A new taper index based on form-factor: application to three bamboo species (*Phyllostachys* spp.)

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Abstract

The form-factor for the stem surface area is directly proportional to the square root of the form-factor for the stem volume, i.e., the square root law of the form-factor. Although the square root law addresses the stems of coniferous trees, the nature of the proportional coefficient of the law has not been discussed. In this study, we demonstrated that the coefficient indicates the stem taper; therefore, it is named “Taper Index based on Form-Factor (TIFF).” We also examined the relationship between the form-factor for the culm surface area and the form-factor for the apparent culm volume of 570 culms across three bamboo species (*Phyllostachys pubescens* Mazel ex Houz., *P. bambusoides* Sieb. et Zucc., and *P. nigra* var. *henonis*). The square root law held true for all three bamboo species. The species-specific TIFF was determined to be 0.873 for *P. pubescens*, 0.897 for *P. bambusoides*, and 0.901 for *P. nigra*. This result implied that *P. pubescens* had a more tapering culm form compared to the other two species, while culm taper was similar between *P. bambusoides* and *P. nigra*. Our findings align with empirical observations of the culm taper of the bamboo species. Intra-species variation in TIFF was considerably small, allowing us to evaluate the species-specific culm taper from a small number of samples. Therefore, we conclude that TIFF provides a simple and useful method for quantifying species-specific culm or stem taper, and facilitates the estimation of merchantable or total volume.

Keywords Apparent culm volume · Culm surface area · *Phyllostachys* spp. · Square root law of form-factor · TIFF (taper index based on form-factor)

Introduction

The stem diameter of a tree decreases gradually from the ground to the top. The rate of decrease in diameter along the stem is called the stem taper (Tenzin et al. 2017; West 2004), and depends on various characteristics, including genetics, silviculture, site, and climatic characteristics (Ikonen et al.

2006; Karlsson 2000; Nigh and Smith 2012; Pukkala et al. 2019; Schneider et al. 2018). Knowledge of the stem taper is indispensable for estimating the total or merchantable volume of trees (Tenzin et al. 2017; Wijenayake et al. 2019). Therefore, an effective way to quantify the stem taper has long been an important topic in ecological and forestry studies (Gregoire 2012; McTague and Weiskittel 2021).

Form-factor, which is defined as a reduction factor of a fictitious cylinder (with a given stem diameter and tree height) to the actual form of the stem, has been used to quantify the stem taper of trees (Tenzin et al. 2017). There are two types of form-factors, i.e., form-factors for stem volume and stem surface area (Gavrikov 2017; Inoue 2005). The form-factor for stem surface area is directly proportional to the square root of the form-factor for stem volume, i.e., the square root law of the form-factor (Inoue 2006). Earlier field measurements have revealed that such a square root law holds true for coniferous trees such as *Cryptomeria japonica* D. Don, *Chamaecyparis obtusa* Endl., and *Larix kaempferi* (Inoue 2006, 2012). It has also been reported

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that intra-species variation in the proportional coefficient was small for each of the three species (Inoue 2006, 2012). As both form-factors indicate degree of taper, the coefficient may be correlated with tree taper, and its value may be uniquely influenced by the tree species. However, the nature of the proportional coefficient of the law has not been discussed. These facts motivated us to investigate the relationship between the form-factors of various plants other than the three coniferous species previously studied.

Bamboo is an interesting target because it has an excurrent stem (culm) with a taper from ground to tip similar to that of coniferous trees (Akinlabi et al. 2017; Liese and Tang 2015). Despite similar tapering, bamboo exhibits four structural traits that distinguish it from coniferous trees (Inoue et al. 2017; Song et al. 2011): (1) most bamboo species have a hollow cylindrical culm, (2) the long hollow cavity is discretely separated by circular disks (i.e., diaphragms), (3) the distribution density of the vascular bundles embedded in the xylem increases from the inner to outer surface of the hollow culm, and (4) the culm does not show secondary growth due to a lack of vascular cambium. These structural differences between trees and bamboos raise the following questions:

1. Does the square root law of form-factor apply to bamboo species?
2. If the law applies, does the proportional coefficient of the square root law differ among bamboo species?
3. Do the proportional coefficients of bamboos vary greatly from those of coniferous trees?

In addition, from a practical viewpoint, it is strongly desired to develop a protocol that can analyze the degree of taper quickly and efficiently using the smallest number of samples as possible. This prompts the following question:

4. How many bamboo culms are sufficient to accurately determine the value of coefficient?

The present study aims to answer these four questions by examining the relationship between form-factor for stem volume and those for stem surface area for three bamboo species. The three species included *Phyllostachys pubescens* Mazel ex Houz., *P. bambusoides* Sieb. et Zucc., and *P. nigra* var. *henonis*. The three bamboo species are the most economically important species in Japan (Inoue et al. 2013; Uchimura 2009). In fact, bamboo forest covers an area of over 150,000 ha in Japan, comprising 99% *P. pubescens* and *P. bambusoides* at a 3:1 ratio (Takano et al. 2017). Although *P. nigra* occupies only 0.4% of the total area, its area exceeds more than 20% of the bamboo forest in a certain district (Inoue et al. 2012). Hence, the three bamboo species have a potential as a plentiful biomass resource (Buckingham et al. 2011; Minami and Saka 2005). In light of the numerous

industrial applications of bamboo, information about culm taper provides a primary mean of estimating apparent culm volume (i.e., the volume including hollow cavity; Kleinn and Morales-Hidalgo 2006). In addition, once the apparent culm volume is determined, the woody culm volume (i.e., the volume of woody walls of culms; Kleinn and Morales-Hidalgo 2006) can be estimated by multiplying the apparent culm volume by the reduction factor (e.g., García and Kleinn 2010; Inoue et al. 2011a, 2019).

In the present study, we initially clarify the geometric meaning of the proportional coefficient of the square root law of the form-factor. Since the coefficient is shown to be strongly related to the taper strength, we name it “Taper Index based on Form-Factor (TIFF).” Subsequently, we analyze the relationship between the two form-factors to examine whether the square root law of the form-factor applies to the three bamboo species. Afterwards, we determine the species-specific TIFF for each of the three species and compare them with those of coniferous trees reported in previous studies (Inoue 2006, 2012). Finally, we present the results of Monte Carlo simulations, which enables us to estimate the number of sample culms required for accurately determining the proportional coefficient.

Materials and methods

Conceptions to evaluate stem taper

Various methods or approaches have been proposed for the evaluation of the stem taper of a tree (Gregoire 2012; McTague and Weiskittel 2021). Among them, we introduce two methods for the evaluation of stem taper, which is necessary for deriving a new taper index.

One of the traditional methods used to evaluate stem taper is based on a phenomenological model defined by Kunze’s equation (Kunze 1873 after Li et al. 2021), which is:

$$y^2 = ax^b \quad (1)$$

where y is the radius of the stem at position x from the tip, and a and b are constants. The power exponent b (called the form exponent; hereafter, denoted as FE) serves as a measure of the stem taper. For instance, FE=3 corresponds to a neiloid, FE=2 a cone, FE=1 a paraboloid, and FE=0 a cylinder (Osawa 1993). In most excurrent trees, FE values range from 1 to 2, indicating that their shape is between a paraboloid and a cone (McTague and Weiskittel 2021).

Another way of quantifying stem taper is using the form-factor. The form-factor for the stem volume (λ_i) is given by:

$$\lambda_i = \frac{v}{\pi d_i^2 h / 4} \quad (2)$$

Here, v is the stem volume, d_i is the stem diameter at relative height i (relative distance from the tree tip; $i=0$ at the tip and $i=1$ at the base), and h is the tree height. Equation 2 indicates that λ_i is the ratio of the volume of a tree stem to the volume of the fictitious cylinder, whose diameter and height are equal to the d_i and h of the tree, respectively. As the stem taper becomes stronger, the value of λ_i decreases. Note that λ_i will change under different relative height indices of i ; it is common to select the i that corresponds to breast height (Tenzin et al. 2017).

By contrast, the form-factor for the stem surface area (κ_i) of a tree is defined as:

$$\kappa_i = \frac{s}{\pi d_i h} \quad (3)$$

with the ratio of the stem surface area s to the side area of the fictitious cylinder (Inoue 2005). Similar to λ_i , a high value of κ_i indicates a weak taper of the tree stem.

A new taper index based on form-factor

First, we derive the square root law of the form-factor as follows: Eliminating d_i from Eqs. 2 and 3, we have:

$$\kappa_i = k \sqrt{\lambda_i} \quad (4)$$

where k is the coefficient defined by:

$$k = \sqrt{\frac{s^2}{4\pi v h}} \quad (5)$$

It is noteworthy that the expression of k in Eq. 5 is independent of the relative height i , although both the values of λ_i and κ_i are dependent on i . Therefore, only one unique value of k is required to establish the square root law of Eq. 4 for the stem (i.e., at an arbitrary choice of i).

Next, we identified that the coefficient k can be written as a function of the FE as defined in Eq. 1 (Inoue 2006). If the stem taper of a given tree obeys Eq. 1, the stem volume (v) is represented by:

$$v = \pi \int_0^h y^2 dx = \frac{a\pi}{b+1} h^{b+1} \quad (6)$$

Then, the form-factor for stem volume λ_i is represented by:

$$\lambda_i = \frac{v}{\pi d_i^2 h / 4} = \frac{4ah^b}{d_i^2(b+1)} \quad (7)$$

Similarly, the stem surface area (s) is an integral with respect to x as follows:

Fig. 1 Relationship between form exponent (FE) and taper index based on form-factor (TIFF). The solid line indicates Eq. 11 in this text, which was derived by assuming that the culm taper obeys Kunze's equation

$$s = \pi \int_0^h y \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx \quad (8)$$

Note that in general coniferous trees, the change rate in stem radius with distance from the tip, dy/dx , is considerably smaller than a unit. This allows us to omit the term dy/dx in Eq. 8 (Inoue 2004); using this approximation, s becomes:

$$s = \frac{4\sqrt{a\pi}}{b+2} h^{b/2+1} \quad (9)$$

The validity of this approximation is supported by a previous study (Takao and Minowa 1993), which showed that in the calculation of s , there is only a 0.1% difference between applying and not applying this approximation. Using the result of Eq. 9, κ_i can be represented as:

$$\kappa_i = \frac{s}{\pi d_i h} = \frac{4\sqrt{a}h^{b/2}}{\pi d_i(b+2)} \quad (10)$$

Eliminating h and d_i from Eqs. 7 and 10 yields Eq. 4, and the proportional coefficient k is given by:

$$k = \frac{2\sqrt{b+1}}{b+2} \quad (11)$$

Equation 11 reveals that the proportional coefficient k only depends on FE (b). Therefore, the coefficient k measures the degree of taper, as an alternative to the commonly used index of FE. Based on this finding, we label the proportional

coefficient k the “Taper Index based on Form-Factor (TIFF).”

Figure 1 illustrates the relationship between FE and k (TIFF), as described in Eq. 11. As previously discussed, FE for the excurrent trees takes a value in the range of 1 to 2 (McTague and Weiskittel 2021). From Fig. 1, we can see that the relationship between FE and TIFF is almost linear in this range of FE. The FE represents the taper throughout a stem (Osawa 1993). Therefore, k also indicates the stem taper and it follows from Eq. 11, where $k=0.8$ corresponds to a neiloid (FE = 3), $k \approx 0.866$ a cone (FE = 2), $k \approx 0.943$ a paraboloid (FE = 1), and $k = 1$ a cylinder (FE = 0). The larger value of k , the less tapered the stem.

Study sites

Data on the three bamboo species (*P. pubescens*, *P. bambusoides*, and *P. nigra*) were collected from specimens on Kyushu Island, western Japan. The *Phyllostachys pubescens* and *P. bambusoides* data were collected from Mt. Toshima, Kumamoto City, Kumamoto Prefecture (32°49'N, 130°48'E, 80–90 m a.s.l.). The average annual temperature and rainfall between 1991 and 2020 at the nearest observatory was 17.2 °C and 2,007 mm, respectively. The soil type was classified as ando soil. The stands of *P. pubescens* and *P. bambusoides* were located beside each other along an approach to a shrine. The stands had been used for upland farming until a few years ago, they are now turning into bamboo stands because of the planting of bamboos and subsequent abandonment.

The *Phyllostachys nigra* data was collected from two sites: the campus of Fukuoka University of Education, located in Munakata City, Fukuoka Prefecture (FUE; 33°48'N, 130°36', 50 m a.s.l.) and the Ito campus of Kyushu University in Fukuoka City, Fukuoka Prefecture (ITO; 33°36'N, 130°13'E, 60 m a.s.l.). The average annual temperature and rainfall recorded at the nearest observatories were, respectively, 15.9 °C and 1,665 mm for FUE and 16.4 °C and 1,701 mm for ITO. The soil type was brown forest soil for FUE, and gley soil or brown forest soil for ITO. Both stands were dense with closed canopies without other vegetation on the forest floor.

Measurements

Healthy, living culm samples were felled at ground level after the growing (elongation) period from October to January. The sampling period was selected because in Asia the bamboo shoots of these species emerge in spring and their elongations take less than three months to attain the maximum culm height after completing culm elongation (Yen 2016; Zhao-hua et al. 2005). The number of samples collected was 200 for *P. pubescens* and 200 for *P. bambusoides*, whereas the number of samples for *P. nigra* were 60 in FUE and 110 in ITO. After felling the culms, the culm height and diameter at breast height were measured using a tape measure and a digital caliper, respectively. The culm diameter at 10 successive points (equal intervals of one-tenth of the measured culm height) were also measured using a caliper. The diameters were measured in two orthogonal directions, and the average was used in our analysis. Note that the data was previously used in another study (Inoue et al. 2011b, 2012, 2013, 2018; Inoue and Suga 2009; Shima et al. 2016; Suga et al. 2011). A general description of the sample culms is provided in Table 1.

Analysis methods

The apparent culm volume (v) and culm surface area (s) for each sample were determined using the sectional measurement method (Inoue and Sakamoto 2017; West 2004). The results were used to calculate the normal form-factors ($\lambda_{0.1}, \lambda_{0.2}, \dots, \lambda_{1.0}$ and $\kappa_{0.1}, \kappa_{0.2}, \dots, \kappa_{1.0}$) and the breast height form-factors (λ_b and κ_b) (Inoue 2005). In theory, Eq. 4 is correct for every relative height (Inoue 2006). Therefore, to examine whether the square root law applies for each species considered, the relationship between λ_i and κ_i at every relative height and breast height was collectively fitted by the following equation using the reduced major axis (RMA) regression after log-transformation (Niklas 1994):

$$\log \kappa_i = \log \alpha + \beta \log \lambda_i \quad (12)$$

where α and β are coefficients. The standard error (SE) of β with the RMA regression was equal to that obtained using

Table 1 General description of sample culms ($n=570$)

Species	Site	Number of samples	Culm height (m) ^a	Diameter at breast height (cm) ^a	Apparent culm volume (dm ³) ^a	Culm surface area (m ²) ^a
<i>P. pubescens</i>	Toshima	200	14.1 ± 2.5	9.2 ± 2.7	47.4 ± 29.4	2.5 ± 1.0
<i>P. bambusoides</i>	Toshima	200	11.1 ± 3.0	5.0 ± 1.7	15.4 ± 13.3	1.3 ± 0.7
<i>P. nigra</i>	Munakata	60	7.3 ± 2.0	2.9 ± 1.1	3.6 ± 3.1	0.5 ± 0.3
	Ito	110	6.2 ± 1.5	2.7 ± 0.8	2.4 ± 1.7	0.4 ± 0.2

^aMean ± S.D

the ordinal least squares method (LSM) regression (Zhang et al. 2005). Therefore, a Student's t-test was used to examine whether the coefficient β was different from the theoretical value (i.e., $\beta = 1/2$). If the coefficient β is not significantly different from $1/2$, then the species- and site-specific TIFF was determined by fitting Eq. 4 to the relationship between λ_i and κ_i using LSM regression (Inoue 2006, 2012).

The individual-specific TIFF and FE values were calculated for each sample. The individual-specific TIFF was determined using the same procedure for determining the species-specific TIFF. The individual-specific FE was obtained by fitting Kunze's equation into the relationship between the distance from the tip and the culm radius for each sample using the RMA regression after log-transformation. The coefficients of variation for the TIFF and FE for each species were calculated as a measure of intra-species variation.

We then performed Monte Carlo simulations (bootstrap method) to evaluate the number of samples required to accurately determine the species-specific TIFF and FE. The sample culms ($N = 2\text{--}100$) were randomly selected from the original data ($n = 200$ for each of *P. pubescens* and *P. bambusoides*, and $n = 170$ for *P. nigra*) without replacement. By repeating the selection 10,000 times, the probability density function (PDF) of the average was approximated based on the normal distribution. The coefficient of variation (CV) was also calculated using the average and standard deviation of the PDF. The change in CV with a sample size (N) was analyzed.

All statistical analyses were performed using R version 4.1.0 (R Core Team 2021), and $P < 0.05$ was considered statistically significant.

Results

The square root law of form-factor for bamboo species

Figure 2 depicts the relationship between λ_i and κ_i for the three bamboo species examined. The RMA regression analysis showed that the power exponent of β was 0.500 (95% confidence interval: 0.499, 0.500) for all three species (*P. pubescens*, *P. bambusoides*, and *P. nigra*) and for both sites (FUE and ITO). The results indicated that the value of β was not significantly different from $1/2$ for all species and sites ($P > 0.05$). Therefore, the square root law of the form-factor prevailed for the three bamboo species examined in this study.

Species- and site-specific TIFF

When the power exponent is fixed at $1/2$, the relationships between λ_i and κ_i for each species and site can be expressed by the following equations (Table 2):

$$P. \textit{pubescens} : \kappa_i = 0.873\sqrt{\lambda_i} \quad (R^2 = 0.999, P < 0.001) \quad (13)$$

$$P. \textit{bambusoides} : \kappa_i = 0.897\sqrt{\lambda_i} \quad (R^2 = 0.999, P < 0.001) \quad (14)$$

$$P. \textit{nigra} \text{ (FUE)} : \kappa_i = 0.906\sqrt{\lambda_i} \quad (R^2 = 0.999, P < 0.001) \quad (15)$$

Fig. 2 Relationship between normal form-factors for culm volume (λ_i) and those for culm surface area (κ_i) of the three bamboo species examined. The solid line was fitted by Eq. 4 in this text

Table 2 Summary of the square root law of form-factor statics

Species	Site	Number of samples	Coefficient $\pm$ SE	95% CI	R^2 ^a
<i>P. pubescens</i>	Toshima	200	0.873 ± 0.001	0.872, 0.873	0.999***
<i>P. bambusoides</i>	Toshima	200	0.897 ± 0.001	0.896, 0.899	0.999***
<i>P. nigra</i>	Munakata	60	0.906 ± 0.002	0.902, 0.909	0.999***
	Ito	110	0.898 ± 0.002	0.895, 0.902	0.999***
	All	170	0.901 ± 0.002	0.898, 0.905	0.999***

^a ***: $P < 0.001$

$$P. \textit{nigra} \text{ (ITO)} : \kappa_i = 0.898\sqrt{\lambda_i} \quad (16)$$

$$(R^2 = 0.999, P < 0.001)$$

There was no significant difference in the TIFF of *P. nigra* between the FUE and ITO samples ($P > 0.05$). Therefore, the relationship between λ_i and κ_i for *P. nigra* at both sites was collectively fitted using Eq. 4, which resulted in

$$P. \textit{nigra} : \kappa_i = 0.901\sqrt{\lambda_i} \quad (17)$$

$$(R^2 = 0.999, P < 0.001)$$

The species-specific TIFF was significantly smaller in *P. pubescens* than the other two species ($P < 0.001$ for both *P. bambusoides* and *P. nigra*). Conversely, there was no statistically significant difference identified between the TIFF of *P. bambusoides* and *P. nigra* ($P > 0.05$).

Individual-specific TIFF

Figure 3 provides the frequency distributions of individual-specific TIFF and FE results. For all three species, TIFF produced a relatively narrow histogram with a high peak. Conversely, FE were scattered over a wide range when compared to the TIFF distributions. The coefficients of variation of the individual-specific TIFF and FE were, respectively, 1.578% and 9.976% for *P. pubescens*, 1.224% and 13.177% for *P. bambusoides*, and 1.174% and 10.920% for *P. nigra*.

Figure 4 shows the changes in the coefficient of variation for TIFF and FE (CV_{TIFF} and CV_{FE}) with the number of samples (N) analyzed using the Monte Carlo simulation technique. For all species, CV_{TIFF} gradually decreased from approximately 1% when $N=2$ to 0.1% when $N=100$, with increasing N . When $N > 20$, the change in CV_{TIFF} with N (dCV_{TIFF}/dN) was less than 0.05%, and CV_{TIFF} was almost stable at 0.25%. In contrast, CV_{FE} decreased with N from 7 to 9% when $N=2$ to 1% when $N=100$. Although dCV_{FE}/dN

Fig. 3 Frequency distributions of the individual-specific taper index based on form-factors (TIFF) and form exponent (FE)

Fig. 4 Changes in the coefficient of variation of taper index based on form-factor (TIFF) and form exponent (FE) with the number of samples. The circles, triangles, and crosses indicate *Phyllostachys pubescens* Mazel ex Houz., *P. bambusoides* Sieb. et Zucc., and *P. nigra* var. *henonis*, respectively

reached less than 0.05% when $N > 70$, the stable CV_{FE} was approximately five times larger than that of the stable CV_{TIFF} . Even when N was increased to 100, the CV_{FE} was more than 1%, which was comparable to the CV_{TIFF} when $N = 2$.

Discussion

In this study, the relationship between λ_i and κ_i was investigated using three bamboo species (*Phyllostachys* spp.). As expected, the square root law of the form-factor held true for the bamboo. However, the effectiveness of the proportional coefficient k (TIFF) as the taper index remains to be clarified. In particular, if the intra-species variation was smaller in the TIFF than in the FE, the TIFF would be beneficial for quantifying the species-specific taper of the bamboo culms or tree stems from a small number of samples. The focus of the following discussion is the effectiveness of TIFF.

Whether TIFF varies among the bamboo species?

The species-specific TIFF obtained in this study ranged from 0.873 (*P. pubescens*) to 0.901 (*P. nigra*). This result indicates that the three bamboo species exhibit the culm form ranging from a paraboloid to a cone. The TIFF obtained for *P. pubescens* was significantly smaller than that of the other two species. The TIFF values were comparable between *P. bambusoides* and *P. nigra*. As discussed previously, a smaller TIFF value indicates a more taper form. Hence, *P. pubescens* has a more tapering culm form when compared to the other two species, while the culm taper is similar between *P. bambusoides* and *P. nigra*. The relationships between relative height and relative radius in the three bamboo species examined are shown in Fig. 5. Despite the scatter, our data demonstrates the inter-species variation in culm taper between *P. pubescens* and two other species. This finding is also supported by the empirical observations of culm taper in the three bamboo species (Ota 1950).

Fig. 5 Relationship between relative height and relative radius of the three bamboo species examined

Previous studies have shown that *P. bambusoides* and *P. nigra* share a common two-way volume equation, whereas *P. pubescens* requires a separate species-specific equation (Inoue et al. 2011b, 2012; Suga et al. 2011). Similar results were obtained by models that predicted the culm surface area using the diameter at breast height and culm height (Inoue et al. 2018). The incompatibility of the models between *P. pubescens* and the other two species could be attributed to the inter-species variation in the culm taper (Inoue et al. 2011b, 2012, 2018). The difference in the culm taper between the bamboo species as expected from TIFF is in accordance with this speculation. For these reasons, TIFF is an appropriate measure for quantifying the species-specific culm taper of bamboo.

Our data indicates that no significant difference in the site-specific TIFF was identified in *P. nigra* (see Eqs. 15 and 16), whereas the culm height was significantly different between specimens from FUE and ITO ($P < 0.05$; Table 1). In contrast, a previous study suggested that the culm form of *P. bambusoides* may become tapering when the culm size is small due to low site productivity (Inoue et al. 2011b). If the culm taper for a given species is influenced by site productivity, the site-specific TIFF for *P. nigra* might be different between the locations of FUE and ITO. At present, there is not enough data to clarify the reasons for this inconsistency. Further investigations on TIFF using data collected from various sites with different productivities are required in the future, to clarify the effect of site productivity on culm taper.

Is TIFF different between trees and bamboos?

The species-specific TIFF was 0.873 for *P. pubescens*, 0.897 for *P. bambusoides*, and 0.901 for *P. nigra*. Previous studies reported that the species-specific TIFF was 0.873 for both *C. japonica* and *C. obtusa* (Inoue 2006), and 0.885 for *L. kaempferi* (Inoue 2012). Consequently, we quantified that *P. nigra* and *P. bambusoides* possessed the least tapering form, followed by *L. kaempferi* and the other three species (*P. pubescens*, *C. japonica*, and *C. obtusa*). Although TIFF varies with species, the range in TIFF for the bamboo considerably overlaps that for the coniferous species. Therefore, the stem (culm) taper predicted by TIFF is comparable between coniferous trees and bamboo species despite their structural differences (Inoue et al. 2017; Song et al. 2011).

A possible approach to clarifying why bamboo and conifers have similar tapers is by understanding the structural mechanics of bamboo culms. The mechanical constraint that determines the stem (culm) taper of various plants has been studied widely (Niklas 1994). Recently, the structural mechanics of the unique structures of bamboo culms have been investigated. Bamboos effectively control their internode length (Inoue et al. 2017) as well as other geometric parameters, such as culm diameter and culm wall thickness,

in accordance with lightweight and high-strength design concepts (Shima et al. 2016). They also maximize their flexural rigidity by optimally regulating the radial gradation of the vascular bundle distribution (Sato et al. 2017). Furthermore, the combination of the uneven internode length and the graded distribution of vascular bundles contributes to bending resistance by forming a nearly optimal shape (Sato et al. 2016). Progress in such research assists in our understanding of why the bamboo and coniferous trees have a similar taper.

How many sample culms are necessary to accurately determine TIFF?

The Monte Carlo simulation revealed that the CV_{TIFF} becomes stable when $N > 20$. This result indicates that when assessing the species-specific culm taper using TIFF, a minimum of 20 sample culms needs to be collected, but beyond 20 culms is not required for an accurate result. Conversely, a larger sample size is required to reach a stable CV_{FE} ($N > 70$). For the same N , CV_{TIFF} was consistently smaller than CV_{FE} . Hence, TIFF will be more convenient to use than FE, as it requires less time and effort to collect sample culms for accurately quantifying the species-specific culm taper.

Recently, many studies have examined the effects of genetic, silviculture, site, and climatic characteristics on the stem taper of trees using the taper equations. The effects on stem taper are not easily ascertained by observations for the taper equations, so a graphical approach has been used in these studies (e.g., Nigh and Smith 2012; Pukkala et al. 2019; Schneider et al. 2018). However, a graphical method is not quantitative but qualitative. Although FE is the simplest measure for quantifying stem taper, it requires a large number of sample trees due to larger intra-species variations. In contrast, TIFF allows the quantification of the stem taper using a small number of samples. Therefore, TIFF is a more useful quantitative technique than FE for examining the effects of genetic, silviculture, site, and climatic characteristics on the stem taper.

Information about stem taper enables the estimation of upper stem diameter at any given height and obtaining stem volume (or apparent culm volume) following integration into the taper equation (McTague and Weiskittel 2021). The apparent culm volume of bamboos can also be converted into woody culm volume by multiplying the apparent culm volume by the reduction factor (García and Kleinn 2010; Inoue et al. 2011a, 2019). Our data demonstrated that the new taper index, i.e., TIFF, can well explain inter-species variation in culm taper in the three bamboo species examined in the present study. Furthermore, if TIFF can also be applied to coniferous trees with varying stem tapering, it can be used to predict their merchantable and total stem volume. Therefore, TIFF is a useful measure for quantifying stem

taper in coniferous trees as well as culm taper in bamboo species. Progress in forestry and ecological studies by applying TIFF is expected in future.

Conclusions

In this study, the relationship between λ_i and κ_i was examined for three bamboo species (*Phyllostachys* spp.). The results showed that the square root law of the form-factor was consistent for the bamboo species, and the species-specific TIFF could successfully explain the inter-species variations in the culm taper. The intra-species variation in TIFF was considerably small, suggesting that TIFF allows the successful evaluation of species-specific culm taper using a small number of sample culms. In conclusion, TIFF is useful for quantifying the species-specific culm taper, and is expected to be applied in various ecological and forestry studies.

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