



# Investment disputes and their explicit role in option market uncertainty and overall risk instability

Zdeněk Drábek<sup>1</sup> · Miloš Kopa<sup>2</sup> · Matúš Maciak<sup>2</sup> · Michal Pešta<sup>2</sup> · Sebastiano Vitali<sup>3</sup>

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## Abstract

We propose a methodological approach for capturing and analyzing the impacts of investment disputes on option markets. A dispute submission typically brings in unspecified uncertainty and additional risk. The implied volatility of options is shown to reflect such effects. However, nontrivial caution and nonstandard statistical techniques are needed to analyze them appropriately. Artificial options with a constant (over time) maturity are introduced to emphasize these effects. A panel data representation of artificial implied volatility smiles is used to ensure the overall model flexibility, transparency, and its practical interpretability. Finally, a stochastically valid changepoint detection procedure is adopted to reveal significant impacts of an investment dispute on the overall riskiness and the stock price evolution. The results show significant impacts of the first tribunal meeting and the first procedural order of the disputes under consideration.

**Keywords** Implied volatility · Investment disputes · Artificial options · Panel data · Sparsity · Changepoint detection

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✉ Michal Pešta  
michal.pesta@mff.cuni.cz

Zdeněk Drábek  
zdenek.drabek1@gmail.com

Miloš Kopa  
milos.kopa@mff.cuni.cz

Matúš Maciak  
matus.maciak@mff.cuni.cz

Sebastiano Vitali  
sebastiano.vitali@unibg.it

<sup>1</sup> CERGE-EI, Charles University, Politických vězňů 7, 11121 Prague, Czech Republic

<sup>2</sup> Faculty of Mathematics and Physics, Department of Probability and Mathematical Statistics, Charles University, Sokolovská 49/83, 18675 Prague, Czech Republic

<sup>3</sup> Department of Economics, University of Bergamo, Via dei Caniana 2, 24127 Bergamo, Italy

## 1 Introduction

Foreign direct investments (FDI) represent a major factor of global economic growth. On the other hand, they are also associated with a rising number of investment disputes which are a major concern for policy makers and foreign investors. Disputes typically involve private foreign investors and governments in host countries. The subject of disputes is usually an intervention by host country governments or the investors activity, which may lead to externalities that are not covered but legal provisions of the appropriate international treaty. The breaches will be perceived by the Parties as causing them injury from the foreign investment operations. Some of those interventions may lead to direct expropriation of assets of the investors. However, the most government interventions are of indirect expropriation types—such as changes in taxes or subsidies that affect a financial performance of the investors. Rights and obligations of both Parties are spelled out and agreed in bilateral investment treaties (BIT) between the host and home countries for FDI.

When the investors or governments perceive that their rights from the relevant BIT have been violated they first respond by expressing to the host country government their complaints and, if they receive no satisfactory reply, they litigate. BIT provide a dispute resolution mechanism—an international arbitration—which normally takes place in some third country. Once the arbitration is setup, arbiters typically make attempts to reconcile the two Parties and, indeed, around 30 percent of all investment disputes have so far been settled amicably and out of court. If no amicable settlement is reached, the arbitration continues until the final Decision on the dispute is taken by the arbiters. The whole process is closely followed not only by the legal teams of the Parties but also by specialized media, professional organizations, politicians, and, most importantly, by markets. Unfortunately, there are no explicit studies describing, statistically analyzing, and explaining how the markets react to such disputes. The existing literature rather concentrates on a much broader question concerning the effects of BIT on FDI flows. Pioneering attempts to examine market responses to investment disputes are, for instance, in Baruník et al. (2021) and Brada et al. (2021) where the effect on the shares of the companies involved in an investment dispute is investigated. In particular, Baruník et al. (2021) examine 46 disputes that involve large cap companies of all economic sectors and they show that the investment disputes lead to abnormal share fluctuations and that the outcomes of the disputes substantially influence the volatility of the stock in the next future. Brada et al. (2021) carry out a meta-analysis of 74 studies showing almost zero effects of international investment agreements for the protection of the foreign investors. The only other relevant paper is Bellak and Leibrecht (2021) in which the authors study the determinants of disputes rather than the impact on market prices. In particular, there is no financial literature examining direct effects of investment disputes on the option prices nor the implied volatilities. There are some papers that relate the changes in the option markets with other types of announcements. For instance, Donders et al. (2000) and Dubinsky et al. (2019) investigate the impact of earning announcements on implied volatility (IV) and other characteristics of the option market but they a priori assume that the dates of the announcements are well

known in advance. Similarly, Pástor and Veronesi (2013) or Kelly et al. (2016) analyze the effects of various political events but, again, with a priori known dates of the events. Clearly, the idea to link political, economic, or social events with the option market uncertainty is not new. However, to our best knowledge, this paper introduces the first explicit methodological (theoretical and empirical) study of the stochastic relationship between the investment disputes and the corresponding option market uncertainty.

The absence of studies investigating the impact of investment disputes on option markets is unfortunate. Disputes create a great deal of uncertainty for investors as there are many unknowns—such as jurisprudence in resolving the dispute, selection of arbiters, matters related to jurisdictions, location of arbitration, or the arbitration process itself. The manner in which those disputes play out is rather random, results are difficult to predict, they may take a long time, they may not be enforced, etc. The transmission of information into the markets is also imperfect. The markets are known to react differently in different stages of the dispute settlement process—see Baruník et al. (2021)—and the disputes cannot be, in general, seen as a win-win situation as some observers usually believe.

A proper stochastic and financial analysis of the relationship between the investment disputes and option prices is, therefore, very much needed. Our manuscript contributes to the existing literature by (1) presenting the first valid statistical methodology of properly assessing the significance of market changes due to an undergoing arbitration; (2) analyzing the explicit connections between the implied volatility changes of European-style options and the underlying investment disputes (the first information about the dispute coming from the first tribunal meeting and the first procedural order); (3) providing a mechanism to understand which types of structural changes of the implied volatility smiles are expected in particular.

The implied volatility itself can be estimated by using many different approaches—see, for instance, Britten-Jones and Neuberger (2000), Skiadopoulos et al. (2000), Fengler (2012), Chen et al. (2018), Jang and Lee (2019), or Parhizgari and Padungaksawasdi (2021) but none of them is fully capable of capturing the investment dispute effects properly. Therefore, we develop a unique methodology based on three consecutive steps. Firstly, the IV smiles of artificial options with a fixed (time to) maturity are constructed. The main reason why we do not rely on “raw” (observed) IV smiles is that they are changing day by day as the maturity is approaching. Moreover, important dispute related patterns (exogenous effects in general) within the IV smile evolution are dominated by the natural IV behaviour and, therefore, they are overlooked (meaning that the variability of the so-called “getting-close-to-maturity” effect is typically much higher than the magnitudes of the changes due to various external causes). In contrast, the artificial IV smiles seem to be stationary over time showing no specific volatility related to approaching maturities. Second, a panel data representation of the overall IV surface is used. We believe, that this provides a transparent and straightforward way not only to model the implied volatility over time but, also, to capture the structural complexity and the overall time dynamics of the market. Moreover, this representation complies with the financial theory of arbitrage-free markets (see Kahale 2004) as it allows to implicitly impose some underlying structure. The idea is similar as in Dumas et al.

(1998) or Borovkova and Permana (2009) but our model goes beyond as it allows for more flexibility and robustness while preserving the same structural integrity. The proposed panel data representation can be seen as an alternative to the well-known parametric or semiparametric local polynomial smoothing estimators—see, for instance, Fengler (2005), Benko et al. (2007), Fengler (2012), or Kopa et al. (2017). However, unlike all the aforementioned approaches, our method guarantees a consistent IV surface estimation even in situations when sudden changes (change-points and structural breaks) occur over time. This is generally not true for standard smoothing techniques which are well-known to be inconsistent at least in some neighbourhoods of such irregularity points. Third, a valid statistical test—suitable for the market conditions and the corresponding IV data—is used to detect significant effects of the considered investment dispute. The test itself is not new from the theoretical perspective (see Maciak et al. 2020b or Maciak et al. 2020a), but it has never been used in studies of the implied volatility of this kind.

The methodology is illustrated using real data examples of three well known world-wide companies undergoing investment disputes (selected from a large set of companies in order to ensure a selection as representative as possible). The results show that significant changes of the implied volatility occur in the days around the first tribunal meeting and/or the first procedural order. Moreover, these changes are clearly twofold: (i) the whole implied volatility smile drops down after the event and (ii) the implied volatility smile becomes more flat and stabilize after the tribunal meeting. The overall uncertainty gradually increases before and substantially decreases (drops) after the first tribunal meeting and/or the first procedural order issue. This nicely complies with the results in Donders et al. (2000) or Dubinsky et al. (2019) where, however, the dates of changes (announcements) are known in advance. Our “change-point framework” allows for more general situations with the dates being left unknown.

The rest of the paper continuous as follows: The essential theoretical background is provided in Sect. 2. Section 3 gives a brief description of the real data used for the empirical illustration. This includes both—the investment dispute data and the option price data. Section 4 gives a detailed description of the empirical results and, finally, Sect. 5 concludes.

## 2 Theoretical framework

In order to analyze market responses related to some undergoing arbitration, a general three-stage approach is proposed. In the first step, implied volatilities of artificial options having always a constant (over time) 35 days maturity are constructed. The maturity of 35 days is the shortest possible in order to be able to interpolate the observed data.<sup>1</sup> In the second stage, the artificial (time dependent) IV surface is

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<sup>1</sup> The options are written regularly, every 30–35 days depending on weekends and holidays, thus, 35 days is roughly the shortest possible maturity which allows to capture the dependence of IVs on strike in the best way. The maturity is not related to the observation (follow-up) period in which the evolution of IV smiles is analyzed.

estimated and fully represented in terms of a panel data model where each parameter in the model has its own specific (structural) meaning related directly to the volatility smile, its analytical curvature, or its time dynamics. This offers a very intuitive and straightforward description of the overall IV surface complexity. Thus, the results can be directly interpreted and discussed in terms of the IV structure itself and its changes over time. The estimation is based on a robust semi-parametric modelling framework and, therefore, the obtained IV representation is flexible and robust. It is not sensitive to outliers and various market anomalies—such as a bid-ask spread, discrete ticks in price, non-synchronous trading, etc. Finally, in the third stage, a formal statistical inference in terms of a consistent changepoint detection test is performed. Detected changepoints are linked with the corresponding changes in the implied volatility structure and specific dates of the given investment dispute. In addition, a consistent estimate for the changepoint location(s)—if the null hypothesis of no change is rejected—is automatically obtained. One could argue that usually there is some prior knowledge about possible changepoint locations explicitly assumed from the investment dispute dates and, thus, it could be more appropriate to use more common two-sample or more-sample techniques—analogously, as in Kelly et al. (2016) or Dubinsky et al. (2019). However, the main reason why we rather use the changepoint framework is that the specific investment dispute dates may not necessarily correspond with the actual changes reflected by the market. For instance, some expectations about the arbitration outcome can cause an earlier change or, alternatively, there can be also some delays in the market response. The proposed methodology can be, therefore, effectively used to also assess such anticipations and delays. Fundamental theoretical details for all three stages—the artificial implied volatility construction, the time-dependent panel data IV representation, and the formal changepoint test—are provided next. The whole methodology is later illustrated on three typical real data examples in Sect. 4.

## 2.1 Stage 1: artificial implied volatilities

The key task is to use a raw (observed) implied volatility dataset of European-style options denoted as  $\{z_{itm}; i = 1, \dots, N; t = 1, \dots, T; m = 1, \dots, M\}$ , where  $i$  stands for the option's strike label,  $t$  is the observing day, and  $m$  is the maturity dataset index, and to construct a new artificial dataset  $\{Y_{it}; i = 1, \dots, N; t = 1, \dots, T\}$  such that the new data will report, for each strike  $i$  and each observing day  $t$ , an implied volatility value  $Y_{it}$  of an artificial option having always a constant maturity of  $K = 35$  days. This can be practically achieved by applying a simple linear interpolation where for each day  $t \in \{1, \dots, T\}$  two options that have their corresponding maturities immediately before and immediately after the expiration  $t + K$  are used to define the artificial option implied volatility

$$Y_{it} = \frac{\frac{1}{(t+K)-t_b} z_{itm_b} + \frac{1}{t_a-(t+K)} z_{itm_a}}{\frac{1}{(t+K)-t_b} + \frac{1}{t_a-(t+K)}}, \quad (1)$$

where  $m_b$  is the maturity index of the first option expiring before the time  $t + K$  (at the time  $t_b$ ) and  $m_a$  is the maturity index of the first option expiring after the time

$t + K$  (at the time  $t_a$ ). The interpolation in (1) acts as a very specific smoother which only smooths IV variability due to the natural market dynamics (such as the options pay-off structure and consecutive maturities) while it fully preserves changes due to all kinds of external causes. Some empirical illustrations are given in Sect. 3 and more algorithmic details together with a sensitivity analysis<sup>2</sup> on different values of  $K$  can be found in Maciak and Vitali (2023).

## 2.2 Stage 2: time-dependent IV smile representation

Recall, that  $Y_{it}$  now represents a strike specific and time specific volatility of the artificial option of some underlying asset where  $i \in \{1, \dots, N\}$  denotes the given strike value  $x_i \in \mathcal{D}$  and  $t \in \{1, \dots, T\}$  stands for the time within some fixed observation period. In our particular situations, the values  $\{Y_{it}\}_{i,t=1}^{N,T}$  represent the implied volatilities for the artificial options with the constant maturity of 35 days constructed according to (1). Some market analysts would argue that these volatilities can be directly used for a formal statistical test of no significant change within the set of time-evolving panels  $\{Y_{it}\}_{t=1}^T$  for  $i = 1, \dots, N$ , but there are at least two good arguments why not to do so and why the intermediate panel data parametrization step is very useful. Firstly, considering the financial econometrics perspective, it is a well-known fact that the observed IV values do not necessarily comply with the financial theory on arbitrage-free markets (which is, unfortunately, also the case for the artificial options having a constant maturity of  $K$  days). Second, bearing in mind the statistical point of view, there is no implicit (analytic) structure explicitly present with respect to the strikes/panels  $i = 1, \dots, N$ . Therefore, there is also no straightforward way to interpret changepoints being detected and tested significant. Thus, no transparent interpretation and no clear conclusions regarding the underlying IV smiles—their specific curvature and time dynamics—can be made. The approach proposed in this paper deals with both of these objections: The arbitrage-free conditions are implemented in terms of a shape constrained minimization where for each  $t \in \{1, \dots, T\}$  the implied volatilities  $\{Y_{it}\}_{i=1}^N$  are smoothed by a convex IV smile and, moreover, the whole IV structure is fully represented by a set of carefully defined parameters  $\{\beta_t \in \mathbb{R}^n; t = 1, \dots, T\}$  where each parameter has its own structural (transparent) meaning. Therefore, unlike any change in the artificial IV values  $\{Y_{it}\}_{i,t=1}^{N,T}$ , the change in the parameters  $\{\beta_t \in \mathbb{R}^n; t = 1, \dots, T\}$  can be directly linked with the corresponding structural instability of the whole underlying IV surface. Therefore, conclusions and practical decisions can be made in a straightforward way.

From the mathematical point of view, a formal stochastic model can be expressed within a simple linear regression framework as

<sup>2</sup> Reasonable values of  $K$  should be in between 30–40 days. Small values tend to undersmooth the natural market dynamics but important structural breaks are still preserved. On the other hand, large values of  $K$  slightly oversmooth the natural market dynamics but the most important changepoints—although not all of them—can be still recovered. Thus, there is a typical statistical trade-off when determining the value of  $K$  for constructing the artificial options and the value of  $K = 35$  seems to be a reasonable compromise—see Maciak and Vitali (2023) for further details.

$$Y_{it} = \mathbf{x}_i^T \boldsymbol{\beta}_t + \varepsilon_{it}, \quad \text{for } i = 1, \dots, N \text{ and } t = 1, \dots, T, \tag{2}$$

where  $\boldsymbol{\beta}_t = (\beta_{t1}, \dots, \beta_{tn})^T \in \mathbb{R}^n$  are the vectors of unknown parameters which may develop over time for  $t \in \{1, \dots, T\}$  and  $\mathbf{x}_i = (\varphi_1(x_i), \dots, \varphi_n(x_i))^T$  is a spline expansion over the strike domain  $\mathcal{D} \subseteq \mathbb{R}$  evaluated for the given strike  $x_i \in \mathcal{D}$ . The model conveniently combines the idea of semi-parametric modelling techniques common for option pricing problems (see, for instance, Fengler 2005) with some robustness in terms of the conditional median estimation, and regularized estimation principles with a data-driven changepoint detection—as firstly proposed in Harchaoui and Lévy-Leduc (2010) and further elaborated in Ciuperca and Maciak (2019) and Ciuperca and Maciak (2020). Classical truncated splines can be considered: For instance,  $\varphi_1(x) = 1$  stands for the intercept term;  $\varphi_2(x) = x$  and  $\varphi_3(x) = x^2$  stand for the linear and quadratic terms; Finally,  $\varphi_l(x) = (x - x_{l-3})_+^2$  for  $l = 4, \dots, n \equiv N + 2$  where  $(\cdot)_+$  denotes a positive part of its argument are quadratic corrections over the domain  $\mathcal{D}$ . It is assumed, without any loss of generality, that the available strikes  $x_i \in \mathcal{D}$  for  $i = 1, \dots, N$  are all ordered, such that  $x_i < x_j$  for  $i < j$ . The error vectors  $\varepsilon_i = [\varepsilon_{i1}, \dots, \varepsilon_{iT}]^T$  in (2) are assumed to be independently distributed across the panels  $i \in \{1, \dots, N\}$ . Otherwise, no specific dependence structure nor any form of stationarity is assumed. In particular, the elements of  $[\varepsilon_{i1}, \dots, \varepsilon_{iT}]^T$  are not assumed to be independent nor identically distributed which is indeed needed to properly model the dependence of the IV smiles over time.

The time evolution of  $\{\boldsymbol{\beta}_t\}_{t=1}^T$  is assumed to be sparse in a sense that  $\boldsymbol{\beta}_t = \boldsymbol{\beta}_{t-1}$  for almost all  $t \in \{2, \dots, T\}$  but some few exceptions—unknown changepoints. For situations where  $\boldsymbol{\beta}_{\tilde{t}} \neq \boldsymbol{\beta}_{\tilde{t}-1}$  for some  $\tilde{t} \in \{2, \dots, T\}$  the underlying IV structure changes between two consecutive trading days  $\tilde{t} - 1$  and  $\tilde{t}$  in order to reflect the market behaviour and, moreover, the given parametric representation provides a direct connection between the unknown parameters and the IV structure itself—either its analytical curvature, its time dynamics or both. For instance, a positive (negative) difference of the first vector elements  $\beta_{\tilde{t}1} - \beta_{(\tilde{t}-1)1}$  gives the magnitude of the overall shift upwards (downwards) between two trading days  $(\tilde{t} - 1)$  and  $\tilde{t}$ .

Similarly, the difference  $\beta_{\tilde{t}2} - \beta_{(\tilde{t}-1)2}$  describes a change in the linear term before and after the change. It can be appropriately interpreted as a counter clockwise turn of the IV smile after the change if the difference is positive (IV decreases for low strikes but erects for high strikes) and, respectively, a clockwise turn of the IV smile after the change for negative magnitudes (IV flattens for high strikes and increases for low strikes). The quadratic terms  $\beta_3, \dots, \beta_n$  may seem to be not that much relevant as sudden changes in the second derivative are not immediately visible for a naked human eye.

On the other hand, these terms are still important as they are responsible for modeling the overall convexity of the IV smiles: In particular, positive changes in the quadratic terms stand for more convex IV smile (in the corresponding subdomain), while negative changes stand for less convexity (again in the corresponding part of the domain  $\mathcal{D}$ ).

The model in (2) can be estimated as a convex optimization problem formulated as

$$\min_{\beta_t \in \mathbb{R}^n} \sum_{i=1}^N \sum_{t=1}^T \rho_\tau \left( Y_{it} - \mathbf{x}_i^\top \beta_t \right) + N \lambda_N \sum_{t=2}^T \|\beta_t - \beta_{t-1}\|_2 \tag{3}$$

with respect to the arbitrage-free (linear) constraints

$$C\beta_t \geq \mathbf{0}, \quad \forall t = 1, \dots, T, \tag{4}$$

where the inequality sign above holds element-wise. First of all, the standard quantile check function  $\rho_\tau(u) = \tau(u - \mathbb{1}_{\{u < 0\}})$  with  $\tau = 0.5$  (conditional median) is used to induce some robustness. Second, there are two different regularization sources playing their particular roles in (3) and (4). The group lasso penalty in the second term in (3) controls for the overall sparsity—the number of changes occurring within the parametric IV representation over time respectively. Denoting  $\{\hat{\beta}_t\}_{t=1}^T$  as the solution of (3) and (4), then for  $\lambda_N = 0$  there will be  $\hat{\beta}_t \neq \hat{\beta}_{t-1}$  for all  $t \in \{2, \dots, T\}$  and the IV surface will adaptively change each day to reflect the market conditions instantly. On the other hand, for  $\lambda_N \rightarrow \infty$ , there will be  $\hat{\beta}_t = \hat{\beta}_{t-1}$ , again for all  $t \in \{2, \dots, T\}$  and, thus, no changes over time are present in the underlying IV smiles over time. Another reasonable (i.e., theoretically justified) choice is, for instance,  $\lambda_N = N^{-1} \cdot (\log N)^{1/2}$  which, under some technical assumptions, ensures the consistency of the parameter estimates  $\hat{\beta}_t$  for  $t = 1, \dots, T$  (for more technical details we refer to Maciak (2019)). The second source of the regularization in the minimization problem above is implicitly present within the linear constraints in (4) where

$$C = \begin{bmatrix} \varphi_1''(x_1) & \varphi_2''(x_1) & \dots & \varphi_n''(x_1) \\ \varphi_1''(x_2) & \varphi_2''(x_2) & \dots & \varphi_n''(x_2) \\ \vdots & \vdots & \ddots & \vdots \\ \varphi_1''(x_N) & \varphi_2''(x_N) & \dots & \varphi_n''(x_N) \end{bmatrix}$$

denotes the matrix of the second derivatives of the functional basis functions  $\varphi_j(x_i)$ , for  $j = 1, \dots, n$ , evaluated at the given strikes  $x_i \in \mathcal{D}$ , for  $i = 1, \dots, N$ . The linear convexity constraints in (4) ensure the overall IV smile structure over time—the arbitrage-free validity.

To summarize the second stage of the proposed methodology, the artificial IV values  $Y_{it}$  are used to estimate the underlying time dependent IV structure which always obeys the arbitrage-free conditions. However, unlike all other common smoothing techniques used for the IV estimation, the artificial IV structure estimated in terms of (3) and (4) may not be smooth nor even continuous. Moreover, unlike the IV values  $\{Y_{it}\}_{i,t=1}^{N,T}$  themselves, where no implicit structure is a priori present and potential change-points can be only interpreted on a strike-specific basis, the proposed panel-data representation offers a direct and very intuitive connection between the estimated parameters and the structural changes of the whole time-dependent IV structure. A formal statistical test can be now easily applied to make a valid inference.



### 2.3 Stage 3: significant changepoint detection

The estimated parameters  $\hat{\beta}_t = (\hat{\beta}_{t1}, \dots, \hat{\beta}_{tm})^\top$  for  $t = 1, \dots, T$  and the corresponding true values  $\beta_t = (\beta_{t1}, \dots, \beta_{tm})^\top \in \mathbb{R}^n$ , again for  $t = 1, \dots, T$ , can be also given into a mutual relationship by postulating the panel data (changepoint) model

$$\hat{\beta}_{jt} = \beta_j + \delta_j \mathbb{1}\{t > \tau\} + \tilde{\varepsilon}_{ij}, \quad (5)$$

for  $j = 1, \dots, n$  and  $t = 1, \dots, T$ . In case of the spline bases discussed in Sect. 2.2 it would hold that  $n = N + 2$ . The values  $\beta_j \in \mathbb{R}$ , for  $j = 1, \dots, n$ , may be understood as panel specific mean parameters (i.e., true parameter values) which, in a full correspondence with the sparsity principle and the lasso regularized estimation in Stage 2, are assumed to be the same before the change and, also, after the change. The changepoint, occurring at some unknown time point  $\tau \in \{1, \dots, T\}$ , is common for all panels (i.e., the same arbitration event affects the market) however, the panel specific changepoint magnitudes,  $\delta_j \in \mathbb{R}$  may be different (thus, the severity of specific effects depends on the price of the asset—the strike) allowing also for situations where only some proportions of the panels is subject of the change (meaning that  $\delta_j = 0$  for some  $j \in \{1, \dots, n\}$ ). Each panel specific mean parameter equals to  $\beta_j$  before the change and it equals to  $\beta_j + \delta_j$  after the change. The panel-specific disturbances  $\tilde{\varepsilon}_j = [\tilde{\varepsilon}_{1j}, \dots, \tilde{\varepsilon}_{Tj}]$  are assumed to be zero mean  $\alpha$ -mixing sequences of random errors. Thus, the errors  $\{\tilde{\varepsilon}_j\}_{j=1}^n$  are neither independent nor identically distributed.

The strong mixing condition among the panels reflects the fact that more distant strike prices have less dependent implied volatility. Moreover, there is no prescribed form of any stationarity assumed within the panels and some heteroscedasticity across the panels is also allowed. This accounts for the situations where option intrinsic values for smaller strikes are expected to have smaller volatility. From the practical point of view, a statistically significant change can be detected by a statistical test of the null hypothesis

$$H_0 : \tau = T, \quad (6)$$

which reflects the situation that there is no changepoint within the observation period  $t = 1, \dots, T$ , against a general alternative

$$H_1 : \tau < T \quad \exists k \in \{0, \dots, n\} \text{ such that } \delta_k \neq 0. \quad (7)$$

There are many different approaches proposed to perform the test of the null hypothesis in (6) against the alternative in (7). Usually different asymptotic assumptions are imposed on the number of panels  $n \in \mathbb{N}$  and the length of the follow-up period  $T \in \mathbb{N}$ .

Bearing in mind arguments of econometricians and financial agents, it is not realistic to assume that  $T \rightarrow \infty$  as the observation period is typically limited. Practitioners usually prefer to rather focus on some shorter periods in order to limit the effects of some additional causes which are of no interest. On the other hand, the price of the underlying asset—the quoted strikes—may be theoretically assumed to be

arbitrarily dense and, therefore, it is assumed that  $n \rightarrow \infty$ . Therefore, a proper test statistic needs to be proposed to correctly handle mutually dependent and generally non-stationary panels (as the strikes are allowed to be arbitrarily dense and, moreover, the implied volatility for two neighbouring strikes is assumed to be more similar than the volatility of two distant strikes) while assuming a relatively very short follow-up period which is usually limited to a few days only. For computational reasons we use the test statistic proposed in Maciak et al. (2018) and Maciak et al. (2020a). The idea is based on a self-normalization principle together with the assumption of dependent and non-stationary panels and possibly an extremely short follow-up period. The test statistic, which does not require the estimation of any nuisance parameters for the errors' variance due to the self-normalization principle, is defined as

$$S_n(T) = \sum_{t=1}^{T-1} \frac{\mathcal{L}_n^2(t, T)}{\sum_{s=1}^t \mathcal{L}_n^2(s, t) + \sum_{s=t}^{T-1} \mathcal{R}_n^2(s, t)},$$

where  $\mathcal{L}_n(s, t) := \sum_{j=1}^n \sum_{r=1}^s (\hat{\beta}_{rj} - \bar{\beta}_{ij})$  and  $\mathcal{R}_n(s, t) := \sum_{i=1}^n \sum_{r=s+1}^T (\hat{\beta}_{rj} - \tilde{\beta}_{ij})$  are cumulative sums of partial residuals. Moreover,  $\bar{\beta}_{ij}$  denotes the average of the first  $t$  observations in the panel  $j$  and  $\tilde{\beta}_{ij}$  is the average of the last  $T - t$  observations in the panel  $j$ , i.e.,  $\bar{\beta}_{ij} = \frac{1}{t} \sum_{s=1}^t \hat{\beta}_{sj}$  and  $\tilde{\beta}_{ij} = \frac{1}{T-t} \sum_{s=t+1}^T \hat{\beta}_{sj}$ . Under the null hypothesis of no change and some mild technical assumptions it can be proved that

$$S_n(T) \xrightarrow[n \rightarrow \infty]{\mathcal{D}} S(T), \tag{8}$$

where

$$S(T) = \sum_{t=1}^{T-1} \frac{(X_t - \frac{t}{T} X_T)^2}{\sum_{s=1}^t (X_s - \frac{s}{t} X_t)^2 + \sum_{s=t}^{T-1} (Z_s - \frac{T-s}{T-t} Z_t)^2}$$

for  $Z_t := X_T - X_t$  where  $(X_1, \dots, X_T)^\top$  is a multivariate normal random vector with a zero mean vector and the covariance matrix

$$\Lambda = \lim_{n \rightarrow \infty} \frac{1}{n} \text{Var} \left\{ \sum_{j=1}^n \left( \sum_{s=1}^1 \tilde{\epsilon}_{sj}, \dots, \sum_{s=1}^T \tilde{\epsilon}_{sj} \right)^\top \right\}$$

Alternatively, under the hypothesis in (7), the test statistics  $S_n(T)$  converges, in probability, to infinity as the number of panels increases,  $n \rightarrow \infty$ . Therefore, the change-point test based on the test statistic  $S_n(T)$  is consistent and it can be properly used to detect significant effects of the undergoing arbitration in the IV smiles and, thus, the overall market riskiness.

The bootstrap extensions is proposed in Maciak et al. (2020a) to effectively mimic the asymptotic distribution of  $S(T)$  in (8). The moving block bootstrap (MBB) is

used because the model assumes mutually dependent panels. There are also other changepoint related statistical tests considering the self-normalized test statistics, cf. Betken (2016) and Shao (2011) but the theoretical assumptions do not suit the situations being considered in this paper.

Finally, for practical applicability of the proposed method one needs to obtain some reasonable estimate for the existing changepoint location. Indeed, if the test is rejected, the test statistic  $\mathcal{S}_n(T)$  does not provide any information where the changepoint should be located. An estimator proposed in Pešta et al. (2020) can be used here. Unlike under estimators suggested in the literature, it does not suffer of any boundary issues when the actual changepoint is located at the beginning or at the end of the observation period. Formally, the changepoint location estimate is given as

$$\hat{\tau}_n = \arg \max_{t=1, \dots, T} \mathcal{U}_n(t), \quad (9)$$

where

$$\mathcal{U}_n(t) = \begin{cases} \frac{1}{t(T-t)} \sum_{j=1}^n \sum_{u=1}^t \sum_{v=t+1}^T (\hat{\beta}_{uj} - \hat{\beta}_{vj})^2 & t < T; \\ \frac{2}{(T-1)^2} \sum_{j=1}^n \sum_{v=2}^T \sum_{u=1}^{v-1} (\hat{\beta}_{uj} - \hat{\beta}_{vj})^2 & t = T. \end{cases}$$

For the consistency of the estimator in (9) together with some further technical details we refer to Pešta et al. (2020).

### 3 Empirical analysis: data description

This section illustrates a practical application of the whole methodology proposed above. Empirical investigations of explicit effects of three arbitration (selected in a way that they are different but rather typical and representative in some sense—see the description below) related to three world-wide leading companies are provided. For sake of a more comprehensive market analysis other companies with more-or-less similar results were also considered (for instance, Edenred, Hochtief, Saint-Gobain, Telefónica, Tenaris, and others) but, for obvious brevity reasons, they are not all discussed in detail here. Nevertheless, the whole methodology can be analogously applied to analyze the effects of any other arbitration if suitable underlying data (as described below) are available.

#### 3.1 Investment dispute data

Three well known companies relatively recently involved in investment disputes are selected (Erste Group AG, Raiffaisen Bank AG, and Veolia). The primary focus is on the implied volatility evolution during the days of (around) the first tribunal meeting and the date of the first procedural order. Both events are considered to be

**Table 1** Important dates for the investment disputes of the three considered companies

Case No	Claimant	Registration	First tribunal meeting	First procedural order	Conclusion
ARB/17/49	Erste Group	December 29, 2017	August 10, 2018	August 20, 2018	July 15, 2021
ARB/17/34	Raiffeisen Bank	September 15, 2017	June 25, 2018	July 19, 2018	August 6, 2021
ARB/18/36	Veolia	October 16, 2018	March 15, 2019	March 19, 2019	March 29, 2019

important external effects that may have some (significant) impact on the market. Specific dates together with the registration day and the conclusion day are summarized in Table 1.

Erste group against Croatia (ARB 17/49) is a typical investment dispute (concluded recently, in 2021). The first tribunal meeting and the first procedural order took both place in August 2018 and the claimant faced no other dispute in 2018. The considered observation period is taken from July 20, 2018 to September 3, 2018 which gives 32 trading days in total. The second example is Raiffeisen Bank against Croatia (ARB17/34) which is again a relatively long dispute (with a slightly slower progress) but the claimant was also active in another pending dispute at the same time (ADHOC/15/1 against Poland). The observation period is from June 4, 2018 to August 2, 2018, thus, there are 44 trading days in total. Finally, Veolia against the Gabonese Republic is a very fast dispute. The Tribunal rendered its award embodying the parties settlement agreement on March 29, 2019, just a few days after the first tribunal meeting and the procedural order. The observation period is from February 22, 2019 to April 1, 2019 (27 trading days altogether). At the same time, Veolia was also a claimant in two other disputes (ARB/16/3 against Lithuania and ARB/18/20 against Italy) but neither an order nor a decision of these disputes were issued during the considered observation period.

### 3.2 Raw option data

The option data—implied volatilities for the call options quoted in the EUREX Deutschland market—are downloaded from Thomson Reuters Datastream for all three companies mentioned above. For each company the corresponding implied volatilities are in a dataset  $\{z_{itm}; i = 1, \dots, N; t = 1, \dots, T; m = 1, \dots, M\}$  where  $i$  stands for the option's strike label,  $t$  is the observing day, and  $m$  is the maturity. Considering Erste Group, the available strikes of the call options span from 30.00 Euro up to 40.00 Euro with a step of 1.00 Euro, i.e.,  $N = 11$  strikes. Since these options are emitted once a month, there are  $M = 3$  available (consecutive) maturities (August 16, September 20, and October 18) related to the observation period. The follow-up period goes from July 20, 2018 to September 3, 2018 ( $T = 32$  trading days). For Raiffaisen Bank, the available strikes are spanned from 23.00 Euro up to 34.00 Euro, again with the step of 1.00 or 2.00 Euro ( $N = 11$  strikes in total); There are  $M = 3$  available maturities (July 19, August 16, and September 20) in summer

2018. The follow-up period starts in June 4, 2018 and ends on August 2, 2018 ( $T = 44$  trading days in total). Finally, for Veolia, the available strikes are spanned from 15.50 Euro up to 23.00 Euro with a steps of 0.50 Euro (in total, there are  $N = 16$  strikes); Again, there are  $M = 3$  considered maturities (March 14, April 17, and May 16), all in 2019. The follow-up period goes from February 22, 2019 till March 29, 2019 ( $T = 27$  trading days).

### 3.3 Artificial implied volatilities

The constant maturity of  $K = 35$  days is used to construct the implied volatilities of artificial options. For example, considering Erste Group, the first day from the observation period ( $t = 1$ ) is July 20, 2018. Thus, the artificial option will expire in  $t + 35$ , i.e., August 24, 2018 and the two options that must be considered for the interpolation in (1) are the options with the maturity August 16, 2018 (denoted by  $m_b$ ) and the maturity September 20, 2018 (denoted by  $m_a$ ). The distance between the artificial maturity (August 24, 2018) and the maturity of the first option (before) is  $b = (t + 35) - m_b = 8$  days and the distance between the artificial maturity and the maturity of the second option (after) is  $a = m_a - (t + 35) = 27$  days. Therefore, the equation in (1) takes the explicit form

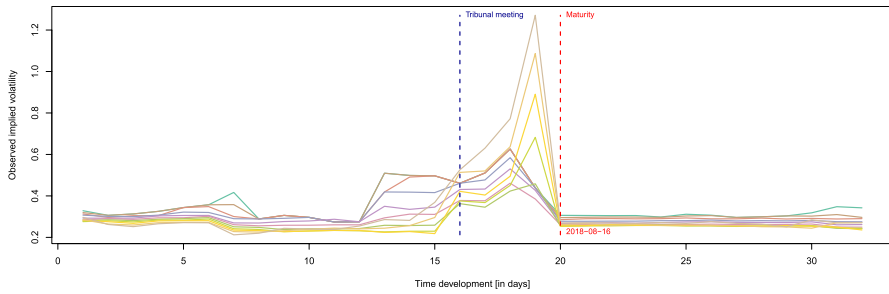
$$Y_{it} = \frac{\frac{1}{8}z_{itm_b} + \frac{1}{27}z_{itm_a}}{\frac{1}{8} + \frac{1}{27}}. \quad (10)$$

The procedure is repeated for all strikes  $i \in \{1, \dots, N\}$  and for all trading days  $t \in \{1, \dots, T\}$  from the observation period of each underlying asset. Considering Erste Group, the average artificial implied volatility (over time and available strikes) in the follow-up period is 0.2708 (within the overall range [0.2249, 0.3547]). Similarly, for Raiffeisen Bank, the average artificial implied volatility is 0.3668 (within the range [0.3025, 0.5025]). Finally, for Veolia, the average artificial volatility is 0.1941 (within the range [0.1400, 0.3101]).

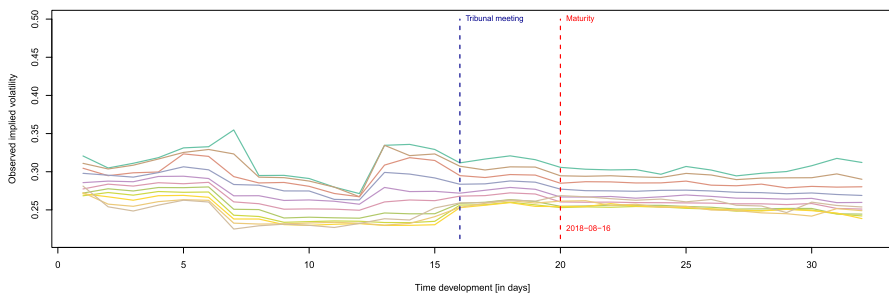
## 4 Empirical analysis: application and results

According to the methodology described in Sect. 2 there are three consecutive steps. The interpolated volatilities of the artificial options with the constant 35 days maturity are constructed in the first stage (see Figs. 1, 2, and 3 for illustration). Let us recall, that there is yet no implicit structure nor any IV smile profile assumed within the given volatility values at any time (as clearly illustrated in the figures, for instance, by overlapping curves—strike profiles or panels respectively). Nevertheless, there is some obvious volatility dynamics observed over time in all three figures.

In the second stage, the interpolated volatilities of the artificial options for each company are represented and estimated using the panel data model in (2). The strike domain  $\mathcal{D}$  is defined by the minimum and maximum quoted strike and the



(a) Implied volatility panels for Erste Group



(b) Interpolated volatility of artificial options for Erste Group

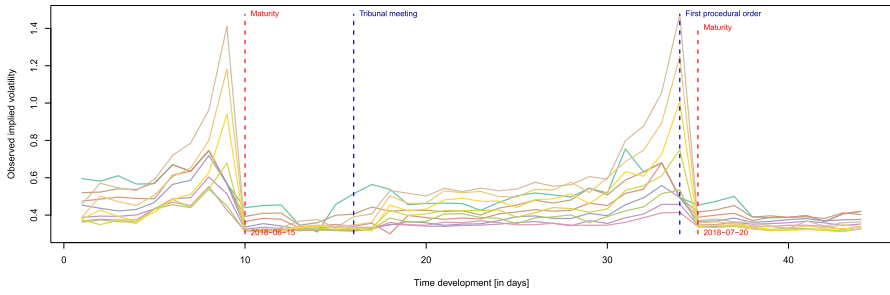
**Fig. 1** Strike specific panels ( $N = 11$ ) of implied volatilities for Erste Group: Raw market data in the top panel and the corresponding interpolated volatilities of the artificial options with the constant  $K = 35$  days maturity in the bottom panel. The same follow-up period of  $T = 32$  trading days is considered for both situations. Vertical lines represent the date of the first tribunal meeting (blue) and the date of the original options maturity (red) (Colour figure online)

minimization problem in (3) is solved under the arbitrage-free constraints in (4). Individual minimization problems are solved for each company separately and the estimated company specific IV surfaces are visualized in Fig. 4. Recall, that the estimated surfaces are not smooth nor continuous in general. However, they can be always intuitively described by the corresponding sets of the estimated parameters.

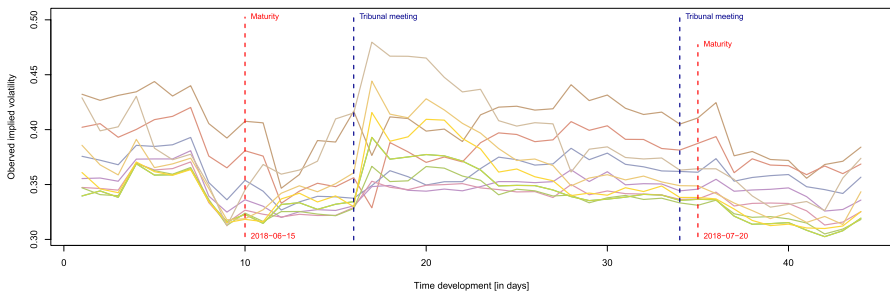
Finally, in the last but not least stage of the proposed methodology, a formal statistical test of the null hypothesis of no changepoint in the estimated surface is performed. The alternative hypothesis is specified by (7). The test statistic  $\mathcal{S}_n(T)$  is utilized and the bootstrap algorithm based on 10.000 Monte Carlo resamples is adopted to mimic the asymptotic distribution of the test statistic under the null hypothesis. The critical level of 5% is considered for all statistical tests. Final results are summarized—considering each company separately—in the following subsections.

#### 4.1 Erste group

Let us start with the implied volatility of the call options of Erste Group which are specific due to the fact that some obvious instabilities are observed around



(a) Implied volatility panels for Raiffeisen Bank

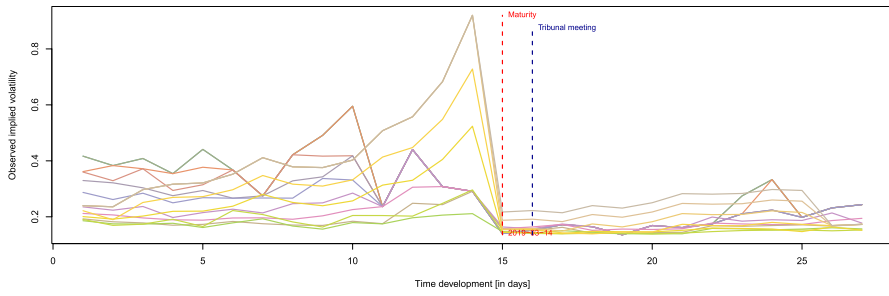


(b) Interpolated volatility of artificial options for Raiffeisen Bank

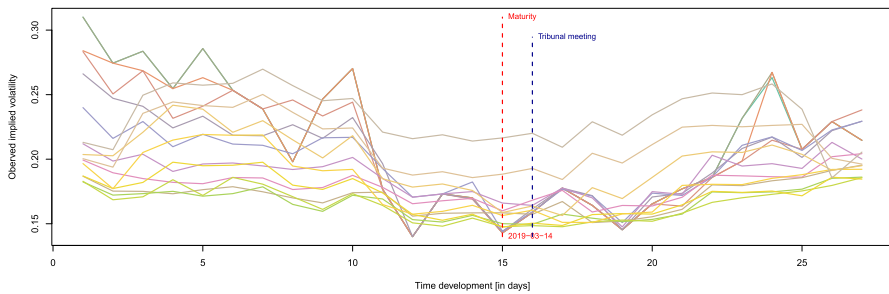
**Fig. 2** Strike specific panels ( $N = 11$ ) of implied volatilities for Raiffeisen Bank: Raw market data in the top panel and the corresponding interpolated volatilities of the artificial options with the constant  $K = 35$  days maturity in the bottom panel. The same follow-up period of  $T = 44$  trading days is considered for both situations. Vertical lines represent the dates of the first tribunal meeting and the first procedural order (blue) and the dates of the original options maturities (red) (Colour figure online)

the day of the first tribunal meeting—see Fig. 1. Considering the whole observation period with the length of  $T = 32$ , the formal statistical test of the null hypothesis of no significant changepoint in the overall IV surface is rejected (test statistic  $\mathcal{S}_n(T) = 3.9734$ ; critical value: 3.1552;  $p$ -value: 0.0001). The corresponding changepoint estimate constructed in terms of (9) yields  $\hat{\tau}_{\text{Erste}} = 16$ , which is August 10, 2018. In other words, the IV smile before (and including also August 10, 2018) is statistically significantly different than the IV smile after.

Consequently, the observation period can be now split into two parts—before the change with the corresponding length of  $T_{\text{before}} = 16$  and after the change with  $T_{\text{after}} = 28$ . The same statistical test of no changepoint is applied again for both fragments separately. While the null hypothesis is not rejected for the follow-up period after the first change, the null hypothesis is rejected for the observation period before the change. Therefore, there is another significant changepoint detected (test statistic  $\mathcal{S}_n(T) = 21.7099$ ; critical value: 6.4504;  $p$ -value:  $< 0.0001$ ) with the corresponding estimate  $\hat{\tau}_{\text{Erste};2} = 14$ , which is August 7, 2018.



(a) Implied volatility panels for Veolia

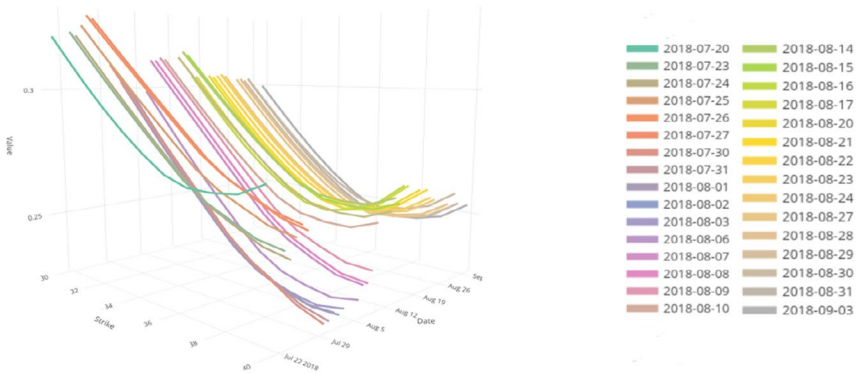


(b) Interpolated volatility of artificial options for Veolia

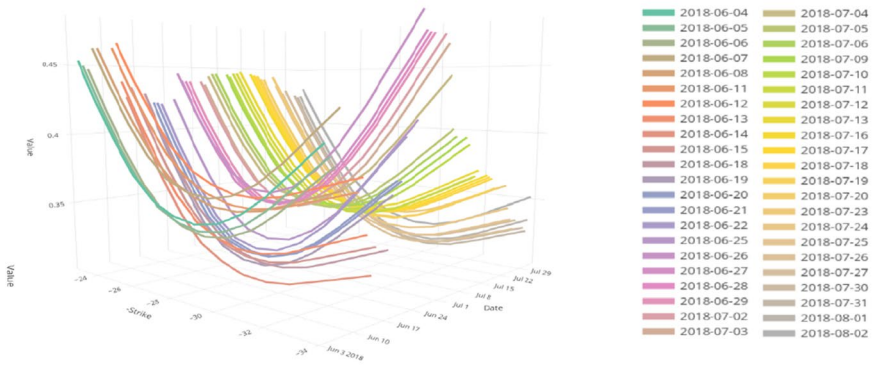
**Fig. 3** Strike specific panels ( $N = 16$ ) of implied volatilities for Veolia: Raw market data in the top panel and the corresponding interpolated volatilities of the artificial options with the constant  $K = 35$  days maturity in the bottom panel. The same follow-up period of  $T = 27$  trading days is considered in both situations. Vertical lines represent the date of the first tribunal meeting (blue) and the date of the original options maturity (red) (Colour figure online)

Using now the linkage between the estimated parameter profiles in Fig. 5 and the overall structure of the IV surface itself, the effect of significant changes can be directly described and easily interpreted. It can be clearly observed that the significant changes in the IV smiles over time are caused by the linear term (middle panel in Fig. 5). The overall volatility gradually increases before the first tribunal meeting and it immediately drops down after the meeting—August 10, 2018 (reflected by the changes in the intercept parameter, the top panel in Fig. 5). Even more importantly, the volatility sharpens and flattens for both, high and low strikes (significant changes in the linear term). There are also some changes observed among the quadratic terms, however, they are all roughly at the same scale as the overall variability (lower panel). Generally speaking, the IV smile moves upwards when the time progresses towards the first detected changepoint and the overall convexity of the smile increases. After the second changepoint (August 10, 2018), the overall IV smile slightly decreases (the drop in the intercept term in the top panel in Fig. 5) but the convexity of the smile changes dramatically—introducing almost a linear shape of the IV smile after the

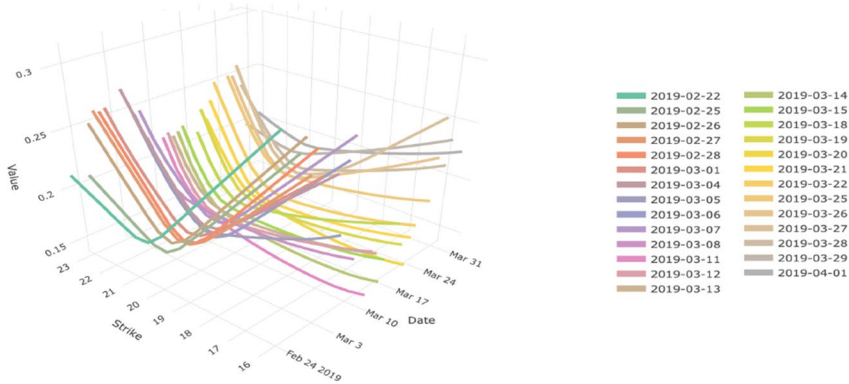




(a) Erste Group | Observation period:  $T = 32$  days

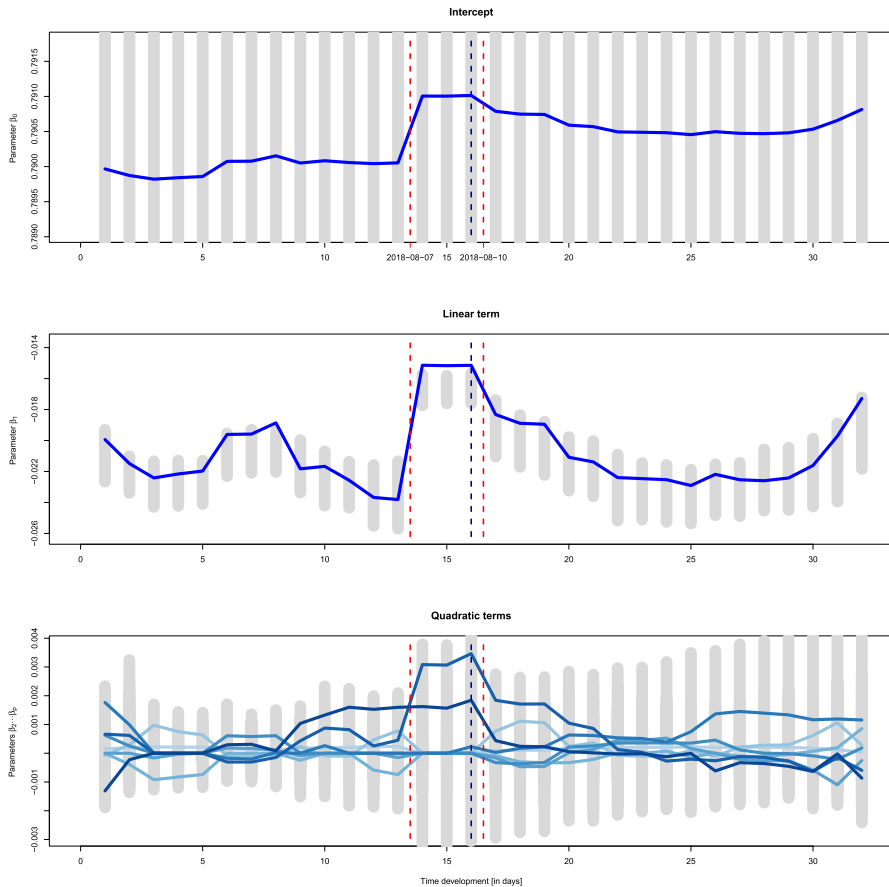


(b) Raiffeisen Bank | Observation period:  $T = 44$  days



(c) Veolia | Observation period:  $T = 27$  days

**Fig. 4** The estimated IV smiles and their time evolution within the given follow-up period as described by the model in (2). The overall IV structures always obey the financial theory on the arbitrage-free market (i.e., the convexity of the IV smiles at any time point  $t \in \{1, \dots, T\}$ ) but some obvious instabilities are clearly observed as the estimated surfaces are not generally smooth or even continuous over time

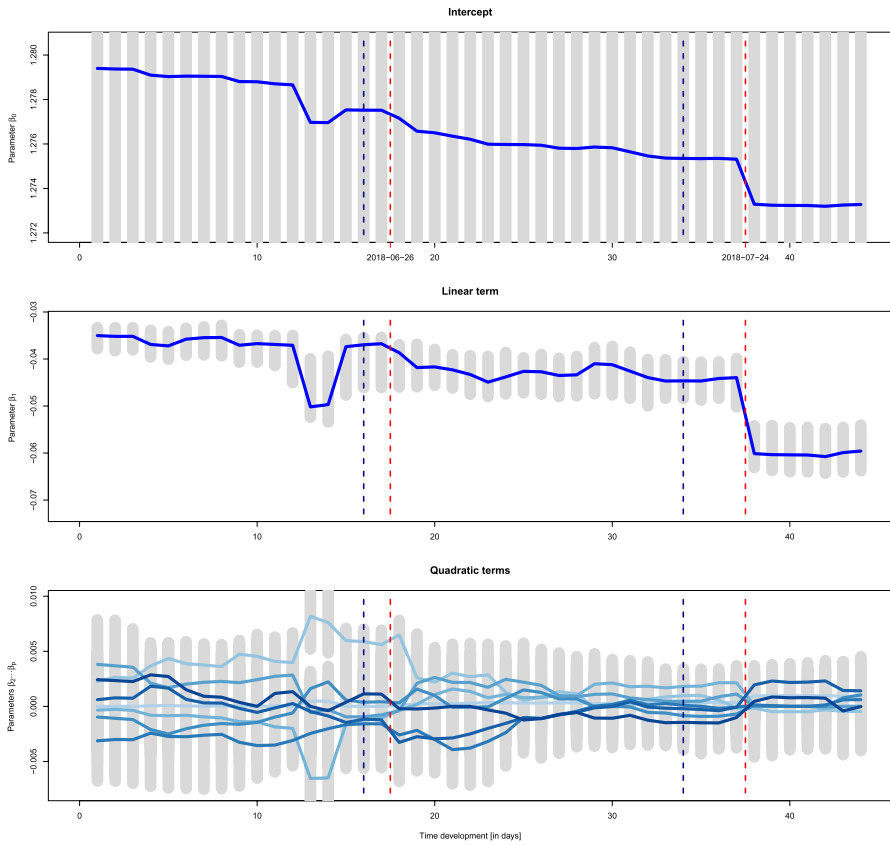


**Fig. 5** The time dependent profiles for the estimated parameters of the overall IV structure for Erste Group (the intercept parameter profile  $\{\beta_{11}\}_t$  in the top panel, the slope parameter profile  $\{\beta_{12}\}_t$  in the middle panel, and the quadratic profiles  $\{\beta_{2j}\}_t$  for  $j = 3, \dots, n$  in the lower panel—light blue profiles correspond with the curvature of the smile for small strikes while dark blue profiles reflect the quadratic curvature of the smile for high strikes). The estimated parametric profiles are given together with the error bounds (*in gray*) obtained as a minimum and maximum value over 1000 Monte Carlo bootstrap resamples. The detected significant changepoints are visualized by the *red vertical lines*. The corresponding dates are given in the top panel. The *blue vertical line* represent the day of the first tribunal meeting (August 10, 2018) (Colour figure online)

changepoint. Moreover, the whole IV smile becomes very stable after this point. This also corresponds with the observations from the top panel in Fig. 4.

## 4.2 Raiffeisen bank

The situation is quite analogous also for the implied volatility of the artificial options of Raiffeisen Bank where, again, two significant changepoints are detected. The whole observation period with the length of  $T = 44$  trading days is considered for the



**Fig. 6** The time dependent profiles for the estimated parameters of the overall IV structure for Raiffeisen Bank (the intercept parameter profile  $\{\beta_{11}\}_t$  in the top panel, the slope parameter profile  $\{\beta_{12}\}_t$  in the middle panel, and the quadratic profiles  $\{\beta_j\}_t$  for  $j = 3, \dots, n$  in the lower panel—light blue profiles correspond with the curvature of the smile for small strikes while dark blue profiles reflect the quadratic curvature of the smile for high strikes). The estimated parametric profiles are given together with the error bounds (in gray) obtained as a minimum and maximum value over 1000 Monte Carlo bootstrap resamples. The detected significant changepoints are visualized by the red vertical lines. The corresponding dates are given in the top panel. The blue vertical lines represent the first tribunal meeting (June 25, 2018) and the first procedural order (July 19, 2018) (Colour figure online)

statistical test of no significant change within the panels of the estimated parameters. The null hypothesis is rejected (test statistic  $S_n(T) = 7.4801$ ; critical value: 5.0524;  $p$ -value: 0.0376) and the corresponding changepoint estimate, again constructed in terms of (9), yields  $\hat{\tau}_{\text{Raiffeisen}} = 37$ , which is July 24, 2018. Repeating the whole testing procedure on both fragments (right and left from the estimated changepoint) another significant changepoint is detected ( $S_n(T) = 11.8254$ ; critical value: 4.3878;  $p$ -value:  $< 0.0001$ ). The estimated changepoint location is  $\hat{\tau}_{\text{Raiffeisen}} : 2 = 17$ , which is June 26, 2018, a day after the first tribunal meeting—Fig. 6.

Similarly as before, it is obvious, that the most relevant change (the biggest magnitude relative to the underlying variability) is caused by the linear term (middle

panel in Fig. 6). Some systematic (rather minor) changes can be also seen with respect to the quadratic terms (lower panel) where, similarly as before, all estimated parameters have a tendency to get close to zero after the change (resulting in a flat IV smile after the change). Finally, there is also some minor overall drop of the whole IV smile—particularly after the first detected changepoint (July 24, top panel in Fig. 6). Let us remind, that from the statistical point of view it is not the magnitude of the change itself that matters but the magnitude must be always compared to a relative variability of the given estimate.

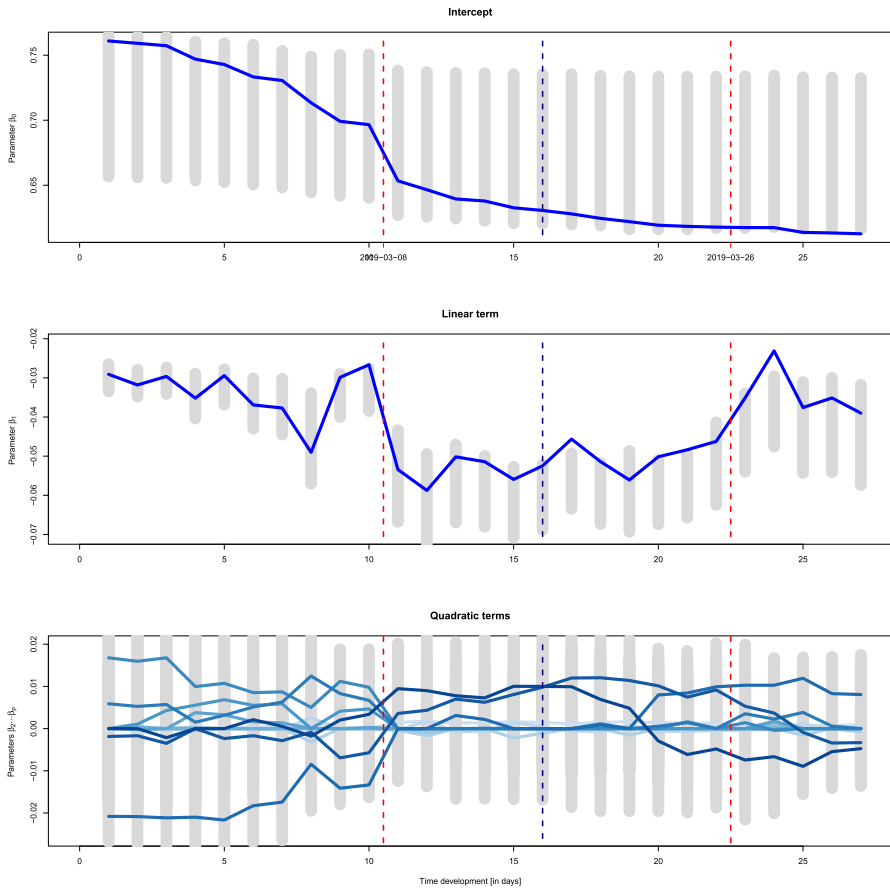
### 4.3 Veolia

Finally, there are also two changepoints being detected for Veolia. Firstly, considering the whole follow-up period, the proposed test statistic detects a significant change at  $\hat{\tau}_{\text{veolia}} = 10$  which corresponds with the weekend before the first tribunal meeting ( $\mathcal{S}_n(T) = 16.8919$ ; critical value: 12.6082;  $p$ -value:  $< 0.0001$ ). After splitting the follow-up period into two parts—before and after the detected changepoint—there is another changepoint being tested significant on March 25, 2019 ( $\mathcal{S}_n(T) = 7.7297$ ; critical value: 6.7709;  $p$ -value: 0.0108). The estimated parameter profiles in Fig. 7 can be again used to draw an interpretation of the detected changes. Despite the fact that the overall market uncertainty seems to generally decrease over the whole observation period (given by the decreasing profile for the intercept parameter in the top panel in Fig. 7), the IV smiles significantly flatten at the days around the first tribunal meeting (March 15, 2019) and the first procedural order (March 19, 2019) which can be seen from the significant drop in the slope parameter and the consecutive increase after the second detected changepoint. Similarly as before, the quadratic terms show some obvious instability before the first tribunal meeting while getting relatively stable after the first procedural order and, particularly, around the day of the arbitration conclusion (March 29, 2019).

### 4.4 Summary

Structural changes of the implied volatility smile in days around the first tribunal meeting and the first procedural order were clearly detected in all considered cases while a proper stochastic model was used for an assessment and statistical inference. The results confirm a general expectation that the investment disputes may have significant effects on the implied volatility of options of claimants. Qualitative conclusions are also very similar to those described in Donders et al. (2000) or Dubinsky et al. (2019) regarding the effects of earning announcements, however, with a priori given dates. Finally, the following specific conclusions were observed in particular cases:

- Erste Group—two significant effects of the undergoing dispute were detected. The first on August 10, 2018, which is exactly the day of the first tribunal session. The change corresponds to the fact that the increasing uncertainty about



**Fig. 7** The time dependent profiles for the estimated parameters of the overall IV structure for Veolia (the intercept parameter profile  $\{\beta_{11}\}_t$  in the top panel, the slope parameter profile  $\{\beta_{12}\}_t$  in the middle panel, and the quadratic profiles  $\{\beta_{ij}\}_t$  for  $j = 3, \dots, n$  in the lower panel—light blue profiles correspond with the curvature of the smile for small strikes while dark blue profiles reflect the quadratic curvature of the smile for high strikes). The estimated parametric profiles are given together with the error bounds (in gray) obtained as a minimum and maximum value over 1000 Monte Carlo bootstrap resamples. The detected significant changepoints are visualized by the red vertical lines. The corresponding dates are given in the top panel. The blue vertical line represents the date of the first tribunal meeting (March 15, 2019) (Colour figure online)

the first tribunal meeting (and its impact on the dispute) is stabilized after the meeting. The second effect in terms of some sudden uncertainty increase just a few days before the meeting was detected some days before, August 7, 2018. As expected, the stock is considered risky when the outcome is unknown and it becomes less risky and relatively quite stable after the meeting outcome.

- Raiffeisen Bank—two significant effects of the undergoing dispute are detected again. The first one occurs just a few days after the first procedural order (July 19, 2018) and, thus, there is some detected delay between the event itself and the

corresponding market reaction. An obvious change can be observed in all parametric terms of the IV representation. However, the most statistically relevant is the one which corresponds with the decrease in the linear term. This means that the IV smile flattens for higher strikes after the change and, additionally, the whole IV smile also drops down after the break. In general, the market becomes less risky and the IV smile evolution is relatively stable after the first procedural order. The second detected changepoint seems to be related to the first tribunal meeting as it occurs on the next day, June 26, 2018. Again, the market does not react immediately and some natural delay is observed.

- Veolia—again, two significant structural breaks are detected. The first one is estimated at the weekend before the first tribunal meeting while the second one occurs after the first procedural order. Since this dispute was relatively short, only one tribunal meeting was held and it is, therefore, not surprising that this meeting also changed the evolution of IV surface instantly—actually right before the arbitration was concluded (March 29, 2019). The most evident is the break in the linear term which again suggests that the whole IV smile flattens and also drops down around the days of the considered investment dispute and the stock becomes less risky, similarly to the previous case.

## 5 Conclusions

The paper proposes a solid methodological approach to analyse changes of the implied volatility smile caused by an underlying investment dispute. In particular, the paper focuses on the period around the first tribunal meeting and the first tribunal decision but an analogous approach can be considered for different periods as well. The main goal of the paper was to identify whether, when, and in which way the IV smiles change when reacting to various dispute related events. The following three-stage procedure can be generally applied to reach the goal:

1. The artificial options and their implied volatilities with a constant (over time) maturity are constructed for every day from the follow-up period and each considered strike;
2. A panel data representation based on a robust semiparametric, sparse, and regularized estimation is applied in order to mathematically describe, intuitively characterize, and properly interpret the artificial IV structure—its analytical curvature and the time dynamics in particular;
3. A consistent statistical test is used to detect significant changes in the estimated panel data representation while the detected changepoints can be straightforwardly interpreted in terms of time-specific and structural-specific effects directly linked with particular events of the undergoing arbitration.

The main contributions of this manuscript are also three-fold: 1) To our best knowledge, we propose the first statistically valid methodological approach to properly assess the significance of exogenous market changes while accounting for the

time and space dependence of the market data; 2) We show a significant impact of the undergoing arbitration on the corresponding option market, and specific dispute events are directly related with the underlying (significant) market changes; 3) A straightforward mechanism to understand specific type and structure of the expected IV smile changes is provided and explained in the paper. Finally, we also believe, that the proposed methodology is straightforward, theoretically transparent, easy to apply in practice.

In all three considered data cases, the IV smiles significantly changed after/around the first tribunal meeting. Moreover, the detected changes are very much the same in all three cases: the overall IV smile dropped down and it became more flat. This means that the market considered the underlying stocks less risky after the changepoint, i.e., the probability of losses was decreased. In particular, the IV smiles of Veolia changes slightly before the day of the first tribunal meeting. Since the dispute was closed 14 days later, the first tribunal meeting was also the last one and the respondent made an agreement with the claimant rather than continue the dispute. The IV smile of Raiffeisen did not react on the first tribunal meeting immediately but on the day when the first procedural order was issued. This is probably due to the fact that the outcome of the first tribunal meeting was not known before the first procedural order was issued. Perhaps the most interesting behaviour of the IV smile was observed in the case of the Erste Group dispute. The test detected two significant changes—the first one just a day after the first tribunal meeting and the second one a day before the tribunal issued the first procedural order. It shows that the uncertainty related to the first tribunal outcome was increasing before the first tribunal meeting and it remained high for a few days after. The uncertainty dropped down and stabilized a day before the first procedural order was issued—the market anticipated the decision. Other companies (with a sufficiently long history and high granularity in the strikes and maturities) have been also considered for the analysis with the outcomes analogous to those presented in this paper.

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