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ERRATUM

Erratum to: Generalizing the Kantorovich Metric to Projection Valued Measures

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There is an error in the proof of Theorem 2.15 in the article Generalizing the Kantorovich Metric to Projection Valued Measures [1]. The result is still true. The theorem is restated below:

Theorem 2.15 [1] *The map $\Phi : P(X) \to P(X)$ given by*

$$E(\cdot) \mapsto \sum_{i=0}^{N-1} S_i E\big(\sigma_i^{-1}(\cdot)\big) S_i^*$$

is a Lipschitz contraction in the ρ metric.

The following is a corrected version of the part of the proof which contains the error. The author wants to acknowledge Krystal Taylor (University of Minnesota) for identifying the error.

The following argument replaces the argument which begins on the line after Claim 2.16. Let $E, F \in P(X)$. Recall that $r = \max_{0 \le i \le N-1}\{r_i\}$, where r_i is the Lipschitz constant associated to σ_i, and note that $0 < r < 1$. Choose $\phi \in \mathrm{Lip}_1(X)$, and $h \in \mathcal{H}$ with $\|h\| = 1$. Then

$$\left|\left\langle\left(\int \phi d\Phi(E) - \int \phi d\Phi(F)\right)h, h\right\rangle\right|$$

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$$= \left|\left\langle\left(\int \phi d\Phi(E)\right)h, h\right\rangle - \left\langle\left(\int \phi d\Phi(F)\right)h, h\right\rangle\right| = \left|\int_X \phi d\Phi(E)_{h,h} - \int_X \phi d\Phi(F)_{h,h}\right|$$

$$= \left|\sum_{i=0}^{N-1}\int_X \phi dE_{S_i^*h,S_i^*h}\big(\sigma_i^{-1}(\cdot)\big) - \sum_{i=0}^{N-1}\int_X \phi dF_{S_i^*h,S_i^*h}\big(\sigma_i^{-1}(\cdot)\big)\right|$$

$$= \left|\sum_{i=0}^{N-1}\int_X (\phi\circ\sigma_i) dE_{S_i^*h,S_i^*h} - \sum_{i=0}^{N-1}\int_X (\phi\circ\sigma_i) dF_{S_i^*h,S_i^*h}\right|$$

$$= \left|\sum_{i=0}^{N-1}\left(\int_X (\phi\circ\sigma_i) dE_{S_i^*h,S_i^*h} - \int_X (\phi\circ\sigma_i) dF_{S_i^*h,S_i^*h}\right)\right|$$

$$= \left|\sum_{i=0}^{N-1} r\left(\int_X \left(\frac{\phi\circ\sigma_i}{r}\right) dE_{S_i^*h,S_i^*h} - \int_X \left(\frac{\phi\circ\sigma_i}{r}\right) dF_{S_i^*h,S_i^*h}\right)\right|$$

$$\leq r\left(\sum_{i=0}^{N-1}\left|\int_X \left(\frac{\phi\circ\sigma_i}{r}\right) dE_{S_i^*h,S_i^*h} - \int_X \left(\frac{\phi\circ\sigma_i}{r}\right) dF_{S_i^*h,S_i^*h}\right|\right)$$

$$= r\left(\sum_{i=0}^{N-1}\left|\left\langle\left(\int \left(\frac{\phi\circ\sigma_i}{r}\right) dE - \int \left(\frac{\phi\circ\sigma_i}{r}\right) dF\right)S_i^*h, S_i^*h\right\rangle\right|\right)$$

$$\leq r\left(\sum_{i=0}^{N-1}\left\|\int \left(\frac{\phi\circ\sigma_i}{r}\right) dE - \int \left(\frac{\phi\circ\sigma_i}{r}\right) dF\right\| \|S_i^*h\|^2\right).$$

Note that the function $\frac{\phi\circ\sigma_i}{r} \in \mathrm{Lip}_1(X)$ for all $0 \leq i \leq N-1$. Hence

$$r\left(\sum_{i=0}^{N-1}\left\|\int \left(\frac{\phi\circ\sigma_i}{r}\right) dE - \int \left(\frac{\phi\circ\sigma_i}{r}\right) dF\right\| \|S_i^*h\|^2\right)$$

$$\leq r\rho(E,F)\left(\sum_{i=0}^{N-1}\big\langle S_i^*h, S_i^*h\big\rangle\right) = r\rho(E,F)\left(\sum_{i=0}^{N-1}\big\langle S_iS_i^*h, h\big\rangle\right)$$

$$= r\rho(E,F)\left\langle\left(\sum_{i=0}^{N-1} S_iS_i^*\right)h, h\right\rangle = r\rho(E,F)\langle h, h\rangle = r\rho(E,F).$$

Therefore

$$\left\|\int \phi d\Phi(E) - \int \phi d\Phi(F)\right\| \leq r\rho(E,F).$$

Since ϕ is an arbitrary element of $\mathrm{Lip}_1(X)$,

$$\rho\big(\Phi(E), \Phi(F)\big) \leq r\rho(E,F).$$

This proves that Φ is a Lipschitz contraction in the ρ metric on $P(X)$.

References

1. Davison, T.: Generalizing the Kantorovich metric to projection-valued measures. Acta Appl. Math. (2014). doi:10.1007/s10440-014-9976-y