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ERRATUM

Erratum to: Symplectic Dolbeault operators on Kähler manifolds

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The Hermitian structure used for a flag manifold G/T in Sect. 4 is not a Kähler structure unless G is a product of $SU(2)$'s. This is because the connection used is the canonical connection on the homogeneous space G/T, which has non-zero torsion since $[\mathfrak{t}^{\perp}, \mathfrak{t}^{\perp}] \not\subseteq \mathfrak{t}$. Indeed, using the metric to lower indices, the torsion at eT is the 3-form given by

$$(X, Y, Z) \mapsto -g_0([X, Y], Z), \quad X, Y, Z \in \mathfrak{t}^{\perp} \simeq T_{eT}G/T \quad [1].$$

Such a structure on G/T turns it into what is called a "Kähler with torsion (KT)" manifold. The general results and constructions established in the previous sections carry over unchanged to KT manifolds except for Proposition 3 which states that the symplectic Dirac operators D and $\tilde{D}$ are formally self-adjoint. For connections with torsion and parallel complex structure, a sufficient condition for D and $\tilde{D}$ to be self-adjoint is the vanishing of the torsion vector field, defined by

$$\mathcal{T} = \sum_{j=1}^{n} \mathbf{T}(a_j, b_j),$$

where $\mathbf{T}$ is the torsion of ∇ and $\{a_1, \ldots, a_n, b_1, \ldots, b_n\}$ is a symplectic frame [2]. In the case of flag manifolds, a symplectic basis at eT is proportional to $\{Z_\alpha, Z_{-\alpha}\}$ where Z_α is a root vector for α. Since $\mathbf{T}(Z_\alpha, Z_{-\alpha}) = -[Z_\alpha, Z_{-\alpha}]_{\mathfrak{t}^{\perp}} = 0$, we see that $\mathcal{T}$ vanishes. Thus the results concerning flag manifolds are correct using the symplectic Dolbeault operators corresponding to their KT structures.

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