Sparse Reconstruction and Damage Imaging Method Based on Uniform Sparse Sampling

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ABSTRACT The full wavefield detection method based on guided waves can efficiently detect and locate damages relying on the collection of large amounts of wavefield data. The acquisition process by scanning laser Doppler vibrometer (SLDV) is generally time-consuming, which is limited by Nyquist sampling theorem. To reduce the acquisition time, full wavefield data can be reconstructed from a small number of random sampling point signals combining with compressed sensing. However, the random sampling point signals need to be obtained by adding additional components to the SLDV system or offline processing. Because the random sparse sampling is difficult to achieve via the SLDV system, a new uniform sparse sampling strategy is proposed in this paper. By using the uniform sparse sampling coordinates instead of the random spatial sampling point coordinates, sparse sampling can be applied to SLDV without adding additional components or offline processing. The simulation and experimental results show that the proposed strategy can reduce the measurement locations required for accurate signal recovery to less than 90% of the Nyquist sampling grid, and the damage location error is within the minimum half wavelength. Compared with the conventional jittered sampling strategy, the proposed sampling strategy can directly reduce the sampling time of the SLDV system by more than 90% without adding additional components and achieve the same accuracy of guided wavefield reconstruction and damage location as the jittered sampling strategy. The research results can greatly improve the efficiency of damage detection technology based on wavefield analysis.

KEY WORDS Scanning laser Doppler vibrometer, Lamb waves, Compressed sensing, Wavefield sparse reconstruction, Damage imaging

1. Introduction

Lamb waves have received considerable attention in the fields of non-destructive testing (NDT) and structural health monitoring (SHM) because they are capable of traveling relatively long distances with low attenuation and sensitive to internal damage in plate-like structures [1–3]. As the most traditional Lamb waves detection method, distributed sensor arrays are often used to record acoustic emissions [4–8], which include single-ring [9], two-dimensional square arrays [10] and more complicated configurations [11]. While the above arrays can effectively obtain signals, the following disadvantages and limitations still exist in practical applications: (1) The sensors are usually connected to the specimen by cables, which easily influence wave propagation. Moreover, sensors and cables are vulnerable to

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damage. (2) The array arrangement is fixed and the operation is not flexible. (3) It is necessary to compensate the temperature effect for improving the quality of received signals. (4) Due to the limitation of the spatial distribution scale of the sensor, it is difficult to recognize damage accurately. In order to overcome these shortcomings, application of full wavefield detection techniques is investigated [12]. The full wavefield detection technology can effectively detect delamination [13], fatigue cracks [14] and notches [15], relying on the collection of large amounts of information. The wavefield is generally acquired by point-by-point measurement devices such as the scanning laser Doppler vibrometer (SLDV) [16]. The main limitation of wavefield-based acquisition process is generally time-consuming. There are several reasons: (1) A large number of points require multiple measurements in average to improve the signal-to-noise ratio, which greatly increases the sampling time. (2) A waiting time needs to be set for each measurement to avoid spatial aliasing. In addition, a large number of sampling data also bring difficulties to data operation and transmission. Therefore, it is necessary to reduce and compress large amounts of sampling data to reduce the acquisition time.

As an excellent signal reconstruction strategy, compressed sensing (CS) has attached much attention in the guided wave based NDT and SHM [17]. Based on the sparse prior of Lamb waves in a specified base, some sparse reconstruction algorithms have been developed to realize the reconstruction of the dispersion curve [18], wavefield [19], amplitude [20], and to locate damage [21]. Harley and Moura [18, 22] extracted the wave number sparsity in the frequency wave number domain to reconstruct the dispersion relation. Perelli et al. [23] proposed an alternative minimization algorithm based on compressed sensing, which improves the accuracy of damage location by accurately predicting the propagation path of guided waves. Levine et al. [21] and Levine and Michaels [24] proposed a block sparse reconstruction method based on compression sensing. By assuming that each pixel has a corresponding multi-dimensional linear scattering model, the guided wavefield was expressed as a superposition of scattering sources, and the compressed sensing algorithm was used to solve the scattering source containing damage information, so as to achieve damage location. Ianni et al. [25] discussed the sparse representation of Lamb wavefield under different basis functions and realized the reconstruction of wavefield by using the compressed sensing. Park and Sohn [26] developed a laser ultrasonic wavefield reconstruction technology by combining compression sensing and super-resolution algorithm. The proposed method reduced the number of measurement points needed for ultrasonic field imaging by more than 90% and was verified by experiments on aluminum plates with single hole damage. Levine and Michaels [27] used a sparsely distributed sensor array to pick up wavefield signals and realized the location of multiple defects in the metal plate through the compressed sensing method. Mesnil and Ruzzene [28] combined the compressed sensing algorithm and the sparsity of the damage in the spatial distribution; the full wavefield data of the sparse measurement points were recovered.

The wavefield reconstruction method proposed by Ianni and Mesnil can reconstruct the wavefield from few spatial measurement points accurately and realize damage imaging. However, sparse points are selected offline in the application of the above methods, which is limited by the fact that the SLDV system cannot directly select random sparse signals. Because the SLDV system is difficult to achieve random sparse sampling, this paper proposes a new sampling strategy by replacing the random spatial measuring point coordinates with uniform sparse sampling coordinates, so that sparse sampling can be applied to SLDV without offline processing. The Lamb wave field data of interesting area can be effectively obtained by the wave field reconstruction scheme based on the SLDV uniform sparse sampling, which has great significance for improving the execution efficiency of the damage detection technology based on wavefield analysis.

This paper is organized in the following manner. After the introduction, the second section presents the wavefield sparse reconstruction framework, which is described in three parts: (1) compressed sensing theory; (2) construction of CS equation in Lamb wavefield; and (3) wavefield reconstruction and damage imaging. In the third section, a simulation model is constructed, and wavefield reconstruction and damage imaging are performed using the proposed method. In addition, the reconstruction accuracy and damage location accuracy of random sparse sampling (taking jittered sampling as an example) and uniform sparse sampling are compared. Following that, the fourth section presents the PZT excitation/SLDV receiving experimental platform, and the effects of two sampling methods on damage location, reconstruction accuracy and calculation time are compared. Finally, the fifth section concludes the paper with a brief summary.

2. Lamb Wavefield Sparse Reconstruction Framework

2.1. Compressed Sensing Theory

Compressed sensing is also called compressed sampling [29], its main idea is to reduce the dimension of the signal by using the condition of sparse signal and then solve the optimization problem to obtain the original signal. Applications of compressed sensing include image processing [30], magnetic resonance imaging [31], radar [32], etc

The compressed sensing problem can be described mathematically as follows:

$$\mathbf{y} = \Phi \mathbf{x} \quad (1)$$

where the sparsity of one-dimensional signal $\mathbf{x} \in \mathbb{R}^{N \times 1}$ is K . That is, $\mathbf{x}$ contains only K nonzero values. Observation matrix $\Phi \in \mathbb{R}^{M \times N}$, $M < N$. $\mathbf{y} \in \mathbb{R}^{M \times 1}$ is the linear observation value of the original signal $\mathbf{x}$.

The original signal $\mathbf{x}$ can be expressed sparsely on a certain basis $\Psi \in \mathbb{R}^{N \times P}$:

$$\mathbf{x} = \Psi \boldsymbol{\alpha} \quad (2)$$

where $\boldsymbol{\alpha} \in \mathbb{R}^{P \times 1}$ is the sparse representation of x . Equation (1) can be transformed into:

$$\mathbf{y} = \mathbf{A} \boldsymbol{\alpha} \quad (3)$$

where $\mathbf{A} = \Phi \Psi$, $\mathbf{A} \in \mathbb{R}^{M \times P}$ is denoted as the “sensing matrix”. The equations $\mathbf{y} = \Phi \mathbf{x}$ and $\mathbf{y} = \mathbf{A} \boldsymbol{\alpha}$ can be collectively referred to as the “CS equation”.

Therefore, if we want to construct a CS equation, the following two conditions need to be met: (1) The original signal must be sparse. If necessary, sparse representation can be performed on a specific basis. (2) There is a linear relationship between the measured signal and the original signal. In this paper, in order to make the guided wavefield meet the above two conditions, first of all, we make the following assumptions: in the actual structure, the structural features that can actively generate guided waves or passively scatter guided waves are collectively referred to as “real sources”, and the excitation source, damage and boundary are all real sources, so the number of real sources in the interested area is usually limited. By dividing the interested area into N equidistant pixels, and assuming all these pixels as the source (i.e. “potential source”), the real source is sparse in the interested area full of potential sources. Secondly, according to the linear propagation process of the guided waves in the frequency domain, the signal of the measurement point in the frequency domain can be regarded as the linear superposition of the guided wave signals that generated by all sources in the interested area. According to the above work, the compression sensing problem of the guided wave field can be constructed.

2.2. Construction of CS Equation in Lamb Wavefield

Assuming that the source number is s ($s = 1, 2, \dots, S$), the measurement point number is m ($m = 1, 2, \dots, M$), $M < S$. Since it is impossible to know the scattering of defects in all directions in advance, it is assumed that all potential sources scatter uniformly in all directions. According to the Lamb wave propagation equation [2], the signal $y_m(t)$ received by measuring point m in the frequency domain $y_m(f)$ can be expressed as:

$$y_m(f) = \sum_{s=1}^S \frac{1}{\sqrt{d_{m,s}}} v_s^{(\mu)}(f) \exp[-ik_{m,s}^{(\mu)}(f) d_{m,s}] \quad (4)$$

where f represents the frequency; $d_{m,s}$ represents the distance m from s ; $v_s^{(\mu)}(f)$ is the excitation function of the potential source s in mode μ ; $k_{m,s}^{(\mu)}(f)$ represents the wave number dispersion data of mode μ in the s to m direction. Since the energy is mainly diffused in the geometric form in the isotropic plate, only the geometric attenuation is considered in this paper [33]. Considering all measuring point signals, Eq. (4) can be rewritten as:

$$\mathbf{y}^{(\mu)}(f) = \sum_{\mu=1}^{N^{(\mu)}} \mathbf{H}^{(\mu)}(f) \mathbf{v}^{(\mu)}(f) \quad (5)$$

Fig. 1. Diagram of coordinate settings of two sampling points and potential source points (red circles: measurement points; blue dots: potential sources). (Color figure online)

where $N^{(\mu)}$ is the mode number; $\mathbf{y}^{(\mu)}(f)$ represents the spectrum set of measured signals on all measuring points: $\mathbf{y}^{(\mu)}(f) = [y_1^{(\mu)}, \dots, y_M^{(\mu)}]^T(f)$; $\mathbf{v}^{(\mu)}(f)$ represents the spectrum set of excitation function of assumed source: $\mathbf{v}^{(\mu)}(f) = [v_1^{(\mu)}, \dots, v_S^{(\mu)}]^T(f)$. The elements in row m and column s of the sensing matrix represent the wavefield transfer function from the potential source s to the measuring point m :

$$H_{m,s}^{(\mu)}(f) = \frac{1}{\sqrt{d_{m,s}}} \exp(-ik_{m,s}^{(\mu)}(f)d_{m,s}) \quad (6)$$

The excitation frequencies considered in this paper are below the A_1 and S_1 mode cutoff frequencies in order to facilitate computation, so that only the A_0 and S_0 modes propagate in the specimen. Accordingly, for $N^{(\mu)} = 2$, Eq. (5) can be rewritten as:

$$\mathbf{y}(f) = H^{(A_0)}(f)\mathbf{v}^{(A_0)}(f) + H^{(S_0)}(f)\mathbf{v}^{(S_0)}(f) = \mathbf{H}(f)\mathbf{v}(f) \quad (7)$$

where $\mathbf{H}(f) \in \mathbb{R}^{M \times SN^{(\mu)}}$ is the sensing matrix.

In order to accurately solve the CS equation, the sensing matrix needs to meet the RIP criterion [34]. This means that uniformity and randomness must be considered in the sampling process. Conventional sparse sampling points are generated by jittered sampling [35], as shown in Fig. 1a. Such methods require offline processing of the data collected by SLDV to obtain random sampling point data. In order to make the SLDV sparse sampling easier to implement, this paper proposes a uniform sparse sampling method. As shown in Fig. 1b, this method sparsely collects measurement point data and assumes that potential sources are randomly distributed in the region of interest. Therefore, the SLDV system can not only easily set the coordinates of the measurement points, but also ensure that the sensing matrix meets the incoherent requirements.

After the coordinates of the sampling point are determined, the SLDV system is used to collect signals. Equation (7) can be solved by the basis pursuit denoising (BPDN) algorithm [36]:

$$\min \|\mathbf{v}(f)\|_1 \quad \text{s.t.} \quad \|\mathbf{y}(f) - \mathbf{H}(f)\mathbf{v}(f)\|_2^2 \leq \varepsilon(f) \quad (8)$$

where $\varepsilon(f)$ is a variable related to the noise level of the signal. This article takes $\varepsilon(f) = \|\mathbf{y}(f)\|_2 / 2$ based on the actual research results [28]. $\mathbf{v}(f)$ contains the amplitude and phase information of the signal in the reconstructed area, which can be used to locate the damage.

2.3. Wavefield Reconstruction and Damage Imaging

After obtaining the excitation function $\mathbf{v}(f)$, the sensing matrix $\mathbf{A}'$ between the potential source and the reconstruction point in the reconstruction area is constructed, then the wavefield signal in the

reconstruction area can be expressed as:

$$\mathbf{W}(f) = \mathbf{A}'(f)\mathbf{v}(f) \quad (9)$$

Hence the full wavefield data $\mathbf{W}(t)$ at any time can be obtained by inversing Fourier transform $\mathbf{W}(f)$. It should be noted that the reconstruction area of wavefield can be selected arbitrarily in theory. In order to evaluate the reconstruction effect of wavefield, the coherence coefficient RC is used to evaluate the reconstruction accuracy of wavefield.

$$\text{RC}(r, f) = \frac{|S_{Ms, Rc}(r, f)|^2}{S_{Ms}(r, f) S_{Rc}(r, f)} \quad (10)$$

where f is frequency, $S_{Ms}(r, f)$ is the self-power spectral density function of the original wavefield, $S_{Rc}(r, f)$ is the self-power spectral density function of the reconstructed wavefield, and $S_{Ms, Rc}(r, f)$ is the mutual power spectral density function of the reconstructed wavefield and the original wavefield.

The specific imaging index calculation method is defined by:

$$\text{index}_s(x, y) = \frac{1}{f_2 - f_1} \int_{f_1}^{f_2} |v_s^{(\mu)}(x, y, f)|, df \quad (11)$$

where $\text{index}_s(x, y)$ is the potential source imaging index of mode μ at point s , and f_1 and f_2 are the start and end frequencies of the calculated frequency band. The selection of excitation frequency in this paper only includes A_0 and S_0 modes, because A_0 mode-guided waves dominate the measurement signal in the SLDV measurement environment, only the source imaging index of A mode part of the signal is subsequently analyzed.

The sampling compressive rates ξ_{SCR} of the reconstructed wavefield can be expressed with respect to the Nyquist sampling rate as:

$$\xi_{\text{SCR}} = 100\% \times \left(1 - \frac{M}{M_{\text{Nyq}}}\right) \quad (12)$$

where M_{Nyq} is the number of measurements according to Nyquist theorem.

3. Numerical Simulation

3.1. Simulation Model and Parameter Setting

An aluminum plate $400 \times 400 \text{ mm}^2$ wide and 1 mm thick was investigated numerically using the commercial FEM software COMSOL. Lamb waves were generated by an equivalent force line with the diameter of 10 mm at the center of the plate. The grid density and time step were selected according to the convergence at the maximum frequency. Figure 2a shows an aluminum plate model with a single damage through the hole circular damage with the diameter of 10 mm, in which the center of excitation source and damage coordinates are, respectively, (20, 20) and (80, 100) under the local coordinates of the construction. The measurement area is $120 \times 120 \text{ mm}$, and the reconstruction area is $200 \times 200 \text{ mm}$. As shown in Fig. 2b, c, the excitation signal is a 150 kHz, 5-cycle Hann-windowed tone burst, in which (b) is the time domain signal and (c) is the spectrum. It can be seen from the spectrum that the energy of the excitation signal is mainly concentrated in the range of 90–210 kHz, so 90–210 kHz is taken as the calculation frequency band. The spatial sampling points should be set to half of the minimum wavelength without distortion to obtain the full wavefield data according to Nyquist sampling theorem. The minimum wavelength in the calculated band is 6.32 mm at 210 kHz according to the dispersion relation. Hence, $M_{\text{Nyq}} = 4,096$ is the theoretical minimum number of reconstruction area required to properly sample the wavefield according to Nyquist sampling theorem. In order to verify the effectiveness of the proposed method, the measurement points and potential sources are 350 points and 4,096 points, respectively, corresponding to a sampling compressive ratio of $\xi_{\text{SCR}} = 91.5\%$. Specifically, the positions of the measuring points are uniformly distributed, and the sources are assumed to be generated by the random jittered method. For comparison, the number of assumed sources is also set to 4,096 in the jittered sampling and the sampling compression ratio is the same as uniform sparse sampling.

Fig. 2. Aluminum plate model (unit: mm) and excitation signal

3.2. Damage Imaging and Wavefield Reconstruction

The data collected by the two sampling methods were brought into the CS equation for solution, and the distribution diagram of imaging index of source function were obtained, as shown in Fig. 3, in which (a) and (b) are the imaging results corresponding to jittered sampling, (c) and (d) are the imaging results corresponding to uniform sparse sampling. From the 2D source function imaging (a) and (c), it can be seen that there are a large number of low amplitude pseudo sources in the measurement area in addition to the location of the excitation source and damage. The imaging results are analyzed as follows: The solution of the CS equation itself is an optimization algorithm. What is obtained is the optimal solution that satisfies the constraints rather than the exact solution. Therefore, in the solution process, the non-structural feature points may be regarded as sound sources. However, most of the energy in the wavefield is generated by real structural features (excitation source, damage, etc.); the amplitude in the non-structural part of the source imaging will not be too high, which does not affect the identification of the real sound source in the wavefield. It can also be seen from (b) and (d) that the amplitude $index(x, y, A_0)$ of the real sound source position is significantly higher than that of other positions. The amplitude $index(x, y, A_0)$ near the center of the excitation source is the highest and reaches 1, followed by the amplitude at the damage position, which is about 0.1. The amplitude $index(x, y, A_0)$ at the non-sound source position is mostly 0. According to the above imaging results, the excitation source and damage can be located, and the results are shown in Table 1.

It can be seen from Table 1 that the location error of the excitation source calculated by jittered sampling data is 0 mm, and the damage location error is about 2.8 mm; while the location error of the excitation source calculated by uniform sparse sampling is 0.9 mm, the damage location error is 1.8 mm. The errors of the two sampling methods are within the minimum half-wavelength, which shows that both methods can identify and locate the sound source with high accuracy.

The wavefield can be reconstructed by substituting the source function into the guided wave propagation equation [Eq. (9)]. Figure 4 shows the snapshots of reconstructed wavefields and original wavefield at 72.0 μ s, in which (a) is the original wavefield, and (b) and (c) are reconstructed wavefields by jittered sampling and uniform sparse sampling, respectively. From the reconstructed wavefields (b) and (c), it can be observed that the incident wavefield and damage scattering wavefield are generated by the excitation source. Compared with the original wavefield, there are noise points in the reconstructed wavefield. The reason is that the error of the source function makes the reconstructed wavefield different. In addition, it can be observed that the scattering of the real damage is not uniform in all directions. Since all potential sources are assumed to scatter the Lamb waves uniformly, this factor also makes the reconstructed wavefield different from the original wavefield. The accuracy of reconstructed wavefield is evaluated by the correlation coefficient $RC(f)$.

As shown in the $RC-f$ curve of Fig. 5, the corresponding values of the two sparse sampling methods can be kept above 0.9 in the calculation frequency band (90–210 kHz), which shows that the reconstructed wavefield is highly consistent with the original wavefield. The corresponding value of frequency

Fig. 3. 2D and 3D source images of jittered sampling and uniform sparse sampling

Table 1. Acoustic source localization results of two methods

Sampling strategy	Source type	Real location	Estimated location	Error
Jittered sampling	Actuator	(20, 20)	(20, 20)	0
	Damage	(80, 100)	(78, 98)	2.8
Uniform sparse sampling	Actuator	(20, 20)	(20.9, 20.2)	0.9
	Damage	(80, 100)	(78.8, 98.6)	1.8

Fig. 4. Snapshots of original wavefield and reconstructed wavefields ($t = 72.0 \mu s$)

points outside the calculated frequency band is lower. Since most of the energy of the wavefield is concentrated in the calculated frequency band, the inaccuracy outside the calculated frequency can be ignored.

(a) RC - f curve corresponding to jittered sampling (b) RC - f curve corresponding to uniform sparse sampling

Fig. 5. The RC - f curve of jittered sampling and uniform sparse sampling

(a) PZT excitation/SLDV sensing platform

(b) Measurement and reconstruction area map(unit: mm)

Fig. 6. PZT excitation/SLDV sensing platform

4. Experimental Validation

4.1. Experiment Setup

The PZT excitation/SLDV receiving experimental platform is shown in Fig. 6a. SLDV is a psv-500 produced by Polytec company. A 6,061-aluminum plate being $1,000 \times 1,000$ mm wide and 1 mm thick was attached with piezoelectric transducers (PZT disk's diameter being 10 mm, and thickness being 1 mm), the damage is through hole circular damage with the diameter of 10 mm. The excitation signal is a five peak modulated sine signal with a center frequency of 150 kHz. During the sampling process, the sampling frequency $f_s = 5.12$ MHz and the sampling time is $100 \mu\text{s}$; each point is repeated 20 times in average to improve the signal-to-noise ratio. The detailed setting of the specific measurement area and the potential source area are shown in Fig. 6b. The measurement area is a rectangle of 81.5×81.5 mm, the potential source area is a rectangle of 200×200 mm, and the wavefield reconstruction area is assumed to coincide with the source area. The sampling compression rate is $\xi_{SCR} = 91.5\%$, which is the same as the simulation data. In the experiment, the jittered sampling strategy needs to be processed offline, so the full wavefield needs to be collected in advance. Since the location of the excitation source is not in the measurement area, this experiment can be used to verify the effect of wavefield reconstruction and source imaging when the sound source is outside the measurement area.

Fig. 7. 2D and 3D source images of uniform sparse sampling and jittered sampling

Table 2. Damage location results and errors

Sampling type	Real location	Estimated location	Error (mm)
Jittered sampling	(98, 136)	(100.0, 136.0)	2.0
Uniform sparse sampling		(99.8, 135.66)	1.8

4.2. Damage Imaging Analysis of Two Sampling Methods

Figure 7 shows the source function imaging results of jittered sampling and uniform sparse sampling. As can be seen from the 2D source function imaging (a) and (c), there are a large number of low-amplitude pseudo sources in the measurement area in addition to the location of the excitation source and damage. Because the solution of the compressed sensing equation itself is an optimization process, the optimal solution is not an exact solution. Therefore, in addition to the excitation source and damage, there are other points that are not zero. It can be concluded from the 3D source function imaging (b) and (d) that the errors of the damage location results based on jittered sampling and uniform sparse sampling are 2.0 and 1.8 mm, respectively; the location error values of which are both within the minimum half-wavelength. The location error corresponding to uniform sparse sampling is slightly lower than that corresponding to jittered sampling.

4.3. Wavefield Reconstruction Analysis of Two Sampling Methods

Figure 8 shows the snapshot of reconstructed wavefields and original wavefield at $62.3 \mu\text{s}$, in which (a) is the original wavefield, and (b) and (c) are reconstructed wavefields by jittered sampling and uniform sparse sampling data, respectively. It can be seen from the original measured wavefield that the A_0 mode incident wavefield generated by the excitation source reflects after encountering the damage. In addition, it can be seen from (b) and (c) that the reconstruction effects of the two sampling methods are highly restored to the original wavefield, and differences can be obvious observed at the damage center. The specific reconstruction quality evaluation is analyzed through the coherence coefficient $RC(f)$.

The accuracy and the computational efficiency of wavefield reconstruction are further compared by two sparse sampling methods. Figure 9 shows the relationship between the frequency and the coherence

Fig. 8. Snapshots of original measured wavefield and reconstructed wavefields ($t = 62.3 \mu\text{s}$)

Fig. 9. RC- f curve comparison between jittered sampling and uniform sparse sampling

coefficient RC. It can be seen from Fig. 9 that the RC values of the two sparse sampling methods can be kept above 0.9 in the calculation frequency band (90–210 kHz), which shows that the reconstructed wavefield is highly consistent with the original wavefield. In addition, the reconstruction accuracy is not high outside the calculation band, and there is no obvious difference between the two sampling. Figure 10 shows the relationship between the RC of the central frequency and the measurement point m under two kinds of sampling strategy. It can be concluded that when $m > 100$, the corresponding values of the two sampling strategies tend to be stable, the reconstruction accuracy RC (f_0) of both can be kept above 0.9, which indicate that the reconstruction effect of both sampling strategies has been improved and tended to be stable with the increase of sampling points. From the perspective of data acquisition efficiency, the jittered sampling strategy needs to be processed offline in the experiment, so the full wavefield data needs to be collected in advance. From the perspective of data acquisition time, it took 2,223 s to collect data using the traditional jittered sampling method, while the uniform sparse sampling method proposed in this paper only took 187 s. The reason is that the uniform sparse sampling method can be directly applied to the SLDV system without additional control elements, which means that the data acquisition time can be induced by more than 90% according to the $\xi_{\text{SCR}} = 91.5\%$ used in the experiment, which greatly improves the efficiency of data acquisition.

5. Conclusions

In order to solve the time-consuming problem of traditional full wavefield acquisition, a Lamb wavefield reconstruction method based on uniform sparse sampling strategy is proposed in this paper. The simulation and experimental results show that uniform sparse sampling strategy can realize wavefield reconstruction accurately with less than 90% Nyquist sampling points. In addition, it can directly reduce more than 90% sampling time of the SLDV system without adding other control elements, and can achieve the same wavefield reconstruction and damage location accuracy as the conventional jittered sampling strategy. The research results can greatly improve the efficiency of the full wavefield

Fig. 10. RC (f_0)- M curve comparison between jittered sampling and uniform sparse sampling

pickup, and further improve the possibility of practical application of the wavefield reconstruction method in engineering.

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