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Preface

Control and Observation of Nonlinear Control Systems with Applications to Medicine and Space Mechanics

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Control theory, a modern area that addresses the need for a strong interplay between the mathematical sciences and other sciences and engineering, is at the intersection of theoretical and experimental research. On one side, theoretical results lay the foundations of the proper geometric concepts and notions that are intrinsic to control systems. On the other side, implementations of these notions and concepts to concrete problems build a bridge from theory to applications. This collection of articles brings together both directions, and is representative of the ideas that were presented at the workshop on *Control and Observation of Nonlinear Control Systems with Application to Medicine Including MRI and Cancer Models, Aerospace Systems, and Others* that took place in September of 2013 at the University of Hawai‘i at Mānoa.

This collection contains six applied-oriented articles and two theoretical ones. Below, we describe each contribution in more details.

Over the last two decades, geometric control has proved to be determining in analyzing and solving questions coming from a large spectrum of applications; in particular, one can observe a significant impact from the introduction of intrinsic objects from optimal control over more traditional procedures used experimentally. This is a common ground of the contributions by Bonnard et al., Chyba et al., and Ledzewicz et al. They are centered around the role that singular extremals play in the optimal synthesis. While singular extremals are very well-known by experts in optimal control, experimentalists are usually not very familiar with these mathematical objects and therefore typically do not include them in the development of their procedures.

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1. The contrast problem in nuclear magnetic resonance (NMR) treated in the paper by Bonnard et al. entitled *Geometric and Numerical Methods in the Contrast Imaging Problem in Nuclear Magnetic Resonance* is a perfect illustration of the benefit of using singular extremals to produce a much improved contrast in the images versus a traditional approach. The paper presented in this issue focuses on the numerical aspects; more precisely, it presents a combination of numerical techniques—direct, indirect and Linear Matrix Inequality (LMI) techniques—to the contrast problem in NMR. The direct method used the Bocop software, the indirect method the Hampath software and LMI techniques are used to estimate a global optimum. The algorithms are fully described and numerical simulations presented and discussed in the different cases.
2. In *Optimal Geometric Control Applied to the Protein Misfolding Cyclic Amplification Process*, Chyba et al. use geometric control for a new approach to the Protein Misfolding Cyclic Amplification (PMCA) process designed to accelerate replication process for prions in a laboratory environment. The goal is to maximize the final density of a polymerization model representing the prion evolution where the control is the sonication intensity. The authors show that the maximum Perron eigenvalue of the matrix describing the system with constant control is a singular control that provides improved alternative procedures to the commonly bang-bang experimental one (48 hours of alternation between 30 minutes portion of zero sonication and pulses of 45 seconds of maximum sonication). Additionally, in the three dimensional case the conjugate points are computed and the singularities of the singular flow are analyzed in relation with singularity and feedback classification.
3. The paper *A 3-Compartment Model for Chemotherapy of Heterogeneous Tumor Populations* by Ledzewicz et al. focuses on a chemotherapy model for the treatment of a heterogeneous tumors by a single therapeutic agent. The model introduces three subpopulations of cells depending on their sensitivity to the therapeutic agents and the chemotherapy is applied over a fixed time interval. Two treatment regimes are compared: the first is the maximum tolerated dose where the patient receives a constant, high concentration dose of the therapeutic agent over the treatment interval; the second treatment regime is a time-varying one, with lower concentration doses of the therapeutic agent that arise as singular controls in the optimal control problem. The authors conclude that for heterogeneous tumor populations, a more modulated approach that varies the dose rates of the drugs may be more beneficial than the classical maximum tolerated dose approach pursued in medical practice.

An additional contribution to the medical field in this special issue is the paper *Identifiability Properties for Inverse Problems in EEG Data Processing with Observability and Optimization Issues* by J. Leblond. This paper introduces several inverse problems coming from functional neuroimaging. More precisely, given only a finite number of electromagnetic measurements (electric potential, current density flow) taken non-invasively outside of the head (e.g. on the scalp), it examines how to reconstruct a 3D map of the potential and sources that generate the surface measurements, or also a 3D map of the conductivity. The major challenge is that typically, inverse problems arising in such a case are severely ill-posed. The author focuses on the case of a finite (but unknown) number of dipolar sources within the brain and reviews some recent methods for estimating the number as well as positions and moments of such sources. Both theoretical and constructive aspects are described, while numerical illustrations are provided.

The paper by Bonnard et al. entitled *Time Versus Energy in the Averaged Optimal Coplanar Kepler Transfer towards Circular Orbits* discusses the important question of orbital transfer in space mechanics. It provides a comparison of the energy minimization and time minimal transfer towards coplanar circular orbits using averaging techniques in the case of low propulsion. The energy case is associated to a 2D Riemannian metric versus a Finsler metric in the time minimal case with a non smooth Hamiltonian. The main result is that the elliptic domain is geodesically convex in the time minimal case but not in the energy case. The optimal solutions are described in both cases.

Finally, on the theoretical side we have two additional contributions. First, the paper by Chekroun et al., *Finite-Horizon Parameterizing Manifolds, and Applications to Suboptimal Control of Nonlinear Parabolic PDEs*, proposes a new approach to derive a low-dimensional suboptimal controller to optimal control problems of dissipative partial differential equations. The authors introduce a new parameterizing strategy for slaving the fast modes with the slow modes by constructing an efficient backward-forward system. This approach has the potential for many applications in both the theory of control and its engineering applications. The second theoretical paper *Quantum Perfect State Transfer in a 2D Lattice* by S. Post is based on previous works on the applications of a two-variable generalization of Krawtchouk polynomials in relation with the trinomial distribution with applications to Quantum Theory. The main contribution is the formulation of necessary bounds on the parameters of the model and a sufficient condition on the ratios of the frequencies of the Hamiltonian to realize a perfect state transfer. The remaining of the article is devoted to compute explicit transfer, in particular in some resonant cases.

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