	16.63450780	16.70253500	18.32371450	0.26270000
Present study (FHO)	16.07152201	16.16353730	16.69908957	0.21014841

Table 23 Best results of different approaches for the multiple disk clutch brake problem

	Deb and Srinivasan (2006)	Eskandar et al. (2012)	Rao et al. (2011)	Ferreira et al. (2018)	Present study (FHO)
Best	0.47040000	0.31365600	0.31365661	0.31365600	0.26092412
r_1	70.00000000	70.00000000	70.00000000	70.00000000	70.67750992
r_0	90.00000000	90.00000000	90.00000000	90.00000000	92.43421461
t	1.50000000	1.00000000	1.00000000	1.00000000	1.00000106
F	1000.00000000	910.00000000	810.00000000	830.00000000	908.65087360
Z	3.00000000	3.00000000	3.00000000	3.00000000	2.00048183
$g_1(x)$	0.00000000	0.00000000	0.00000000	0.00000000	− 1.75670470
$g_2(x)$	− 22.00000000	− 24.00000000	− 24.00000000	− 24.00000000	− 25.49927407
$g_3(x)$	− 0.90050000	− 0.90948000	− 0.91942781	− 0.91743800	− 0.91849788
$g_4(x)$	− 9.79060000	− 9.80942900	− 9830.37109400	− 9.82618300	− 9.98544504
$g_5(x)$	− 7.89470000	− 7.89469600	− 7894.69659000	− 7.89469700	− 9.82141618
$g_6(x)$	− 3.35270000	− 2.23142100	− 0.70201320	− 0.17385500	− 14.98390798
$g_7(x)$	− 60.62500000	− 49.76874900	− 37706.25000000	− 40.11875000	− 89415.90309757
$g_8(x)$	− 11.64730000	− 12.76857800	− 14.29798680	− 14.82614500	− 0.01609202

r1 inner radius, *t* thickness of the disk, *r0* outer radius, *Z* number of friction surfaces, *F* actuating force

Table 24 Statistical results for the multiple disk clutch brake problem considering different approaches

Approaches	Best	Mean	Worst	Std-dev
Eskandar et al. (2012)	0.31365600	0.31365600	0.31365600	0.00000000
Rao et al. (2011)	0.31365700	0.32716620	0.39207100	0.67000000
Ferreira et al. (2018)	0.31365600	0.31365600	0.31365600	0.00000000
Present study (FHO)	0.26092412	0.253219791	0.274830333	0.012228977

Table 25 Optimum design sections of the 15-story benchmark structure

Groups	HPSACO (Kaveh & Talatahari, 2009)	HBB–BC (Kaveh and Talatahari 2010a)	ICA (Kaveh and Siamak Talatahari 2010b)	DE (Talatahari et al. 2015)	ES–DE (Talatahari et al. 2015)	FHO (Present study)
1	W21X111	W24X117	W24X117	W21X122	W18X106	W24X104
2	W18X158	W21X132	W21X147	W33X141	W36X150	W33X152
3	W10X88	W12X95	W27X84	W14X82	W12X79	W16X77
4	W30X116	W18X119	W27X114	W30X108	W27X114	W24X104
5	W21X83	W21X93	W14X74	W30X108	W30X90	W14X74
6	W24X103	W18X97	W18X86	W12X79	W10X88	W14X90
7	W21X55	W18X76	W12X96	W14X61	W18X71	W14X61
8	W27X114	W18X65	W24X68	W18X71	W18X65	W18X65
9	W10X33	W18X60	W10X39	W6X25	W8X28	W6X20
10	W18X46	W10X39	W12X40	W24X62	W12X40	W14X43
11	W21X44	W21X48	W21X44	W21X48	W21X48	W21X44
Weight (kN)	426.36	434.54	417.47	423.83	415.06	390.87

HPSACO Hybrid algorithm based on particle swarm, ant colony and harmony search algorithms, *HBB–BC* Hybrid big bang-big crunch and particle swarm optimization algorithms, *ICA* Imperialist Competitive Algorithm, *DE* Differential Evolution, *ES–DE* Hybrid eagle strategy algorithm with differential evolution

10 Structural optimization

Metaheuristic algorithms are one type of method to solve various optimization problems in different fields regardless of the possible multi-level complexity of these problems, which presents great challenges. Accordingly, the capability of these algorithm should be evaluated by means of difficult optimization problems, including the optimizing real-size structural frame designs of buildings. To perform a comparative investigation of such challenges, two benchmark structural frame design optimization problems, namely 15-story and 24-story buildings with steel wide flange sections as structural elements, were selected for further investigation. The first design example has 3 bays and 15 stories with a total number of 105 structural elements combined in 11 design groups. The second design problem has 3 bays with 24 stories with a total number of 268 structural elements combined into 20 design groups (Davison and Adams 1974).

The results of the weight optimization procedure for the considered benchmark structural frames of 15- and 24- stories utilizing FHO are provided in Tables 25 and 26, in which the results of the other algorithms are also presented for comparative purposes. It is obvious that FHO, again, produced outstanding results in dealing with these complex problems.

Table 26 Optimum design sections of the 24-story benchmark structure

Groups	ACO (Camp et al. 2005)	HS (Degertekin, 2008)	IACO (Kaveh and Siamak Talatahari 2010a)	ICA (Kaveh and Siamak Talatahari 2010b)	DE (Talatahari et al. 2015)	ES–DE (Talatahari et al. 2015)	FHO (Present study)
1	W30X90	W30X90	W30X99	W30X90	W30X90	W30X90	W30X90
2	W8X18	W10X22	W16X26	W21X50	W21X48	W21X55	W21X48
3	W24X55	W18X40	W18X35	W24X55	W21X44	W21X48	W21X48
4	W8X21	W12X16	W14X22	W8X28	W27X129	W10X45	W18X46
5	W14X145	W14X176	W14X145	W14X109	W14X176	W14X145	W14X159
6	W14X132	W14X176	W14X132	W14X159	W14X120	W14X109	W14X120
7	W14X132	W14X132	W14X120	W14X120	W14X132	W14X99	W14X109
8	W14X132	W14X109	W14X109	W14X90	W14X132	W14X145	W14X74
9	W14X68	W14X82	W14X48	W14X74	W14X109	W14X109	W14X68
10	W14X53	W14X74	W14X48	W14X68	W14X53	W14X48	W14X43
11	W14X43	W14X34	W14X34	W14X30	W14X61	W14X38	W14X30
12	W14X43	W14X22	W14X30	W14X38	W14X30	W14X30	W14X34
13	W14X145	W14X145	W14X159	W14X159	W14X99	W14X99	W14X99
14	W14X145	W14X132	W14X120	W14X132	W14X132	W14X132	W14X109
15	W14X120	W14X109	W14X109	W14X99	W14X109	W14X109	W14X99
16	W14X90	W14X82	W14X99	W14X82	W14X74	W14X68	W14X109
17	W14X90	W14X61	W14X82	W14X68	W14X82	W14X68	W14X74
18	W14X61	W14X48	W14X53	W14X48	W14X82	W14X68	W14X61
19	W14X30	W14X30	W14X38	W14X34	W14X48	W14X61	W14X38
20	W14X26	W14X22	W14X26	W14X22	W14X82	W14X22	W14X22
Weight (kN)	980.63	956.13	967.33	946.25	997.56	945.15	910.72

ACO Ant colony optimization, *HS* Harmony search algorithm, *IACO* Improved ant colony optimization

11 Conclusions

In this study, the FHO is proposed as a novel metaheuristic algorithm motivated by the foraging behavior of whistling kites, black kites, and brown falcons. These birds follow specific actions to catch prey, including setting fire to prey-dominant areas, and thus are called Fire Hawks. To evaluate the performance of this algorithm, a total of 233 mathematical functions with dimensions varying from 2 to 100 were utilized as test functions, while a maximum number of 150,000 function evaluations and 100 optimization runs were considered in the optimization process. A total of 10 alternative metaheuristic algorithms were utilized for comparative purposes, and well-known statistical analyses were also conducted to further verify the capability of FHO. In addition, two of the latest Competitions on Evolutionary Computation (CEC), i.e. CEC 2020 on bound constraint problems and CEC 2020 on real-world optimization problems, were considered for the performance evaluation of FHO in addition to complete computational cost and complexity analyses. The results of this research are summarized as follows:

- FHO outranks the compared algorithms in the selected mathematical test functions, by converging to the predefined tolerance of the global best in a faster and more efficient way.

- FHO is able to converge to the global best of the mathematical test functions by requiring a lower number of objective function evaluations, which proves its efficiency from a computational point-of-view.
- The W and MW statistical test results prove that FHO provides lower values for mean and summation of ranks in a two-by-two comparison, which demonstrates the ability of this algorithm from statistical view point.
- FHO outranks the other algorithms in terms of the mean of ranks in the KW test, whereby FHO has a lower mean, proving it comparable capability with the alternative methods.
- FHO ranks third among seven of the well-established metaheuristic algorithms of the CEC 2020 completion on bound constraints, which is a step forward for the future challenges, while further study is suggested to improve FHO with a better ranking.
- By evaluating the results of FHO in dealing with the CEC 2020 real-size optimization problems, it is proved that FHO has the ability to yield acceptable results in these cases.
- FHO provides better and even more outstanding results in dealing with the structural engineering design problems compared to the state-of-the-art algorithms from CEC 2020.

For future research, FHO can be implemented in different fields of optimization by considering complex constraint problems, while improving the general formulation of this algorithm.

Appendix

See Table 27.

Table 27 The mean of function evaluations for metaheuristic algorithms considering mathematical test functions

No.	Alternative metaheuristic algorithms										
	BIA	BBBC	BOA	CPA	CSS	GSO	JA	RFO	TWO	WDO	FHO
F_1	1.50E+05	1.50E+05	1.46E+05	2.52E+04	1.50E+05	2.61E+03	1.50E+05	1.50E+05	1.41E+05	7.87E+03	8.00E+03
F_2	6.30E+04	2.25E+04	1.33E+03	2.15E+03	3.88E+03	1.53E+05	1.50E+05	1.50E+05	3.15E+04	6.06E+03	4.10E+03
F_3	5.00E+01	1.50E+05	1.12E+05	1.50E+05	1.50E+05	1.53E+05	1.50E+05	1.50E+05	1.50E+05	1.50E+05	1.50E+05
F_4	1.50E+05	1.50E+05	3.31E+04	1.97E+04	1.50E+05	2.76E+03	1.50E+05	1.50E+05	1.36E+05	1.41E+05	7.19E+03
F_5	1.50E+05	1.50E+05	1.50E+05	1.98E+04	7.41E+04	1.53E+05	1.50E+05	1.50E+05	5.76E+04	1.50E+05	8.91E+03
F_6	1.49E+05	1.50E+05	1.50E+05	1.54E+04	3.40E+04	5.00E+04	1.50E+05	1.50E+05	1.35E+05	1.50E+05	1.45E+04
F_7	1.37E+05	1.50E+05	4.63E+03	9.93E+03	3.06E+04	1.53E+05	1.50E+05	1.50E+05	5.59E+04	1.50E+05	3.92E+03
F_8	1.50E+05	1.50E+05	8.98E+04	1.50E+05	1.50E+05	1.53E+05	1.50E+05	1.50E+05	1.50E+05	1.50E+05	1.50E+05
F_9	1.50E+05	1.50E+05	8.31E+04	1.50E+04	1.48E+05	1.34E+03	1.50E+05	1.50E+05	7.83E+04	6.05E+03	5.82E+03
F_{10}	1.50E+05	1.50E+05	5.98E+04	1.49E+04	1.48E+05	1.40E+03	1.50E+05	1.50E+05	7.80E+04	4.54E+03	5.41E+03
F_{11}	1.50E+05	1.50E+05	1.50E+05	2.38E+04	1.44E+05	1.34E+03	1.50E+05	1.50E+05	7.54E+04	4.56E+03	8.08E+03
F_{12}	1.47E+05	1.50E+05	1.50E+05	1.64E+04	3.92E+04	1.53E+05	1.50E+05	1.50E+05	6.22E+04	1.50E+05	6.05E+03
F_{13}	1.50E+05	1.50E+05	1.50E+05	1.50E+05	1.50E+05	1.53E+05	1.50E+05	1.50E+05	1.50E+05	1.50E+05	1.50E+05
F_{14}	1.02E+05	1.36E+05	1.44E+05	2.45E+04	1.15E+04	1.53E+05	1.49E+05	1.50E+05	2.21E+04	1.21E+05	4.95E+04
F_{15}	1.46E+05	5.00E+01	1.50E+05	5.20E+01	5.95E+02	3.06E+02	1.00E+02	3.66E+02	3.10E+02	1.57E+02	2.23E+03
F_{16}	1.50E+05	1.50E+05	1.50E+05	1.65E+04	1.42E+05	1.53E+05	1.50E+05	1.50E+05	8.77E+04	7.41E+03	5.32E+03
F_{17}	1.50E+05	1.50E+05	6.14E+04	1.50E+05	1.50E+05	1.53E+05	1.50E+05	1.50E+05	1.50E+05	1.50E+05	1.50E+05
F_{18}	1.46E+05	1.50E+05	5.69E+03	1.05E+04	6.74E+04	9.82E+02	1.50E+05	1.50E+05	5.70E+04	3.32E+03	4.43E+03
F_{19}	3.77E+03	3.95E+04	2.24E+04	1.08E+03	1.96E+03	1.53E+05	1.41E+05	1.50E+05	1.79E+04	9.53E+03	3.77E+03
F_{20}	5.00E+01	1.28E+05	1.03E+05	6.43E+03	1.50E+05	3.06E+02	1.50E+05	1.50E+05	1.04E+05	1.50E+05	9.53E+03
F_{21}	1.50E+05	1.50E+05	1.03E+05	1.50E+05	1.50E+05	1.12E+05	1.50E+05	1.50E+05	1.50E+05	1.50E+05	1.49E+05
F_{22}	1.50E+05	1.50E+05	1.66E+03	1.50E+05	1.50E+05	1.53E+05	1.50E+05	1.50E+05	1.47E+05	1.50E+05	1.46E+05
F_{23}	1.50E+05	1.50E+05	2.27E+03	1.50E+05	1.50E+05	1.53E+05	1.50E+05	1.50E+05	1.50E+05	1.50E+05	1.50E+05
F_{24}	1.50E+05	1.50E+05	1.50E+05	1.50E+05	1.50E+05	1.53E+05	1.50E+05	1.50E+05	1.50E+05	1.50E+05	1.50E+05
F_{25}	1.50E+05	1.50E+05	1.50E+05	4.43E+04	9.47E+04	6.79E+04	1.50E+05	1.50E+05	7.04E+04	1.50E+05	1.42E+04

Table 27 (continued)

No.	Alternative metaheuristic algorithms										
	BIA	BBBC	BOA	CPA	CSS	GSO	JA	RFO	TWO	WDO	FHO
F_{26}	1.50E+05	1.50E+05	1.50E+05	1.46E+05	1.35E+05	1.53E+05	1.50E+05	1.50E+05	1.46E+05	1.50E+05	1.50E+05
F_{27}	1.30E+04	4.27E+03	3.80E+04	9.48E+02	2.14E+03	1.53E+05	1.37E+05	1.50E+05	1.53E+04	3.80E+03	2.41E+03
F_{28}	1.50E+05	1.50E+05	1.40E+05	1.33E+04	1.50E+05	7.49E+04	1.50E+05	1.50E+05	7.53E+04	1.50E+05	7.92E+03
F_{29}	1.30E+05	1.47E+05	1.50E+05	1.14E+04	1.50E+05	1.53E+05	1.50E+05	1.50E+05	4.84E+04	1.39E+05	5.50E+03
F_{30}	1.50E+05	1.50E+05	5.36E+04	1.21E+04	5.43E+04	1.08E+03	1.50E+05	1.50E+05	6.40E+04	3.66E+03	6.32E+03
F_{31}	1.40E+05	1.50E+05	1.50E+05	1.03E+04	3.01E+04	1.53E+05	1.50E+05	1.50E+05	5.54E+04	1.50E+05	3.96E+03
F_{32}	1.50E+05	1.50E+05	1.50E+05	1.79E+04	5.57E+04	1.53E+05	1.50E+05	1.50E+05	7.08E+04	1.50E+05	8.24E+03
F_{33}	1.50E+05	1.50E+05	1.50E+05	1.50E+05	1.50E+05	1.53E+05	1.50E+05	1.50E+05	1.50E+05	1.50E+05	1.50E+05
F_{34}	1.50E+05	1.50E+05	1.50E+05	1.34E+04	1.18E+05	1.53E+05	1.50E+05	1.50E+05	6.63E+04	1.50E+05	4.88E+03
F_{35}	3.14E+04	1.14E+05	5.30E+03	1.69E+03	2.55E+03	1.53E+05	1.24E+05	1.49E+05	1.51E+04	2.36E+04	7.16E+03
F_{36}	1.50E+05	1.50E+05	1.50E+05	1.97E+04	6.75E+04	1.53E+05	1.50E+05	1.50E+05	9.80E+04	1.50E+05	1.48E+04
F_{37}	1.08E+02	1.50E+05	1.52E+04	8.90E+02	1.22E+03	1.53E+05	1.36E+05	1.50E+05	1.03E+04	3.79E+03	1.29E+03
F_{38}	1.50E+05	1.50E+05	1.50E+05	1.50E+05	1.50E+05	1.53E+05	1.50E+05	1.50E+05	1.50E+05	1.50E+05	1.50E+05
F_{39}	1.50E+05	1.50E+05	1.59E+04	1.50E+05	1.50E+05	1.53E+05	1.50E+05	1.50E+05	1.50E+05	1.50E+05	1.50E+05
F_{40}	1.50E+05	1.50E+05	1.50E+05	3.31E+04	7.22E+04	3.40E+04	1.50E+05	1.50E+05	5.69E+04	1.50E+05	1.17E+04
F_{41}	1.50E+05	1.50E+05	5.35E+03	1.50E+05	1.50E+05	1.53E+05	1.50E+05	1.50E+05	1.50E+05	1.50E+05	1.50E+05
F_{42}	1.29E+05	1.46E+05	1.25E+04	6.10E+03	6.84E+03	1.53E+05	1.50E+05	1.50E+05	3.15E+04	1.40E+05	9.70E+03
F_{43}	1.29E+05	1.50E+05	6.85E+04	1.77E+04	3.19E+04	7.62E+02	1.50E+05	1.50E+05	5.38E+04	3.15E+03	7.28E+03
F_{44}	6.50E+01	1.50E+05	9.31E+03	1.50E+05	1.50E+05	1.53E+05	1.50E+05	1.50E+05	1.50E+05	1.50E+05	1.50E+05
F_{45}	2.49E+03	3.04E+04	3.40E+03	3.79E+02	2.44E+02	3.12E+02	2.29E+03	9.65E+04	7.76E+02	1.87E+02	4.68E+02
F_{46}	1.78E+04	1.50E+05	1.97E+04	1.83E+03	5.60E+03	1.53E+05	1.49E+05	1.50E+05	2.06E+04	7.23E+04	3.87E+03
F_{47}	1.84E+04	1.50E+05	1.50E+05	9.12E+04	1.50E+05	1.53E+05	1.50E+05	1.50E+05	1.29E+05	1.50E+05	1.49E+05
F_{48}	1.80E+04	1.50E+05	1.50E+05	2.03E+04	1.49E+05	1.53E+05	1.50E+05	1.50E+05	1.31E+05	1.50E+05	1.49E+05
F_{49}	8.50E+01	1.50E+05	1.24E+04	1.50E+05	1.50E+05	1.53E+05	1.50E+05	1.50E+05	1.50E+05	1.50E+05	1.50E+05

Table 27 (continued)

No.	Alternative metaheuristic algorithms										
	BIA	BBBC	BOA	CPA	CSS	GSO	JA	RFO	TWO	WDO	FHO
F_{50}	1.50E+05	1.50E+05	1.06E+04	1.50E+05	1.50E+05	1.53E+05	1.50E+05	1.50E+05	1.50E+05	1.50E+05	1.50E+05
F_{51}	3.26E+03	6.25E+02	6.30E+02	3.27E+02	2.62E+02	1.82E+03	4.63E+03	9.68E+04	1.36E+03	3.43E+02	6.85E+02
F_{52}	8.52E+04	7.83E+04	3.17E+04	1.28E+03	2.62E+04	9.46E+02	1.50E+05	1.50E+05	1.49E+05	9.31E+04	1.15E+04
F_{53}	1.48E+05	1.50E+05	1.50E+05	1.65E+04	5.40E+04	1.53E+05	1.50E+05	1.50E+05	1.47E+05	1.50E+05	1.54E+04
F_{54}	5.00E+01	1.50E+05	3.46E+03	1.50E+05	1.50E+05	3.06E+02	1.50E+05	1.50E+05	1.50E+05	1.50E+05	1.50E+05
F_{55}	1.50E+05	1.50E+05	5.74E+04	3.29E+04	2.46E+04	7.99E+02	1.50E+05	1.50E+05	6.12E+04	1.15E+05	1.51E+04
F_{56}	1.49E+05	1.50E+05	1.50E+05	2.02E+04	1.50E+05	8.77E+04	1.50E+05	1.50E+05	1.50E+05	1.50E+05	1.52E+04
F_{57}	1.50E+05	1.50E+05	5.74E+04	3.29E+04	2.46E+04	7.99E+02	1.50E+05	1.50E+05	6.12E+04	1.15E+05	1.51E+04
F_{58}	1.50E+05	1.50E+05	1.50E+05	4.25E+04	1.50E+05	8.87E+04	1.50E+05	1.50E+05	1.48E+05	1.50E+05	1.61E+04
F_{59}	1.50E+05	1.50E+05	1.88E+04	6.53E+04	1.50E+05	1.52E+03	1.50E+05	1.50E+05	1.47E+05	6.02E+04	5.60E+04
F_{60}	1.50E+05	1.50E+05	1.00E+04	1.50E+05	1.50E+05	1.53E+05	1.50E+05	1.50E+05	1.50E+05	1.50E+05	1.50E+05
F_{61}	1.50E+05	1.50E+05	1.00E+02	2.71E+04	1.50E+05	1.53E+05	1.50E+05	1.50E+05	7.74E+04	1.50E+05	1.98E+04
F_{62}	1.50E+05	1.50E+05	2.22E+04	1.99E+04	1.25E+05	1.53E+05	1.50E+05	1.50E+05	6.10E+04	1.50E+05	1.19E+04
F_{63}	1.18E+05	1.44E+05	1.84E+04	8.47E+04	7.37E+03	1.53E+05	6.90E+04	1.49E+05	8.66E+03	1.14E+05	6.12E+04
F_{64}	1.50E+05	1.50E+05	7.21E+04	1.89E+04	1.50E+05	1.35E+05	1.50E+05	1.50E+05	8.39E+04	4.80E+03	5.94E+03
F_{65}	1.50E+05	1.50E+05	7.12E+04	1.80E+04	1.50E+05	1.28E+05	1.50E+05	1.50E+05	7.85E+04	4.44E+03	5.44E+03
F_{66}	1.50E+05	1.50E+05	9.06E+04	4.07E+03	1.50E+05	2.72E+03	1.50E+05	1.50E+05	1.50E+05	1.50E+05	1.70E+04
F_{67}	1.42E+05	2.07E+04	1.12E+05	2.44E+03	1.34E+05	4.87E+02	1.50E+05	1.50E+05	2.72E+04	1.75E+03	4.86E+03
F_{68}	1.33E+05	5.09E+04	5.16E+04	6.49E+03	1.32E+05	5.32E+02	1.50E+05	1.50E+05	3.40E+04	2.36E+03	6.42E+03
F_{69}	1.50E+05	1.50E+05	1.50E+05	1.50E+05	1.50E+05	1.53E+05	1.50E+05	1.50E+05	1.50E+05	1.50E+05	1.50E+05
F_{70}	1.20E+05	4.07E+03	5.29E+04	2.17E+03	9.87E+04	1.53E+05	1.35E+05	1.49E+05	1.49E+04	1.46E+03	3.72E+03
F_{71}	1.50E+05	1.50E+05	1.50E+05	3.52E+04	1.50E+05	1.53E+05	1.50E+05	1.50E+05	1.41E+05	1.50E+05	1.25E+04
F_{72}	5.00E+01	1.50E+05	4.59E+03	2.00E+04	1.50E+05	1.53E+05	1.50E+05	1.50E+05	9.93E+04	1.50E+05	7.09E+03
F_{73}	5.00E+01	1.50E+05	6.38E+03	1.50E+05	1.50E+05	3.06E+02	1.50E+05	1.50E+05	1.50E+05	1.50E+05	1.50E+05

Table 27 (continued)

No.	Alternative metaheuristic algorithms										
	BIA	BBBC	BOA	CPA	CSS	GSO	JA	RFO	TWO	WDO	FHO
F_{74}	5.00E+01	2.57E+04	5.02E+03	2.38E+03	3.83E+03	3.06E+02	1.50E+05	1.50E+05	1.17E+05	1.47E+05	8.38E+03
F_{75}	5.00E+01	1.26E+05	2.24E+04	6.43E+03	1.47E+04	3.06E+02	1.50E+05	1.50E+05	9.91E+04	1.50E+05	9.53E+03
F_{76}	1.23E+05	1.46E+05	3.72E+03	7.93E+04	3.04E+04	1.53E+05	1.50E+05	1.50E+05	4.97E+04	1.49E+05	1.47E+04
F_{77}	1.50E+05	1.50E+05	5.78E+04	1.23E+04	6.46E+04	9.03E+02	1.50E+05	1.50E+05	1.29E+05	7.38E+04	1.20E+04
F_{78}	1.50E+05	1.50E+05	2.43E+03	1.50E+05	1.50E+05	1.53E+05	1.50E+05	1.50E+05	1.50E+05	1.50E+05	1.50E+05
F_{79}	1.50E+05	1.50E+05	1.50E+05	2.71E+04	1.50E+05	1.53E+05	1.50E+05	1.50E+05	1.41E+05	1.50E+05	2.22E+04
F_{80}	7.85E+01	1.50E+05	1.23E+04	9.04E+03	6.21E+04	3.06E+02	1.50E+05	1.50E+05	5.34E+04	1.50E+05	3.60E+03
F_{81}	4.11E+03	1.50E+05	4.22E+04	2.04E+04	1.50E+05	2.08E+03	1.50E+05	1.50E+05	1.21E+05	6.91E+03	8.72E+03
F_{82}	1.60E+02	1.50E+05	2.77E+04	2.27E+04	1.50E+05	4.13E+02	1.50E+05	1.50E+05	1.19E+05	6.67E+03	7.00E+03
F_{83}	5.40E+01	1.49E+05	1.81E+04	1.46E+04	1.11E+05	3.06E+02	1.50E+05	1.50E+05	1.38E+05	1.50E+05	1.63E+04
F_{84}	1.49E+05	1.50E+05	9.33E+02	1.20E+04	1.29E+04	1.10E+03	1.50E+05	1.50E+05	6.73E+04	3.93E+03	6.19E+03
F_{85}	1.50E+05	1.50E+05	1.50E+05	3.48E+04	1.50E+05	1.53E+05	1.50E+05	1.50E+05	1.46E+05	1.50E+05	1.44E+04
F_{86}	1.49E+05	1.50E+05	1.50E+05	3.16E+04	1.50E+05	1.53E+05	1.50E+05	1.50E+05	1.50E+05	1.50E+05	1.09E+04
F_{87}	7.48E+04	3.51E+03	6.31E+03	9.81E+02	1.50E+05	1.53E+05	1.45E+05	1.44E+05	1.35E+04	9.09E+02	1.25E+03
F_{88}	2.73E+03	2.35E+04	1.31E+04	2.15E+04	6.55E+03	1.53E+05	1.50E+05	1.50E+05	2.65E+04	8.79E+04	2.38E+03
F_{89}	2.71E+04	2.73E+04	1.22E+04	9.76E+02	9.46E+02	1.53E+05	1.41E+05	1.50E+05	1.26E+04	9.95E+02	1.39E+03
F_{90}	1.49E+05	1.50E+05	7.17E+04	1.41E+04	1.50E+05	1.29E+03	1.50E+05	1.50E+05	7.94E+04	4.49E+03	4.81E+03
F_{91}	1.50E+05	1.50E+05	1.50E+05	3.39E+04	2.61E+04	1.53E+05	1.50E+05	1.50E+05	6.28E+04	1.50E+05	1.10E+04
F_{92}	2.09E+02	1.50E+05	1.50E+05	1.03E+05	1.50E+05	3.06E+02	1.50E+05	1.50E+05	1.18E+05	1.50E+05	1.41E+04
F_{93}	1.50E+05	1.50E+05	3.99E+04	1.50E+05	1.50E+05	1.53E+05	1.50E+05	1.50E+05	1.50E+05	1.50E+05	1.50E+05
F_{94}	1.50E+05	1.50E+05	1.50E+05	4.06E+04	3.53E+04	1.53E+04	1.50E+05	1.50E+05	1.41E+05	1.50E+05	1.71E+04
F_{95}	1.50E+05	1.50E+05	1.50E+05	1.50E+05	1.50E+05	1.53E+05	1.50E+05	1.50E+05	1.50E+05	1.50E+05	1.50E+05
F_{96}	1.49E+05	7.25E+04	1.50E+05	2.56E+03	2.07E+04	9.71E+04	1.50E+05	1.50E+05	5.77E+04	1.50E+05	1.25E+04
F_{97}	6.75E+01	5.15E+01	9.66E+04	8.45E+01	7.00E+02	1.53E+05	2.32E+02	2.00E+02	6.07E+02	2.68E+02	3.55E+03

Table 27 (continued)

No.	Alternative metaheuristic algorithms										
	BIA	BBBC	BOA	CPA	CSS	GSO	JA	RFO	TWO	WDO	FHO
F_{98}	1.50E+05	1.50E+05	1.50E+05	1.39E+05	5.58E+04	1.53E+05	1.50E+05	1.50E+05	1.49E+05	1.50E+05	2.72E+04
F_{99}	1.50E+05	1.50E+05	1.50E+05	1.50E+05	1.39E+05	7.46E+04	1.50E+05	1.50E+05	1.37E+05	1.50E+05	2.38E+04
F_{100}	1.50E+05	1.05E+05	9.90E+03	3.45E+04	1.50E+05	2.31E+03	1.50E+05	1.50E+05	3.74E+04	4.61E+03	7.57E+03
F_{101}	5.75E+01	1.50E+05	1.50E+05	9.48E+04	1.50E+05	1.53E+05	1.50E+05	1.50E+05	1.50E+05	1.50E+05	1.00E+05
F_{102}	1.49E+05	1.50E+05	1.50E+05	1.49E+05	1.07E+05	1.53E+05	1.45E+05	1.50E+05	8.95E+04	1.38E+05	3.92E+04
F_{103}	1.43E+05	1.49E+05	1.50E+05	1.18E+05	1.34E+05	1.53E+05	1.50E+05	1.50E+05	1.50E+05	1.50E+05	2.72E+04
F_{104}	6.59E+03	1.34E+05	1.50E+05	4.76E+03	6.13E+04	1.53E+05	1.45E+05	1.50E+05	8.45E+03	9.30E+03	7.01E+03
F_{105}	1.50E+05	1.50E+05	1.50E+05	1.50E+05	1.50E+05	1.53E+05	1.50E+05	1.50E+05	1.50E+05	1.50E+05	1.50E+05
F_{106}	1.09E+05	1.32E+05	9.93E+04	1.97E+04	1.70E+04	1.53E+05	1.50E+05	1.50E+05	3.76E+04	1.22E+05	1.99E+04
F_{107}	1.12E+05	1.47E+05	7.65E+04	2.38E+04	1.39E+04	1.53E+05	1.50E+05	1.50E+05	3.97E+04	1.45E+05	1.41E+04
F_{108}	1.50E+05	1.50E+05	1.50E+05	1.50E+05	1.50E+05	1.53E+05	1.50E+05	1.50E+05	1.50E+05	1.50E+05	1.49E+05
F_{109}	5.15E+01	1.50E+05	1.50E+05	1.50E+05	1.50E+05	1.53E+05	1.50E+05	1.50E+05	1.50E+05	1.50E+05	1.50E+05
F_{110}	7.45E+01	1.50E+05	4.13E+03	1.08E+05	1.50E+05	1.53E+05	1.50E+05	1.50E+05	1.49E+05	1.50E+05	9.49E+04
F_{111}	1.50E+05	1.35E+05	8.74E+03	9.22E+04	2.96E+04	1.53E+05	1.25E+05	1.50E+05	8.61E+03	1.28E+05	1.16E+04
F_{112}	1.50E+05	1.50E+05	1.50E+05	1.50E+05	1.50E+05	1.53E+05	1.50E+05	1.50E+05	1.50E+05	1.50E+05	1.50E+05
F_{113}	6.96E+04	1.02E+05	9.32E+04	1.24E+04	3.46E+04	1.53E+05	1.50E+05	1.50E+05	1.20E+05	1.50E+05	5.12E+04
F_{114}	1.50E+05	1.50E+05	1.50E+05	1.14E+05	1.50E+05	1.53E+05	1.50E+05	1.50E+05	8.25E+04	1.50E+05	1.83E+04
F_{115}	1.09E+02	1.40E+05	1.50E+05	5.01E+04	1.33E+05	1.53E+05	1.50E+05	1.50E+05	1.13E+05	1.35E+05	1.31E+05
F_{116}	5.05E+01	1.25E+05	1.50E+05	1.53E+04	3.14E+04	3.06E+02	1.50E+05	1.50E+05	3.02E+04	1.38E+05	1.01E+04
F_{117}	1.50E+05	1.50E+05	1.50E+05	1.50E+05	1.50E+05	1.53E+05	1.50E+05	1.50E+05	1.50E+05	1.50E+05	1.27E+05
F_{118}	1.50E+05	1.50E+05	1.36E+05	1.50E+05	1.50E+05	8.45E+04	1.50E+05	1.50E+05	1.50E+05	2.21E+04	5.05E+02
F_{119}	1.50E+05	1.50E+05	9.15E+04	1.50E+05	1.50E+05	4.00E+04	1.50E+05	1.50E+05	1.50E+05	5.14E+04	5.36E+02
F_{120}	1.50E+05	1.50E+05	8.49E+04	1.50E+05	1.50E+05	8.60E+04	1.50E+05	1.50E+05	8.37E+04	1.50E+05	1.60E+02
F_{121}	1.50E+05	1.50E+05	5.43E+04	1.28E+05	1.50E+05	8.59E+04	1.50E+05	1.50E+05	6.59E+04	8.66E+03	2.00E+02

Table 27 (continued)

No.	Alternative metaheuristic algorithms										
	BIA	BBBC	BOA	CPA	CSS	GSO	JA	RFO	TWO	WDO	FHO
F_{122}	1.50E+05	3.91E+04	1.50E+05	1.37E+05	1.50E+05	8.58E+04	1.50E+05	1.50E+05	2.74E+04	3.48E+03	3.92E+01
F_{123}	1.50E+05	1.50E+05	1.32E+04	1.50E+05	1.50E+05	9.31E+04	1.50E+05	1.50E+05	1.50E+05	1.50E+05	1.46E+05
F_{124}	5.00E+01	1.50E+05	1.86E+04	1.49E+05	1.50E+05	1.53E+05	1.50E+05	1.50E+05	1.50E+05	1.50E+05	1.50E+05
F_{125}	1.50E+05	1.50E+05	1.50E+05	1.50E+05	1.50E+05	1.53E+05	1.50E+05	1.50E+05	1.50E+05	1.50E+05	1.50E+05
F_{126}	1.50E+05	1.50E+05	1.50E+05	1.50E+05	1.50E+05	1.27E+05	1.50E+05	1.50E+05	1.50E+05	1.50E+05	1.50E+05
F_{127}	1.50E+05	1.50E+05	2.92E+04	1.45E+05	1.50E+05	8.59E+04	1.50E+05	1.50E+05	7.58E+04	1.00E+04	3.47E+02
F_{128}	1.50E+05	1.50E+05	8.28E+04	1.50E+05	1.50E+05	8.59E+04	1.50E+05	1.50E+05	1.50E+05	2.26E+04	1.16E+02
F_{129}	1.50E+05	1.50E+05	5.39E+04	1.42E+05	1.50E+05	8.59E+04	1.50E+05	1.50E+05	6.10E+04	7.38E+03	1.37E+02
F_{130}	1.50E+05	1.50E+05	8.72E+04	1.50E+05	1.50E+05	8.61E+04	1.50E+05	1.50E+05	1.50E+05	4.81E+04	9.92E+02
F_{131}	1.50E+05	1.50E+05	3.26E+04	1.50E+05	1.50E+05	8.74E+04	1.50E+05	1.50E+05	1.50E+05	1.50E+05	1.36E+05
F_{132}	1.50E+05	1.50E+05	1.50E+05	1.50E+05	1.50E+05	1.26E+05	1.50E+05	1.50E+05	1.50E+05	1.50E+05	1.50E+05
F_{133}	5.00E+01	1.50E+05	1.50E+05	9.16E+04	1.50E+05	3.06E+02	1.50E+05	1.50E+05	1.50E+05	1.49E+05	8.00E+00
F_{134}	5.00E+01	1.50E+05	1.50E+05	9.36E+04	1.50E+05	3.06E+02	1.50E+05	1.50E+05	1.50E+05	1.50E+05	8.00E+00
F_{135}	1.50E+05	1.50E+05	1.50E+05	1.50E+05	1.50E+05	1.53E+05	1.50E+05	1.50E+05	1.50E+05	1.50E+05	1.50E+05
F_{136}	7.48E+04	1.50E+05	9.19E+04	6.23E+04	1.50E+05	3.06E+02	1.50E+05	8.50E+04	7.32E+04	1.65E+04	7.54E+01
F_{137}	1.50E+05	1.50E+05	1.50E+05	1.50E+05	1.50E+05	3.06E+02	1.50E+05	1.50E+05	1.50E+05	1.50E+05	4.70E+02
F_{138}	1.50E+05	1.50E+05	5.56E+04	1.50E+05	1.50E+05	8.59E+04	1.50E+05	1.50E+05	1.50E+05	1.59E+04	3.86E+02
F_{139}	1.50E+05	1.50E+05	8.57E+04	1.50E+05	1.50E+05	1.05E+05	1.50E+05	1.50E+05	1.50E+05	1.50E+05	2.41E+02
F_{140}	1.50E+05	1.50E+05	8.65E+04	1.50E+05	1.50E+05	1.12E+05	1.50E+05	1.50E+05	1.50E+05	1.50E+05	2.67E+02
F_{141}	1.50E+05	1.50E+05	5.62E+04	8.18E+04	1.41E+05	8.59E+04	1.50E+05	1.50E+05	1.50E+05	1.11E+04	7.00E+01
F_{142}	1.50E+05	1.50E+05	8.95E+04	1.50E+05	1.50E+05	8.58E+04	1.50E+05	1.50E+05	1.50E+05	1.50E+05	3.34E+02
F_{143}	1.50E+05	1.50E+05	1.50E+05	1.50E+05	1.50E+05	1.53E+05	1.50E+05	1.50E+05	1.50E+05	1.50E+05	1.50E+05
F_{144}	1.50E+05	1.50E+05	1.50E+05	1.50E+05	1.50E+05	1.50E+05	1.50E+05	1.50E+05	1.50E+05	1.50E+05	1.50E+05
F_{145}	1.50E+05	1.50E+05	1.50E+05	1.50E+05	1.50E+05	1.43E+05	1.50E+05	1.50E+05	1.50E+05	1.50E+05	1.50E+05

Table 27 (continued)

No.	Alternative metaheuristic algorithms										
	BIA	BBBC	BOA	CPA	CSS	GSO	JA	RFO	TWO	WDO	FHO
F_{146}	1.50E+05	1.50E+05	1.50E+05	1.50E+05	1.50E+05	8.60E+04	1.50E+05	1.50E+05	1.50E+05	1.50E+05	3.73E+02
F_{147}	1.50E+05	1.50E+05	5.43E+04	1.50E+05	1.50E+05	8.59E+04	1.50E+05	1.50E+05	6.23E+04	8.35E+03	1.67E+02
F_{148}	1.50E+05	1.50E+05	1.50E+05	7.68E+04	1.50E+05	8.58E+04	1.50E+05	1.50E+05	4.25E+04	5.43E+03	1.76E+02
F_{149}	1.70E+04	1.47E+05	2.43E+04	4.19E+03	1.50E+05	8.59E+04	1.50E+05	1.50E+05	5.84E+04	6.57E+03	1.84E+02
F_{150}	1.50E+05	1.50E+05	1.50E+05	1.50E+05	1.50E+05	1.53E+05	1.50E+05	1.50E+05	1.50E+05	1.50E+05	1.50E+05
F_{151}	1.50E+05	1.50E+05	1.37E+05	1.50E+05	1.50E+05	8.60E+04	1.50E+05	1.50E+05	1.50E+05	2.52E+04	8.71E+02
F_{152}	1.50E+05	1.50E+05	1.37E+05	1.50E+05	1.50E+05	8.60E+04	1.50E+05	1.50E+05	1.50E+05	2.58E+04	3.31E+02
F_{153}	1.50E+05	1.50E+05	1.50E+05	1.50E+05	1.50E+05	8.60E+04	1.50E+05	1.50E+05	1.50E+05	2.56E+04	8.91E+02
F_{154}	9.89E+04	4.99E+04	2.87E+04	6.39E+04	1.50E+05	8.58E+04	1.50E+05	1.50E+05	2.91E+04	3.69E+03	7.47E+01
F_{155}	1.50E+05	1.50E+05	1.50E+05	1.50E+05	1.50E+05	1.53E+05	1.50E+05	1.50E+05	1.50E+05	1.50E+05	1.50E+05
F_{156}	1.28E+02	1.50E+05	1.50E+05	1.50E+05	1.50E+05	3.06E+02	1.50E+05	1.50E+05	1.50E+05	1.50E+05	1.50E+05
F_{157}	1.50E+05	1.50E+05	8.66E+04	8.63E+04	1.50E+05	1.53E+05	1.50E+05	5.40E+03	1.50E+05	1.95E+04	1.03E+02
F_{158}	1.50E+05	3.06E+04	8.70E+03	1.34E+05	1.50E+05	8.58E+04	1.50E+05	1.50E+05	4.03E+04	2.97E+03	1.15E+02
F_{159}	1.50E+05	6.78E+04	8.78E+03	1.50E+05	1.50E+05	8.58E+04	1.50E+05	1.50E+05	4.47E+04	3.50E+03	1.39E+02
F_{160}	1.50E+05	6.80E+04	1.50E+05	1.50E+05	1.50E+05	8.58E+04	1.50E+05	1.50E+05	4.37E+04	2.89E+04	2.09E+02
F_{161}	5.00E+01	1.50E+05	1.50E+05	1.50E+05	1.44E+05	8.58E+04	1.50E+05	5.31E+03	1.50E+05	3.34E+04	1.22E+01
F_{162}	1.50E+05	1.50E+05	1.50E+05	1.50E+05	1.50E+05	8.64E+04	1.50E+05	1.50E+05	1.50E+05	1.50E+05	3.90E+03
F_{163}	1.50E+05	1.50E+05	8.64E+04	1.50E+05	1.50E+05	8.59E+04	1.50E+05	1.50E+05	1.50E+05	1.33E+04	2.63E+02
F_{164}	1.50E+05	1.50E+05	7.29E+04	1.50E+05	1.50E+05	8.62E+04	1.50E+05	1.50E+05	1.50E+05	1.50E+05	9.93E+03
F_{165}	1.50E+05	1.50E+05	1.50E+05	1.50E+05	1.50E+05	1.53E+05	1.50E+05	1.50E+05	1.50E+05	1.50E+05	1.50E+05
F_{166}	1.50E+05	1.50E+05	8.28E+03	1.50E+05	1.50E+05	1.53E+05	1.50E+05	3.67E+03	1.50E+05	1.37E+05	1.31E+02
F_{167}	1.50E+05	1.50E+05	1.50E+05	1.50E+05	1.50E+05	1.42E+05	1.50E+05	1.50E+05	1.50E+05	1.50E+05	1.50E+05
F_{168}	1.50E+05	1.50E+05	1.50E+05	1.50E+05	1.50E+05	8.58E+04	1.50E+05	1.50E+05	1.50E+05	1.49E+05	1.62E+02
F_{169}	1.50E+05	1.50E+05	1.50E+05	1.50E+05	1.50E+05	3.06E+02	1.50E+05	1.50E+05	1.50E+05	1.50E+05	2.77E+04

Table 27 (continued)

No.	Alternative metaheuristic algorithms										
	BIA	BBBC	BOA	CPA	CSS	GSO	JA	RFO	TWO	WDO	FHO
F_{170}	5.00E+01	5.00E+01	1.00E+02	5.00E+01	4.80E+01	3.06E+02	1.00E+02	5.00E+01	5.00E+01	5.00E+01	8.00E+00
F_{171}	2.86E+03	5.00E+01	4.93E+04	5.00E+01	6.39E+02	3.06E+02	5.25E+04	1.06E+03	1.82E+02	1.02E+04	5.71E+03
F_{172}	1.50E+05	1.50E+05	1.50E+05	1.50E+05	1.50E+05	9.19E+04	1.50E+05	1.50E+05	1.50E+05	1.50E+05	2.09E+02
F_{173}	1.50E+05	1.50E+05	1.50E+05	1.50E+05	1.50E+05	3.06E+02	1.50E+05	1.50E+05	1.50E+05	1.50E+05	6.67E+04
F_{174}	1.50E+05	1.50E+05	1.50E+05	1.50E+05	1.50E+05	1.53E+05	1.50E+05	1.50E+05	1.50E+05	1.50E+05	1.50E+05
F_{175}	1.50E+05	1.50E+05	8.61E+04	1.50E+05	1.50E+05	8.58E+04	1.50E+05	1.50E+05	1.50E+05	1.50E+05	3.35E+02
F_{176}	1.50E+05	1.50E+05	1.37E+05	1.50E+05	1.50E+05	8.89E+04	1.50E+05	1.50E+05	1.50E+05	2.15E+04	5.04E+02
F_{177}	1.50E+05	1.50E+05	9.47E+04	1.50E+05	1.50E+05	5.84E+04	1.50E+05	1.50E+05	1.50E+05	3.09E+04	6.33E+02
F_{178}	1.50E+05	1.50E+05	8.50E+04	1.50E+05	1.50E+05	8.90E+04	1.50E+05	1.50E+05	1.50E+05	1.50E+05	1.79E+02
F_{179}	1.50E+05	1.50E+05	5.45E+04	1.50E+05	1.50E+05	8.89E+04	1.50E+05	1.50E+05	7.04E+04	8.48E+03	2.21E+02
F_{180}	1.50E+05	7.02E+04	1.50E+05	1.50E+05	1.50E+05	8.89E+04	1.50E+05	1.50E+05	1.44E+05	3.38E+03	4.58E+01
F_{181}	1.50E+05	1.50E+05	1.37E+04	1.50E+05	1.50E+05	9.64E+04	1.50E+05	1.50E+05	1.50E+05	1.50E+05	1.47E+05
F_{182}	5.00E+01	1.50E+05	1.88E+04	1.50E+05	1.50E+05	1.53E+05	1.50E+05	1.50E+05	1.50E+05	1.50E+05	1.50E+05
F_{183}	1.50E+05	1.50E+05	1.50E+05	1.50E+05	1.50E+05	1.53E+05	1.50E+05	1.50E+05	1.50E+05	1.50E+05	1.50E+05
F_{184}	1.50E+05	1.50E+05	1.50E+05	1.50E+05	1.50E+05	6.74E+04	1.50E+05	1.50E+05	1.50E+05	1.50E+05	1.50E+05
F_{185}	1.50E+05	1.50E+05	3.08E+04	1.50E+05	1.50E+05	8.74E+04	1.50E+05	1.50E+05	7.92E+04	9.81E+03	3.84E+02
F_{186}	1.50E+05	1.50E+05	8.53E+04	1.50E+05	1.50E+05	8.89E+04	1.50E+05	1.50E+05	1.50E+05	1.58E+04	1.28E+02
F_{187}	1.50E+05	1.50E+05	5.42E+04	1.50E+05	1.50E+05	8.89E+04	1.50E+05	1.50E+05	1.50E+05	7.46E+03	1.59E+02
F_{188}	1.50E+05	1.50E+05	1.50E+05	1.50E+05	1.50E+05	8.89E+04	1.50E+05	1.50E+05	1.50E+05	5.49E+04	1.47E+03
F_{189}	1.50E+05	1.50E+05	3.21E+04	1.50E+05	1.50E+05	8.89E+04	1.50E+05	1.50E+05	1.50E+05	1.50E+05	1.40E+05
F_{190}	1.50E+05	1.50E+05	1.50E+05	1.50E+05	1.50E+05	1.32E+05	1.50E+05	1.50E+05	1.50E+05	1.50E+05	1.50E+05
F_{191}	5.05E+01	1.50E+05	1.50E+05	1.44E+05	1.50E+05	3.06E+02	1.50E+05	1.50E+05	1.50E+05	1.50E+05	8.00E+00
F_{192}	5.05E+01	1.50E+05	1.50E+05	1.46E+05	1.50E+05	3.06E+02	1.50E+05	1.50E+05	1.50E+05	1.50E+05	8.00E+00
F_{193}	1.50E+05	1.50E+05	1.50E+05	1.50E+05	1.50E+05	1.53E+05	1.50E+05	1.50E+05	1.50E+05	1.50E+05	1.50E+05

Table 27 (continued)

No.	Alternative metaheuristic algorithms										
	BIA	BBBC	BOA	CPA	CSS	GSO	JA	RFO	TWO	WDO	FHO
F_{194}	1.08E+05	1.50E+05	1.50E+05	1.08E+05	1.50E+05	3.06E+02	1.50E+05	1.49E+05	1.45E+05	1.29E+04	8.13E+01
F_{195}	1.50E+05	1.50E+05	1.50E+05	1.50E+05	1.50E+05	3.06E+02	1.50E+05	1.50E+05	1.50E+05	1.50E+05	4.90E+02
F_{196}	1.50E+05	1.50E+05	7.88E+04	1.50E+05	1.50E+05	8.89E+04	1.50E+05	1.50E+05	1.50E+05	1.61E+04	4.28E+02
F_{197}	1.50E+05	1.50E+05	8.62E+04	1.50E+05	1.50E+05	1.07E+05	1.50E+05	1.50E+05	1.50E+05	1.50E+05	2.62E+02
F_{198}	1.50E+05	1.50E+05	8.67E+04	1.50E+05	1.50E+05	1.12E+05	1.50E+05	1.50E+05	1.50E+05	1.50E+05	2.84E+02
F_{199}	1.50E+05	1.50E+05	5.63E+04	1.19E+05	1.50E+05	8.89E+04	1.50E+05	1.50E+05	1.49E+05	1.11E+04	7.38E+01
F_{200}	1.50E+05	1.50E+05	7.74E+04	1.50E+05	1.50E+05	8.89E+04	1.50E+05	1.50E+05	1.50E+05	1.42E+05	3.26E+02
F_{201}	1.50E+05	1.50E+05	1.50E+05	1.50E+05	1.50E+05	1.53E+05	1.50E+05	1.50E+05	1.50E+05	1.50E+05	1.50E+05
F_{202}	1.50E+05	1.50E+05	1.50E+05	1.50E+05	1.50E+05	1.51E+05	1.50E+05	1.50E+05	1.50E+05	1.50E+05	1.50E+05
F_{203}	1.50E+05	1.50E+05	1.50E+05	1.50E+05	1.50E+05	1.48E+05	1.50E+05	1.50E+05	1.50E+05	1.50E+05	1.50E+05
F_{204}	1.50E+05	1.50E+05	1.50E+05	1.50E+05	1.50E+05	8.89E+04	1.50E+05	1.50E+05	1.50E+05	1.50E+05	4.12E+02
F_{205}	1.50E+05	1.50E+05	5.46E+04	1.50E+05	1.50E+05	8.89E+04	1.50E+05	1.50E+05	1.50E+05	8.04E+03	1.92E+02
F_{206}	1.50E+05	1.50E+05	1.50E+05	1.46E+05	1.50E+05	8.89E+04	1.50E+05	1.50E+05	1.33E+05	5.38E+03	1.88E+02
F_{207}	2.56E+04	1.44E+05	2.44E+04	5.94E+03	1.47E+05	8.89E+04	1.50E+05	1.50E+05	6.01E+04	6.75E+03	2.19E+02
F_{208}	1.50E+05	1.50E+05	1.50E+05	1.50E+05	1.50E+05	1.53E+05	1.50E+05	1.50E+05	1.50E+05	1.50E+05	1.50E+05
F_{209}	1.50E+05	1.50E+05	1.37E+05	1.50E+05	1.50E+05	8.89E+04	1.50E+05	1.50E+05	1.50E+05	2.47E+04	9.53E+02
F_{210}	1.50E+05	1.50E+05	1.37E+05	1.50E+05	1.50E+05	8.89E+04	1.50E+05	1.50E+05	1.50E+05	2.60E+04	3.40E+02
F_{211}	1.50E+05	1.50E+05	1.50E+05	1.50E+05	1.50E+05	8.74E+04	1.50E+05	1.50E+05	1.50E+05	2.53E+04	9.49E+02
F_{212}	1.38E+05	8.87E+04	2.89E+04	1.48E+05	1.50E+05	8.89E+04	1.50E+05	1.50E+05	1.50E+05	3.75E+03	8.12E+01
F_{213}	1.50E+05	1.50E+05	1.50E+05	1.50E+05	1.50E+05	1.53E+05	1.50E+05	1.50E+05	1.50E+05	1.50E+05	1.50E+05
F_{214}	1.31E+02	1.50E+05	1.50E+05	1.50E+05	1.50E+05	3.06E+02	1.50E+05	1.50E+05	1.50E+05	1.50E+05	1.50E+05
F_{215}	1.50E+05	1.50E+05	8.67E+04	1.48E+05	1.50E+05	1.53E+05	1.50E+05	1.31E+04	1.50E+05	2.98E+04	1.23E+02
F_{216}	1.50E+05	6.45E+04	8.49E+03	1.50E+05	1.50E+05	8.89E+04	1.50E+05	1.50E+05	1.35E+05	2.94E+03	1.17E+02
F_{217}	1.50E+05	1.45E+05	8.58E+03	1.50E+05	1.50E+05	8.89E+04	1.50E+05	1.50E+05	1.32E+05	3.37E+03	1.36E+02

Table 27 (continued)

No.	Alternative metaheuristic algorithms										
	BIA	BBBC	BOA	CPA	CSS	GSO	JA	RFO	TWO	WDO	FHO
F_{218}	1.50E+05	1.48E+05	1.50E+05	1.50E+05	1.50E+05	8.89E+04	1.50E+05	1.50E+05	1.26E+05	2.00E+04	3.10E+02
F_{219}	5.10E+01	1.50E+05	1.50E+05	1.50E+05	1.50E+05	8.89E+04	1.50E+05	1.44E+04	1.50E+05	7.42E+04	9.20E+00
F_{220}	1.50E+05	1.50E+05	1.50E+05	1.50E+05	1.50E+05	8.89E+04	1.50E+05	1.50E+05	1.50E+05	1.50E+05	4.49E+03
F_{221}	1.50E+05	1.50E+05	8.67E+04	1.50E+05	1.50E+05	8.89E+04	1.50E+05	1.50E+05	1.50E+05	1.32E+04	3.21E+02
F_{222}	1.50E+05	1.50E+05	1.17E+05	1.50E+05	1.50E+05	8.91E+04	1.50E+05	1.50E+05	1.50E+05	1.50E+05	1.32E+04
F_{223}	1.50E+05	1.50E+05	1.50E+05	1.50E+05	1.50E+05	1.53E+05	1.50E+05	1.50E+05	1.50E+05	1.50E+05	1.50E+05
F_{224}	1.50E+05	1.50E+05	7.49E+03	1.50E+05	1.50E+05	1.53E+05	1.50E+05	8.51E+03	1.50E+05	1.38E+05	1.51E+02
F_{225}	1.50E+05	1.50E+05	1.50E+05	1.50E+05	1.50E+05	1.44E+05	1.50E+05	1.50E+05	1.50E+05	1.50E+05	1.48E+05
F_{226}	1.50E+05	1.50E+05	1.50E+05	1.50E+05	1.50E+05	8.74E+04	1.50E+05	1.50E+05	1.50E+05	1.50E+05	1.71E+02
F_{227}	1.50E+05	1.50E+05	1.50E+05	1.50E+05	1.50E+05	3.06E+02	1.50E+05	1.50E+05	1.50E+05	1.50E+05	8.29E+04
F_{228}	5.00E+01	5.00E+01	1.00E+02	5.00E+01	4.80E+01	3.06E+02	1.00E+02	5.00E+01	5.00E+01	5.00E+01	8.00E+00
F_{229}	4.65E+02	5.00E+01	8.72E+03	5.00E+01	5.28E+01	3.06E+02	1.00E+02	4.78E+02	5.00E+01	2.20E+02	1.13E+01
F_{230}	1.50E+05	1.50E+05	1.50E+05	1.50E+05	1.50E+05	8.57E+04	1.50E+05	1.50E+05	1.50E+05	1.50E+05	2.13E+02
F_{231}	1.50E+05	1.50E+05	1.50E+05	1.50E+05	1.50E+05	3.06E+02	1.50E+05	1.50E+05	1.50E+05	1.50E+05	1.12E+05
F_{232}	1.50E+05	1.50E+05	1.50E+05	1.50E+05	1.50E+05	1.53E+05	1.50E+05	1.50E+05	1.50E+05	1.50E+05	1.50E+05
F_{233}	1.50E+05	1.50E+05	8.61E+04	1.50E+05	1.50E+05	9.04E+04	1.50E+05	1.50E+05	1.50E+05	1.50E+05	3.87E+02

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