An extended TOPSIS method based on ordered fuzzy numbers for group decision making

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Abstract

Multiple criteria decision making methods have become very popular in recent years and are frequently applied in many real-life situations. One of the most commonly used is the Technique for Order Preference by Similarity to Ideal Solution. Its original version is based on the information provided by the decision maker as exact numerical values. However, in some real-life situations, the decision maker may not be able to precisely express the value of the ratings of alternatives with respect to criteria or else he/she uses linguistic expressions. In such situations he/she may use other data formats, such as: interval numbers, fuzzy numbers, ordered fuzzy numbers, hesitant fuzzy sets, intuitionistic fuzzy sets and other. On the other hand, the increasing complexity of the decision problems analysed makes it less feasible to consider all the relevant aspects of the problems by a single decision maker. As a result, many real-life problems are discussed by a group of decision makers. The aim of this paper and its main contribution is to present a new approach for ranking of alternatives for group decision making using the Technique for Order Preference by Similarity to Ideal Solution method based on ordered fuzzy numbers. This is an alternative to methods that use different forms of averages for the aggregation of the individual matrices into a collective matrix. In the proposed approach aggregation is not needed and all individual decision information of decision makers is taken into account in determining the ranking of alternatives and the selecting the best one. The key stage of this method is the transformation of the decision matrices provided by the decision makers into matrices of alternatives. A matrix corresponding to an alternative is composed of its assessments with respect to all criteria, performed by all the decision makers. A numerical example illustrates the proposed approach.

Keywords MCDM · Ordered fuzzy numbers · TOPSIS · Group decision making

1 Introduction

Multiple criteria decision making (MCDM) methods have become very popular in recent years and are frequently applied in many real-life situations [for more information see, e.g.,

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Behzadian et al. (2012) and Abdullah and Adawiyah (2014)]. One of the most popular and widely applied MCDM methods is the Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) proposed by Hwang and Yoon (1981). The basic idea of this method is fairly straightforward. It uses two reference points: the so-called positive ideal solution (PIS) and negative ideal solution (NIS) as benchmarks. The selected alternative is that which has both the shortest distance from the PIS and the longest distance from the NIS. The PIS is a solution that maximizes all the benefit criteria and minimizes all the cost criteria, whereas the NIS is a solution that maximizes all the cost criteria and minimizes all the benefit criteria.

The classical TOPSIS method is based on the information provided by the decision maker (DM) or expert as exact numerical values. However, in some real-life situations, the DM may not be able to precisely express the value of the ratings of alternatives with respect to criteria or else he/she uses linguistic expressions. In such situations, when evaluations are based on unquantifiable, incomplete, or unobtainable information, the DM may use other data formats, such as: interval numbers (Jahanshahloo et al. 2006a; Yue 2011), fuzzy numbers (Chen 2000; Jahanshahloo et al. 2006b), hesitant fuzzy sets (Senvar et al. 2016), intuitionistic fuzzy sets (Boran et al. 2009) and others. Recently, Roszkowska and Kacprzak (2016) have developed a new approach for TOPSIS in which information provided by the DM is represented in the form of ordered fuzzy numbers (OFNs) which are well suited to handle incomplete and uncertain knowledge.

This new approach to MCDM problems has been considered, so far, in a few papers only, among others in Roszkowska and Kacprzak (2016), Rudnik and Kacprzak (2017) and Kacprzak (2017). The first application of OFNs for TOPSIS was presented at the Sixth Podlasie Conference on Mathematics in Białystok, Poland, by Kacprzak and Roszkowska (2014), and discussed in detail in Roszkowska and Kacprzak (2016). In that paper, the authors evaluated alternatives with respect to criteria using linguistic expressions, in which linguistic terms were quantified on a scale given in advance. The scale was extended to include intermediate values, such as “*more than i* ” or “*less than $i + 1$* ”, expressed by trapezoidal OFNs, together with “ *i* ” and “ *$i + 1$* ”, where $i \in \mathbb{N}$. The authors showed that the extended TOPSIS using OFNs can better distinguish among alternatives as compared with the classical TOPSIS which uses crisp values, and with the extended TOPSIS which uses (convex) fuzzy numbers (CFNs). The extended TOPSIS with OFNs has been also used to solve a real-life problem of discrete flow control in a manufacturing system (Rudnik and Kacprzak 2017). The authors have tested this method in a flow control system and compared it to the classical TOPSIS and to other simple control methods. As a result, they concluded that the extended TOPSIS with OFNs is better suited for the analysed case than the classical TOPSIS or than most of other methods considered. Kacprzak (2017) developed a method for obtaining criteria weights based on the concept of Shannon entropy when data are in the form of OFNs. The proposed approach allows to obtain, for each criterion, its weight in the form of an OFN. Moreover, it has also been shown that the obtained criteria weights satisfy certain conditions similar to those for real numbers: for instance, they can be normalized and sum up to 1.

On the other hand, the increasing complexity of the decision problems analysed makes it less feasible to consider all the relevant aspects of the problem by a single decision maker or expert. Therefore, many real-life problems are considered by a group of decision makers (or experts), resulting in Group Decision Making (GDM), which is an interesting and important part of today’s decision science and research area. In the literature, a majority of researchers used aggregation methods for all individual decisions made by all DMs (usually in the form of decision matrices) to construct a collective decision (also in the form of a collective decision matrix). This collective decision is the starting point for the ranking of alternatives or the

selection of the best one. In such a situation the MCDM problem for GDM reduces to a classical MCDM problem and can lead to the loss of some important information.

One of the most commonly used methods of aggregation, in MCDM methods such as TOPSIS, is arithmetic mean (Chen 2000; Wang and Chang 2007; Roszkowska and Kacprzak 2016). On the one hand, this type of aggregation of individual decisions is also used in practice, e.g., in certain sports, such as snowboard slopestyle or halfpipe. Each participant is evaluated by a group of referees (as DMs) and the average of the referees' scores is taken as the final result for each participant. On the other hand, due to this method of aggregation of individual information, some significant information related to the individual decisions of DMs may be omitted. The PIS and NIS, as benchmarks of the classical TOPSIS method, are expressed by a vector. For this reason, the TOPSIS methods for GDM based on the aggregation of the individual decisions made by each DM are limited to comparisons of vectors of alternatives (with respect to criteria) with the vector of PIS and NIS. This comparison cannot reflect the DMs' individual decisions, which are expressed by a decision matrix. In order to better explain these limitations, we present a simple example. Let us consider a group of two decision makers $\{DM_1, DM_2\}$ who evaluate three alternatives $\{A_1, A_2, A_3\}$ using a scale of positive trapezoidal OFNs [see Eq. (4)], given in advance: $\{(1, 2, 3, 4), (2, 3, 4, 5), (3, 4, 5, 6), (4, 5, 6, 7), (5, 6, 7, 8), (6, 7, 8, 9), (7, 8, 9, 10)\}$. Their evaluations of the alternatives with respect to the j th criterion are [see Eq. (26)]: $x_{1j}^1 = (4, 5, 6, 7)$, $x_{2j}^1 = (3, 4, 5, 6)$ and $x_{3j}^1 = (2, 3, 4, 5)$ for DM_1 and $x_{1j}^2 = (4, 5, 6, 7)$, $x_{2j}^2 = (5, 6, 7, 8)$ and $x_{3j}^2 = (6, 7, 8, 9)$ for DM_2 . The aggregation results (using arithmetic mean) are all equal to $v_{1j} = v_{2j} = v_{3j} = (4, 5, 6, 7)$. This means that the corresponding elements of PIS and NIS [using Eqs. (7) and (8) in formulas (21) and (22)] are: $v_j^+ = v_j^- = (4, 5, 6, 7)$. It follows that the distances of each alternative A_i for $i = 1, 2, 3$ to PIS and NIS obtained using Eq. (6) are equal to 0, i.e. $d(v_{1j}, v_j^+) = d(v_{2j}, v_j^+) = d(v_{3j}, v_j^+) = 0$ and $d(v_{1j}, v_j^-) = d(v_{2j}, v_j^-) = d(v_{3j}, v_j^-) = 0$. This means that the j th criterion has no influence on the ranking of the alternatives [see Eqs. (23), (24) and (25)] and can be omitted. Therefore we can conclude that such an averaged result does not reflect the discrepancies between the individual decisions (preferences of DMs) and that using such averaged information may lead to an incorrect final decision.

The aim of this paper and its main contribution is to present a new approach for ranking of alternatives for group decision making using the TOPSIS method based on ordered fuzzy numbers. This is an alternative to methods that use different forms of averages for the aggregation of the individual matrices into a collective matrix. In the proposed approach aggregation is not needed and all individual decision information of DMs is taken into account in determining the ranking of alternatives and selecting the best one. The key stage of this method is the transformation of the decision matrices provided by the decision makers into matrices of alternatives. A matrix corresponding to an alternative is composed of its assessments with respect to all criteria, performed by all the decision makers. Since all individual decision matrices are normalized beforehand with respect to the type of criterion, the positive ideal solution in this approach is a matrix composed of maximal assessments, and the negative ideal solution is a matrix composed of minimal assessments. The distances of alternatives from the PIS and the NIS, in contrast to the classic TOPSIS and to the method based on the aggregation of the individual decisions made by each DM, are distances between matrices. Using the coefficient of relative closeness of each alternative to the positive ideal solution, a ranking of alternatives is created and the best one is indicated.

The rest of the paper is organized as follows. In Sect. 2 basic definitions and notations of ordered fuzzy numbers are introduced. In Sect. 3 the classical TOPSIS method and its

fuzzy extension based on ordered fuzzy numbers are presented. The proposed approach and a numerical example are described in Sects. 4 and 5, respectively. Section 6 is devoted to the comparison of the proposed approach with other, similar approaches. Finally, concluding remarks and suggestions for further research are in Sect. 7.

2 Ordered fuzzy numbers

In this section, some definitions related to the model of ordered fuzzy numbers used in the paper are briefly presented. The model of ordered fuzzy numbers, as an extension of the notion of (convex) fuzzy numbers, was introduced and developed by Kosiński and his two coworkers Prokopowicz and Ślęzak (2002) in a series of papers (Kosinski et al. 2002, 2003, 2004; Kosiński 2006). The main goal guiding the authors was to overcome the limitations of the CFNs model resulting primarily from the definition of arithmetic operations on these numbers, an increase of fuzziness during the operations, and a lack of an opposite element with respect to addition and an inverse element with respect to multiplication. Due to that, the simplest fuzzy arithmetic equation $A + X = C$ or $A \cdot X = C$, where A and C are given fuzzy numbers, has no fuzzy solution X in general. Arithmetic operations in the OFNs model are similar to the operations on real numbers, which are a special case of OFNs.

Definition 1 (Kosinski et al. 2004; Chwastyk and Kosinski 2013). An ordered fuzzy number¹ A is an ordered pair $A = (f_A, g_A)$ of continuous functions $f_A, g_A : [0, 1] \rightarrow \mathbb{R}$.

The set of all OFNs will be denoted by $\mathfrak{R}$. The elements of the OFN A are called: f_A —the up part and g_A —the down part. To conform to the classical notation of fuzzy numbers, the independent variable of both functions f_A and g_A will be denoted by y , while their values, by x (Fig. 1a). The continuity of both functions f_A and g_A implies that their images are bounded intervals, called UP_A and $DOWN_A$, respectively (Fig. 1a), described by their endpoints as follows: $UP_A = [f_A(0), f_A(1)]$ and $DOWN_A = [g_A(1), g_A(0)]$.

In general, the functions f_A and g_A of the OFN A need not be invertible as functions of the variable y ; only continuity is required in Definition 1. But we can assume, additionally, that (Kosiński 2006):

- (A1) f_A is increasing and g_A is decreasing,
- (A2) $f_A \leq g_A$ (pointwise),

and we can also define a function of the variable x on the interval $[f_A(1), g_A(1)]$ with constant value equal to 1. Then the membership function of the ordered fuzzy number A can be defined as follows (Fig. 1b)

$$\mu_A(x) = \begin{cases} 0 & \text{if } x < f_A(0) \\ f_A^{-1}(x) & \text{if } f_A(0) \leq x \leq f_A(1) \\ 1 & \text{if } f_A(1) \leq x \leq g_A(1) \\ g_A^{-1}(x) & \text{if } g_A(1) \leq x \leq g_A(0) \\ 0 & \text{if } g_A(0) < x. \end{cases} \quad (1)$$

Using the characteristic point in (1), an OFN A can be written as $A = (f_A(0), f_A(1), g_A(1), g_A(0))$, where $f_A(0) \leq f_A(1) \leq g_A(1) \leq g_A(0)$. Figure 1c shows the

¹ In our paper, we use the term “Ordered Fuzzy Numbers” which was proposed by Professor Kosiński and his colleagues Prokopowicz and Ślęzak in (2002). After the death of Professor Kosiński, to commemorate and honour him, these numbers are often called “Kosinski Fuzzy Numbers” (Prokopowicz and Pedrycz 2015).

Fig. 1 **a** An OFN A , **b** an OFN A with its membership function, **c** the arrow denotes the order of the inverted functions and the orientation of OFN A

Fig. 2 **a** An OFN (f_A, g_A) with a positive orientation, **b** an OFN (g_A, f_A) with a negative orientation

membership function of an OFN A with an extra arrow denoting the orientation (the order of the inverse functions f_A^{-1} and g_A^{-1}). In this case, the application of the OFNs model is no different from the application of the CFNs model. But let us note that the pair of continuous functions $(g_A, f_A) = (g_A(0), g_A(1), f_A(1), f_A(0))$, where $g_A(0) \geq g_A(1) \geq f_A(1) \geq f_A(0)$, determines a different OFN than the pair (f_A, g_A) . Figure 2 shows that although the two curves have an identical shape, the corresponding membership functions determine two different OFNs differing in orientation. Using the orientation of OFNs, the set $\mathfrak{N}$ can be divided into two subsets:

- numbers with a positive orientation, if the direction of the OFNs is the same as the x -axis (Fig. 2a),
- numbers with a negative orientation, if the direction of the OFNs is opposite (Fig. 2b).

Arithmetic operations on OFNs are defined as pairwise operations on their functions f and g .

Definition 2 (Kosiński 2006). Let $A = (f_A, g_A)$, $B = (f_B, g_B)$ and $C = (f_C, g_C)$ be OFNs. The arithmetic operations: addition ($C = A + B$), subtraction ($C = A - B$), multiplication ($C = A \cdot B$) and division ($C = A/B$) are defined in $\mathfrak{N}$ as follows

$$\forall y \in [0, 1][f_C(y) = f_A(y) * f_B(y) \text{ and } g_C(y) = g_A(y) * g_B(y)] \quad (2)$$

where $*$ $\in \{+, -, \cdot, /\}$ and A/B is defined when $f_B \neq 0$ and $g_B \neq 0$ for each $y \in [0, 1]$.

Fig. 3 Improper OFNs with a membership curve (or relation) instead of a membership function

Since real numbers are a special case of OFNs, they can be represented in $\mathfrak{R}$ as follows. Let $r \in \mathbb{R}$ and let r' be a constant function, i.e., $r'(y) = r$ for all $y \in [0, 1]$. Then $r^*(y) = (r', r')$ is the OFN which represents the real number r in $\mathfrak{R}$. Now we can define the multiplication of an ordered fuzzy number $A = (f_A, g_A)$ by a real number r .

Definition 3 (Kosiński 2006). Let $A = (f_A, g_A) \in \mathfrak{R}$ and $r \in \mathbb{R}$. Multiplication of an ordered fuzzy number A by a real number r is defined by the formula

$$\forall y \in [0, 1][r \cdot A = (r \cdot f_A(y), r \cdot g_A(y))]. \quad (3)$$

Here it is worth mentioning that if the functions f_A and/or g_A are not invertible (Fig. 3a) or assumption (A2) is not satisfied (Fig. 3b), we can obtain the so-called improper OFNs (Kosiński et al. 2003). Instead of the membership function, we can define the membership curve (or relation) in the xy -plane, consisting of the functions f_A and g_A (as functions of the variable x) and a segment of the line $y = 1$ over the interval $[f_A(1), g_A(1)]$ (Fig. 3).

Moreover, these type of improper OFNs can appear when arithmetic operations on OFNs are performed. Let us consider two numerical examples:

- the sum of two OFNs $A = (1 + y, 6 - y)$ and $B = (7 - y, 1 + 4y)$, $y \in [0, 1]$ is equal to $A + B = (8, 7 + 3y)$ and is an improper OFN (Fig. 4a),
- the sum of another pair of OFNs $A = (1 + y, 5 - y)$ and $B = (8 - 3y, 1 + 3y)$, $y \in [0, 1]$ is equal to $A + B = (9 - 2y, 6 + 2y)$ which is also an improper OFN (Fig. 4b).

In formula (1) of the membership function of the OFN A there are four characteristic real numbers: $f_A(0)$, $f_A(1)$, $g_A(1)$ and $g_A(0)$. If the functions f_A and g_A are linear, these four numbers uniquely describe A as follows (Fig. 5)

$$A = (f_A(0), f_A(1), g_A(1), g_A(0)). \quad (4)$$

The number A is called a positive trapezoidal OFN if $0 < f_A(0) \leq f_A(1) < g_A(1) \leq g_A(0)$ or $f_A(0) \geq f_A(1) > g_A(1) \geq g_A(0) > 0$ (if $f_A(1) = g_A(1)$, it is called a positive triangular OFN).

The representation of the form (4) allows us to quickly perform arithmetic operations on trapezoidal OFNs using these characteristic points. Let $A = (f_A(0), f_A(1), g_A(1), g_A(0))$ and $B = (f_B(0), f_B(1), g_B(1), g_B(0))$ be positive trapezoidal OFNs. The arithmetic operations on these numbers are defined by the formula

$$A \diamond B = (f_A(0) \diamond f_B(0), f_A(1) \diamond f_B(1), g_A(1) \diamond g_B(1), g_A(0) \diamond g_B(0)) \quad (5)$$

Fig. 4 OFNs A and B and their sum $A + B$ which is an improper OFN

Fig. 5 A trapezoidal OFN A (a pair of linear functions) with characteristic points

Fig. 6 Special OFNs which will be used in the numerical example

where $\diamond \in \{+, -, *, /\}$.

In the numerical example we will use three special types of OFNs (Roszkowska and Kacprzak 2016). For $i \in \mathbb{N}$, the first one is the real number $A_1 = (i, i, i, i)$, which will express the term “exactly i ” (Fig. 6a). The other OFNs are $A_2 = (i, i, \frac{2i+1}{2}, i+1)$ and $A_3 = (i+1, i+1, \frac{2i+1}{2}, i)$, expressing the terms “more than i ” (Fig. 6b) and “less than $i+1$ ” (Fig. 6c), respectively.

In some fuzzy MCDM methods, including fuzzy TOPSIS, it is necessary to measure the distance between fuzzy numbers, and to perform max and min operations on them.

Definition 4 Let $A = (f_A(0), f_A(1), g_A(1), g_A(0))$ and $B = (f_B(0), f_B(1), g_B(1), g_B(0))$ be two positive trapezoidal OFNs. Then the distance calculated by the vertex method and the max and min operations are defined as

$$d(A, B) = \sqrt{\frac{1}{4}[(f_A(0) - f_B(0))^2 + (f_A(1) - f_B(1))^2 + (g_A(1) - g_B(1))^2 + (g_A(0) - g_B(0))^2]}, \quad (6)$$

$$\max(A, B) = (\max\{f_A(0), f_B(0)\}, \max\{f_A(1), f_B(1)\}, \max\{g_A(1), g_B(1)\}, \max\{g_A(0), g_B(0)\}), \quad (7)$$

$$\min(A, B) = (\min\{f_A(0), f_B(0)\}, \min\{f_A(1), f_B(1)\}, \min\{g_A(1), g_B(1)\}, \min\{g_A(0), g_B(0)\}). \quad (8)$$

3 The TOPSIS method based on OFNs

In this section the classical TOPSIS method and its fuzzy extension based on OFNs are presented. Let us assume that the decision maker has to choose one of m possible alternatives described by n criteria. The rating of alternative A_i ($i = 1, \dots, m$) with respect to criterion C_j ($j = 1, \dots, n$) is denoted by x_{ij} . The set of criteria is divided into two subsets: benefit criteria (greater value is better) denoted by B and cost criteria (lower value is better) denoted by C . Let $w = (w_1, w_2, \dots, w_n)$ be the vector of criteria weights.

The original TOPSIS method assumes that the rating x_{ij} of the alternatives with respect to the criteria, as well as the criteria weights w_j , are expressed precisely by real numbers. It consists of the following steps (Hwang and Yoon 1981).

Step 1 Determination of the decision matrix X

$$X = [x_{ij}]_{m \times n} \quad (9)$$

where $x_{ij} \in \mathbb{R}$.

Step 2 Calculation of the normalized decision matrix R using vector normalization

$$R = [r_{ij}]_{m \times n} \quad (10)$$

where $r_{ij} = \frac{x_{ij}}{\sqrt{\sum_{k=1}^m x_{kj}^2}}$.

Step 3 Calculation of the weighted normalized matrix V by multiplying the columns of the normalized decision matrix R by the associated weights $w_j \in \mathbb{R}$ satisfying

$$\sum_{j=1}^n w_j = 1$$

$$V = [v_{ij}]_{m \times n} \quad (11)$$

where $v_{ij} = r_{ij} \cdot w_j$.

Step 4 Determination of the positive ideal solution A^+

$$A^+ = (v_1^+, v_2^+, \dots, v_n^+) = \left\{ \left(\max_i v_{ij} | j \in B \right), \left(\min_i v_{ij} | j \in C \right) \right\} \quad (12)$$

and the negative ideal solution A^-

$$A^- = (v_1^-, v_2^-, \dots, v_n^-) = \left\{ \left(\min_i v_{ij} | j \in B \right), \left(\max_i v_{ij} | j \in C \right) \right\}. \quad (13)$$

Step 5 Calculation of the Euclidean distances of each alternative A_i from the positive ideal solution A^+

$$d_i^+ = \sqrt{\sum_{j=1}^n (v_{ij} - v_j^+)^2} \quad (14)$$

and from the negative ideal solution A^-

$$d_i^- = \sqrt{\sum_{j=1}^n (v_{ij} - v_j^-)^2}. \quad (15)$$

Step 6 Calculation of the relative closeness of each alternative A_i to the positive ideal solution A^+

$$RC_i = \frac{d_i^-}{d_i^+ + d_i^-}. \quad (16)$$

Step 7 Ranking of the alternatives A_i according to their relative closeness to the ideal solutions A^+ (the larger the value of RC_i the better the alternative A_i). The best alternative is the one with the largest value of RC_i .

In real-life decision making problems it is usually difficult to express evaluations precisely using real numbers, due to a lack of knowledge and data or to subjective and imprecise expert judgments. In such situations, instead of exact numbers, OFNs can be used. The fuzzy TOPSIS method based on positive trapezoidal OFNs proposed by Roszkowska and Kacprzak (2016) consists of the following steps.

Step 1 Define the fuzzy decision matrix X

$$X = [x_{ij}]_{m \times n} \quad (17)$$

where $x_{ij} = (f_{x_{ij}}(0), f_{x_{ij}}(1), g_{x_{ij}}(1), g_{x_{ij}}(0))$ is a positive trapezoidal OFN representing the rating of alternative A_i with respect to attribute C_j .

Step 2 Construct the normalized fuzzy decision matrix R using linear normalization

$$R = [r_{ij}]_{m \times n} \quad (18)$$

where

$$r_{ij} = \begin{cases} \left(\frac{f_{x_{ij}}(0)}{\max_i g_{x_{ij}}(0)}, \frac{f_{x_{ij}}(1)}{\max_i g_{x_{ij}}(0)}, \frac{g_{x_{ij}}(1)}{\max_i g_{x_{ij}}(0)}, \frac{g_{x_{ij}}(0)}{\max_i g_{x_{ij}}(0)} \right) & \text{if } j \in B \\ \left(\frac{\min_i g_{x_{ij}}(0)}{f_{x_{ij}}(0)}, \frac{\min_i g_{x_{ij}}(0)}{f_{x_{ij}}(1)}, \frac{\min_i g_{x_{ij}}(0)}{g_{x_{ij}}(1)}, \frac{\min_i g_{x_{ij}}(0)}{g_{x_{ij}}(0)} \right) & \text{if } j \in C \end{cases}. \quad (19)$$

Step 3 Construct the weighted normalized fuzzy matrix V by multiplying the columns of the normalized fuzzy decision matrix R by the associated weights $w_j \in \mathbb{R}$ satisfying

$$\sum_{j=1}^n w_j = 1$$

$$V = [v_{ij}]_{m \times n} \quad (20)$$

where

$$\begin{aligned} v_{ij} &= r_{ij} \cdot w_j = (f_{(x_{ij})}(0), f_{(x_{ij})}(1), g_{(x_{ij})}(1), g_{(x_{ij})}(0)) \cdot w_j \\ &= (f_{(x_{ij})}(0) \cdot w_j, f_{(x_{ij})}(1) \cdot w_j, g_{(x_{ij})}(1) \cdot w_j, g_{(x_{ij})}(0) \cdot w_j). \end{aligned}$$

Step 4 Determine the positive ideal solution as follows

$$A^+ = (v_1^+, v_2^+, \dots, v_n^+) \quad (21)$$

where $v_j^+ = \max_i v_{ij}$ ($j = 1, \dots, n$) and the negative ideal solution

$$A^- = (v_1^-, v_2^-, \dots, v_n^-) \quad (22)$$

where $v_j^- = \min_i v_{ij}$ ($j = 1, \dots, n$).

Step 5 Calculate the distances of each alternative A_i ($i = 1, \dots, m$) from the positive ideal solution A^+

$$d_i^+ = \sum_{j=1}^n d(v_{ij}, v_j^+) \quad (23)$$

and from the negative ideal solution A^-

$$d_i^- = \sum_{j=1}^n d(v_{ij}, v_j^-). \quad (24)$$

Step 6 Calculate the relative closeness of alternative A_i to the ideal solution A^+

$$RC_i = \frac{d_i^-}{d_i^+ + d_i^-}. \quad (25)$$

Step 7 Rank the alternatives A_i and select the one with the largest value of RC_i .

4 The proposed approach

An extended TOPSIS based on OFNs for GDM consists of the following four main stages presented in Fig. 7.

1. *Problem description stage*, consisting of the determination of the MCDM problem, the determination of the group of decision makers and the selection of all the feasible alternatives and important criteria.
2. *Preparation stage*, consisting of the construction of fuzzy decision matrices, their normalization, and the calculation of weighted normalized fuzzy decision matrices for each DM.
3. *Transformation stage*, consisting of the construction of normalized fuzzy decision matrices for each alternative based on the weighted normalized fuzzy decision matrices for each DM.
4. *Selection stage*, consisting of the determination of the positive ideal solution and the negative ideal solution, the computation of the total score for each alternative, rank ordering them and the selection of the best one.

4.1 Problem description

Consider an MCDM problem for group decision making; for instance, the example presented in the next section: hiring a secretary for a company. Let $A = \{A_1, A_2, \dots, A_m\}$ ($m \geq 2$) be a discrete set of m feasible alternatives (candidates), $C = \{C_1, C_2, \dots, C_n\}$ ($n \geq 2$) be a finite set of criteria, $w = (w_1, w_2, \dots, w_n)$ be the vector of criteria weights, such that $0 \leq w_j \leq 1$ and $\sum_{j=1}^n w_j = 1$. Moreover, let $DM = \{DM_1, DM_2, \dots, DM_K\}$ ($K \geq 2$) be a group of decision makers.

Fig. 7 The conceptual framework of the proposed approach

4.2 Data preparation

In the process of group decision making, the DMs are asked to rate alternatives with respect to criteria. In many cases—when our knowledge of the analysed subject is incomplete, or the available data are inaccurate, or when the ratings are expressed linguistically—OFNs can be used. In that case, each decision maker DM_k ($k = 1, 2, \dots, K$) provides a decision matrix of the form

$$X^k = [x_{ij}^k]_{m \times n} = \begin{matrix} & C_1 & C_2 & \cdots & C_n \\ \begin{matrix} A_1 \\ A_2 \\ \vdots \\ A_m \end{matrix} & \begin{bmatrix} x_{11}^k & x_{12}^k & \cdots & x_{1n}^k \\ x_{21}^k & x_{22}^k & \cdots & x_{2n}^k \\ \vdots & \vdots & \ddots & \vdots \\ x_{m1}^k & x_{m2}^k & \cdots & x_{mn}^k \end{bmatrix} \end{matrix} \quad (26)$$

where $x_{ij}^k = (f_{x_{ij}^k}(0), f_{x_{ij}^k}(1), g_{x_{ij}^k}(1), g_{x_{ij}^k}(0))$ is a positive trapezoidal OFN representing the rating of alternative A_i with respect to criterion C_j provided by decision maker DM_k .

Remark 1 The orientation of the OFN can be used to indicate the type of the criterion. In the case of a cost criterion, the OFN has a negative orientation (the lower the value, the better). In the case of a benefit criterion, the OFN has a positive orientation (the higher the value, the better). In our example in the next section the orientation is used to express the terms “more than i ” and “less than i ”.

Next, in order to ensure comparability of criteria, the fuzzy decision matrix X^k ($k = 1, \dots, K$) is normalized. The normalized fuzzy decision matrix

$$Y^k = [y_{ij}^k]_{m \times n} = \begin{matrix} & C_1 & C_2 & \cdots & C_n \\ \begin{matrix} A_1 \\ A_2 \\ \vdots \\ A_m \end{matrix} & \begin{bmatrix} y_{11}^k & y_{12}^k & \cdots & y_{1n}^k \\ y_{21}^k & y_{22}^k & \cdots & y_{2n}^k \\ \vdots & \vdots & \ddots & \vdots \\ y_{m1}^k & y_{m2}^k & \cdots & y_{mn}^k \end{bmatrix} \end{matrix} \quad (27)$$

can be calculated using the following formulas

$$y_{ij}^k = \begin{cases} \left(\frac{f_{x_{ij}^k}(0)}{\max_i g_{x_{ij}^k}(0)}, \frac{f_{x_{ij}^k}(1)}{\max_i g_{x_{ij}^k}(0)}, \frac{g_{x_{ij}^k}(1)}{\max_i g_{x_{ij}^k}(0)}, \frac{g_{x_{ij}^k}(0)}{\max_i g_{x_{ij}^k}(0)} \right) & \text{if } j \in B \\ \left(\frac{\min_i g_{x_{ij}^k}(0)}{f_{x_{ij}^k}(0)}, \frac{\min_i g_{x_{ij}^k}(0)}{f_{x_{ij}^k}(1)}, \frac{\min_i g_{x_{ij}^k}(0)}{g_{x_{ij}^k}(1)}, \frac{\min_i g_{x_{ij}^k}(0)}{g_{x_{ij}^k}(0)} \right) & \text{if } j \in C \end{cases} \quad (28)$$

Using the vector of criteria weights $w = (w_1, w_2, \dots, w_n)$, the weighted normalized fuzzy decision matrix is calculated for each DM_k ($k = 1, 2, \dots, K$)

$$V^k = [v_{ij}^k]_{m \times n} = \begin{matrix} & C_1 & C_2 & \cdots & C_n \\ \begin{matrix} A_1 \\ A_2 \\ \vdots \\ A_m \end{matrix} & \begin{bmatrix} v_{11}^k & v_{12}^k & \cdots & v_{1n}^k \\ v_{21}^k & v_{22}^k & \cdots & v_{2n}^k \\ \vdots & \vdots & \ddots & \vdots \\ v_{m1}^k & v_{m2}^k & \cdots & v_{mn}^k \end{bmatrix} \end{matrix}, \quad (29)$$

where $v_{ij}^k = y_{ij}^k \cdot w_j = (f_{y_{ij}^k}(0) \cdot w_j, f_{y_{ij}^k}(1) \cdot w_j, g_{y_{ij}^k}(1) \cdot w_j, g_{y_{ij}^k}(0) \cdot w_j)$.

4.3 Transformation of the weighted normalized fuzzy decision matrices for all DMs into normalized fuzzy decision matrices for all alternatives

The matrices V^k ($k = 1, 2, \dots, K$) form the basis for the construction of weighted normalized fuzzy decision matrices for each alternative A_i ($i = 1, 2, \dots, m$)

$$A^i = [v_{ij}^k]_{K \times n} = \begin{matrix} & C_1 & C_2 & \cdots & C_n \\ \begin{matrix} DM_1 \\ DM_2 \\ \vdots \\ DM_K \end{matrix} & \begin{bmatrix} v_{i1}^1 & v_{i2}^1 & \cdots & v_{in}^1 \\ v_{i1}^2 & v_{i2}^2 & \cdots & v_{in}^2 \\ \vdots & \vdots & \ddots & \vdots \\ v_{i1}^K & v_{i2}^K & \cdots & v_{in}^K \end{bmatrix} \end{matrix}. \quad (30)$$

4.4 Rank ordering of the alternatives and the selection of the best one

Matrices A^i ($i = 1, 2, \dots, m$) constitute the basis for the construction of a ranking of the alternatives and the selection of the best one. The positive ideal solution A^+ is determined as follows

$$A^+ = [v_j^{k+}]_{K \times n} = \begin{matrix} & C_1 & C_2 & \cdots & C_n \\ \begin{matrix} DM_1 \\ DM_2 \\ \vdots \\ DM_K \end{matrix} & \begin{bmatrix} v_1^{1+} & v_2^{1+} & \cdots & v_n^{1+} \\ v_1^{2+} & v_2^{2+} & \cdots & v_n^{2+} \\ \vdots & \vdots & \ddots & \vdots \\ v_1^{K+} & v_2^{K+} & \cdots & v_n^{K+} \end{bmatrix} \end{matrix}. \quad (31)$$

where $v_j^{k+} = \max_i v_{ij}^k$ ($j = 1, 2, \dots, n; k = 1, 2, \dots, K$), and the negative ideal solution A^- is determined as follows

$$A^- = [v_j^{k-}]_{K \times n} = \begin{matrix} & C_1 & C_2 & \cdots & C_n \\ \begin{matrix} DM_1 \\ DM_2 \\ \vdots \\ DM_K \end{matrix} & \begin{bmatrix} v_1^{1-} & v_2^{1-} & \cdots & v_n^{1-} \\ v_1^{2-} & v_2^{2-} & \cdots & v_n^{2-} \\ \vdots & \vdots & \ddots & \vdots \\ v_1^{K-} & v_2^{K-} & \cdots & v_n^{K-} \end{bmatrix} \end{matrix}. \quad (32)$$

where $v_j^{k-} = \min_i v_{ij}^k$ ($j = 1, 2, \dots, n; k = 1, 2, \dots, K$). Next, the distances of each alternative A_i represented by matrix A^i ($i = 1, \dots, m$) from PIS

$$d_i^+ = \sum_{k=1}^K \sum_{j=1}^n d(v_{ij}^k, v_j^{k+}) \quad (33)$$

and from NIS

$$d_i^- = \sum_{k=1}^K \sum_{j=1}^n d(v_{ij}^k, v_j^{k-}) \quad (34)$$

are calculated. Using these distances, the relative closeness coefficients RC_i to PIS for each alternative A_i is calculated

$$RC_i = \frac{d_i^-}{d_i^- + d_i^+}. \quad (35)$$

According to the descending values of RC_i , all alternatives A_i ($i = 1, \dots, m$) are ranked and the best one is selected.

Remark 2 Note that if there is only one DM, i.e. $K = 1$, then the proposed approach is equivalent to the extended TOPSIS method proposed by Roszkowska and Kacprzak (2016).

Remark 3 Note that if we take into account the form of the matrices A^i ($i = 1, 2, \dots, m$), the positive ideal solution A^+ and the negative ideal solution A^- , the proposed approach can be regarded as a simultaneous application of the extended TOPSIS for each of the DMs represented by the corresponding rows of these matrices.

4.5 The procedure of the proposed approach

To sum up, the steps of the proposed approach for group decision making are as follows.

- Step 1* Construction of the fuzzy decision matrix $X^k = [x_{ij}^k]_{m \times n}$ for each decision maker DM_k ($k = 1, \dots, K$) in the form (26).
- Step 2* Normalization of the fuzzy decision matrices $X^k = [x_{ij}^k]_{m \times n}$ into matrices $Y^k = [y_{ij}^k]_{m \times n}$ using the Eq. (28).
- Step 3* Calculation of the weighted normalized fuzzy decision matrices $V^k = [v_{ij}^k]_{m \times n}$ using the Eq. (29), for the given vector of criteria weights $w = (w_1, w_2, \dots, w_n)$.
- Step 4* Transformation of the weighted normalized fuzzy decision matrices $V^k = [v_{ij}^k]_{m \times n}$ for all DMs, using Eq. (30), into normalized fuzzy decision matrices for all alternatives $A^i = [v_{ij}^k]_{K \times n}$ ($i = 1, \dots, m$).
- Step 5* Determination of the positive ideal solution $A^+ = [v_j^{k+}]_{K \times n}$ and the negative ideal solution $A^- = [v_j^{k-}]_{K \times n}$, using Eqs. (31) and (32), respectively.
- Step 6* Calculation of the distances of each alternative A_i represented by matrix A^i ($i = 1, \dots, m$), from PIS and from NIS, using Eqs. (33) and (34), respectively.
- Step 7* Calculation of the relative closeness coefficients to PIS for each alternative A_i ($i = 1, \dots, m$), using Eq. (35).
- Step 8* Rank ordering of the alternatives and the selection of the best one.

5 Numerical example

In this section, we examine a numerical example taken from Roszkowska and Kacprzak (2016), where the authors used arithmetic mean to aggregate the decision matrices of DMs; we use the proposed approach.

Let us consider the following situation: a company intends to hire a secretary and five candidates (alternatives) $\{A_1, A_2, A_3, A_4, A_5\}$ are available for evaluation. A committee of five decision makers $\{DM_1, DM_2, DM_3, DM_4, DM_5\}$ have conducted interviews to select the most suitable candidate. They have considered five benefit criteria $\{C_1, C_2, C_3, C_4, C_5\}$, where C_1 —emotional steadiness, C_2 —oral communication skills, C_3 —personality, C_4 —past experience, C_5 —self-confidence, with the weight vector $w =$

Fig. 8 The hierarchical structure of the considered decision problem

Table 1 Linguistic variables for the ratings of the alternatives and their representation by OFNs

Linguistic variables	Description	OFN
Very Poor	1	(1, 1, 1, 1)
<i>More than Very Poor</i>	$M(1)$	(1, 1, 1.5, 2)
<i>Less than Poor</i>	$L(2)$	(2, 2, 1.5, 1)
Poor	2	(2, 2, 2, 2)
<i>More than Poor</i>	$M(2)$	(2, 2, 2.5, 3)
<i>Less than Average</i>	$L(3)$	(3, 3, 2.5, 2)
Average	3	(3, 3, 3, 3)
<i>More than Average</i>	$M(3)$	(3, 3, 3.5, 4)
<i>Less than Good</i>	$L(4)$	(4, 4, 3.5, 3)
Good	4	(4, 4, 4, 4)
<i>More than Good</i>	$M(4)$	(4, 4, 4.5, 5)
<i>Less than Very Good</i>	$L(5)$	(5, 5, 4.5, 4)
Very Good	5	(5, 5, 5, 5)

(0.3, 0.3, 0.2, 0.1, 0.1). The hierarchical structure of this decision problem is shown in Fig. 8.

The DMs have used linguistic variables to rate the candidates (alternatives) with respect to the criteria (see the first column of Table 1) and their evaluations using descriptions of linguistic variables are shown in Table 2. Next, these descriptions of linguistic variables are expressed as positive trapezoidal OFNs (see the third column of Table 1). Because all the criteria are on the same ordinal scale, normalization is not needed. Using the vector $w = (0.3, 0.3, 0.2, 0.1, 0.1)$ of criteria weights, the weighted normalized fuzzy decision matrix is calculated for each DM (see Table 3). Next, these matrices are transformed into the weighed normalized fuzzy decision matrices for each alternative (see Table 4). Using these matrices, the positive ideal solution A^+ and the negative ideal solution A^- are determined (see Table 5). Finally, the distances of each alternative from the positive ideal solution d_i^+ and the negative ideal solution d_i^- , as well as the relative closeness coefficients RC_i are calculated; finally, the alternatives are rank ordered (see Table 6)

$$A_1 < A_4 < A_2 < A_5 < A_3$$

where $<$ means “inferior to”. Hence, alternative (candidate) A_3 should be selected.

Table 2 Decision matrices provided by decision makers

<i>DMs</i>	<i>ALT.</i>	<i>CRITERIA</i>				
		<i>C</i> ₁	<i>C</i> ₂	<i>C</i> ₃	<i>C</i> ₄	<i>C</i> ₅
<i>DM</i> ₁	<i>A</i> ₁	3	<i>L</i> (3)	5	<i>L</i> (5)	<i>M</i> (2)
	<i>A</i> ₂	<i>M</i> (3)	<i>L</i> (3)	5	<i>M</i> (4)	<i>L</i> (3)
	<i>A</i> ₃	3	<i>L</i> (4)	5	<i>M</i> (4)	<i>M</i> (2)
	<i>A</i> ₄	<i>L</i> (3)	<i>L</i> (3)	5	<i>M</i> (4)	<i>M</i> (2)
	<i>A</i> ₅	<i>M</i> (3)	<i>L</i> (3)	5	<i>M</i> (4)	<i>M</i> (2)
<i>DM</i> ₂	<i>A</i> ₁	3	3	5	<i>M</i> (4)	<i>M</i> (2)
	<i>A</i> ₂	3	<i>M</i> (3)	5	<i>M</i> (4)	<i>M</i> (2)
	<i>A</i> ₃	3	<i>M</i> (3)	5	<i>M</i> (4)	<i>M</i> (2)
	<i>A</i> ₄	3	<i>M</i> (3)	5	<i>M</i> (4)	<i>M</i> (2)
	<i>A</i> ₅	<i>L</i> (4)	<i>L</i> (3)	5	<i>M</i> (4)	<i>M</i> (2)
<i>DM</i> ₃	<i>A</i> ₁	<i>M</i> (3)	<i>M</i> (3)	<i>L</i> (5)	5	<i>M</i> (2)
	<i>A</i> ₂	3	<i>M</i> (3)	<i>L</i> (5)	5	<i>M</i> (2)
	<i>A</i> ₃	<i>L</i> (3)	<i>M</i> (3)	<i>L</i> (5)	5	<i>M</i> (2)
	<i>A</i> ₄	3	<i>M</i> (3)	<i>L</i> (5)	5	<i>M</i> (2)
	<i>A</i> ₅	<i>M</i> (3)	<i>L</i> (3)	<i>L</i> (5)	5	<i>M</i> (2)
<i>DM</i> ₄	<i>A</i> ₁	<i>M</i> (3)	<i>M</i> (2)	<i>L</i> (5)	<i>M</i> (4)	2
	<i>A</i> ₂	3	3	<i>L</i> (5)	<i>M</i> (4)	2
	<i>A</i> ₃	3	3	<i>L</i> (5)	<i>M</i> (4)	2
	<i>A</i> ₄	<i>M</i> (3)	3	<i>L</i> (5)	<i>M</i> (4)	2
	<i>A</i> ₅	<i>M</i> (3)	3	<i>L</i> (5)	<i>M</i> (4)	2
<i>DM</i> ₅	<i>A</i> ₁	<i>M</i> (3)	3	<i>L</i> (5)	<i>L</i> (5)	2
	<i>A</i> ₂	3	3	<i>L</i> (5)	<i>L</i> (5)	2
	<i>A</i> ₃	3	3	<i>L</i> (5)	<i>L</i> (5)	2
	<i>A</i> ₄	<i>M</i> (3)	3	<i>L</i> (5)	<i>L</i> (5)	2
	<i>A</i> ₅	<i>M</i> (3)	3	<i>L</i> (5)	<i>L</i> (5)	2

Now we perform sensitivity analysis, which is essential in evaluating the influence of changing the weights of evaluation criteria on the ranking of alternatives. One element of the weight vector w will be changed, the rest will be adjusted, and the resulting ranking of alternatives will be analyzed. Let w_j^m be the modified weight w_j of the j th criterion, for $j = 1, 2, 3, 4, 5$. The remaining weights need to be adjusted and for this purpose the weight w_h of the h th criterion ($h \neq j$) is calculated using the formula

$$w_h^m = \frac{1 - w_j^m}{1 - w_j} \cdot w_h.$$

Table 7 presents the obtained results, where **CW**—the current criterion weight, **SR** - the range of changes of those criterion weights, for which the current ranking does not change (the same ranking), **SBA** - the range of changes of those criterion weights, for which the best alternative does not change (the same best alternative).

Based on the results presented in Table 7, we can conclude that the obtained ranking of alternatives is not a stable solution. For criteria C_1 and C_2 , even a small change of the weights (e.g. equal to 0.0036) may result in a new ranking of alternatives and another best alternative.

Table 3 Weighted normalized decision matrices of decision makers

DM_s	ALT_s	CRITERIA				
		C_1	C_2	C_3	C_4	C_5
DM_1	A_1	(0.90,0.90,0.90,0.90)	(0.90,0.90,0.75,0.60)	(1.00,1.00,1.00,1.00)	(0.50,0.50,0.45,0.40)	(0.20,0.20,0.25,0.30)
	A_2	(0.90,0.90,1.05,1.20)	(0.90,0.90,0.75,0.60)	(1.00,1.00,1.00,1.00)	(0.40,0.40,0.45,0.50)	(0.30,0.30,0.25,0.20)
	A_3	(0.90,0.90,0.90,0.90)	(1.20,1.20,1.05,0.90)	(1.00,1.00,1.00,1.00)	(0.40,0.40,0.45,0.50)	(0.20,0.20,0.25,0.30)
	A_4	(0.90,0.90,0.75,0.60)	(0.90,0.90,0.75,0.60)	(1.00,1.00,1.00,1.00)	(0.40,0.40,0.45,0.50)	(0.20,0.20,0.25,0.30)
	A_5	(0.90,0.90,1.05,1.20)	(0.90,0.90,0.75,0.60)	(1.00,1.00,1.00,1.00)	(0.40,0.40,0.45,0.50)	(0.20,0.20,0.25,0.30)
DM_2	A_1	(0.90,0.90,0.90,0.90)	(0.90,0.90,0.90,0.90)	(1.00,1.00,1.00,1.00)	(0.40,0.40,0.45,0.50)	(0.20,0.20,0.25,0.30)
	A_2	(0.90,0.90,0.90,0.90)	(0.90,0.90,1.05,1.20)	(1.00,1.00,1.00,1.00)	(0.40,0.40,0.45,0.50)	(0.20,0.20,0.25,0.30)
	A_3	(0.90,0.90,0.90,0.90)	(0.90,0.90,1.05,1.20)	(1.00,1.00,1.00,1.00)	(0.40,0.40,0.45,0.50)	(0.20,0.20,0.25,0.30)
	A_4	(0.90,0.90,0.90,0.90)	(0.90,0.90,1.05,1.20)	(1.00,1.00,1.00,1.00)	(0.40,0.40,0.45,0.50)	(0.20,0.20,0.25,0.30)
	A_5	(1.20,1.20,1.05,0.90)	(0.90,0.90,0.75,0.60)	(1.00,1.00,1.00,1.00)	(0.40,0.40,0.45,0.50)	(0.20,0.20,0.25,0.30)
DM_3	A_1	(0.90,0.90,1.05,1.20)	(0.90,0.90,1.05,1.20)	(1.00,1.00,0.90,0.80)	(0.50,0.50,0.50,0.50)	(0.20,0.20,0.25,0.30)
	A_2	(0.90,0.90,0.90,0.90)	(0.90,0.90,1.05,1.20)	(1.00,1.00,0.90,0.80)	(0.50,0.50,0.50,0.50)	(0.20,0.20,0.25,0.30)
	A_3	(0.90,0.90,0.75,0.60)	(0.90,0.90,1.05,1.20)	(1.00,1.00,0.90,0.80)	(0.50,0.50,0.50,0.50)	(0.20,0.20,0.25,0.30)
	A_4	(0.90,0.90,0.90,0.90)	(0.90,0.90,1.05,1.20)	(1.00,1.00,0.90,0.80)	(0.50,0.50,0.50,0.50)	(0.20,0.20,0.25,0.30)
	A_5	(0.90,0.90,1.05,1.20)	(0.90,0.90,0.75,0.60)	(1.00,1.00,0.90,0.80)	(0.50,0.50,0.50,0.50)	(0.20,0.20,0.25,0.30)
DM_4	A_1	(0.90,0.90,1.05,1.20)	(0.60,0.60,0.75,0.90)	(1.00,1.00,0.90,0.80)	(0.40,0.40,0.45,0.50)	(0.20,0.20,0.20,0.20)
	A_2	(0.90,0.90,0.90,0.90)	(0.90,0.90,0.90,0.90)	(1.00,1.00,0.90,0.80)	(0.40,0.40,0.45,0.50)	(0.20,0.20,0.20,0.20)
	A_3	(0.90,0.90,0.90,0.90)	(0.90,0.90,0.90,0.90)	(1.00,1.00,0.90,0.80)	(0.40,0.40,0.45,0.50)	(0.20,0.20,0.20,0.20)
	A_4	(0.90,0.90,1.05,1.20)	(0.90,0.90,0.90,0.90)	(1.00,1.00,0.90,0.80)	(0.40,0.40,0.45,0.50)	(0.20,0.20,0.20,0.20)
	A_5	(0.90,0.90,1.05,1.20)	(0.90,0.90,0.90,0.90)	(1.00,1.00,0.90,0.80)	(0.40,0.40,0.45,0.50)	(0.20,0.20,0.20,0.20)
DM_5	A_1	(0.90,0.90,1.05,1.20)	(0.90,0.90,0.90,0.90)	(1.00,1.00,0.90,0.80)	(0.40,0.40,0.45,0.50)	(0.20,0.20,0.20,0.20)
	A_2	(0.90,0.90,1.05,1.20)	(0.90,0.90,0.90,0.90)	(1.00,1.00,0.90,0.80)	(0.40,0.40,0.45,0.50)	(0.20,0.20,0.20,0.20)
	A_3	(0.90,0.90,0.90,0.90)	(0.90,0.90,0.90,0.90)	(1.00,1.00,0.90,0.80)	(0.50,0.50,0.45,0.40)	(0.20,0.20,0.20,0.20)
	A_4	(0.90,0.90,1.05,1.20)	(0.90,0.90,0.90,0.90)	(1.00,1.00,0.90,0.80)	(0.50,0.50,0.45,0.40)	(0.20,0.20,0.20,0.20)
	A_5	(0.90,0.90,1.05,1.20)	(0.90,0.90,0.90,0.90)	(1.00,1.00,0.90,0.80)	(0.50,0.50,0.45,0.40)	(0.20,0.20,0.20,0.20)

Table 4 Weighted normalized decision matrices of alternatives

ALT.	DM_s	CRITERIA				
		C_1	C_2	C_3	C_4	C_5
A_1	DM_1	(0.90,0.90,0.90,0.90)	(0.90,0.90,0.75,0.60)	(1.00,1.00,1.00,1.00)	(0.50,0.50,0.45,0.40)	(0.20,0.20,0.25,0.30)
	DM_2	(0.90,0.90,0.90,0.90)	(0.90,0.90,0.90,0.90)	(1.00,1.00,1.00,1.00)	(0.40,0.40,0.45,0.50)	(0.20,0.20,0.25,0.30)
	DM_3	(0.90,0.90,1.05,1.20)	(0.90,0.90,1.05,1.20)	(1.00,1.00,0.90,0.80)	(0.50,0.50,0.50,0.50)	(0.20,0.20,0.25,0.30)
	DM_4	(0.90,0.90,1.05,1.20)	(0.60,0.60,0.75,0.90)	(1.00,1.00,0.90,0.80)	(0.40,0.40,0.45,0.50)	(0.20,0.20,0.20,0.20)
	DM_5	(0.90,0.90,1.05,1.20)	(0.90,0.90,0.90,0.90)	(1.00,1.00,0.90,0.80)	(0.50,0.50,0.45,0.40)	(0.20,0.20,0.20,0.20)
A_2	DM_1	(0.90,0.90,1.05,1.20)	(0.90,0.90,0.75,0.60)	(1.00,1.00,1.00,1.00)	(0.40,0.40,0.45,0.50)	(0.30,0.30,0.25,0.20)
	DM_2	(0.90,0.90,0.90,0.90)	(0.90,0.90,1.05,1.20)	(1.00,1.00,1.00,1.00)	(0.40,0.40,0.45,0.50)	(0.20,0.20,0.25,0.30)
	DM_3	(0.90,0.90,0.90,0.90)	(0.90,0.90,1.05,1.20)	(1.00,1.00,0.90,0.80)	(0.50,0.50,0.50,0.50)	(0.20,0.20,0.25,0.30)
	DM_4	(0.90,0.90,0.90,0.90)	(0.90,0.90,0.90,0.90)	(1.00,1.00,0.90,0.80)	(0.40,0.40,0.45,0.50)	(0.20,0.20,0.20,0.20)
	DM_5	(0.90,0.90,0.90,0.90)	(0.90,0.90,0.90,0.90)	(1.00,1.00,0.90,0.80)	(0.50,0.50,0.45,0.40)	(0.20,0.20,0.20,0.20)
A_3	DM_1	(0.90,0.90,0.90,0.90)	(1.20,1.20,1.05,0.90)	(1.00,1.00,1.00,1.00)	(0.40,0.40,0.45,0.50)	(0.20,0.20,0.25,0.30)
	DM_2	(0.90,0.90,0.90,0.90)	(0.90,0.90,1.05,1.20)	(1.00,1.00,1.00,1.00)	(0.40,0.40,0.45,0.50)	(0.20,0.20,0.25,0.30)
	DM_3	(0.90,0.90,0.75,0.60)	(0.90,0.90,1.05,1.20)	(1.00,1.00,0.90,0.80)	(0.50,0.50,0.50,0.50)	(0.20,0.20,0.25,0.30)
	DM_4	(0.90,0.90,0.90,0.90)	(0.90,0.90,0.90,0.90)	(1.00,1.00,0.90,0.80)	(0.40,0.40,0.45,0.50)	(0.20,0.20,0.20,0.20)
	DM_5	(0.90,0.90,0.90,0.90)	(0.90,0.90,0.90,0.90)	(1.00,1.00,0.90,0.80)	(0.50,0.50,0.45,0.40)	(0.20,0.20,0.20,0.20)
A_4	DM_1	(0.90,0.90,0.75,0.60)	(0.90,0.90,0.75,0.60)	(1.00,1.00,1.00,1.00)	(0.40,0.40,0.45,0.50)	(0.20,0.20,0.25,0.30)
	DM_2	(0.90,0.90,0.90,0.90)	(0.90,0.90,1.05,1.20)	(1.00,1.00,1.00,1.00)	(0.40,0.40,0.45,0.50)	(0.20,0.20,0.25,0.30)
	DM_3	(0.90,0.90,0.90,0.90)	(0.90,0.90,0.90,0.90)	(1.00,1.00,0.90,0.80)	(0.50,0.50,0.50,0.50)	(0.20,0.20,0.25,0.30)
	DM_4	(0.90,0.90,0.90,0.90)	(0.90,0.90,0.90,0.90)	(1.00,1.00,0.90,0.80)	(0.40,0.40,0.45,0.50)	(0.20,0.20,0.20,0.20)
	DM_5	(0.90,0.90,0.90,0.90)	(0.90,0.90,0.90,0.90)	(1.00,1.00,0.90,0.80)	(0.50,0.50,0.45,0.40)	(0.20,0.20,0.20,0.20)
A_5	DM_1	(0.90,0.90,1.05,1.20)	(0.90,0.90,0.90,0.90)	(1.00,1.00,1.00,1.00)	(0.40,0.40,0.45,0.50)	(0.20,0.20,0.25,0.30)
	DM_2	(0.90,0.90,1.05,1.20)	(0.90,0.90,1.05,1.20)	(1.00,1.00,1.00,1.00)	(0.40,0.40,0.45,0.50)	(0.20,0.20,0.25,0.30)
	DM_3	(0.90,0.90,0.90,0.90)	(0.90,0.90,1.05,1.20)	(1.00,1.00,0.90,0.80)	(0.50,0.50,0.50,0.50)	(0.20,0.20,0.25,0.30)
	DM_4	(0.90,0.90,1.05,1.20)	(0.90,0.90,0.90,0.90)	(1.00,1.00,0.90,0.80)	(0.40,0.40,0.45,0.50)	(0.20,0.20,0.20,0.20)
	DM_5	(0.90,0.90,1.05,1.20)	(0.90,0.90,0.90,0.90)	(1.00,1.00,0.90,0.80)	(0.50,0.50,0.45,0.40)	(0.20,0.20,0.20,0.20)
A_5	DM_1	(0.90,0.90,1.05,1.20)	(0.90,0.90,0.90,0.90)	(1.00,1.00,0.90,0.80)	(0.50,0.50,0.45,0.40)	(0.20,0.20,0.20,0.20)
	DM_2	(0.90,0.90,1.05,1.20)	(0.90,0.90,0.75,0.60)	(1.00,1.00,1.00,1.00)	(0.40,0.40,0.45,0.50)	(0.20,0.20,0.25,0.30)
	DM_3	(1.20,1.20,1.05,0.90)	(0.90,0.90,0.75,0.60)	(1.00,1.00,1.00,1.00)	(0.40,0.40,0.45,0.50)	(0.20,0.20,0.25,0.30)
	DM_4	(0.90,0.90,1.05,1.20)	(0.90,0.90,0.75,0.60)	(1.00,1.00,0.90,0.80)	(0.50,0.50,0.50,0.50)	(0.20,0.20,0.25,0.30)
	DM_5	(0.90,0.90,1.05,1.20)	(0.90,0.90,0.90,0.90)	(1.00,1.00,0.90,0.80)	(0.40,0.40,0.45,0.50)	(0.20,0.20,0.20,0.20)

Table 5 Positive ideal solution and negative ideal solution

PIS/NIS	ALT.	CRITERIA				
		C_1	C_2	C_3	C_4	C_5
A^+	DM_1	(1.5,1.5,1.5,1.5)	(1.5,1.5,1.5,1.5)	(1.0,1.0,1.0,1.0)	(0.5,0.5,0.5,0.5)	(0.5,0.5,0.5,0.5)
	DM_2	(1.5,1.5,1.5,1.5)	(1.5,1.5,1.5,1.5)	(1.0,1.0,1.0,1.0)	(0.5,0.5,0.5,0.5)	(0.5,0.5,0.5,0.5)
	DM_3	(1.5,1.5,1.5,1.5)	(1.5,1.5,1.5,1.5)	(1.0,1.0,1.0,1.0)	(0.5,0.5,0.5,0.5)	(0.5,0.5,0.5,0.5)
	DM_4	(1.5,1.5,1.5,1.5)	(1.5,1.5,1.5,1.5)	(1.0,1.0,1.0,1.0)	(0.5,0.5,0.5,0.5)	(0.5,0.5,0.5,0.5)
	DM_5	(1.5,1.5,1.5,1.5)	(1.5,1.5,1.5,1.5)	(1.0,1.0,1.0,1.0)	(0.5,0.5,0.5,0.5)	(0.5,0.5,0.5,0.5)
A^-	DM_1	(0.3,0.3,0.3,0.3)	(0.3,0.3,0.3,0.3)	(0.2,0.2,0.2,0.2)	(0.1,0.1,0.1,0.1)	(0.1,0.1,0.1,0.1)
	DM_2	(0.3,0.3,0.3,0.3)	(0.3,0.3,0.3,0.3)	(0.2,0.2,0.2,0.2)	(0.1,0.1,0.1,0.1)	(0.1,0.1,0.1,0.1)
	DM_3	(0.3,0.3,0.3,0.3)	(0.3,0.3,0.3,0.3)	(0.2,0.2,0.2,0.2)	(0.1,0.1,0.1,0.1)	(0.1,0.1,0.1,0.1)
	DM_4	(0.3,0.3,0.3,0.3)	(0.3,0.3,0.3,0.3)	(0.2,0.2,0.2,0.2)	(0.1,0.1,0.1,0.1)	(0.1,0.1,0.1,0.1)
	DM_5	(0.3,0.3,0.3,0.3)	(0.3,0.3,0.3,0.3)	(0.2,0.2,0.2,0.2)	(0.1,0.1,0.1,0.1)	(0.1,0.1,0.1,0.1)

Table 6 The distances of each alternative from the positive ideal solution d_i^+ and the negative ideal solution d_i^- , the relative closeness coefficients RC_i and the ranking order R of alternatives

ALT.	d_i^+	d_i^-	RC_i	R
A_1	7.92747	12.45687	0.61110	5
A_2	7.82153	12.50202	0.61515	3
A_3	7.77392	12.55192	0.61754	1
A_4	7.87258	12.50432	0.61365	4
A_5	7.82670	12.60470	0.61693	2

Table 7 Ranges of criterion weights which do not affect the current solutions

Criterion	CW	SR	SBA
C_1	0.3	[0.2845,0.3036]	[0.0000,0.3036]
C_2	0.3	[0.2964,0.3170]	[0.2964,1.0000]
C_3	0.2	[0.0000,1.0000]	[0.0000,1.0000]
C_4	0.1	[0.0000,0.2795]	[0.0000,0.4582]
C_5	0.1	[0.0000,0.2290]	[0.0000,0.2494]

Taking into account the ranges of weight changes for criteria C_4 and C_5 which do not affect the ranking and selection of the best alternative we can see that they are much larger (equal to at least 0.229) than those for criteria C_1 and C_2 . Moreover, we can see that the weight of criterion C_3 does not affect either the ranking of alternatives or the selection of the best one. The sensitivity analysis for the criteria is presented in Fig. 9. Figure 9a shows how the weights of the remaining criteria change when the weight of the chosen criterion changes. Figure 9b shows how the ranking of alternatives changes when the weight of the chosen criterion changes.

Fig. 9 Sensitivity analysis for the criteria with respect to the ranking of alternatives

Fig. 10 The hierarchical structure of the classical TOPSIS

6 Comparison of the proposed approach with other and similar approaches

The proposed approach will be compared with other methods. Figures 10, 11 and 12 show the hierarchical structure of the classical TOPSIS (Hwang and Yoon 1981), the TOPSIS for group decision making with aggregation of individual decision matrices, and the proposed approach, respectively. Table 8 illustrates the differences and similarities between the proposed approach (PA), the classical TOPSIS (CT), and the TOPSIS for group decision making with aggregation of individual decision matrices using arithmetic mean based on OFNs (AT) proposed by Roszkowska and Kacprzak (2016).

Now we will compare the proposed approach with methods which aggregate the individual weighted normalized decision matrixes $V^k = [v_{ij}^k]$ into an aggregated collective matrix $V = [v_{ij}]$, which is the starting point for the ranking of the alternatives or the selection of the best one, based on data from the example in Sect. 5 (Table 2). For the comparison we use the following aggregation methods:

- **AGG1** - arithmetic mean (Chen 2000; Wang and Chang 2007; Roszkowska and Kacprzak 2016), defined by

$$v_{ij} = \frac{1}{K} \sum_{k=1}^K v_{ij}^k = \left(\frac{1}{K} \sum_{k=1}^K f_{v_{ij}^k}(0), \frac{1}{K} \sum_{k=1}^K f_{v_{ij}^k}(1), \frac{1}{K} \sum_{k=1}^K g_{v_{ij}^k}(1), \frac{1}{K} \sum_{k=1}^K g_{v_{ij}^k}(0) \right),$$

Fig. 11 The hierarchical structure of the TOPSIS method for group decision making with aggregation of individual decision matrices

- **AGG2** - geometric mean (Shih et al. 2007; Ye and Li 2009), defined by

$$v_{ij} = \left(\prod_{k=1}^K v_{ij}^k \right)^{\frac{1}{K}} = \left(\left(\prod_{k=1}^K f_{v_{ij}^k}(0) \right)^{\frac{1}{K}}, \left(\prod_{k=1}^K f_{v_{ij}^k}(1) \right)^{\frac{1}{K}}, \left(\prod_{k=1}^K g_{v_{ij}^k}(1) \right)^{\frac{1}{K}}, \left(\prod_{k=1}^K g_{v_{ij}^k}(0) \right)^{\frac{1}{K}} \right),$$

- **AGG3** - modified arithmetic mean (Shemshadi et al. 2011; Nadaban et al. 2016), defined by

$$v_{ij} = \left(\min_k f_{v_{ij}^k}(0), \frac{1}{K} \sum_{k=1}^K f_{v_{ij}^k}(1), \frac{1}{K} \sum_{k=1}^K g_{v_{ij}^k}(1), \max_k g_{v_{ij}^k}(0) \right),$$

- **AGG4** - modified geometric mean (Ding 2011; Chang et al. 2009; Hatami-Marbini and Kangi 2017), defined by

$$v_{ij} = \left(\min_k f_{v_{ij}^k}(0), \left(\prod_{k=1}^K f_{v_{ij}^k}(1) \right)^{\frac{1}{K}}, \left(\prod_{k=1}^K g_{v_{ij}^k}(1) \right)^{\frac{1}{K}}, \max_k g_{v_{ij}^k}(0) \right),$$

- **AGG5** - weighted mean (Kacprzak 2019), defined by

$$v_{ij} = \sum_{k=1}^K \lambda_k v_{ij}^k = \left(\sum_{k=1}^K \lambda_k f_{v_{ij}^k}(0), \sum_{k=1}^K \lambda_k f_{v_{ij}^k}(1), \sum_{k=1}^K \lambda_k g_{v_{ij}^k}(1), \sum_{k=1}^K \lambda_k g_{v_{ij}^k}(0) \right),$$

where λ_k is the weight of the k th DM.

Fig. 12 The hierarchical structure of the proposed TOPSIS method for group decision making

Table 9 shows the distance of each alternative A_i from the positive ideal solution d_i^+ and the negative ideal solution d_i^- , as well as the relative closeness coefficients RC_i and ranking R of the alternatives using the proposed approach and different aggregation methods. The last column, denoted by J , consists of the normalized (summing up to 1) values of the relative closeness coefficients of each alternative to the ideal solution, which allows to highlight the differences between the final scores of the alternatives. Next, Table 10 and Fig. 13 show the ranking of the alternatives. Let us note that the proposed approach and the aggregation methods using arithmetic mean and geometric mean give the same ranking of the alternatives

$$A_1 < A_4 < A_2 < A_5 < A_3.$$

Moreover, taking into account column J in Table 9 and Fig. 13b, we can notice that the methods PA, AGG1 and AGG2 give a similar final score for each alternative. The aggregation methods AGG3 and AGG4 also give the same ranking of the alternatives

$$A_1 < A_5 < A_3 < A_4 < A_2$$

Table 8 The differences and similarities between the proposed approach (PA), the classical TOPSIS (CT) and TOPSIS for group decision making with aggregation of individual decision matrices using arithmetic mean based on OFNs (AT)

Characteristic	CT	AT	PA
Number of decision makers	One	More than one	More than one
Main information	Decision matrix X expressed by real numbers	Decision matrices X^k of each DM expressed by OFNs	Decision matrices X^k of each DM expressed by OFNs
Weights of criteria	Given	Given	Given
PIS	The best alternative expressed by a vector of real numbers	The best alternative expressed by a vector of OFNs	The best alternative expressed by a matrix of OFNs
NIS	The worst alternative expressed by a vector of real numbers	The worst alternative expressed by a vector of OFNs	The worst alternative expressed by a matrix of OFNs
Distance of each alternative to PIS/NIS	Distance between two vectors	Distance between two vectors	Distance between two matrices
The relative closeness coefficients	$RC_i = \frac{d_i^-}{d_i^- + d_i^+}$	$RC_i = \frac{d_i^-}{d_i^- + d_i^+}$	$RC_i = \frac{d_i^-}{d_i^- + d_i^+}$
Final decision	Ranking of alternatives	Ranking of alternatives	Ranking of alternatives

but different from that given by the methods considered earlier. These methods swap the order of alternatives A_4 and A_5 and alternatives A_2 and A_3 . The aggregation methods AGG5 give the following ranking of alternatives

$$A_1 < A_4 < A_2 < A_3 < A_5$$

and swap the order of alternatives A_3 and A_5 as compared to the methods PA, AGG1 and AGG2. This means that the final ranking order of the alternatives and the choice of the best one (in our example, the successful candidate) depend on the method used. Taking into account column J in Table 9 and Fig. 13b, we can notice that the aggregation methods using the modified arithmetic mean and the modified geometric mean result in a fairly high score of alternatives A_2 and A_4 and a fairly low final score of the remaining alternatives in comparison with the other methods analysed (which result in more diverse final scores).

Moreover, using the same data (from the example presented in Sect. 5, Table 2), we will compare the application of the CFNs model and the real numbers (RNs) model with the application of the OFNs model in the proposed approach. For this purpose, the negative orientation of OFNs in the decision matrices (Table 2) will be changed to positive orientation. For instance, the linguistic variables “less than i ” of the form $L(i) = (i, i, \frac{2i-1}{2}, i-1)$ will be replaced by fuzzy numbers of the form $(i-1, \frac{2i-1}{2}, i, i)$. In this situation, the use of the OFNs model is no different from the application of the CFNs model, while the RNs model will be obtained by the application of a defuzzification method in the decision matrices (Table 2). We use the very commonly used and well-known method called centre of gravity (CoG) defined by

$$\phi_{CoG}(A) = \frac{\int_0^1 \frac{f_A(y)+g_A(y)}{2} \cdot |f_A(y) - g_A(y)| dy}{\int_0^1 |f_A(y) - g_A(y)| dy}$$

which for the trapezoidal OFN $A = (f_A(0), f_A(1), g_A(1), g_A(0))$ has the form

Table 9 The comparison of the results using the proposed approach and different aggregation methods

Method	ALT.	d_i^+	d_i^-	RC_i	R	J
PA	A_1	7.92747	12.45687	0.61110	5	0.19877
	A_2	7.82153	12.50202	0.61515	3	0.20009
	A_3	7.77392	12.55192	0.61754	1	0.20087
	A_4	7.87258	12.50432	0.61365	4	0.19960
	A_5	7.82670	12.60470	0.61693	2	0.20067
AGG1	A_1	1.56138	2.47635	0.61330	5	0.19886
	A_2	1.54374	2.48579	0.61689	3	0.20002
	A_3	1.53475	2.49801	0.61943	1	0.20084
	A_4	1.54972	2.48302	0.61571	4	0.19964
	A_5	1.54367	2.50605	0.61882	2	0.20064
AGG2	A_1	1.58222	2.45487	0.60808	5	0.19865
	A_2	1.56204	2.46777	0.61238	3	0.20006
	A_3	1.55055	2.48321	0.61561	1	0.20111
	A_4	1.57294	2.45910	0.60989	4	0.19925
	A_5	1.55982	2.49218	0.61505	2	0.20093
AGG3	A_1	1.53340	2.60479	0.62945	5	0.19618
	A_2	1.42215	2.66798	0.65230	1	0.20330
	A_3	1.47031	2.58998	0.63788	3	0.19881
	A_4	1.42756	2.66406	0.65110	2	0.20293
	A_5	1.47179	2.59097	0.63774	4	0.19877
AGG4	A_1	1.53986	2.59888	0.62794	5	0.19608
	A_2	1.42705	2.66398	0.65118	1	0.20334
	A_3	1.47510	2.58590	0.63676	3	0.19884
	A_4	1.43303	2.65953	0.64984	2	0.20292
	A_5	1.47625	2.58740	0.63672	4	0.19882
AGG5	A_1	1.56243	2.47623	0.61313	5	0.19886
	A_2	1.54586	2.48401	0.61640	3	0.19992
	A_3	1.53872	2.49429	0.61847	2	0.20059
	A_4	1.54723	2.48608	0.61639	4	0.19992
	A_5	1.54342	2.50597	0.61885	1	0.20071

Table 10 The comparison of the rankings of alternatives based on the relative closeness coefficients using the proposed method and different aggregation methods

AGG	RANKING - RC_i
PA	$A_1 < A_4 < A_2 < A_5 < A_3$
AGG1	$A_1 < A_4 < A_2 < A_5 < A_3$
AGG2	$A_1 < A_4 < A_2 < A_5 < A_3$
AGG3	$A_1 < A_5 < A_3 < A_4 < A_2$
AGG4	$A_1 < A_5 < A_3 < A_4 < A_2$
AGG5	$A_1 < A_4 < A_2 < A_3 < A_5$

Fig. 13 The comparison of the rankings of alternatives using the proposed approach and different aggregation methods based on: **a** RC_i , **b** J

$$\phi_{CoG}(A) = \frac{[f_A(0)]^2 + [f_A(1)]^2 + f_A(0)f_A(1) - [g_A(1)]^2 - [g_A(0)]^2 - g_A(1)g_A(0)}{3(f_A(0) + f_A(1) - g_A(1) - g_A(0))}.$$

The weakness of the CoG method for OFNs is that it is not sensitive to the orientation of OFNs. For this reason, we will also use two simple methods sensitive to the orientation of OFNs:

- first of maximum (FOM) defined by

$$\phi_{FOM}(A) = f_A(1)$$

- last of maximum (LOM) defined by

$$\phi_{LOM}(A) = g_A(1).$$

The real numbers obtained using these defuzzification methods will be written as OFNs as follows: $A = (\phi_{df}(A), \phi_{df}(A), \phi_{df}(A), \phi_{df}(A))$, where $df \in \{CoG, FOM, LOM\}$. The results obtained using the proposed approach are presented in Tables 11 and 12 and in Fig. 14. If we compare OFNs and CFNs in the proposed approach (Table 11 and Fig. 14), we can see that when we disregard the orientation of OFNs, for instance by using CFNs, we obtain the same final scores of the alternatives, that is, the same ranking of the alternatives. This is due to the method of calculation of the distance between fuzzy numbers (see formula (6)) and to the determination of PIS and NIS in the numerical example (see Table 5). Moreover, if we use the RNs obtained from OFNs using CoG, we can see that the alternatives A_3 and A_5 are not distinguished ($\approx$ means “equivalent”), and the same result is obtained when FOM is applied. Furthermore, LOM results in the same final score for all the alternatives: it does not distinguish between the alternatives at all. The results of LOM and FOM are due to the fact that when OFNs are used, we have

$$\phi_{FOM}(M(i)) = \phi_{FOM}(L(i))$$

and

$$\phi_{LOM}(M(i)) = \phi_{LOM}(L(i + 1)).$$

Finally, we use sensitivity analysis to compare the proposed method with the TOPSIS method with aggregation of individual matrices into a collective matrix using arithmetic mean (Roszkowska and Kacprzak 2016). The results are presented in Table 13 and Fig. 15.

From Tables 7 and 13 we can conclude that the results obtained for criteria C_1 , C_2 and C_3 are almost identical. Taking into account the ranges of changes of the weight of criterion

Table 11 The comparison of the results using OFNs, CFNs and RNs in the proposed approach

Method	ALT.	d_i^+	d_i^-	RC_i	R	J
OFNs	A_1	7.92747	12.45687	0.61110	5	0.19877
	A_2	7.82153	12.50202	0.61515	3	0.20009
	A_3	7.77392	12.55192	0.61754	1	0.20087
	A_4	7.87258	12.50432	0.61365	4	0.19960
	A_5	7.82670	12.60470	0.61693	2	0.20067
CFNs	A_1	7.92747	12.45687	0.61110	5	0.19877
	A_2	7.82153	12.50202	0.61515	3	0.20009
	A_3	7.77392	12.55192	0.61754	1	0.20087
	A_4	7.87258	12.50432	0.61365	4	0.19960
	A_5	7.82670	12.60470	0.61693	2	0.20067
RNs						
CoG	A_1	7.65000	12.35000	0.61750	5	0.19893
	A_2	7.58333	12.41667	0.62083	3	0.20000
	A_3	7.53889	12.46111	0.62306	1	0.20072
	A_4	7.60556	12.39444	0.61972	4	0.19964
	A_5	7.53889	12.46111	0.62306	1	0.20072
FOM	A_1	8.00000	12.00000	0.60000	5	0.19512
	A_2	7.70000	12.30000	0.61500	3	0.20000
	A_3	7.50000	12.50000	0.62500	1	0.20325
	A_4	7.80000	12.20000	0.61000	4	0.19837
	A_5	7.50000	12.50000	0.62500	1	0.20325
LOM	A_1	7.55000	12.45000	0.62250	1	0.20000
	A_2	7.55000	12.45000	0.62250	1	0.20000
	A_3	7.55000	12.45000	0.62250	1	0.20000
	A_4	7.55000	12.45000	0.62250	1	0.20000
	A_5	7.55000	12.45000	0.62250	1	0.20000

Table 12 The comparison of the rankings of alternatives based on the relative closeness coefficients using OFNs, CFNs and RNs in the proposed approach

Method	RANKING - RC_i
OFNs	$A_1 < A_4 < A_2 < A_5 < A_3$
CFNs	$A_1 < A_4 < A_2 < A_5 < A_3$
RNs	
CoG	$A_1 < A_4 < A_2 < A_3 \approx A_5$
FOM	$A_1 < A_4 < A_2 < A_3 \approx A_5$
LOM	$A_1 \approx A_2 \approx A_3 \approx A_4 \approx A_5$

C_4 we can see that the proposed approach gives a much larger range for both the same ranking (**SR**) and the same best alternative (**SBA**) than the method proposed in Roszkowska and Kacprzak (2016). This means that the resulting ranking of alternatives is a more stable solution when the proposed approach is used. For criterion C_5 the proposed approach gives a slightly smaller range of weight changes than the method proposed in Roszkowska and

Fig. 14 The comparison of the rankings of alternatives using OFNs, CFNs and RNs in the proposed approach based on: **a** RC_i , **b** J

Table 13 Ranges of criterion weights which do not affect the current solutions for the TOPSIS method with aggregation of individual matrices into a collective matrix using arithmetic mean (Roszkowska and Kacprzak 2016)

Criterion	CW	SR	SBA
C_1	0.3	[0.2838, 0.3035]	[0.0000, 0.3035]
C_2	0.3	[0.2965, 0.3178]	[0.2965, 1.0000]
C_3	0.2	[0.0000, 1.0000]	[0.0000, 1.0000]
C_4	0.1	[0.0000, 0.2147]	[0.0000, 0.3395]
C_5	0.1	[0.0000, 0.2439]	[0.0000, 0.2667]

Kacprzak (2016), both for the ranking of alternatives (**SR**) and the selection of the best one (**SBA**).

7 Conclusion

In this paper an extended TOPSIS method based on OFNs for GDM problems has been presented. Most papers in the literature aggregate the individual decision matrices provided by the DMs into a collective decision matrix which is the starting point for the ranking of alternatives or the selection of the best one, using arithmetic mean, geometric mean or their modifications. In such a situation the MCDM problem for GDM reduces to a classical MCDM problem. Moreover, such an averaged result does not reflect the discrepancies between the individual assessments or the preferences of the DMs. By contrast, in the proposed approach, all individual decision data of the DMs are taken into account in determining the ranking of alternatives and the selection of the best one. This is because the distances of alternatives from the PIS and the NIS are distances between matrices.

The numerical example has shown that the proposed approach based on OFNs—as compared with other methods of aggregation of individual decision matrices of each DM based on OFNs and the omission of orientation of OFNs (which is equivalent to using CFNs) or their transformation into RNs (using selected defuzzification methods)—can give a different final result, both as regards the ranking of alternatives and the selection of the best one.

One of the very important steps in MCDM methods is finding appropriate weights of criteria. In future research methods of the determination of criteria weights (subjective, provided by each decision maker and objective, obtained from decision matrices using the entropy method, as well as integrated subjective and objective) for GDM using the OFNs model will be investigated.

Fig. 15 Sensitivity analysis for the criteria with respect to the ranking of alternatives for the TOPSIS method with aggregation of individual matrices into a collective matrix using arithmetic mean (Roszkowska and Kacprzak 2016)

Moreover, the proposed approach is specific to OFNs. In the future, we will continue working on other types of data, such as: real numbers, intervals numbers, hesitant fuzzy sets, interval-valued intuitionistic fuzzy information, and others.

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