The long-term impact of trade with firm heterogeneity

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Abstract

This paper explores the welfare effects of openness in a setting with firm heterogeneity and country asymmetry and presents results in terms of the well-known formula from Arkolakis et al. (Am Econ Rev 102(1):94–130, 2012). By allowing agents to save and the economy to grow, new channels for the welfare effects of openness appear, since firm selection affects the value of accumulated savings and the average efficiency of the economy, and therefore its future growth rate. Country asymmetry yields differentiated, and in some cases opposite, results between countries. In line with the empirical literature, net welfare effects in each region depend on countries' specific conditions and losses may occur. A numerical exercise fits the model to the UK and EU economies to show the magnitude and direction that each effect can take if trade barriers increase between them. It is shown that welfare losses for UK consumers can be greatly underestimated if asymmetries and dynamics are removed from the analysis.

Keywords Firm heterogeneity · Expanding varieties · Asymmetric countries · Welfare · Trade liberalization

JEL Classification F12 · F15 · O40

1 Introduction

The emergence and development of the heterogeneous firm trade model (HFTM) that followed the seminal work of Melitz (2003), has provided great insights on the mechanisms through which trade openness affects economies, highlighting within-industry resource reallocation. A standard result in the HFTM is the increase in welfare for consumers of all countries involved, following trade liberalization. In Melitz (2003) and Helpman et al. (2004), openness provides incentives for the most efficient

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firms in the industry to export and expand while low-productivity firms are forced out by tougher competition, in a process of firm selection. As a result, resources move from the latter firms to the former, average efficiency in the industry increases and welfare rises due to the subsequent reduction of the price index. The HFTM has been extensively used over the last decade to account for many previously unexplained facts in the trade literature. However, most extensions have remained either static in nature, or assume symmetric countries.

This paper contributes to the trade and growth literature by presenting a North-South model with heterogeneous firms and growth. The welfare effects of freer trade may not be as straightforward when simple dynamics are introduced into the analysis, as new static and dynamic effects can be brought up by firm selection, adding to the well-known reduction in prices. Allowing agents to save resources implies that their consuming possibilities can be modified by freer trade, if firm selection affects the activity towards which savings are directed. In the present model, savings are devoted to creating the knowledge demanded by final producers. This means that firm selection, having an impact on the average efficiency of the economy, affects saving returns and consumers' income. Openness yields dynamic effects when it impacts the growth rate of the economy. Firm selection affects the average amount of knowledge required by firms to develop goods, and this alters the amount of resources that must be devoted to research activities and therefore affects future growth. Effects are differentiated between countries when technological asymmetries are allowed in the model. Although effects go in the same direction for countries when asymmetries are small, large asymmetries yield effects going in opposing directions between economies, at least in the short-term. In all cases analyzed here, net welfare results depend on parameter values representing countries' characteristics, a conclusion often underlined by the empirical research on welfare effects of trade liberalization (see for example Winters et al. 2004, Wacziarg and Welch 2008, or Caliendo and Parro 2015). Moreover, this paper highlights the possibility of a negative effect on the growth rate. This happens when firm selection increases average efficiency in production and this translates into greater knowledge requirements for further product developments that slows down growth. The possibility that resource reallocation may harm future growth, even when creating short-term productivity gains, has been stressed in recent empirical works as an important source of income divergence between countries in Latin America or Africa, and the developed world (see for example Daude and Fernández-Arias 2010 or de Vries et al. 2015).

To provide a sense of magnitude for each of these effects, a numerical exercise is performed where the main parameters of the model are set to fit trade between the UK and the EU. Welfare results can then be compared with those estimating the effects on consumers that an increase in trade barriers between these two economies (Brexit) can have. A small increase in trade costs between the two regions yields welfare losses to consumers on both sides of the deal of around 11% of the change, when asymmetries or dynamic effects are not considered. When the discounted value of a permanent change in the growth rate in each economy is considered, the dynamic effect largely dominates the static effect, as its magnitude can be substantially larger. The loss that consumers from the UK experience is therefore

much larger than the initial number, since the effect is negative in that economy. The dynamic effect is positive for consumers of the EU in the present model since product development is enhanced rather than hindered in that economy, so the net welfare effect turns positive for that region.

The model presented here introduces expanding product variety growth as in Grossman and Helpman (1991) into an asymmetric country setting where firms are heterogeneous. Among the papers introducing dynamics into the HFTM, the closest to this paper are Baldwin and Robert-Nicoud (2008) and Ourens (2016), where expanding varieties growth is inserted into a HFTM à la Melitz (2003) in a context of country symmetry.¹ Other works with similar proposals are Gustafsson and Segerstrom (2010) and Dinopoulos and Unel (2011). As opposed to these contributions the model in the present paper allows for country asymmetries.² As a result, the present paper explores welfare effects in a context that is closer to reality, and allows for differentiated effects to happen across trade partners.

Previous works allowing for differences in size and technology between countries are Demidova (2008), Baldwin and Forslid (2010) and Demidova and Rodríguez-Clare (2013) respectively. Models in those works are static so they lack the largest welfare channels explored in the present paper. Welfare results in all these papers are driven entirely by movements in the price index. Changes in saving decisions or in the growth rates are not explored. Moreover, in cases where the only asymmetry allowed is in size (as in Baldwin and Forslid 2010), there is no room for welfare losses, while this is possible in the present model.

In a very influential work, Arkolakis et al. (2012, ACR hereafter) explain that welfare outcomes of freer trade in a static HFTM with Pareto distributed firms (HFTM+P) can be written as a function of two observables: the import penetration ratio and the elasticity of imports with respect to variable trade costs. The present paper presents results in terms of that formula to make them comparable with the literature. Results from this simple dynamic setting differ greatly from those in static models, even when countries are assumed to be symmetric. While in Melitz (2003) consumers from all countries involved experience welfare gains from a price fall, in the present model this effect is enhanced by an increase in expenditure due to higher returns from households' savings. However, firm selection increases the average requirement for knowledge thus increasing the cost of new varieties. The resulting negative dynamic effect alters long-term welfare outcomes dependent on country characteristics represented here by exogenous parameters. A symmetric setting

¹ Ourens (2018) also presents a framework with asymmetric countries and expanding variety growth, however the structure of the model is quite different from the one proposed here, as it imposes homogeneous firms and presents a different utility function. More importantly, that paper's focus is on how terms of trade can change in the presence of uneven product creation, so trade shocks, which are at the core of the analysis presented here, are not explored.

² Another stream of dynamic HFTMs has adopted a somewhat different approach. Atkeson and Burstein (2010) allow differentiated-good producers to invest in innovations making growth the result of the firm's strategic decisions. This choice, also followed by others such as Burstein and Melitz (2013) or Alessandria and Choi (2014), yields rich firm dynamics that, although interesting, are unnecessary for the purposes of this work. So far this stream of the literature has not been expanded to asymmetric countries. This could be a subject for future work.

suffices to highlight that the dynamic feature of the present model brings about new channels for welfare effects absent in static models. Asymmetries in technological development between countries add an extra layer of heterogeneity that interacts with dynamics in the model affecting welfare outcomes. A small technological lag in the South relative to the North still yields positive static effects in both regions but their magnitude differ, being greater in the North. Moreover, even though growth rates converge to a lower value in the long term they can differ in the short term. When the technological gap is large, the South may experience reversed firm selection, making prices rise and expenditure fall. The negative static effect in the South contrasts with a positive effect in the North. Finally, openness endogenously exacerbates specialization in the industry with comparative advantages of each region.

The present paper is also related to other works evaluating welfare outcomes in dynamic models with heterogeneous firms such as Perla et al. (2015), Sampson (2016) and Naito (2017). In Perla et al. (2015), firms' entry costs are expressed in terms of labor and when liberalization raises wages it pushes up the entry cost, reducing the number of varieties available and producing an unequivocally static loss. The dynamic effect in their model is always positive, as openness improves the distribution of potential technologies firms can have, so the net effect depends on the parametrization of the model. In the model presented here, entry costs are expressed in terms of knowledge, and the signs of the dynamic and static effects are the exact opposite as those in Perla et al. (2015). In this sense, the contribution of this paper is to show how the signs of these effects depend on the way the free entry condition is specified.³ Sampson (2016) explores welfare effects in the case where the productivity distribution of firms is positively affected by knowledge spillovers. The resulting dynamic selection introduces a new source of gains from trade previously unexplored. The present paper can be seen as complementary to his in that, acknowledging the value of endogenizing the distribution of firms, it shows that introducing simple dynamics it is possible to obtain richer welfare effects than the static model, even with an exogenous distribution of firms.

In this respect, the model proposed here is closer to that in Naito (2017), where a general framework with heterogeneous firms and growth is proposed. The present paper proposes the following contributions upon that work. First, the case of Pareto-distributed firms is discussed in full. This particular case is of key importance, as is the one on which the theoretical debate on the net welfare effects of trade with firm heterogeneity has focused on (see ACR and Melitz and Redding 2015). Presenting welfare results for the model with Pareto-distributed firms enables comparison with previous results on these papers and contributes to this literature. I show exactly how one can move from the ACR framework to a framework with dynamics and asymmetric economies. Moreover, focusing exclusively on the Pareto case enables closed form solutions, which come very handy when analyzing the effect of trade shocks in the model. In heterogeneous firm trade models, it is often the case that intuition is lost and models give results that seem obscure to the reader. In the present model,

³ An evaluation of how both mechanisms may interact in theory and which prevails empirically for given countries' characteristics should be subject for future research.

comparative statics are easy to follow and help intuition build up, gaining transparency. Moreover, by providing a quantitative sense of the magnitude of the effects, this paper provides a further contribution to this literature.⁴

The rest of the paper is organized as follows. Sect. 2 presents the model identifying the main assumptions made. Section 3 defines the Balanced Growth Path analyzed here and presents closed-form solutions for all endogenous variables under the assumption that firms' productivity is Pareto-distributed. Section 4 presents the long-term welfare results of increasing openness. First, it presents results assuming country symmetry in order to contrast these to those arising in well-known static frameworks and to show the new effects brought up by the dynamic setting proposed here. Then, country asymmetry is allowed for to show how the dynamic feature of the model interacts with countries asymmetries in welfare evaluations. For both cases, results are compared to those in ACR, highlighting the differences. Section 5 presents a numerical exercise that makes the model fit a potential increase in trade costs between the UK and the EU, and presents welfare results of changing trade costs between the two economies. Finally, Sect. 6 concludes.

2 The theory

The continuous time model is composed of two economies (North and South) indexed by $i = N, S$ differing only in size and degree of technological development. Each economy features two types of final good producers: one sector (denoted M) produces differentiated goods and sells them to consumers under monopolistic competition; the other one (sector C) produces a homogeneous good under perfect competition. The homogeneous good is freely traded in the world market. This imposes factor price equalization between countries as long as the C -good is produced in both: wages can be then normalized to 1. Economies also feature an R&D sector in charge of producing the knowledge (denoted K) that enables the production of new M -products. Each economy is endowed with a fixed amount of labor, distributed among the three productive activities $L_i = L_{K,i} + L_{M,i} + L_{C,i}$.

The timing of production in the M -sector is similar to that proposed in Melitz (2003). First, differentiated good producers buy blueprints for a new variety. Then they discover the productivity ($1/a$) they could have if they engage production of that new variety. With this information, firms evaluate whether it is profitable for them to incur the extra costs of serving the domestic market, or even exporting at a greater cost. The fact that profits increase with productivity implies that there will always be a firm with marginal selling cost a_D (a_X) indifferent between producing for the domestic (foreign) market or not.

⁴ Notice that the approach followed in this paper differs from the recent macro oriented literature that can be found in Ravikumar et al. (2019) or Alessandria et al. (2018). Those works provide quantitative analysis including transitions to the equilibrium, but do not obtain the closed-form solutions allowing the present paper to analytically track the different effects.

2.1 Consumers

Consumers in i make three choices. First, they decide their optimal expenditure level $E_i(t)$. Then, they decide expenditure in each of the two types of final goods, i.e. chose $E_{M,i}(t)$ and $E_{C,i}(t)$ with $E_i(t) = E_{M,i}(t) + E_{C,i}(t)$. Finally, they establish how to split their consumption of M -goods among the different varieties available at each t . Welfare at t is the present value of future consumption:

$$U_i(t) = \int_t^{\infty} e^{-\rho(s-t)} \ln(Q_i(s)) ds \quad (1)$$

where $\rho > 0$ is the rate of pure time preference, $Q_i(t) = E_i(t)/P_i(t)$ is real consumption and $P_i(t)$ is the aggregate price index at t . At every t , consumers from i maximize (1) subject to a standard budget constraint. The optimality conditions that arise from this problem are a transversality condition and an Euler equation $\dot{E}_i(t)/E_i(t) = \iota(t) - \rho$.

Consumers' upper tier preferences are Cobb-Douglas, giving a fixed share $\mu > 0$ of expenditure on M , while the remaining $1 - \mu$ goes to purchases of the C -good. Lower-tier preferences between varieties of the M -sector are CES. Then the perfect price index in the market with monopolistic competition is

$$P_{M,i}(t) = \left[\int_{\theta \in \Theta_i(t)} p_i(\theta, t)^{1-\sigma} d\theta \right]^{1/(1-\sigma)} \quad (2)$$

where $p_i(\theta, t)$ is the price of variety θ at time t in market i , $\Theta_i(t)$ represents the mass of available varieties in the M -good market of this economy (both produced domestically and imported) at time t , and $\sigma > 1$ is the constant elasticity of substitution between any two varieties of the M -good. Expenditure in each M -variety available $e_i(\theta, t)$ and aggregate expenditure in the M -good at country i are respectively:

$$e_i(\theta, t) = E_{M,i}(t) \left[\frac{p_i(\theta, t)}{P_{M,i}(t)} \right]^{1-\sigma} \quad ; \quad E_{M,i}(t) = \int_{\theta \in \Theta_i(t)} e_i(\theta, t) d\theta \quad (3)$$

2.2 Final good producers

2.2.1 C-sector

Technology of production is the same in both countries $Q_C = L_C$. Perfect competition imposes zero profits in this sector so marginal revenues must equal marginal costs, i.e. $P_{C,i} = 1$. This, combined with the upper-tier preferences in the previous section, gives an aggregate price for domestic consumers at i of $P_i(t) = P_{M,i}(t)^\mu B$, where $B > 0$ is a constant. There are no trade costs for the C -good. Revenues made by C -firms from country i are $R_{C,i}(t) = L_{C,i}(t)$. These revenues come from selling in the domestic and in the foreign markets. Unlike the autarkic or the symmetric

country cases where revenues made by a sector equal domestic expenditure in the goods of that sector, in an asymmetric world with trade $R_{C,i}$ and $E_{C,i}$ are not necessarily equal.

2.2.2 *M*-sector

The structure for the *M*-sector closely follows Baldwin and Robert-Nicoud (2008). At any t , potential entrants need to pay three different sunk costs f_I , f_D and f_X to discover their marginal cost a , serve the domestic market and serve the foreign market, respectively. Each variety is produced by only one firm and each firm has a unique marginal cost, so it is possible to index goods and firms by θ or a indistinctly. Marginal costs are drawn from a country-specific Pareto distribution $g_i(a)$, with cumulative distribution $G_i(a) = (a/a_{m,i})^\alpha$, with $a \in (0, a_{m,i}]$. Scale parameter $a_{m,i} > 0$ in country i resumes information about the state of the technology in that region. For any two countries, prospective entrants to the *M*-sector in the country with a lower a_m have a greater probability of obtaining a cost level below a certain value a^* , $\forall a^* \in (0, a_{m,i}]$. It is assumed that $a_{m,i}$ is high enough to make production non-profitable for the higher values of a 's support. Parameter $\alpha > 0$ is the shape parameter for firm productivities and marginal costs. The shape parameter for firm sizes and operating profits is then $\beta = \frac{\alpha}{\sigma-1}$. As in Melitz and Trefler (2012), it is assumed that $\beta > 1$, as this is a necessary condition for integrals in (6) to converge. The average marginal cost, firm size and operating profit are guaranteed to be finite as a consequence.

Assuming a Pareto distribution is one of the main departures from Naito (2017), and allows the current model to fit in the framework proposed in ACR, and makes the results comparable to those in a large bulk of the literature on welfare effects of trade. Moreover, this choice is very convenient analytically as it yields closed-form solutions for the endogenous variables of the model.⁵

Each period, firms are hit with a negative shock that is large enough to force them out the market with exogenous probability $\delta_i > 0$.⁶ Marginal firms to sell domestically and export have marginal costs equal to $a_{D,i}$ and $a_{X,i}$, respectively. Each firm pays its corresponding sunk costs only once, at the beginning of their activities. As in Baldwin and Robert-Nicoud (2008), all sunk costs are composed by units of knowledge so their value can be written as $f_{v,i}(t) = \kappa_v \cdot P_{K,i}(t)$ for $v = I, D, X$, where κ_v is a fixed quantity of units of knowledge and $P_{K,i}$ is the price of knowledge. It is

⁵ The use of this distribution is customary in the literature although its empirical validation is not undisputed. An incomplete list of papers using this distribution may include Helpman et al. (2004), Melitz and Ottaviano (2008), Baldwin and Robert-Nicoud (2008), Falvey et al. (2011) or Melitz and Trefler (2012). Nevertheless, empirical validation of the distribution is not settled. Axtell (2001) shows that the Pareto distribution is a good fit for sales and employment among the US's firms. Using data for French firms, Combes et al. (2012) present empirical evidence suggesting that the log-normal distribution fits firms' productivity better.

⁶ The exogenous exit rate appears as innocuous when evaluating the effects of openness in the present model. However, introducing firms' exit in a model where all costs are upfront ensures that, in the long term, the distribution of producing and exporting firms are both truncated at their respective endogenous thresholds.

assumed that the κ 's are the same worldwide while $P_{\kappa,i}(t)$ is country specific. Moreover, it is assumed that $\kappa_D < \kappa_X$ reflecting the fact that it is less costly to sell locally than to engage in business abroad.

Per-unit costs to international shipments are modeled as iceberg costs $\tau \geq 1$. Let us denote with m the marginal selling cost a firm with marginal cost a has in a given market. Then, m equals a for sales to the domestic market and τa for sales to the foreign market. This allows us to write the pricing rule in the M -sector as:

$$p(m, t) = \frac{m\sigma}{\sigma - 1} \quad (4)$$

Notice that there are no differences in preference parameters ρ and σ between countries. The pricing rule in (4) means that each M -firm has a mark-up over its sales of $1/\sigma$ in each market so the operating profits a firm with selling cost m makes in market i are $\pi_i(m, t) = s_i(m, t)E_{M,i}(t)/\sigma$, where the share that a firm with marginal selling cost of m has in market i is $s_i(m, t) = [m/h_i(t)]^{1-\sigma}$, with

$$h_i(t)^{1-\sigma} = n_i(t)\hat{m}_i^{1-\sigma} + n_j(t)v_j\tilde{m}_i^{1-\sigma} \quad (5)$$

and

$$\begin{aligned} v_i &= G_i(a_{X,i})/G_i(a_{D,i}) \\ \hat{m}_i^{1-\sigma} &= \int_0^{a_{D,i}} a^{1-\sigma} dG_i(a|a_{D,i}) \\ \tilde{m}_i^{1-\sigma} &= \phi \int_0^{a_{X,j}} a^{1-\sigma} dG_j(a|a_{X,j}) \end{aligned} \quad (6)$$

Here $n_i(t)$ denotes the mass of varieties being produced in country i and $0 \leq \phi = \tau^{1-\sigma} \leq 1$ is a measure of the importance of variable trade costs in the industry (when $\tau \rightarrow \infty$ then $\phi \rightarrow 0$ and when $\tau \rightarrow 1$ then $\phi \rightarrow 1$). $\hat{m}_i$ is the marginal selling cost of the representative firm producing in i for the domestic market, while $\tilde{m}_i$ is the marginal selling cost of the representative foreign firm selling in i . v_i is the proportion of firms from i that serve the foreign market j . Then we can think of h_i as the aggregate marginal selling costs of firms from both countries serving market i . The expression for shares shows that the greater the marginal selling cost a firm has, the lower its share in a given market. A firm's share also decreases with tougher competition in that market (i.e. with a lower h). Notice that (5) implicitly assumes that the set of varieties sold domestically includes the set of exported varieties in both countries. This is formally stated in Assumption 1. Calling $N_i(t) = n_i(t) + v_j n_j(t)$ the total mass of firms that sell in country i , then $h_i N_i^{1/\sigma-1}$ is the marginal selling cost of the representative firm serving that market.⁷

⁷ Expression $h_i^{1-\sigma}$ in the present model is the equivalent to $M_{ii}\hat{\varphi}_{ii}^{\sigma-1}$ in Demidova (2008) and -if symmetry is imposed- to $n\tilde{m}^{1-\sigma}$ in Baldwin and Robert-Nicoud (2008).

2.3 K-sector

This sector produces units of knowledge demanded by firms in the M -sector using labor as sole input. It has the following production function:

$$Q_{K,i}(t) = \frac{n_i(t)}{a_{K,i}} L_{K,i}(t) \quad (7)$$

where $L_{K,i}(t)$ is the amount of labor devoted to the sector in country i and $n_i(t)/a_{K,i}$ is the productivity of its workers. As in the standard expanding variety model, labor productivity in the K -sector is composed by an exogenous parameter $a_{K,i}$ and is affected by spillovers from existing blueprints $n_i(t)$ (see for example Sect. 3.2 in Grossman and Helpman 1991). Perfect competition in the market of knowledge imposes zero profits and therefore $a_{K,i}/n_i(t) = P_{K,i}(t)$ where $P_{K,i}(t)$ is the unitary price of knowledge expressed in units of labor at time t in country i .

2.4 Equilibrium conditions

2.4.1 Cut-off conditions in the M -sector

Firms from country i discount future operating profits at a rate γ_i to be defined later on. The present value of operating profits that a firm from country i and selling cost m gets by serving its domestic market is $s_i(m, t)E_{M,i}(t)/\sigma\gamma_i$, while selling in the foreign market it gets $s_j(m, t)E_{M,j}(t)/\sigma\gamma_j$. After paying the initial fixed cost f_i , firms take producing and exporting decisions comparing these values with the sunk cost required to engage each activity, which must be paid upfront at their country of origin. The cut-off conditions for serving market i are defined as:

$$\frac{s_i(m_{D,i}, t)E_{M,i}(t)}{\sigma\gamma_i} = P_{K,i}(t)\kappa_D \quad ; \quad \frac{s_i(m_{X,j}, t)E_{M,i}(t)}{\sigma\gamma_j} = P_{K,j}(t)\kappa_X \quad (8)$$

where $m_{D,i} = a_{D,i}$ and $m_{X,j} = \tau a_{X,j}$. The second condition imposes that a firm $a_{X,j}$ from country j is indifferent between serving market i or not.

2.4.2 Free-entry conditions in the M -sector

While under symmetry all firms face the same expected operating profits regardless of their country of origin, the same is not true in the asymmetric country case. It is possible that firms from one country enjoy larger average shares than firms from the other. Let us define $d_i(t) = \bar{s}_{i,i}(t)n_i(t)$ as the aggregate share of market i served by domestic firms and $x_i(t) = v_i(t)\bar{s}_{i,j}(t)n_i(t)$ as the aggregate share of market j served by firms from i . Then:

$$d_i(t) = \left[1 + \frac{n_j(t)v_j(t)}{n_i(t)} \left(\frac{\tilde{m}_i}{\hat{m}_i} \right)^{1-\sigma} \right]^{-1} \quad \text{and} \quad x_i(t) = \left[1 + \frac{n_j(t)}{n_i(t)v_i(t)} \left(\frac{\hat{m}_j}{\tilde{m}_j} \right)^{1-\sigma} \right]^{-1} \quad (9)$$

Using (9), it is possible to prove that at any t : $d_i \geq x_i$, and equality would only hold if $a_{D,i} = a_{X,i}$ in both countries and $\phi = 1$ (see Proof 2 in the "Appendix"). So unless we are in the extreme case in which exporting brings about no extra costs, firms from country i always enjoy a greater aggregate share at their domestic market than abroad. By definition $d_i \leq 1$, $x_i \leq 1$ and $d_i + x_j = 1$, $\forall i, j = N, S$. Then, as long as there is some degree of openness ($\phi > 0$), average shares are lower than $1/n_i$, i.e. the average share in autarky.

Free entry in the M -sector implies that, at equilibrium, incentives for entry must be exhausted. The expected value of operating profits for all firms in i must equal the expected sunk costs these firms pay. The free-entry condition (FEC) is:

$$\frac{d_i(t)E_{M,i}(t) + x_i(t)E_{M,j}(t)}{\sigma\gamma_i} = a_{K,i}\bar{\kappa}_i \quad (10)$$

where

$$\bar{\kappa}_i = \frac{\kappa_D G_i(a_{D,i}) + \kappa_X G_i(a_{X,i}) + \kappa_I}{G_i(a_{D,i})} \quad (11)$$

is the expected sunk cost that a producing firm has to pay, expressed in units of knowledge.

2.4.3 Instantaneous equilibrium

The clearing condition in the market for knowledge imposes that, at any t , the production of knowledge must be used in sunk costs of new entrants. This gives:

$$Q_{K,i}(t) = [\dot{n}_i(t) + \delta_i n_i(t)]\bar{\kappa}_i \quad (12)$$

As long as the creation of new knowledge is sufficiently large, the mass of newcomers in the M -sector will more than compensate for the exit of firms and variety growth is positive ($g_i(t) = \dot{n}_i(t)/n_i(t) > 0$). Notice that (12) introduces a major departure from the standard expanding-variety model with homogeneous firms, as in such a model all firms buy the same number of blueprints and the value of that purchase can then be normalized to one. In terms of the present model, this equals imposing $\bar{\kappa}_i = 1$. Allowing for an endogenous, and country-specific, value of $\bar{\kappa}_i$ amounts to introducing an externality in the K -sector, which depends on firm selection and links the value of savings to average efficiency. This adds to the usual spillover in the standard expanding-variety model stemming from the quantity of existing knowledge, which is usually introduced to ensure positive growth in the long run.

Once the number of varieties is established, the rest of the static equilibrium is similar to that in Demidova (2008). Using the cut-off conditions in (8) yields:

$$\frac{a_{X,j}}{a_{D,i}} = \left(\frac{\phi}{T} \frac{P_{K,i}(t)}{P_{K,j}(t)} \right)^{\frac{1}{\sigma-1}} \quad (13)$$

where $T = \kappa_X/\kappa_D > 1$ measures how important the supplementary cost to export is relative to the cost of serving the domestic market. This expression shows that there exist a relationship between the range of varieties being exported to i and the range of varieties being produced in i . As in Demidova (2008), this relationship depends on the degree of openness to trade (in variable and sunk costs), but equation (13) includes a new determinant of the ratio $a_{X,j}/a_{D,i}$, i.e. the relative price of knowledge between countries, that introduces firm selection as a relevant factor here too. The range of exports from j to i , is enhanced by greater openness (i.e. larger ϕ and/or smaller T), lower relative price of knowledge in j (larger $P_{K,i}/P_{K,j}$) and greater average costs among local firms which follows from a larger $a_{D,i}$. Inversely, solving for $a_{D,i}$ the equation shows that greater openness, a larger relative price of knowledge in i and a larger average efficiency of firms exporting to i (smaller $a_{X,i}$), increases average productivity of domestic firms in i .

Using (8) and (10), the cut-off value for domestic production is:

$$a_{D,i} = a_{m,i} \left[\frac{\kappa_I}{\kappa_D} \frac{\beta - 1}{Z_i} \right]^{1/\alpha} \quad (14)$$

where

$$Z_i = \frac{1 - \Omega^2}{1 - \Omega R_i} \quad (15)$$

and $\Omega = \phi^\beta(1/T)^{\beta-1}$ is a measure of openness in the economy that combines the importance of variable (ϕ) and sunk (T) trade costs in a Cobb-Douglas function with increasing returns to scale. Remember that $0 < \phi < 1$ and $T > 1$, therefore $0 < \Omega < 1$. Then, Z_i resumes how the Ricardian comparative advantage, given here by $R_i = [P_{K,j}(t)/P_{K,i}(t)]^\beta [a_{m,j}/a_{m,i}]^\alpha$, affects firm selection. Indeed, the relatively better the technology in country i (i.e. the greater $a_{m,j}/a_{m,i}$ or $P_{K,j}/P_{K,i}$) the higher Z_i and the lower $a_{D,i}$ implying a greater average productivity for local M -firms. Expression (15) helps put forward that Ricardian comparative advantage in the M -sector comes both from a better productivity distribution for M -firms but also from better productivity in the K -sector. It is noteworthy that under country symmetry $R_i = 1$ and so $Z_i = Z_j = Z = 1 + \Omega$. Notice also that when economies are closed ($\Omega \rightarrow 0$) then $Z_i \rightarrow 1$ and Eq. (14) reduces into its equivalent under autarky, so technological asymmetries play no role in firm selection.

In Eq. (14), the cut-off cost to produce for the domestic market depends negatively on the ratio κ_D/κ_I reflecting that the greater the cost of domestic production (relative to the initial sunk cost) the narrower the range of varieties the economy is going to produce. The threshold $a_{D,i}$ also depends positively on $a_{m,i}$ reflecting that a better technology (lower $a_{m,i}$) necessarily yields greater average efficiency in the economy (lower $a_{D,i}$). These results are quite intuitive and resemble what has been obtained under symmetry. What the asymmetric-country expression helps put forward is the effect that openness and technological differences have on this threshold. Indeed, unlike the symmetric country case in which greater openness leads to a monotonic reduction in a_D this is not necessarily the

case when countries are technologically asymmetric. This is analyzed in detail in Sect. 4.2.

Merging Eqs. (13) and (14) yields two useful expressions for $a_{X,i}$:

$$a_{X,i} = a_{m,j} \left[\frac{\kappa_I}{\kappa_X} \frac{\Omega(\beta - 1)}{Z_j} \left(\frac{P_{K,j}(t)}{P_{K,i}(t)} \right)^\beta \right]^{1/\alpha} = a_{m,i} \left[\frac{\kappa_I}{\kappa_X} (\beta - 1) \left(1 - \frac{1}{Z_i} \right) \right]^{1/\alpha} \quad (16)$$

Although a detailed analysis of comparative statics for $a_{X,i}$ is provided in the "Appendix", direct inspection of these expressions shows that the range of varieties exported by country i , depends negatively on the sunk cost to export κ_X relative to the sunk entry-cost κ_I in units of knowledge. Following intuition, if trade costs are at their highest ($\Omega \rightarrow 0$) there are no firms efficient enough to export ($a_{X,i} \rightarrow 0$).

Abundant evidence supports the intuitive idea that only a fraction of producing firms export to foreign markets (see for example Bernard and Jensen 1999 or Eaton et al. 2004). The present model should reflect this and therefore condition $a_{X,i} < a_{D,i}$ is imposed:

Assumption 1 Assume $a_{X,i} < a_{D,i} \forall i = N, S$.

As shown in Proof 3, this entails $Z_i \in [(T + \Omega^2)/T, 1 + T] \forall i = N, S$. Notice the lower bound for Z_i is greater than 1, which means that $Z_i > 1 \forall i$, and guarantees $a_{D,i} > 0$ and $a_{X,i} > 0 \forall i = N, S$ without need of further assumptions.

At the world level the market for final goods clears since the share of a good in production value equals the share of that good in world expenditure. Finally, there is no capital account in this model so trade must be balanced at every t . This means that trade imbalances in the M -sector, which amount to $x_i E_{M,j} - x_j E_{M,i}$ are exactly compensated by an imbalance in the C -sector.

3 Balanced growth path

3.1 Definition and direct implications

The remainder of the paper analyses a Balanced Growth Path that follows the next definition⁸:

Definition 1 A Balanced Growth Path (BGP) in this model is characterized by a fixed allocation of labor among sectors (i.e. constant $L_{K,i}$, $L_{M,i}$ and $L_{C,i}$), constant cut-off values ($a_{D,i}$ and $a_{X,i}$) and aggregate shares (d_i and x_i), and endogenous variables n_i and E_i growing at a constant rate, $\forall i = N, S$.

⁸ Following a different definition for the BGP yields a different result. In particular, removing the requirement of constant d_i and x_i , it is possible to prove that both variables would asymptotically converge to 1 for one country while they converge to zero in the other (see Proof 1). Complete specialization arises at least in one region which violates one of the main assumptions of the present paper.

The fact that cut-off thresholds must be constant at the BGP implies that $\bar{\kappa}_i$, $\hat{m}_i$ and $\tilde{m}_i$ are constant too. This gives

$$\frac{\dot{h}_i(t)}{h_i(t)} = \frac{\frac{\dot{n}_i(t)}{n_i(t)}d_i + \frac{\dot{n}_j(t)}{n_j(t)}x_j}{1 - \sigma} \quad (17)$$

which shows that the aggregate marginal selling cost decreases in time since $\sigma > 1$ and every term in the numerator is positive. The rate at which aggregate marginal costs of firms serving country i are reduced is a weighted average of the rates at which both countries introduce new varieties, where weights are the importance that firms from each country have on market i .

Constant aggregate shares impose that average shares of firms from country i in both markets decrease at the rate at which new varieties are introduced in country i . Using this result it is possible to get $\dot{n}_i/n_i = \dot{n}_j/n_j = \dot{n}/n$, which is a necessary condition for aggregate shares to remain constant over time. This gives the following lemma:

Lemma 1 *At BGP, the M-sector introduces new varieties at the same rate in both countries (i.e. $\dot{n}_i/n_i = \dot{n}_j/n_j = \dot{n}/n$).*

By (17) a direct implication of this result is:

$$\frac{\dot{h}_i(t)}{h_i(t)} = \frac{\dot{h}_j(t)}{h_j(t)} \quad (18)$$

The aggregate marginal selling cost in both economies decreases at the same rate as a consequence of new varieties being introduced at the same rate in both regions.

Since $a_{K,i}$ is an exogenous parameter then the price of knowledge $P_{K,i} = a_{K,i}/n_i$ decreases at the same rate the mass of varieties increases over time (i.e. g_i). Nevertheless notice that, as a second implication from Lemma 1, the ratio n_i/n_j is constant over time which, by (15), implies that Z_i is a constant at BGP.

Finally, expressions for constant $\bar{\kappa}_i$, d_i and x_i are:

$$\bar{\kappa}_i = \frac{\beta \kappa_D Z_i}{(\beta - 1)}; \quad d_i = \left[1 + \frac{a_{K,j}}{a_{K,i}} (Z_j - 1) \right]^{-1}; \quad x_i = \left[1 + \frac{a_{K,j}}{a_{K,i}} (Z_i - 1) \right]^{-1} \quad (19)$$

A larger knowledge requirement for producing domestically (κ_D) pushes up the average amount of knowledge used by firms in any region ($\bar{\kappa}_i$). A greater value of Z_i , which corresponds with a larger average productivity of firms in i (lower $a_{D,i}$), also implies larger requirements for knowledge. In the same line, a larger Z_i yields a greater share of firms from i selling in the foreign market and a lower share of firms from j serving their domestic market. The effects of comparative advantages in the knowledge sector, given by the ratio $a_{K,i}/a_{K,j}$, on aggregate shares, are also very intuitive: the larger the exogenous technological advantage of the K -sector of i , the

larger the market share that firms from that region serve in both the domestic and foreign markets.

3.2 Expenditure, savings and labor allocation

As in Baldwin and Robert-Nicoud (2008) and Ourens (2016), E_i is constant at BGP. By the Euler equation, constant expenditure gives $\rho = \iota$, so the interest rate is constant at BGP and the same in both regions. It is possible to obtain the following expression:

$$E_i = L_i + \iota a_{K,i} \bar{\kappa}_i \quad (20)$$

Equation (20) shows that aggregate expenditure in country i equals the income that domestic consumers obtain from labor (L_i) plus the interests (ι) they get on their savings. Remember that in this model, savings are directed towards the production of knowledge so the value of the stock of savings equals aggregate knowledge purchased by firms ($a_{K,i} \bar{\kappa}_i$). Then, aggregate expenditure equals *permanent income* in a given country. This result contrasts with models like Melitz (2003) where time is set not to play a role in the model ($\rho = \iota = 0$), then $E_i = L_i$ which means permanent expenditure is unaffected by the static reallocation of resources. More importantly, (20) shows that permanent expenditure is affected by firm selection as that process has an impact on the expected value of firms through $\bar{\kappa}_i$. In a model where externalities in the creation of knowledge are independent of firm selection (e.g. $\bar{\kappa}_i = 1$ as in the standard expanding variety model with homogeneous firms) aggregate expenditure would be greater than L_i but still exogenous. On the contrary, in the present model, externalities stemming from firm selection and affecting the expected stock value of firms in country i can have an impact on expenditure levels, and therefore welfare in that country.

Having a path for aggregate expenditure means that the path of savings is fully determined. This gives the resources devoted to the R&D sector and therefore the path of $L_{K,i}$: $L_{K,i} = (\gamma - \rho) a_{K,i} \bar{\kappa}_i$. Allocation of labor in the M -sector is

$$L_{M,i} = (\sigma - 1) \gamma a_{K,i} \bar{\kappa}_i \quad (21)$$

This gives the amount of labor used in the C -sector: $L_{C,i} = L_i - (\gamma \sigma - \rho) a_{K,i} \bar{\kappa}_i$. As is clear from the previous equations, the allocation of the productive resource in this world is constant at BGP but depends on firm selection. In other words, a shock affecting the relative productivity of sectors would change the allocation of resources in the economy. In particular, an increase in the average productivity of the M -sector, which implies a greater knowledge requirement by firms in that sector (greater $\bar{\kappa}$) increases the allocation of labor towards both the M and K -sectors to the detriment of the C -sector.

Notice that $E_{M,i} = \mu E_i$ and $E_{C,i} = (1 - \mu) E_i$ and so aggregate expenditure in both sector C and M are a constant fraction of aggregate expenditure at BGP. Due to Cobb-Douglas preferences between goods, firm selection affecting aggregate expenditure affects expenditure equally in both final products. A constant $E_{C,i}$ at BGP yields constant consumption in the C -sector since the price of the C -good is

one. Given that $E_{M,i}$ is constant, the growth rate of real consumption of the M -good is $g_{Q_{M,i}} = -\dot{h}_i/h_i = (g_i + \delta_i)/(\sigma - 1)$, the same expression that arises under country symmetry. The term on the right-hand side is the rate at which the price index of the M -good decreases over time due to an increasing mass of varieties.

The economy described here has constant cut-off values $a_{D,i}$ and $a_{X,i}$ at any moment in time, as described by (14) and (16). By Definition 1, $\bar{\kappa}_i$, d_i , x_i , g_i , γ , $L_{K,i}$ and $L_{M,i}$ are also constant at BGP. As previously stated, $P_{K,i}$ and n_i are not constant at BGP, but grow at a constant rate which means h_i and $P_{M,i}$ also do. Finally, the discount factor is a constant equal to $\gamma = \rho + g_i + \delta_i$ (see Proof 9).

The remaining expressions for the full solution of the model are:

$$h_i^{1-\sigma}(t) = \frac{\beta a_{D,i}^{1-\sigma} n_i(t)}{(\beta - 1)d_i}; \quad P_{M,i}(t) = \frac{\sigma h_i(t)}{\sigma - 1}; \quad P_i(t) = P_{M,i}(t)^\mu B \quad (22)$$

where $n_i(t) = n_i(s)e^{(t-s)g_i}$, $n_i(s)$ is the mass of producing firms from country i at time s , and the initial value $n_i(0)$ is a parameter of the model. In this expression the growth rate in i is given by:

$$g_i = \frac{\mu d_i Z_i}{\sigma} \left[\frac{L_i}{a_{K,i} \bar{\kappa}_i} + \rho \right] - \rho - \delta_i \quad (23)$$

These expressions can be reduced to their counterparts under autarky by imposing $\Omega = 0$. They can also be transformed into their symmetric-country version by setting $i = j$ (as shown in Sect. 4.1).

3.3 Welfare

Welfare at a moment of time t belonging to the BGP analyzed in this paper, is defined here by equation (1), and is a function of the present value of real consumption. Using that expression and the solutions at BGP it is possible to write:

$$U_i(t) = \frac{1}{\rho} \left[\ln \left(\frac{E_i}{P_i(t)} \right) + \frac{\mu g_{Q_{M,i}}}{\rho} \right] \quad (24)$$

Aggregate welfare at t depends on the level of consumption at that moment ($E_i/P_i(t)$) and its growth over time, represented here by $g_{Q_{M,i}}$, since consumption in the C -good is constant ($g_{Q_{C,i}} = 0$). When evaluating how freer trade affects the level of consumers' welfare, changes in the former are referred to as *static effects* while changes modifying the latter are called *dynamic effects*. Expression (24) helps disentangling three types of welfare effects of trade in this model. First, there is a static effect on prices as freer trade affects $P_i(t)$ at t . In fact, this is the effect that most welfare analysis deals with in static versions of the HFTM. Second, there can also be static effects on expenditure when a change in the degree of openness alters the value of E_i which is constant at BGP. Finally, the only dynamic effect in this model comes from changes in the rate at which prices in the M -sector fall over time, which is determined by the rate at which new varieties are introduced in that sector.

The previous expression also highlights that dynamic effects are less important when consumers assign greater value to the consumption of the traditional C -good (i.e. $\mu \rightarrow 0$). Intuitively, while the price of the M -good decreases as new varieties are introduced in the market period after period, the price of the C -good in terms of units of labor remains constant. Therefore, the more consumers value consumption of the traditional product the lower the impact that decreasing prices in the M -sector have on welfare.

4 The impact of trade openness

This section analyses the effect that freer trade has on welfare over the long term. In the present model, freer trade can be a consequence of lower iceberg costs ($d\phi > 0$), lower sunk-costs for exporting ($d\kappa_X < 0$) or a combination of both. All these possibilities can be synthesized in an increase in the bundle Ω ($d\Omega > 0$). Therefore, welfare effects of trade can be found by differentiating (24) and finding:

$$\frac{dU_i(t)}{d\Omega} = \frac{1}{\rho} \left[\frac{d \ln E_i}{d\Omega} - \frac{d \ln P_i(t)}{d\Omega} + \frac{\mu}{\rho(\sigma - 1)} \frac{dg_i(t)}{d\Omega} \right] \quad (25)$$

As shown in Proof 6, comparative static results stemming from an increase in openness ($d\Omega > 0$) are qualitatively similar to those that arise if only changes in variable costs of international trade are considered ($d\phi > 0$). This makes the results comparable to the literature that mostly considers a reduction of iceberg costs. Analyzing the effects of a reduction in the exporting sunk costs is an additional feature of this work that does not change the main conclusions of the paper. Whenever the effects of a reduction in κ_X yields different results to an increase in ϕ , these differences are explicitly underlined.

When the world economy experiences an openness shock ($d\Omega$), it moves away its initial BGP to a new one through some transition dynamics. To simplify exposition and facilitate comparability with the related literature, I compare the welfare situation at BGP before the shock, with that in the BGP after the shock, ignoring the transitional period. This means that the analysis approximates an evaluation of welfare effects of openness in the long run. The initial mass of varieties is the same in both situations. Although new varieties are likely introduced during the adjustment period, the net creation (or destruction) of varieties during this period depends on the length of the adjustment process. By setting an equal mass of varieties both right before and after the shock, this effect is eliminated from the welfare evaluation. This amounts to setting $dn_i/d\Omega = 0, \forall i = N, S$ in what follows.

The analysis is split in two cases, one with symmetric countries and another one where country asymmetry is permitted. This allows to better highlight the role that dynamic effects play on welfare when firms are different between countries.

4.1 Symmetric countries

A symmetric country setting simplifies greatly the analysis and provides a benchmark for better understanding results under country asymmetry. Moreover, this setting suffices to show some of the key welfare features in the heterogeneous firm model that are absent when homogeneous firms is assumed. By country symmetry $a_{K,i}/a_{K,j} = 1$, $a_{m,i}/a_{m,j} = 1$, $R = 1$, and so $Z = 1 + \Omega$. The fact that the degree of openness is related to Z in such a simple way turns the study very straightforward. Previous analytic solutions get reduced to:

$$a_D = a_m \left[\frac{\kappa_I(\beta - 1)}{\kappa_D(1 + \Omega)} \right]^{1/\alpha}; \quad a_X = a_m \left[\Omega \frac{\kappa_I(\beta - 1)}{\kappa_X(1 + \Omega)} \right]^{1/\alpha} \quad (26)$$

$$\bar{\kappa} = \frac{\beta \kappa_D(1 + \Omega)}{\beta - 1}; \quad d = \frac{1}{1 + \Omega}; \quad x = \frac{\Omega}{1 + \Omega} \quad (27)$$

$$h^{1-\sigma}(t) = \frac{\beta a_D^{1-\sigma} n(t)}{(\beta - 1)d}; \quad P(t) = \left[\frac{\sigma h(t)}{\sigma - 1} \right]^\mu B; \quad (28)$$

$$g = \frac{\mu}{\sigma} \left[\frac{L}{a_K \bar{\kappa}} + \rho \right] - \rho - \delta$$

Then, using the previous expressions for prices, expenditure and the growth rate of varieties gives:

$$\frac{dU}{d\Omega} = \frac{1}{\rho} \left[\underbrace{\frac{\rho a_K \beta \kappa_D}{E(\beta - 1)}}_{\frac{d \ln E}{d\Omega}} - \underbrace{\frac{\mu(1 + \beta)}{\alpha} \frac{d \ln d}{d\Omega}}_{\frac{d \ln P}{d\Omega}} + \frac{\mu}{\rho(\sigma - 1)} \underbrace{\frac{\mu L}{\sigma a_K \bar{\kappa}} \frac{d \ln d}{d\Omega}}_{\frac{dg}{d\Omega}} \right] \quad (29)$$

How does this welfare expression compare to the literature? ACR show that welfare effects of trade in the HFTM+P are a function of two observables: the share of expenditure in domestic goods (which is denoted d here) and the elasticity of imports to changes in international trade costs (which is $-\alpha$). Their well known formula can be written in the present notation as:

$$\frac{d \ln U}{d\Omega} = \frac{-1}{\alpha} \frac{d \ln d}{d\Omega}$$

Comparison between both expressions allows me to state the following result:

Result 1 *Extending the HFTM+P to include simple dynamics, yields new welfare effects, absent in the ACR formula designed within a static framework.*

To prove the above result, the first step is to show that welfare results in a static version of the present model exactly match the ACR formula (see Proof 4 in the "Appendix"). In fact, ignoring parameter μ (which in ACR is set to 1 as there is no outside good), the result for the static effect on prices is exactly the total welfare effect in ACR, scaled up by a factor $1 + \beta$ (remember that $\beta > 0$). The reason for this difference is that, in the dynamic model, the mass of varieties (n) is determined by history so a change in trade costs does not immediately affect it. In ACR, on the other hand, greater openness increases import competition which reduces the mass of existing firms, an effect that attenuates the reduction of prices in the original HFTM+P.

Expressions in (29) show the role that the dynamic feature of the model plays by highlighting a static effect on E and a dynamic effect on g absent in ACR's formula. Results would be exactly those in ACR only in the specific case in which both new effects exactly offset each other. In this respect, the fact that the model introduces new channels for welfare changes does not mean that the overall effect is necessarily different. The next section goes one step further to evaluate how the new effects brought up by the dynamic feature in the model, interplay with an extra layer of heterogeneity often studied in the literature, i.e. country asymmetry.

4.2 Asymmetric countries

Country asymmetry allows for welfare results to differ among countries. In a static context, Demidova (2008) finds that, if the technological gap between countries is small both countries experience welfare gains from trade, but if the gap is too large then the laggard economy can experience losses. Her model features no externalities and all welfare effects are channeled through the price index. This effect is also present in the current model, but it is accompanied here by new effects that can offset or enhance its impact on both economies.

Without loss of generality, let me assume that the North is the technologically advanced country and therefore has a natural comparative advantage in the production of M -goods. In the present model this is imposed by setting $R_N > R_S$. Comparative advantage in the M -good can come from a better technology in the M -sector itself (i.e. $a_{m,N} < a_{m,S}$), a better technology in the K -sector ($a_{K,N} < a_{K,S}$), greater spillovers from learning ($n_N > n_S$) or some combination of them.⁹ Joining this with Assumption 1 and expressions (14) and (16) immediately yields $a_{X,S} < a_{X,N} < a_{D,N} < a_{D,S}$ for every level of openness (Ω).

The different welfare effects that might arise in the asymmetric version of the model are depicted by the following:

⁹ Remember that setting $a_{m,N} < a_{m,S}$ implies that the support of the distribution of marginal costs in the North excludes the highest values compared to that of the South. M -firms from the North have a greater probability of getting a marginal cost below any threshold than those from the South.

$$\frac{dU_i(t)}{d\Omega} = \frac{1}{\rho} \left[\underbrace{\frac{\rho a_{K,i} \beta \kappa_D}{E_i(\beta - 1)} \frac{dZ_i}{d\Omega}}_{\frac{d \ln E_i}{d\Omega}} - \underbrace{\mu \left[\frac{\beta}{\alpha} \frac{d \ln d_i}{d\Omega} - \frac{1}{\alpha Z_i} \frac{dZ_i}{d\Omega} \right]}_{\frac{d \ln P_i}{d\Omega}} \right. \\ \left. + \frac{\mu}{\rho(\sigma - 1)} \underbrace{\frac{\mu d_i}{\sigma} \left[\frac{d \ln d_i}{d\Omega} \frac{E_i Z_i}{a_{K,i} \bar{\kappa}_i} + \rho \frac{dZ_i}{d\Omega} \right]}_{\frac{dg_i}{d\Omega}} \right] \quad (30)$$

As is clear from the previous expressions, the effect that freer trade has on welfare in i depends on how openness (Ω) affects firm selection in that country, which according to (14), depends on $dZ_i/d\Omega$. Then, it is important to consider the following lemma (see Proof 5 in the "Appendix"):

Lemma 2 $dZ_N/d\Omega > 0$ holds for all parameter values, but $dZ_S/d\Omega > 0 \Leftrightarrow R_S > R^* = 2\Omega/(1 + \Omega^2)$ and $dZ_S/d\Omega < 0$ otherwise.

According to this lemma, Z_N always responds positively to trade liberalization, but a positive change in Z_S is only achieved if the level of the technological lag in the South is below a certain threshold R^* . Since R^* depends positively on Ω then for a certain gap level, an infinitesimal increase in Ω from an initially small value may increase Z_S , while a similar change from an initially larger value may reduce Z_S . From now on the analysis is split into two cases: first the case of relatively small technological gap ($R^* < R_S$) and then the case where that gap is large ($R^* > R_S$).

When the technological gap is relatively small, the welfare effects of trade are depicted by the following result:

Result 2 When $R^* < R_S$, $\uparrow \Omega$ yields $\uparrow E_i/P_i \forall i = N, S$, but this effect is relatively larger in N . For both countries g_i falls in the long-term, but g_N and g_S can take divergent paths in the short-term before reaching the new BGP, and a rising g_N in the short-term is possible.

To prove this result, notice that according to Lemma 2, a case in which technological asymmetries are not strong ($R^* < R_S$) is characterized by $dZ_N/d\Omega > 0$ and $dZ_S/d\Omega > 0$. Taking this to the expressions in (30) directly gives the result above.

Intuitively, greater trade openness reduces threshold $a_{D,i}$ in both countries increasing average efficiency. Firm selection operates in both economies in the same direction, as in the symmetric country case. New export opportunities make it easier for firms from both countries to engage in foreign trade, which translates into a reduction of $a_{X,i} \forall i = N, S$. In aggregate terms, d_i decreases and x_i increases for both regions as can be deduced from (19). The average sunk cost paid by firms

from country i ($\bar{\kappa}_i$) increases due to increased profitability in the M -sector in both countries. As in the previous section, the greater value of the stock of knowledge implies that consumers get greater income from their savings, generating a positive static effect on expenditure. The increased efficiency due to firm selection, plus the lower costs of imports, makes the price index fall. Therefore, as in the symmetric country case, greater openness has a positive static effect on real consumption in both countries. What distinguishes the asymmetric country case is the fact that the positive effect is relatively greater in the North. This is easy to prove noting that $dZ_N/d\Omega > dZ_S/d\Omega$.

Regarding the effect that freer trade has on variety growth, it must be pointed out that, as in the symmetric country case, the greater requirement for knowledge by M -firms in both countries is a force detrimental to variety growth. This makes the growth rate at the new BGP lower than the value before the shock. In the short term however, growth rates in each country may jump in different directions. Proof 7 shows that if firms from the North are able to expand their share in the foreign market relatively more than their counterparts from the South, a positive growth rate of northern varieties is possible.¹⁰

When the technological gap is large enough to make $dZ_S/d\Omega < 0$ (i.e. $R_S < R^*$), trade-induced movements of most endogenous variables go in different directions for the North and South. An increase in openness pushes down $a_{D,N}$ increasing average efficiency in the North, but rises $a_{D,S}$ so average efficiency in the laggard economy decreases. The possibility for this inverse firm selection in the South was first documented in Demidova (2008). The analytic solutions in this paper allow me to point out the exact reason that this reverse firm selection can take place in the South: it is actually the result of two opposing effects. On the one hand, a direct effect of new export opportunities push $a_{X,i}$ up, and tougher competition by foreign firms pulls $a_{D,i}$ down, as under country symmetry. But on the other hand, the cut-off value to produce in the domestic market in one country is linked to the cut-off value to export in the other. In plain words, there is an indirect effect on thresholds given by the fact that greater efficiency in domestic production in i reduces export opportunities for firms in j and vice-versa. Inspection of (13) shows that when the technological lag is large, the second effect is strong in the South while it is weak in the North.

Reducing barriers for trade of the M -good in such a context promotes the expansion of firms from the North in both markets. By Proof 6, $a_{X,N}$, d_N and x_N rise, while $a_{X,S}$, d_S and x_S fall.¹¹ Moreover, by (21), the case in which the technological asymmetry is large is the only one that yields an unambiguous reallocation of labor towards the M -sector in the North while the opposite happens in the South. This gives the following result:

¹⁰ The fact that short-term growth rates of both economies are affected differently by a one-time shock in openness highlights the importance of the study of transitional dynamics towards the BGP in order to quantify net welfare effects in both countries. This should be a subject of future work.

¹¹ A case in which $a_{X,S}$ rises is possible where there is a large technological gap. According to Proof 6, this happens if the increase in Ω is to a large extent an outcome of reduction in κ_X . However, this does not affect the results that follow.

Result 3 When $R_S < R^*$, $\uparrow \Omega$ yields $\uparrow L_{M,N}$ and $\downarrow L_{M,S}$.

Increasing specialization in a context of firm heterogeneity is not a new result. In a Heckscher-Ohlin setting, Bernard et al. (2007) find that when countries differ in their relative factor abundance, within-industry resource reallocation following trade openness reinforces each country's comparative advantage. Since in the present model, comparative advantages come from differences in relative productivities, the previous result can be considered the Ricardian version of that conclusion, and shows that for comparative advantages to be reinforced by freer trade in a Ricardian model, the technological asymmetry must be larger than a certain threshold.¹²

Regarding the welfare outcomes in a context of large technological differences the following result is obtained:

Result 4 When $R_S < R^*$, $\uparrow \Omega$ generates $\uparrow E_N$, $\uparrow g_N$, $\downarrow E_S$ and $\downarrow g_S$ in the short-term. In the long-term, $\downarrow g_i \forall i = N, S$ must happen. The movement of P_i depends on how firm selection affects domestic and imported prices in both countries: while domestic prices in N unequivocally fall, the price of imported goods could imply a price increase to consumers, and the opposite happens in S .

Again, the proof involves considering Lemma 2 and the expressions in (30). As established in Proof 6, having $dZ_S/d\Omega < 0$ and $dZ_N/d\Omega > 0$ implies that the average amount of knowledge used by producing M -firms increases in the North, while decreases in the South. This means that returns on savings increase in the North, raising the constant consumption level, while the opposite happens in the South, explaining the effect on expenditure in each country.

Unlike previous cases, the direction of price movements is not straightforward when technological differences are large. Given the expression for prices in (22), inspection of (5) reveals that the effect freer trade has on the price index in country i is a combination of the effects it has on: (a) the average selling costs of domestic firms, (b) the average selling costs of foreign firms exporting to i , and (c) the proportion of foreign firms that reach market i . In the case where countries are symmetric, lower trade costs make a_D fall and raises a_X in both countries, which yields a reduction of prices from local firms, and an increase in the proportion of firms reaching the foreign market. The effect on the average price exporters charge is ambiguous since on the one hand firms with higher marginal costs reach the foreign market, but on the other hand all exporters face lower trading costs. This means that (a) and (c) tend to reduce prices while (b) is indeterminate. However it is easy to show that, since $\beta > 1$, the net effect (b)+(c) makes prices fall. This drives the well-known

¹² This result is independent of the dynamic feature of the model and also arises if growth and savings are assumed away. Okubo (2009) reaches a similar conclusion in a static Ricardian model with a somewhat different setting. In his model with multiple homogeneous-good sectors, some sectors have comparative advantages in one country and some other sectors feature advantages in the other. The result of freer trade is that sectors with Ricardian advantages expand relatively more than the others, but no sector shrinks.

result that, under country symmetry, trade liberalization unambiguously reduces the price index. As is easy to see, the same result is obtained if there is a small technological gap, as all endogenous variables move in the same direction. But when the technological asymmetry is large, the effect that freer trade has on the price index is not determined in both countries. In the North, domestic producers sell at a lower average price, but the effect stemming from foreign firms is not straightforward. While the average selling price of foreign firms is lower (both because of greater efficiency and lower tariff costs), the proportion of exporters among foreign firms gets reduced. In the South the picture is the exact opposite but the effect is still ambiguous. Unlike in the case of a small gap, analytical solutions do not show one effect prevailing over the others when the technological gap is large (see Proof 8).

Finally, a one-time increase in openness in a case of large technological asymmetry undoubtedly pushes growth rates in different directions in the short-term. The immediate response of g_S is to fall, while g_N rises (see Proof 7). As in the previous cases both rates converge in the long-term to a common value, smaller than its value before the shock.

The present section has explored the welfare effects of freer trade in a dynamic model with firm heterogeneity when countries are asymmetric. Effects may differ between countries and can even go in opposite directions when the technological difference is large enough. These findings add new effects to previously found results in static context and highlight the importance of considering country characteristics in assessing the long-term welfare effects of freer trade on an economy.

5 Quantitative results

This section performs a numerical exercise with the aim of quantifying the relative magnitude of the effects highlighted in the previous sections, with respect to those in the existing literature. Parameter values of the two asymmetric economies are chosen to match the relevant characteristics of the UK on one hand, and the remaining 27 EU economies on the other. Results can then relate to a case of particular current relevance: the perspective of the UK leaving the European Union (Brexit). Many reports have been written assessing the repercussions that Brexit could have in terms of welfare changes for both parties involved.¹³ Nevertheless, these evaluations are often constructed upon models assuming country symmetry and static settings.

I follow closely the choices made in the previous literature of parameters that are also present in (30). New parameters are estimated or observed directly in the data. The resulting parameter values are presented in Table 1. As is customary in these exercises, it is important to clarify that the following exercise aims at providing the reader with an assessment of how sizable the effects highlighted in the current paper are in comparison with similar models in the literature. Results are therefore more an illustration than an exact representation of reality, which is often more complex than what can be captured in mathematical models.

¹³ One of the most comprehensive of such works can be found in Dhingra et al. (2017).

Table 1 Parameter set

Parameter	Definition	Value
ρ	Time preference	0.4
$a_{K,UK}$	Cost of developing new M product in UK	0.0003
$a_{K,EU}$	Cost of developing new M product in EU	0.0002
κ_D	Sunk cost to develop new varieties (in knowledge units)	1
κ_X	Sunk cost to export new varieties (in knowledge units)	1.5464
ϕ	Variable costs of exporting	1
α	Shape parameter in Pareto distribution of firms	4.5284
μ	Share of expenditure in M	0.8910
σ	Elasticity of Substitution within M	5.8221
Ω	Degree of openness	0.9738
L_{UK}	Size of UK economy (population)	0.1647
L_{EU}	Size of EU economy (population)	1
d_{UK}	Share of expenditure in domestic goods in UK	0.6571

See “Appendix 2” for detailed methodological choices and sources

While the methodological choices made to reach these parameter values are detailed in the “Appendix 2”, a few remarks should be made at this point. First, only trade in goods is considered, using data for 2015. Excluding services means ignoring a large part of the trade between the EU and the UK, but it enables the use of widely available data to estimate parameter values. Manufactured goods are used as M and agricultural goods as the outside good C , as this seems the most appropriate fit of the model to the selected trading partners. When evaluating a symmetric country setting, country-specific parameters take the corresponding values for the UK. Finally, following the literature, results for welfare effects are presented evaluating infinitesimal increases in the level of openness. This implies that there is no need to integrate expressions in (30), a practice normally followed when evaluating changes from openness to autarky. Brexit would correspond to a reduction of openness, so signs in (30) should be the opposite to interpret that specific situation.

Using the parameter values in Table 1, yields the results in Table 2 for welfare effects in each of the models discussed in this paper. There are a number of salient features among these results. First, using the parametrization in Table 1, an infinitesimal increase in the level of openness yields welfare gains of around 11% that change, according to the ACR formula. Inversely, if economies experience a reduction in openness instead, then we have a welfare loss of that magnitude. Second, evaluating the results obtained under the symmetric country version of the model yields a positive static effect of an increase in openness between the two partners. This is composed of a small decrease in expenditure and a relatively larger price reduction. The size of this net static effect (18%) resembles that obtained with the ACR formula, except it is scaled up by the coefficient $\mu(\beta + 1)$, as explained in Sect. 4.1. As anticipated, the dynamic effect has the opposite sign as the static effect on expenditure. This is a direct result of the simple way the dynamics are modeled. So in this version of the model, an increase in trade openness raises welfare in the

Table 2 Quantitative results

<i>Result following ACR</i>		
$d \ln U / d\Omega$		0.1119
<i>Results with symmetric countries</i>		
$d \ln E / d\Omega$	– 0.0131	
$d \ln P / d\Omega$	– 0.1933	
$dg / d\Omega$	1.2259	
$d \ln U / d\Omega$		1.8662
<i>Results with asymmetric countries</i>		
$d \ln E_{UK} / d\Omega$	0.0865	
$d \ln E_{EU} / d\Omega$	– 0.0195	
$d \ln P_{UK} / d\Omega$	0.0027	
$d \ln P_{EU} / d\Omega$	– 0.9496	
$dg_{UK} / d\Omega$	6.6678	
$dg_{EU} / d\Omega$	– 7.7429	
$d \ln U_{UK} / d\Omega$		7.9097
$d \ln U_{EU} / d\Omega$		–6.6167

Size of effects following an infinitesimal increase in openness (Ω). For symmetric country settings parameter values correspond to the UK

two regions, due both to an immediate gain in consuming possibilities, and accumulated gains brought about by an increase in the growth rate of varieties. Again, a reduction in openness as expected under Brexit, would bring changes in the opposite direction. More importantly, notice that the magnitude of the dynamic effect is quite larger than the static effect. A permanent change in the growth rate of varieties accumulates over time and produces a sizable effect on welfare. The latter observation underlines the importance of considering dynamic models in welfare evaluations of trade shocks, even in a symmetric country context.

Finally, analyzing the more flexible setting that allows for country asymmetries yields the results in the last part of Table 2. Notice that according to the chosen parameter values, the technological difference between countries does not seem large, being favorable to the EU which plays the part of the North in the previous model ($R_{UK} = 0.9967$ while $R_{EU} = 1.0033$). Nevertheless, the technological asymmetry between the two trading partners qualifies as large in terms of Lemma 2, mainly due to a level of openness that is close to unity: Ω is close to 1, so R^* is close to 1 as well ($R^* = 0.9997$), and $R_{UK} < R^*$. This explains why the static effect on expenditure and the dynamic effect go in different directions for the EU and the UK.

Higher openness between regions pushes expenditure up in the UK, while it goes down in the EU. Prices move in the same direction as expenditure in both regions. Combining both effects, consuming possibilities increase in both regions, but the rise is more favorable to EU consumers. The magnitude of the dynamic effect is again larger than the static effect for both economies, and in the asymmetric country case, it appears as even more important than in the previous comparison. While the effect is negative for the EU it is positive for the UK. Therefore, in the case of increasing trade barriers following Brexit, the model predicts that consumers in both

sides experience an immediate loss in their consuming possibilities, but this is worsened by a very sizable dynamic loss to UK consumers driven by the reduction in the rate at which new varieties become available to them. On the contrary, consumers in the EU experience a dynamic gain more than offsetting the static loss, as the growth rate of varieties jumps in that region.

6 Conclusions

This paper presents a simple heterogeneous firm trade model with growth and asymmetric countries. Focus is placed on the welfare effects of freer trade and the results are presented in terms of the well-known formula in Arkolakis et al. (2012). Introducing dynamics into the heterogeneous firm trade model changes welfare considerations greatly with respect to the basic model in Melitz (2003). A symmetric country setting suffices to show that allowing consumers to save and create new future value brings about new channels through which freer trade can affect consumers purchasing power. In particular, this work highlights that dynamic effects (i.e. the impact on the future growth rate) may be negative when firm selection raises the knowledge requirements for developing new varieties. Allowing for country asymmetries adds a further layer of complexity to the analysis. Results show that, even considering a very restricted framework with a simple balanced growth path, differentiated outcomes between countries must appear in the transition towards the new equilibrium. Overall, the sign of the net welfare effect always depends on countries characteristics represented by the parameters of the model. This resembles what the empirical literature on the matter has found. A numerical exercise makes the model fit the potential increase in trade barriers between the UK and EU and shows the large magnitude of the dynamic effect. Moreover, the exercise highlights that welfare losses for UK consumers following an increase in trade barriers with the EU may be severely underestimated by analysis ignoring dynamic effects and country asymmetries.

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Appendix 1: Proofs

Proof 1 Imposing constant aggregate shares in the BGP analyzed here is a requirement for the model not to degenerate into one with full specialization $\square$

Assume the BGP follows the next definition instead of Definition 1:

Definition 2 A Balanced Growth Path (BGP) in this model is characterized by a fixed allocation of labor among sectors (i.e. constant $L_{K,i}$, $L_{M,i}$ and $L_{C,i}$), constant cut-off values ($a_{D,i}$ and $a_{X,i}$) and endogenous variables n_i and E_i growing at a constant rate $\forall i = N, S$.

Notice that the only difference between the BGP in Definition 1 and that in Definition 2 is that the latter allows for $\dot{d} \neq 0$ at some moment in equilibrium. It is possible to prove that a BGP characterized by Definition 2 yields a situation where firms from one country take over the entire global M -market (i.e. if $\dot{d}_i > 0 \Rightarrow d_i \rightarrow 1$, $x_i \rightarrow 1$, $d_j \rightarrow 0$ and $x_j \rightarrow 0$). In such a situation, at least one of the regions of the model becomes fully specialized.

First, notice that if $\dot{d}_i > 0$ at some t at BGP, then $d_i \rightarrow 1$. This follows from (17) and Lemma 1, which give $\dot{d}_i(t) = \left(\frac{\dot{n}_i}{n_i} - \frac{\dot{n}_j}{n_j}\right)(d_i(t) - d_i(t)^2)$. The solution for the previous Bernoulli differential equation is as follows:

$$d_i(t) = \left[1 + me^{-\left(\frac{\dot{n}_i}{n_i} - \frac{\dot{n}_j}{n_j}\right)t} \right]^{-1}$$

where $m \in \mathbb{R}$. The equilibrium conditions impose constant $\frac{\dot{n}_i}{n_i} \forall i$, so the only way to have $\frac{\dot{d}_i}{d_i} > 0$ at some t belonging to the BGP, is if $\frac{\dot{n}_i}{n_i} > \frac{\dot{n}_j}{n_j}$ at that t , which yields $\frac{\dot{d}_i}{d_i} > 0 \forall t$ at BGP. By definition $d_i = 1 - x_j \Rightarrow \dot{d}_i = -\dot{x}_j$, so if $d_i \rightarrow 1$, then $x_j \rightarrow 0$. Finally, notice that the same condition for $\frac{\dot{d}_i}{d_i} > 0$ at some t gives $\frac{\dot{d}_j}{d_j} < 0$ at that t and following a similar reasoning this gives $d_j \rightarrow 0$, then $x_i \rightarrow 1$.

According to this result, a situation in which the stability condition does not hold at some t of the BGP, is characterized by divergent dynamics between countries in the M -sector. Without loss of generality assume $\dot{d}_N > 0$ at some t which can only be true if constant rates of expanding varieties in each country fulfill $\frac{\dot{n}_N}{n_N} > \left| \frac{\dot{s}_{N,N}}{\bar{s}_{N,N}} \right|$. Then M -producers from the N monotonically increase their aggregate market shares in both countries, to the detriment of producers of that industry from the S until, in the limit, the latter are practically expelled from production. $\square$

Proof 2 $d_i \geq x_i \forall i$ $\square$

$$\text{By their definitions in (9), } d_i \geq x_i \Leftrightarrow v_j \left[\frac{\bar{m}_i}{\bar{m}_j} \right]^{1-\sigma} \geq \frac{1}{v_i} \left[\frac{\bar{m}_i}{\bar{m}_j} \right]^{1-\sigma}.$$

$$\text{By (6), } \left[\frac{\hat{m}_i}{\hat{m}_j} \right]^{1-\sigma} = \frac{\int_0^{a_{D,i}} a^{1-\sigma} dG_i(a|a_{D,i})}{\phi \int_0^{a_{X,i}} a^{1-\sigma} dG_i(a|a_{X,i})} = \frac{v_i \int_0^{a_{D,i}} a^{1-\sigma} dG_i(a|a_{D,i})}{\phi \int_0^{a_{X,i}} a^{1-\sigma} dG_i(a|a_{D,i})}.$$

Notice that, since $a^{1-\sigma} > 0 \forall a > 0$ and assuming $a_{X,i} < a_{D,i}$ (see Assumption 1), then $\left[\frac{\hat{m}_i}{\hat{m}_j} \right]^{1-\sigma} = \frac{v_i}{\phi} M$ with $M > 1$.

Following a symmetric reasoning the condition for $d_i \geq x_i$ can be written as

$$\frac{\phi \int_0^{a_{X,j}} a^{1-\sigma} dG_j(a|a_{D,j})}{\int_0^{a_{D,j}} a^{1-\sigma} dG_j(a|a_{D,j})} \leq \frac{1 \int_0^{a_{D,i}} a^{1-\sigma} dG_i(a|a_{D,i})}{\phi \int_0^{a_{X,i}} a^{1-\sigma} dG_i(a|a_{D,i})}$$

This condition always holds as the left-hand side is lower than or equal to one while the right-hand side is greater than or equal to unity. The previous condition shows that strict equality holds only if $a_{D,i} = a_{X,i} \forall i$ and $\phi = 1$. $\square$

Proof 3 $a_{D,i} > a_{X,i} \Leftrightarrow Z_i \in [\frac{T+\Omega^2}{T}, 1+T] \forall i = N, S$ $\square$

Using (14) and (16) it is easy to see that $a_{D,i} > a_{X,i} \Leftrightarrow Z_i < 1+T \forall i = N, S$. Without loss of generality let us assume it is the North that reaches a maximum level of Ricardian comparative advantage R_N^{max} such that $Z_N^{max} = 1+T$. Solving for such a value gives $R_N^{max} = \frac{T+\Omega^2}{\Omega(T+1)}$ which by definition yields $R_S^{min} = \frac{\Omega(T+1)}{T+\Omega^2}$. By definition, this value for R_S^{min} yields $Z_S^{min} = \frac{T+\Omega^2}{T}$.

Then $R_i \in [\frac{\Omega(T+1)}{T+\Omega^2}, \frac{T+\Omega^2}{\Omega(T+1)}]$ and $Z_i \in [\frac{T+\Omega^2}{T}, 1+T] \forall i = N, S$. $\square$

Proof 4 Eliminating dynamics from the model with symmetric countries gives welfare results in line with those in ACR $\square$

First, notice that imposing no product creation ($L_K = 0$) and no savings ($\rho = \iota = 0$ so $E = L$) in the model eliminates any welfare effect on the growth rate and the expenditure level so the static effect on prices is the only effect remaining. Then, notice that with no firm death ($\delta = 0$), this setting yields a constant and endogenous mass of varieties. Indeed, further ignoring the outside sector in the present model, which does not exist in ACR (i.e. setting $\mu = 1$ and $L_C = 0$), gives $L = \sigma \delta a_K \bar{\kappa}$. Assuming $P_K = 1$ to better fit the specification in ACR, $a_K = n$ obtains by definition. Combining these results yields:

$$n = \frac{L}{\sigma \delta \bar{\kappa}}$$

Finally, merging that result with the expressions of $\bar{\kappa}$ and h in the symmetric country case:

$$P = \frac{\sigma}{\sigma - 1} \left[\frac{\sigma \delta \kappa_D}{L} \right]^{\frac{1}{\sigma-1}} a_D$$

The above expression for prices matches the inverse of the welfare expression in Melitz (2003) and therefore gives the same welfare results as in ACR. Indeed, given

that the static price effect on prices is the only active in the current setting, (25) can be rewritten as:

$$\frac{dU}{d\Omega} = -\frac{d \ln P}{d\Omega} = -\frac{d \ln a_D}{d\Omega} = \frac{1}{\alpha(1+\Omega)} = \frac{-1}{\alpha} \frac{d \ln d}{d\Omega} \quad \square$$

Proof 5 Proof of Lemma 2 □

The effect that trade openness (Ω) has on Z_i can be expressed by $\frac{dZ_i}{d\Omega} = \frac{R_i Z_i - 2\Omega}{1 - \Omega R_i}$. The analysis of the sign of this expression can be split in two cases here:

- if $1 - \Omega R_i > 0$, then $\frac{dZ_i}{d\Omega} > 0 \Leftrightarrow Z_i R_i > 2\Omega \Leftrightarrow \frac{1 - \Omega^2}{1/R_i - \Omega} > 2\Omega \Leftrightarrow R_i > \frac{2\Omega}{\Omega^2 + 1}$
- if $1 - \Omega R_i < 0$, then $\frac{dZ_i}{d\Omega} < 0 \Leftrightarrow Z_i R_i < 2\Omega \Leftrightarrow \frac{1 - \Omega^2}{1/R_i - \Omega} < 2\Omega \Leftrightarrow R_i > \frac{2\Omega}{\Omega^2 + 1}$

So in both cases it is possible to conclude that $\frac{dZ_i}{d\Omega} > 0 \Leftrightarrow R_i > R^* = 2\Omega/(\Omega^2 + 1)$.

Notice that R^* is a monotonically increasing function of Ω , and for $\Omega \in (0, 1)$ $R^* \in (0, 1)$ is also true. This explains why under country symmetry, when $R = 1$, we have $R > R^* \forall \Omega$, so $\frac{dZ}{d\Omega} > 0$ holds. Under country asymmetry $R_N > 1$ and $R_S < 1$, so $\frac{dZ_N}{d\Omega} > 0 \forall \Omega$. As shown in to Proof 3, Assumption 1 imposes a lower limit to R_S given by $R_S^{\min} = \frac{\Omega(T+1)}{T+\Omega^2}$. It is easy to show that $T > 1$ implies $R_S^{\min} < R^*$. Then for values of the technological gap that set R_S to the range between $[R_S^{\min}, R^*)$ we are going to have $\frac{dZ_S}{d\Omega} < 0$, while when R_S is in $(R^*, 1]$ we are going to have $\frac{dZ_S}{d\Omega} > 0$. □

Proof 6 Comparative statics of trade effect □

Using (14)-(22) it is possible to derive the effect of greater openness upon each of the main variables. In what follows let us consider rises in the degree of openness ($d\Omega > 0$) stemming from both a reduction of the variable cost ($d\phi > 0$) or in the fixed cost ($d\kappa_X < 0$) indistinctly.

$$\begin{aligned} \frac{da_{D,i}}{d\Omega} &= \frac{-a_{D,i}}{\alpha Z_i} \frac{dZ_i}{d\Omega}, \quad \frac{d\bar{\kappa}_i}{d\Omega} = \frac{\bar{\kappa}_i}{Z_i} \frac{dZ_i}{d\Omega}, \quad \frac{dd_i}{d\Omega} = -d_i^2 \frac{a_{K,j}}{a_{K,i}} \frac{dZ_j}{d\Omega}, \quad \text{and} \\ \frac{dx_i}{d\Omega} &= \frac{x_i^2}{(Z_i - 1)^2} \frac{a_{K,j}}{a_{K,i}} \frac{dZ_i}{d\Omega} \end{aligned}$$

Changes in the degree of openness affect $a_{X,i}$ differently whether they stem from a reduction of the variable cost ($d\phi > 0$) or in the fixed cost ($d\kappa_X < 0$). Indeed,

$$\frac{da_{X,i}}{d\phi} = \frac{a_{X,i} \beta \Omega}{\alpha Z_i (Z_i - 1) \phi} \frac{dZ_i}{d\Omega} \quad \text{and} \quad \frac{da_{X,i}}{d\kappa_X} = \frac{-a_{X,i}}{\alpha \kappa_X} \left[\frac{(\beta - 1) \Omega}{Z_i (Z_i - 1)} \frac{dZ_i}{d\Omega} + 1 \right]$$

From these expressions it is clear that $\frac{dZ_i}{d\Omega} > 0$ is a necessary and sufficient condition for $\frac{da_{D,i}}{d\Omega} < 0$, $\frac{d\bar{k}_i}{d\Omega} > 0$, $\frac{dd_j}{d\Omega} < 0$, $\frac{dx_i}{d\Omega} > 0$ and $\frac{da_{X,i}}{d\phi} > 0$. The condition for $\frac{da_{X,i}}{d\kappa_X} < 0$ is $\frac{dZ_i}{d\Omega} > \frac{Z_i(1-Z_i)}{(\beta-1)\Omega}$ with $\frac{Z_i(1-Z_i)}{(\beta-1)\Omega} < 0$, so $\frac{dZ_i}{d\Omega} > 0$ is sufficient but not necessary. $\square$

Proof 7 Comparative statics of trade effect on the growth rate of varieties $\square$

The effect that a one-time increase in openness (Ω) immediately has on the growth rate of country i (g_i) is given by the following expression:

$$\frac{dg_i}{d\Omega} = \frac{\mu d_i E_i}{\sigma a_{K,i} \bar{k}_i} \left[\left(1 - \frac{L_i}{E_i} \right) \frac{dZ_i}{d\Omega} - d_i Z_i \frac{a_{K,j}}{a_{K,i}} \frac{dZ_j}{d\Omega} \right]$$

The growth rate of local varieties in country i depends on the signs of $\frac{dZ_i}{d\Omega}$ and $\frac{dZ_j}{d\Omega}$. Tracing back the origins of these two effects it is possible to notice that $\frac{dZ_i}{d\Omega}$ is there for two reasons. One (a) is that the growth rate of varieties produced in i is affected by the market share that firms from i cover in the foreign market (represented here by $1 - \frac{L_i}{E_i}$). The other (b) is that, by (19), the average knowledge required by new firms $\bar{k}_i$ changes with freer trade (represented here by $\frac{L_i}{E_i} \frac{dZ_i}{d\Omega}$). By (20), $1 - \frac{L_i}{E_i} > 0$ which means that the first of these effects is always greater than the second. The fact that $\frac{dZ_j}{d\Omega}$ is in the previous expression too is related to the fact that (c) freer trade can hinder growth in i if foreign firms increase their market share in i to the detriment of local firms. When countries are symmetric $\frac{dZ_i}{d\Omega} = \frac{dZ_j}{d\Omega}$ and $d_i Z_i \frac{a_{K,j}}{a_{K,i}} = 1$ so the previous expression reduces to (29) and g decreases with Ω due to the increase in $\bar{k}$ in both countries. By symmetry, (a) and (c) perfectly offset each other. But when countries are asymmetric that might not be the case and that enables $g_S \neq g_N$. Since by Lemma 1 both rates must converge to the same value at the new BGP any divergence must happen only in the short term. By Lemma 2 two cases can be considered:

- The technological gap is relatively small and therefore $\frac{dZ_i}{d\Omega} > 0 \forall i = N, S$. It can be shown that the net effect on growth is negative for the South while for the North its sign depends on the size of the technological gap.
- The technological gap is relatively large and therefore $\frac{dZ_N}{d\Omega} > 0$ and $\frac{dZ_S}{d\Omega} < 0$. In such a case it is easy to see that the growth rate in the South is negatively affected by openness while the growth rate in the North unambiguously increases with freer trade. $\square$

Proof 8 Comparative statics of trade effect on the price index $\square$

By (22), the effect that a one-time increase in openness (Ω) immediately has on the price index of country i (P_i) is given by the following expression:

$$\frac{dP_i}{d\Omega} = -\frac{\mu P_i}{\alpha Z_i} \left[\frac{dZ_i}{d\Omega} + \beta Z_i d_i \frac{a_{K,i}}{a_{K,i}} \frac{dZ_j}{d\Omega} \right]$$

This shows how in cases where technological asymmetries are not large (which by Lemma 2 implies $\frac{dZ_i}{d\Omega} > 0 \forall i = N, S$) the price index unequivocally decreases with freer trade. However, when the technological gap between countries is large (which means $\frac{dZ_N}{d\Omega} > 0$ and $\frac{dZ_S}{d\Omega} < 0$) the effect freer trade has on prices is determined by the following conditions:

- $\frac{dP_N}{d\Omega} < 0 \Leftrightarrow \left| \frac{dZ_N}{d\Omega} \right| > \beta Z_N d_N \frac{a_{K,S}}{a_{K,N}} \left| \frac{dZ_S}{d\Omega} \right|$ and $\frac{dP_N}{d\Omega} > 0$ otherwise,
- $\frac{dP_S}{d\Omega} < 0 \Leftrightarrow \left| \frac{dZ_S}{d\Omega} \right| < \beta Z_S d_S \frac{a_{K,N}}{a_{K,S}} \left| \frac{dZ_N}{d\Omega} \right|$ and $\frac{dP_S}{d\Omega} > 0$ otherwise.

By Proof 5, a case of a large technological gap is one where $R_S \in [R_S^{min}, R^*)$ and when $R_S \rightarrow R^*$ then $\frac{dZ_S}{d\Omega} \rightarrow 0$. Under the previous conditions, it is possible to have falling prices in both countries, even in a situation where the technological gap is large. $\square$

Proof 9 Discount factor $\square$

The value at t of future operating profits for a firm from country i selling in its domestic market and producing with marginal cost a is

$$V_i(a, t) = \int_t^\infty e^{-\rho(s-t)} \pi_i(a, s) ds = \frac{E_{M,i}(t)}{\sigma} \left(\frac{a}{h_i(t)} \right)^{1-\sigma} \int_t^\infty e^{-(\rho+g_i+\delta_i)(s-t)} ds$$

Besides selling to the domestic market, a fraction v_i of firms from i export to j . For them, the present value of operating profits from exporting, which amounts to $\phi V_j(a, t)$, must also be added. Using this and (18), the discount factor for firms from i is

$$\gamma = \rho + g_i + \delta_i$$

so it is the sum of consumers' time preference and the rate at which new varieties are introduced in their economy ($\dot{n}_i/n_i = g_i + \delta_i$). Since both values are equal between economies, it follows that firms in all regions discount the future at the same rate. $\square$

Appendix 2: Sources and methodological choices for the quantitative results

Fitting international trade to the model presented here requires a distinction between sectors M and C . Following the spirit of the model, M is set to represent manufactured products while agricultural products are left to the C sector. To separate the two sectors in trade data, the classification in Ourens (2018) is used. UNCOMTRADE

bilateral international trade flows (SITC-Rev2, 4-digits of aggregation) constitute the main data source for computing the share of imports in manufactured goods to approximate μ , which in the model represents the share of expenditure in goods ($M+C$) devoted to M . The implicit assumption being that the split between M and C in domestic expenditure is equal to that in international trade. To follow the structure of the model, it is assumed that this proportion is the same in both the UK and the EU, and the value corresponding to the UK in the data is considered. This assumption does not seem at odds with the data as these proportions are 0.8915 for EU and 0.8913 for the UK in 2015.

To compute the parameters reflecting technological differences between regions, the Product Space as proposed by Hidalgo et al. (2007) is used. They present a measure of technological proximity in the production of any two goods, based on the probability that both goods are jointly exported by a given country and provide a matrix with such measure in the SITCRev2 classification at 4-digits, using UNCOMTRADE data for the period 1998–2000. Proximity between two goods A and B is interpreted here as the inverse of the cost of developing new product B provided A is being produced. Then, for each region, the cost of developing new products ($a_{k,i} \forall i = UK, EU$) is approximated as the inverse of the average proximity between produced and not produced goods. For each good not produced by one region, the minimum proximity with a produced good is considered. Following the literature, goods produced are those that are exported with a revealed comparative advantage. Capabilities in a given country are probably not closely interacting with those in another, so each country in the EU is taken as a separate cell. Proximity of current production with all non-produced goods is computed at the country level. Then, the minimum of those is taken for the entire EU (0 being a possibility if a country already produces that good).

Parameter σ represents the elasticity of substitution within M -goods in the present model. To approximate its value, the data for elasticities of substitution provided in Broda and Weinstein (2006) is used to compute the average elasticity of manufactured goods mutually traded by the UK and the EU. This gives the value of 5.822081.

The time preference parameter ρ is standard in the macro literature. The standard value of 0.4 is used here (see for example Gourinchas and Parker 2002). This value is assumed to be the same in both regions so no differences between them emerge from it. The same can be said about κ_D which is set to one, while κ_X (the sunk cost of exporting goods) is 54.63% larger, representing the additional costs of exporting goods brought about by Non-Tariff Barriers (NTB). This number is obtained by computing the average NTB in manufactured goods presented in Table A.1 in Dhingra et al. (2017). While the figures in that table are constructed measuring NTBs between the EU and the US, the authors use this information as a good approximation for barriers between the EU and the UK. The degree of openness Ω is computed, as defined in the model, as a bundle of the variable costs of exporting ϕ (representing tariff barriers) and the surplus sunk costs of exporting relative to selling domestically ($T = \kappa_X/\kappa_D$). Current tariff barriers are set to zero ($\phi = 1$) representing the state of variable trade costs between the UK and the EU pre-Brexit.

In the model, α is the shape parameter of the Pareto distribution of firm's productivity, and as is well known, the parameter also represents (-1 times) the elasticity of imports to changes in trade costs. This value is computed as the weighted average trade elasticity among manufacturing good sectors, using Table A.4 in Dhingra et al. (2017), which itself gathers information from Caliendo and Parro (2015).

Regarding d_{UK} which represents the share of expenditure in domestic goods for consumers in the UK, data from the PWT9.0 is used, as reported by Feenstra et al. (2015). $d_{UK} = (1 - c_X)/(1 - c_X + c_M)$ is then computed, where c_X is the share of merchandise exports, and c_M is the share of merchandise imports, at current PPPs. The last year available (2014) is used as benchmark. Finally, the size (in terms of labor) in each economy is computed as the total amount of hours worked in each region in 2014 according to PWT9.0. Since average hours worked in Croatia are missing in the data, that figure is assumed to be close to that of Slovenia. The total number hours worked in the EU and UK (in billions) are 314.9657 and 51.8648 respectively. Normalizing the figure for EU gives $L_{UK}=0.1647$.

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