Investors' perspective on portfolio insurance

Expected utility vs prospect theories

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Abstract

This study supports the use of behavioural finance to explain the popularity of portfolio insurance. Portfolio insurance strategies are important financial solutions sold to institutional and individual investors that protect against downside risk while maintaining some upside valuation potential. The way some of these strategies are engineered has been criticised and portfolio insurance itself blamed for increasing market volatility in depressed markets. Despite this, investors keep on buying portfolio insurance that keeps its solid market share. This study contributes to understand the phenomenon. We compare individual investors' decision using two distinct frameworks: expected utility theory and behavioural theories. Based upon Monte Carlo simulation techniques we compare portfolio insurance strategies against uninformed basic benchmark strategies, for a wide variety of market scenarios. We conclude that cumulative prospect theory may be a viable framework to explain the popularity of portfolio insurance. However, among portfolio insurance strategies, naïve strategies seem to be preferable to most commonly traded strategies.

Keywords Portfolio insurance · Expected utility · Prospect theory · Monte Carlo simulation

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1 Introduction

Portfolio insurance (PI) strategies have been a popular investment alternative for institutional and retail investors since the 1980s. Despite its designation, a PI strategy is not an insurance contract where an investor pays a premium for a risk transference to an insurance carrier to limit losses from adverse market conditions. Instead, it is a dynamic asset allocation strategy.

The two most common PI strategies are option-based portfolio insurance (OBPI) and constant proportion portfolio insurance (CPPI). The OBPI was developed after the seminal article of Black and Scholes (1973), when Leland and Rubinstein (1976) suggested the use of options for hedging portfolios. About a decade later, Perold (1986) and Black and Jones (1987) proposed CPPIs.

From an incumbent market in the late 70s and early 80s, PI investments gained momentum and became important in the asset management industry. Due to falling costs on trading and exuberant product innovation (e.g. structured financial products, index notes, warrants, etc.), PI have also expanded from institutional solutions to retail investments, recommended to individual investors.

The current importance of PI is clear when The New York Times, in November 11th 2015, dedicates its wealth special section to the subject describing the most recently developed strategies (Wasik 2015) or when Forbes, in March 4th 2016, recommends its usage after the strong equity returns on 2016 and the rising fear of losses on the increasing wealth from market returns (Russel 2016). With the opposite view on PI, the Financial Times March 21st 2017 claims that the rise of new forms of portfolio insurance spark fears (Wigglesworth 2017), the Nobel prize winner Prof. Robert Shiller at The Wall Street Journal October 19th 2017 notes that there is a resurgent idea about PI as a solution that was designed to protect investors from falling markets but, instead, it could lead to a new market panic as in 1987 (Shiller 2017) and Bloomberg Opinion in October 25th 2017 calls for caution (Gandel 2017).

Taking the investors' perspective, several studies have questioned the optimality of PI strategies. Using the von Neumann and Morgenstern (1947) classical expected utility theory (EUT) we have, among others, Benninga and Blume (1985), Black and Perold (1992), Cesari and Cremonini (2003), Annaert et al. (2009), or Bertrand and JI (2016). Black and Perold (1992), for instance, documented that a CPPI strategy, with unconstrained borrowing, only maximizes expected utility when investors show hyperbolic absolute risk aversion utility functions. More importantly, Vrecko and Branger (2009) show that averse investors would not choose PI investments over unhedged alternatives, such as a plain investment in the risky asset. EUT is appealingly simple and relies in agents rationality, and it would be very satisfying if its predictions were empirically confirmed. However, after years of effort, it has become clear that basic facts about the aggregate stock market, the cross-section of average returns and individual trading behaviour are not easily understood in this framework (see Samuelson (1977)). Behavioural theories of Tversky and Kahneman (1974), Kahneman and Tversky (1979) and Tversky and Kahneman (1992) have emerged in response to these difficulties, arguing that investors are not fully rational. In fact, experimental results show investors tend to be more averse to losses than

gains, which seem aligned with the PI downside protection profile. See Barberis and Thaler (2003) for an overview on behavioural issues. Vrecko and Branger (2009) and Dichtl and Drobetz (2011) were the first to look at PI investments from a behavioural perspective. More recent related studies are Bernard (2016), Deng and Liu (2017), Silva (2018), and Escobar et al. (2019).

This paper uses Monte Carlo simulations to compare PI strategies, with plain investments on the risky asset or the riskless asset, to find how capable the previously mentioned alternative theories – the traditional EUT, and behavioural prospect and cumulative prospect theories (PT and CPT) – are of explaining the persistent popularity of portfolio insurance. It reproduces some results in the existing literature, but represents a more exhaustive study by considering not only popular PI strategies and non-PI investments, but also naïve PI strategies. With respect to the naïve strategies, it considers in addition to the stop-loss strategy a safety-first strategy first introduced by Costa and Gaspar (2014), that turns out to be the dominant PI strategy in almost all scenarios, even when behavioural theories are considered.

The remaining of the text is organised as follows. A brief literature review on Section 2. Section 3 on methodology, defines the various PI and non-PI strategies and sets notation, the theoretical framework and simulation features. Section 4 presents and discusses the results. Finally, Section 5 concludes summarising the main results and discussing future research.

2 Literature overview

There has been extensive theoretical work on the optimality of portfolio insurance strategies, as well as empirical studies comparing different portfolio strategies against alternative basic investment strategies (i.e. buy-and-hold, mix buy and hold and risk-free allocation). There are also comparative studies using simulation methods that focus on the dominance of portfolio insurance strategies in specific market conditions. For a comprehensive survey see Ho et al. (2010), Lee et al. (2013) or Ho et al. (2013).

The first studies on the evaluation of CPPI and OBPI performance were made by Zhu and Kavee (1988). They find that both strategies capture partial upside and protect on the downside risk. However, this market protection has two costs: explicit (transactions costs) and implicit (returns that are forgone when using the protection). Perold and Sharpe (1988) simulate the performance of four dynamic asset allocation strategies in bull, bear, trendless and volatile markets – namely, the buy-and-hold, constant mix, CPPI, and OBPI; they find that there is no dominance of a particular strategy in all situations. Cesari and Cremonini (2003) use measures for risk, returns, and risk adjusted performance, to compare portfolio insurance strategies. They also find that there are no dominant strategies in any of the simulated market conditions. On the other hand, Ho et al. (2010) find that different strategies are dominant in specific market conditions, depending also on the considered performance measure. Annaert et al. (2009) and Almeida and Gaspar (2013) evaluate the performance of the most common strategies based on block-bootstrap simulation of empirical distributions. The first indicate that a 100% floor value should be preferred

to lower floor values, while the latter indicate that the unhedged CPPI strategy (with $m = 1$) is preferable in terms of stochastic dominance. Zagst and Kraus (2011) and Maalej and Prigent (2016) also analyse conditions for stochastic dominance in PI strategies. Costa and Gaspar (2014) use Monte Carlo simulations and also find naïve strategies generally outperform OBPI, and CPPIs. Dichtl et al. (2017) test for statistical significance of the differences in downside performance risk measures between pairs of PI strategies using a bootstrap-based hypothesis test. They find OBPI and CPPI provide superior downside protection compared to SLPI or more recently developed strategies, such as the TIPP strategy or the dynamic VaR-strategy, that do not seem to provide significant improvements over the plain-vanilla CPPI. Carvalho et al. (2016) further analyse the design problem of CPPIs using conditional Monte Carlo simulations.

Vrecko and Branger (2009) are among the first to consider behavioural theories. The analysis looks at OBPI and CPPI and considers both an EUT investor with constant relative risk aversion (CRRA), and a cumulative prospect theory (CPT) investor. They find that the CRRA investor does not profit from PI and chooses rather low protection levels. The CPT investor, on the other hand, strongly prefers OBPI to CPPI strategies. Both loss aversion, and probability weighting in CPT, turn out to be critical to explain the attractiveness of PI strategies, as utility gains drop sharply if one of these two elements is eliminated. The study of Dichtl and Drobetz (2011) also analyses PI in a behavioural context and find that most PI strategies are the preferred investment strategy for a prospect theory investor.

Our study closely follows the approach of Dichtl and Drobetz (2011), however, we directly compare EUT to both PT and CPT, under a variety of controlled scenarios. In terms of EUT, by considering a second order Taylor approximation we analyse the effect of a wide range of relative risk aversion coefficient levels, in line with the empirical literature, thus covering all possible realistic utility functions.

Furthermore, in terms of investment strategies, in addition to the OBPI, CPPI and Time Invariant Portfolio Protection (TIPP) strategies, we also consider both naïve PI strategies – the Stop Loss Portfolio Insurance (SLPI) and the unleveraged CPPI 1 – and simple benchmark non-PI investments – pure investments in the risky asset or the riskless asset and the 50:50 constant mix. It turns out behavioural theories can indeed explain why investors may opt for PI strategies as opposed to unhedged investments, but interestingly in that context naïve PI strategies are preferable to the market traded strategies. These findings are strengthened by several robust tests.

3 Methodology

3.1 Portfolio insurance strategies

Portfolio insurance (PI) strategies offer investors the ability to limit the downside risk and, simultaneously, to keep the possibility of benefiting from the upside potential of an underlying risk asset or portfolio. Despite the concerns from the literature, and the increasing complexity of risk management decisions – see Brady (1988), Rubinstein (1999) and Tucker (2005) – PI popularity has not decreased. If some PI strategies

Table 1 List of major portfolio insurance strategies

Portfolio Insurance Strategies				
Naïve strategies		Complex strategies		
SLPI	CPPI $m = 1$	OBPI Synthetic Options	CPPI $m > 1$	TIPP Dynamic Floor

became controversial and lost popularity (e.g. CPPIs as discussed in Carvalho et al. (2016)), other have emerged in their place. Pain and Rand (2008) summarises some of the developments until then, from which we highlight various modifications of the original CPPI strategy: from the TIPP proposed by Estep and Kritzman (1988), to the “cushion insurance” of Prigent and Tahar (2005), the dynamics proportions proposed by Chen et al. (2008) or the contingent retracted floor of Lee et al. (2013). Hocquard et al. (2015) proposes alternative strategies with pre-specified distributional properties that present much better results than CPPIs and Bernard (2016) suggests modifications taking the perspective of the pension funds industry and long-term investors. Nowadays investors are, thus, presented with a wide range of strategies, offered from very active distribution channels on retail banking and institutional segments. The implementation of portfolio insurance is possible both using PI strategies commonly traded in the market, and naïve strategies that can be directly implemented by investors, as they require no financial engineering.

Historically, the most popular PI strategies on retail and institutional markets are the synthetic OBPI and the CPPI with a multiplier superior to 1. As plain CPPIs lost popularity, TIPP became also very popular (amongst the array of variants from plain CPPIs). On the naïve side, we have the classical Stop Loss Portfolio Insurance and the more recently suggested safety-first strategy, which turn out to be nothing but a CPPI with a multiplier equal to 1 (CPPI 1). Table 1 summarises the PI strategies under analysis.

3.1.1 Stop Loss Portfolio Insurance (SLPI)

In a SLPI strategy, the investor sets the equivalent to a stop loss order, which is a conditional instruction to sell the risky underlying if its value falls below a given level – the *floor*. In this case, an investor, who allocates the total initial wealth (V_0) in the underlying risky asset, cuts the loss to the predetermined level. If the floor is never reached the SLPI allows full benefit from an upward market. A major problem with this strategy is its path dependency (Rubinstein (1985)): when market falls below the predefined floor portfolio, stock is sold and converted into cash/bonds, which are then held until maturity. From that moment on, there is independence on future market movements until maturity.

The investor position in the risky underlying is held as long as the present value of the floor (F_t) is smaller than the current wealth (portfolio value at time t), i.e. the investor will be 100% invested in the risky underlying as long as $V_t > F_t$ and it will

be 100% invested in the risk free asset otherwise. Thus the exposure to risky underlying (ES_t):

$$ES_t = \begin{cases} V_t, & \text{if } V_t > F_t \\ 0, & \text{if } V_t \leq F_t, \end{cases} \quad (1)$$

where the value of the floor at maturity $F_T = K$ is the guaranteed amount.

If the market value of the risky underlying falls below the floor F_t , the portfolio is sold and converted into the risk-free asset and held until maturity. EB_t denotes the strategy's exposure to the riskless asset. Therefore, at maturity, the value of a SLPI strategy (V_T^{SLPI}), is given by:

$$V_T^{SLPI} = ES_T + EB_T \begin{cases} ES_T = 0 \wedge EB_T = ES_t e^{r(T-t)}, & \text{if } V_t < F_t, \\ ES_T = W_T \wedge EB_T = 0, & \text{if } V_t \geq F_t. \end{cases} \quad (2)$$

3.1.2 Option-Based Portfolio Insurance(OBPI)

Investors, pursuing an OBPI strategy, mimic a portfolio investing in the risky asset, the riskless asset and at-the-money options on the risky asset. Given a floor, the proportions of the risk-free and risky assets depend on the price of at-the-money options at the time of initial investment.¹ OBPIs can be understood as a static approach when it happens that the option can be bought. That is, however, rarely the case for the desired maturities, strikes or underlying (see Lee et al. 2013), so in practice the option is dynamically replicated using only the risky and riskless asset, as proposed by Rubinstein and Leland (1981).

Here we consider a dynamic OBPI using synthetic replication of call options under the Black and Scholes (1973) model. The value of the OBPI strategy is as follows:

$$V_t^{OBPI} = ES_t^{OBPI} + EB_t^{OBPI}, \quad (3)$$

where

$$\begin{cases} ES_t^{OBPI} = qS_t N(d_1), \\ EB_t^{OBPI} = qKe^{-r(T-t)} N(-d_2) + EB_{t=0}^{OBPI}, \end{cases}$$

where C_0 is the price of a call at the money in inception, S_t is the price of the underlying risky asset, K is the strike price (floor), the number of call options is given by

¹ A static OPBI strategy can be implemented either using put or call options (Leland 1980). With the put option, investor holds the underlying risky portfolio and buys a put option with striking price equal to floor. When an investor insures the portfolio with call options, there is a call on the underlying risky asset with striking price equal to the floor and holds risk free (cash/bonds) asset discounted by the risk-free interest rate until maturity.

$$q = \frac{V_{t=0}^{OBPI} - Ke^{-rT}}{C_0},$$

and the initial amount in risk free asset is $EB_{t=0}^{OBPI} = (1 - q)Ke^{-rT}$. The factor $N(x)$ is the cumulative probability distribution function for a standardized normal distribution, and d_1 and d_2 , which are defined by $d_1 = \frac{(\ln(\frac{S}{K}) + (\frac{1}{2}r\sigma^2)T)}{(\sigma\sqrt{T})}$, and $d_2 = d_1 - \sigma\sqrt{T}$, where σ is the standard deviation of the underlying risky asset returns.

3.1.3 Constant Proportion Portfolio Insurance $m > 1$ (CPPI m)

A CPPI strategy can be understood as a portfolio of risky and risk-free assets, where holdings are dynamically rebalanced according to a discrete trading rule – the *mechanism* – in order to achieve at maturity a minimum amount of the initial portfolio.

In each rebalancing date, the difference between the discounted floor and the initial portfolio value ($V_{t=0}$) – the *cushion* (Cu) – must be determined. The amount invested in the risky asset is defined by applying a factor – *multiplier* (m) – to the cushion. As a result, the exposure to the risky asset is simply $ES_t^{CPPI} = m \times Cu_t$. The multiplier is fixed and chosen to reflect the expected performance of the risky asset and investor's preferences. In this strategy, the investor must decide both the floor and the multiplier. Typical multiplier values range from 3 to 7. Thus, when $m > 1$ there is a potential leverage of the investment.

The outcome of the CPPI strategy is the following:

$$V_t^{CPPI} = ES_t^{CPPI} + EB_t^{CPPI}, \quad (4)$$

where

$$\begin{cases} ES_t^{CPPI} = m \times Cu_t \\ EB_t^{CPPI} = V_t^{CPPI} - ES_t^{CPPI} \end{cases},$$

and the floor at any time $t \in [0; T]$ is $K_t = K_T e^{-r(T-t)}$, the cushion is calculated at rebalancing moments $Cu_t = V_t^{CPPI} - K_t$ and m remains fixed until maturity.

3.1.4 Safety First Portfolio Insurance (CPPI 1)

If the multiplier on the CPPI strategy above is set to $m = 1$, its is equivalent to initially putting aside the present value of the floor in the riskless asset and investing only the remaining, i.e. the initial cushion, in the risky asset – this naïve safety-first, CPPI 1 or unleveraged CPPI strategy, was first proposed in Almeida and Gaspar (2013) and Costa and Gaspar (2014) as has surprisingly good performance indicators.

3.1.5 Time Invariant Portfolio Protection (TIPP)

The TIPP is a variant of CPPI proposed by Estep and Kritzman (1988), and incorporates a dynamic absolute floor, which is ratchet up whenever the portfolio value increases. In this way, the investor not only guarantees the standard floor (present value of future guarantee), but can increase it by incorporating intra-period gains,

$$V_t^{TIPP} = ES_t^{TIPP} + EB_t^{TIPP}, \quad (5)$$

where

$$\begin{cases} ES_t^{TIPP} = m \times Cu_t \\ EB_t^{TIPP} = V_t^{TIPP} - ES_t^{TIPP} \end{cases},$$

and the floor at any time $t \in [0; T]$ is

$$K_t^{TIPP} = \begin{cases} V_t^{TIPP} e^{-r(T-t)}, & \text{if } V_t^{TIPP} > K_t \\ K_t e^{-r(T-t)}, & \text{if } V_t^{TIPP} \leq K_t \end{cases}.$$

The cushion is calculated at rebalancing moments $Cu_t = V_t^{TIPP} - K_t^{TIPP}$ and m remains fixed until maturity. Due to this mechanism, at the end of period (T), the TIPP strategy is expected to have a greater percentage of risk-free assets than the plain-vanilla CPPI.

3.2 Theoretical framework

3.2.1 Expected Utility Theory (EUT)

The classical *expected utility theory* (EUT), initially designed by Bernoulli, in 1738, and further developed by von Neumann and Morgenstern (1947) to model economic agents' decisions, is based on preference axioms – completeness, transitivity, continuity and independence – that define decision making of “rational” investors over uncertain prospects. Intuitively, it states individuals do not care directly about monetary values of the outcomes, but about the utility money provides, $U(W)$, and that given the rationality axioms, the investment choice problem is reduced to the goal of maximising the final expected utility, $E[U(W)]$, also known in finance as risk tolerance function (RTF).

Utility functions map between the physical measure of money and the investors' perceived value of money. The most common utility functions used in finance are the exponential, logarithmic, power and iso-elastic, due to its mathematical tractability.

However, even in those cases, the non-linearity of utility functions, make it hard to obtain the RTF function, $E[U(w)]$, in closed-form. Two solutions are possible: (i) numerical evaluation of the RTF (e.g. via Monte Carlo simulations), or (ii) choosing to use second order approximations of utility functions, for which is always possible to obtain an approximation of the RTF in closed-form. A common approximation that results from second-order Taylor approximations is

$$E[U(w)] \approx \bar{R} - \frac{1}{2} RRA(w_0)(\bar{R}^2 + \sigma^2) \quad (6)$$

where w_0 denotes the initial wealth, $\bar{R} = \frac{w-w_0}{w_0}$ the expected final return on the investment and σ the associated volatility, while $RRA(w_0)$ represents the well-known coefficient of relative risk aversion evaluated at initial wealth,

$$RRA(w_0) = -w_0 \frac{U''(w_0)}{U'(w_0)}. \quad (7)$$

In this study we first consider both approaches mentioned above. We start by considering three concrete utility functions $U(w) = \ln(w)$, $U(w) = w$ and $U(w) = w^2$ representing the three basic investor profiles, risk averse, risk neutral and risk loving, respectively. We then use the approximation in (6) to analyse various levels of relative risk aversion, ranging from -1 to 6. The level -1 is the one resulting from our risk loving utility $U(w) = w^2$, the level 0 from risk neutral utility $U(w) = w$, while a relative risk aversion of 1, corresponds to risk averse utility $U(w) = \ln(w)$. While it can be argued the logarithmic utility function is appealing, since log utility maximizes the rate of growth, it results from the empirical literature on relative risk aversion that coefficients may take higher values.

Some of the earliest empirical estimates of RRA coefficients originated in the mid 1970s. Friend and Blume (1975) concluded that RRA generally exceeds unity and is probably greater than 2. Hansen and Singleton (1983) looking at financial investments, yielded RRA estimates ranging from as low as 0.359 to as high as 58.25, although most fell between 0.26 and 2.7. Mankiw (1985) obtained RRA estimates ranging from 2.44 to 5.26 for nondurable consumption and from 1.79 to 3.21 for durable goods consumption. Szpiro (1986) obtained a range of 1.2 to 1.8 using aggregate time-series data on property insurance. More recently we also have the studies of Azar (2010), Grable and Chatterjee (2016), Liu et al. (2016), Conine et al. (2017) or Stark (2019). By considering RRA from -1 to 6 for a realistic range of relative risk aversion coefficients, we prove robustness and consistency of our EUT results.

3.2.2 Prospect Theories

Since the 1950s, experimental work showed that individuals systematically violate EUT axioms when choosing among risky gambles. Since then, alternative theories have been proposed to explain the experimental evidence. In chronological order, the most well-known are: the weighted-utility theory (see Chew and MacCrimmon (1979) and Chew (1983)), the implicit expected utility (see Chew (1989) and Dekel (1986)), the disappointment aversion (see Gul (1991)), the regret theory (Bell (1982) and Loomes and Sugden (1982)), the rank-dependent utility theories (as in Quiggin (1982), Segal (1987), Segal (1989) and Yaari (1987)), and prospect and cumulative prospect theories (from the seminal papers of Kahneman and Tversky (1979) and Tversky and Kahneman (1992)).

Under the *prospect theory* (PT) assumptions, investors assess their decisions against a reference target, in terms of gains and losses. This formulation means that utility is defined over gains and losses rather than over final wealth positions, an idea already discussed in Markowitz (1952). PT investors' value function is a sigmoid curve, with the curve being concave for gains, and convex for losses with a degree of response defined by a factor λ , representing risk-aversion in the domain of gains, and risk-seeking in the domain of losses. This means that, for the same referential absolute variation, the impact of losses is bigger than the impact of gains. Investors weight the probabilities of outcomes, and evidence highlights the fact that events with low probabilities are overweighted.

In its first formulation PT was concerned with simple prospects of the form $(x, p; y, q)$, which had at most two non-zero outcomes. In this prospect an individual receives x with probability p , y with probability q , and nothing with probability $(1 - p - q)$, where $p + q \leq 1$. The decision between the prospects depends on its value (V). The overall value of a prospect is defined in two scales π and v , where π associates with each probability p a decision weight $\pi(p)$, reflecting the impact of p on the value of the prospect, and v assigns each outcome x a number $v(x)$ which reflects a subjective value for the outcome. The outcomes are defined relative to a reference point, which is the zero point of the value scale (i.e. measures losses and gains). That means that V is defined on prospects and v is defined in outcomes. Therefore,

$$V(x, p; y, q) = \pi(p)v(x) + \pi(q)v(y), \quad (8)$$

where x_i being the strategy result, $v(0) = 0$, $\pi(0) = 0$, $\pi(1) = 1$ and when prospects are sure $V(x, 1) = V(x) = v(x)$ and the evaluation of strictly positive or negative prospects follows a different formula.

The theory, originally introduced by Kahneman and Tversky (1979), has been modified since then. Based on additional evidence, Tversky and Kahneman (1992) propose a generalization of PT which can be applied to gambles with more than two outcomes, leading to the *cumulative prospect theory* (CPT). The CPT value function is defined as strictly increasing and assess the negative and positive outcomes of each prospect (f),

$$V(f) = V(f^+) + V(f^-) = \sum_{i=-m}^n \pi_i v(x_i), \quad (9)$$

where $V(f^+) = \sum_{i=-m}^0 \pi_i^- v(x_i)$, $V(f^-) = \sum_{i=0}^n \pi_i^+ v(x_i)$ and $-m \leq i \leq n$.

With this value function, it is possible to evaluate investment strategies prospects (from negative to positive outcomes in relation to a reference point—initial wealth), as in the case of EUT. In this theory, a two-component power valuation function is defined to characterise the curve pattern describing decision behaviour:

$$v(\Delta x_i) = \begin{cases} (\Delta x_i)^\alpha & (\Delta x_i) \geq 0 \\ -\lambda(-\Delta x_i)^\beta & (\Delta x_i) < 0, \end{cases} \quad (10)$$

Table 2 Stock market scenarios

Expected return	Volatility		
	Low: $\sigma = 10\%$	Medium: $\sigma = 20\%$	High: $\sigma = 30\%$
$\mu = 15\%$	Scenario 1	Scenario 4	Scenario 7
$\mu = 10\%$	Scenario 2	Scenario 5	Scenario 8
$\mu = 5\%$	Scenario 3	Scenario 6	Scenario 9

where Δx_i denotes each outcome in relation with the reference point, and $i = 1, \dots, N$, which are ranked in ascending order. Tversky and Kahneman (1992) propose values for the parameters, namely $\alpha \approx \beta \approx 0.88$ for the exponent reflect the diminishing sensitivity and $\lambda \approx 2.25$ for the coefficient of loss aversion, a measure of the relative sensitivity to gains and losses.

Investors are assumed to assign value to all outcomes, not by applying single probabilities as in PT, but instead in CPT, by using weighted probabilities, different for gains and losses,

$$\pi_i = \begin{cases} \pi_i^- = w^-(p_{-m} + \dots + p_i) - w^-(p_{-m} + \dots + p_{i-1}), & 1 - m \leq i \leq 0, \\ \pi_i^+ = w^+(p_i + \dots + p_n) - w^+(p_{i+1} + \dots + p_n), & 0 \leq i \leq n - 1. \end{cases} \quad (11)$$

where

$$w^+(p) = \frac{p^\gamma}{(p^\gamma + (1-p)^\gamma)^{(1/\gamma)}} \text{ and } w^-(p) = \frac{p^\delta}{(p^\delta + (1-p)^\delta)^{(1/\delta)}}. \quad (12)$$

In our analysis, as in Dichtl and Drobetz (2011), we use the valuation function in (10) and consider both $\lambda = 1$, which means that reaction to losses is not different from the reaction to gains, and $\lambda = 2.25$ that differentiates gains and losses. For Tversky and Kahneman (1992) we also use the typical parameter leading to a curvature of the value function that indicate that risk aversion for gains ($\gamma = 0.61$) is more pronounced than risk seeking for losses ($\delta = 0.69$).

In this paper we compare the results obtained from the classical EUT with those obtained from both PT and CPT behavioural frameworks.

3.3 Simulations

We use standard Monte Carlo methods to simulate the evolution of the risky asset for a variety of 9 different scenarios – see Table 2. The continuous compound stock market returns are generated 100,000 times for each scenario, using the Black and Scholes (1973) model, a one-year time horizon with daily rebalancing (252 trading days), a floor at 100%, and a risk-free rate of 5%.²

² In the robustness analysis of Section 4.3 we look at different values, but the results do not change considerably.

The risky asset dynamics are, thus, given by

$$dS_t = \mu S_t dt + \sigma S_t W_t \quad \Leftrightarrow \quad d(\ln S) = (\mu - \frac{\sigma^2}{2})dt + \sigma dW_t, \quad (13)$$

where μ and σ are constant and W is a Wiener process.

The above 9 market scenarios – low, medium and high volatility jointly with bearish, normal and bullish expected returns for the risky asset – allow us to capture a large span of possible market conditions. Extracting real life distributions, using bootstrapped methods (as used for instance in Almeida and Gaspar (2013)) would perhaps be more realistic, but would lead to only one empirical distribution (one scenario), and the results would be specific of a particular time period or underlying asset. By using a standard Monte Carlo setup, under a fully controlled setting, such as that of Black and Scholes (1973) model, we are able to consider the variety of scenarios. In addition, we are able to directly compare our results to those of the previous literature.

Following most empirical studies on PI, our simulations do not consider tax, bid and offer spreads, nor transaction costs on the stock market. Although tax and transaction costs may impact performance and affect final wealth, the purpose of this study is the comparison of the ability of the different frameworks – EUT, PT and CPT – to explain investor preferences across investment strategies, along the same simulated paths. See Leland (1985) for further discussion on the matter. Contrary to the studies of Dichtl and Drobetz (2011), Do and Faff (2004), and Cesari and Cremonini (2003), we do not apply any filter on portfolio shifts (single or cumulative), in order to avoid biased comparisons. The time horizon of the investment period is one year, which is the standard maturity in retail and institutional markets.

The implementation of each PI strategy, follows from the formulas in Section 3.1. Besides PI strategies we also consider simple benchmark non-PI simple investments, such as just investing in the underlying risk asset, or in the riskless asset, or in a 50:50 mix between of both. Given a concrete scenario (and the correspondence 100,000 paths), we start by evaluating the returns' distributional properties and performance of all strategies. Then, we evaluate them from the point of view of an investor with EUT, PT or CPT preferences. Concretely *PI strategies* considered are:

1. Stop Loss Portfolio Insurance (SLPI)
2. Option Based Portfolio Insurance (OBPI)
3. Constant Proportion Portfolio Insurance with $m = 1$ (CPPI 1)
4. Constant Proportion Portfolio Insurance with $m = 3$ (CPPI 3)
5. Time Invariant Portfolio Protection (TIPP)

and the benchmark *non-PI strategies* are:

6. Passive stock market strategy (risky asset)
7. Portfolio with risky and riskless assets (50:50)
8. Risk-free cash market deposit (riskless asset).

Table 3 Distributions of Returns. Descriptive statistics of distributions of returns at maturity for PI and non-PI benchmark strategies. Returns of the risky asset are generated using a GBM and the risk-free rate is set at 5%. Risky daily returns are simulated for a period of one year (252 trading days) and 100,000 paths for each of the 9 scenarios presented in Table 2

Floor = 100%	SLPI	OBPI Synt.	CPPI m=1	CPPI m=3	TIPP	Risky Asset	50:50	Risk Free
Panel A: Mean return p.a. (%)								
S1 $\mu = 15\% \sigma = 10\%$	13.96	12.77	5.66	6.91	5.90	15.00	10.00	5.00
S2 $\mu = 10\% \sigma = 10\%$	9.00	8.28	5.39	5.96	5.51	10.00	7.50	5.00
S3 $\mu = 5\% \sigma = 10\%$	5.08	4.84	5.12	5.12	5.12	5.00	5.00	5.00
S4 $\mu = 15\% \sigma = 20\%$	10.49	9.56	5.66	6.89	5.89	15.00	10.00	5.00
S5 $\mu = 10\% \sigma = 20\%$	7.51	6.61	5.39	5.96	5.52	10.00	7.50	5.00
S6 $\mu = 5\% \sigma = 20\%$	5.09	4.24	5.12	5.11	5.12	5.00	5.00	5.00
S7 $\mu = 15\% \sigma = 30\%$	9.09	7.70	5.67	6.91	5.87	15.00	10.00	5.00
S8 $\mu = 10\% \sigma = 30\%$	6.98	5.40	5.39	5.97	5.51	10.00	7.50	5.00
S9 $\mu = 5\% \sigma = 30\%$	5.11	3.43	5.13	5.13	5.12	5.00	5.00	5.00
Panel B: Volatility p.a. (%)								
S1 $\mu = 15\% \sigma = 10\%$	10.92	8.97	0.53	1.95	0.85	10.00	5.00	0.00
S2 $\mu = 10\% \sigma = 10\%$	9.51	7.90	0.51	1.71	0.85	10.00	5.00	0.00
S3 $\mu = 5\% \sigma = 10\%$	7.59	6.31	0.49	1.48	0.85	10.00	5.00	0.00
S4 $\mu = 15\% \sigma = 20\%$	15.88	13.58	1.08	4.05	1.69	20.00	10.00	0.00
S5 $\mu = 10\% \sigma = 20\%$	13.64	11.97	1.03	3.56	1.68	20.00	10.00	0.00
S6 $\mu = 5\% \sigma = 20\%$	11.50	10.25	0.98	3.10	1.66	20.00	10.00	0.00
S7 $\mu = 15\% \sigma = 30\%$	18.37	17.16	1.63	6.40	2.47	30.00	15.00	0.00
S8 $\mu = 10\% \sigma = 30\%$	16.32	15.55	1.56	5.65	2.46	30.00	15.00	0.00
S9 $\mu = 5\% \sigma = 30\%$	14.26	13.91	1.48	4.95	2.42	30.00	15.00	0.00
Panel C: Skewness								
S1 $\mu = 15\% \sigma = 10\%$	0.31	0.45	0.27	0.86	-0.01	0.00	0.00	-
S2 $\mu = 10\% \sigma = 10\%$	0.79	0.83	0.29	0.89	0.01	0.00	0.00	-
S3 $\mu = 5\% \sigma = 10\%$	1.40	1.31	0.28	0.88	0.02	0.00	0.00	-
S4 $\mu = 15\% \sigma = 20\%$	1.60	1.11	0.57	1.84	0.07	0.00	0.00	-
S5 $\mu = 10\% \sigma = 20\%$	1.98	1.36	0.58	1.95	0.09	0.00	0.00	-
S6 $\mu = 5\% \sigma = 20\%$	2.47	1.62	0.60	2.01	0.15	0.00	0.00	-
S7 $\mu = 15\% \sigma = 30\%$	2.46	1.40	0.89	3.04	0.20	0.00	0.00	-
S8 $\mu = 10\% \sigma = 30\%$	2.80	1.55	0.88	3.16	0.24	0.00	0.00	-
S9 $\mu = 5\% \sigma = 30\%$	3.20	1.69	0.90	3.32	0.29	0.00	0.00	-
Panel D: Excess Kurtosis								
S1 $\mu = 15\% \sigma = 10\%$	-0.87	-0.30	0.14	1.32	-0.02	0.00	0.00	-
S2 $\mu = 10\% \sigma = 10\%$	-0.36	0.25	0.18	1.46	-0.02	0.00	0.00	-
S3 $\mu = 5\% \sigma = 10\%$	1.13	1.64	0.14	1.40	-0.03	0.00	0.00	-
S4 $\mu = 15\% \sigma = 20\%$	1.62	0.99	0.53	5.78	-0.12	0.00	0.00	-
S5 $\mu = 10\% \sigma = 20\%$	3.23	1.98	0.62	6.93	-0.16	0.00	0.00	-
S6 $\mu = 5\% \sigma = 20\%$	5.77	3.39	0.64	7.61	-0.18	0.00	0.00	-
S7 $\mu = 15\% \sigma = 30\%$	5.56	2.29	1.41	15.27	-0.29	0.00	0.00	-
S8 $\mu = 10\% \sigma = 30\%$	7.73	3.09	1.39	17.27	-0.31	0.00	0.00	-
S9 $\mu = 5\% \sigma = 30\%$	10.44	4.18	1.44	19.26	-0.32	0.00	0.00	-

Given the evolution of the risky asset, the implementation of each strategy follows from the formulas in Section 3.1. In relation to CPPI 3 we opted for not allowing the automatic embedded loan, using a constraint on the risk-free asset: $w_{risk-free} = \frac{EB_t^{CPPI}}{V_t^{CPPI}} \in [0, 1]$, as in Benninga (1990), Do (2002), Annaert et al. (2009), and Dichtl and Drobetz (2011):

$$ES_t^{CPPI} = \max[\min(m \times Cu_t, V_t^{CPPI}), 0]. \quad (14)$$

The constraint is included in each simulated path at the end of each trading day, when risky and risk-free asset allocations are recalculated. We also include this in the implementation of the TIPP – which can be seen as a variant of CPPI – so we can compare the strategies.

4 Results

4.1 Performance and statistical results

Table 3 presents descriptive statistics on the distribution of returns of the various investment strategies for the different scenarios. Although the descriptive statistics are clearly not sufficient to conclude on the best portfolio insurance strategy, specially when the focus is on wealth protection or potential wealth increase (Annaert et al. (2009)), it is interesting to point results under several market conditions.

The annual expected return is presented in Panel A. In three bullish market conditions (scenarios 1-2, 4-5 and 7-8), holding the risky asset delivers the highest expected return. Amongst the PI strategies, the SLPI is the one with the highest expected return. The OBPI is better than the unleveraged CPPI ($m = 1$), when market presents high expected returns, which is explained by the lower cost of protection in those scenarios. The TIPP, with the ability to protect intermediary gains, presents the second lowest expected returns, amongst the PI strategies, due to the cost of “cashing in” the intermediary upward movements of the risky asset. However, in depressed markets it tends to present better performance, excluding the unleveraged CPPI. The reason for the dominant results of the passive stock market strategy in bullish markets, compared with any of the PI strategies, lies on the protection cost of downside risk, which is implicit on the latter. In a neutral market (Scenarios 3, 6 and 9), expected returns from the simulated strategies are all very similar, except for the OBPI, which penalizes returns due to volatility. The annual volatilities are presented in Panel B. The results are dependent on volatility parameter we set for the scenarios. TIPP and unleveraged CPPI are the PI strategies with the lowest volatility in all scenarios. Regarding the benchmark strategies, naturally, the 50:50 strategy delivers the lowest volatility (except for the case of risk-free asset). The characteristic of moving up the floor of TIPP limits the dispersion of the returns at maturity, which gives the investor the lowest volatile strategy in all scenarios, except in the case of unleveraged CPPI. A particular feature of the SLPI strategy is that volatility decreases as scenarios

Table 4 Performance Ratios. Sharpe and Sortino ratios, for the returns at maturity of PI and non-PI benchmark strategies. Returns of the risky asset are generated using a GBM and the risk-free rate is set at 5%. Risky daily returns are simulated for a period of one year (252 trading days) and 100,000 paths for each of the 9 scenarios presented in Table 2

Floor = 100%	SLPI	OBPI Synt.Put	CPPI m=1	CPPI m=3	TIPP	Risky Asset	50:50	Risk Free
Panel A: Sharpe								
S1 $\mu = 15\%$ $\sigma = 10\%$	1.27	1.42	10.50	3.51	6.90	1.00	1.00	-
S2 $\mu = 10\%$ $\sigma = 10\%$	0.94	1.04	10.41	3.45	6.43	0.50	0.50	-
S3 $\mu = 5\%$ $\sigma = 10\%$	0.66	0.76	10.39	3.42	5.96	0.00	0.00	-
S4 $\mu = 15\%$ $\sigma = 20\%$	0.66	0.70	5.21	1.69	3.46	0.50	0.50	-
S5 $\mu = 10\%$ $\sigma = 20\%$	0.55	0.55	5.20	1.66	3.26	0.25	0.25	-
S6 $\mu = 5\%$ $\sigma = 20\%$	0.44	0.41	5.18	1.63	3.05	0.00	0.00	-
S7 $\mu = 15\%$ $\sigma = 30\%$	0.49	0.45	3.45	1.07	2.35	0.33	0.33	-
S8 $\mu = 10\%$ $\sigma = 30\%$	0.42	0.34	3.42	1.05	2.21	0.17	0.17	-
S9 $\mu = 5\%$ $\sigma = 30\%$	0.36	0.24	3.42	1.03	2.10	0.00	0.00	-
Panel B: Sortino (MAR = 5%)								
S1 $\mu = 15\%$ $\sigma = 10\%$	12.88	5.59	28.95	12.79	15.06	3.69	4.88	-
S2 $\mu = 10\%$ $\sigma = 10\%$	8.35	3.69	23.99	9.81	12.36	2.09	3.12	-
S3 $\mu = 5\%$ $\sigma = 10\%$	5.00	2.26	19.91	7.56	10.07	0.88	1.77	-
S4 $\mu = 15\%$ $\sigma = 20\%$	14.09	2.37	13.09	6.97	6.86	1.71	2.27	-
S5 $\mu = 10\%$ $\sigma = 20\%$	10.01	1.64	11.75	5.84	6.07	1.06	1.58	-
S6 $\mu = 5\%$ $\sigma = 20\%$	6.82	1.06	10.62	4.88	5.38	0.48	0.97	-
S7 $\mu = 15\%$ $\sigma = 30\%$	9.55	1.29	9.05	5.69	4.90	1.21	1.59	-
S8 $\mu = 10\%$ $\sigma = 30\%$	7.25	0.91	8.27	4.85	4.43	0.76	1.13	-
S9 $\mu = 5\%$ $\sigma = 30\%$	5.33	0.58	7.62	4.16	4.04	0.36	0.71	-

change from more unstable market conditions to less volatile market (in relation to expected returns), due to the lower probability of activating stop loss orders. The skewness measure is presented in Panel C. Due to the characteristics of PI strategies, we obtained the expect positive skewness. The SLPI tends to present the highest skewness, as the loss cutting feature make these distributions very right tailed. However, in bullish, and volatile markets (bigger likelihood of stop loss activation), the leveraged CPPI ($m = 3$) presents the most right tailed returns distribution. Finally, the excess kurtosis results are presented in Panel D. As expected, return distributions of the benchmark non-PI strategies have zero excess kurtosis as returns follow a normal distribution. The TIPP return distributions are near mesokurtic in all scenarios, due to the fact that a dynamic floor limits deviations. In scenarios 1 to 3 (low volatility), in particular, the SLPI is also near mesokurtic (due to lower likelihood of hitting the floor), thus becoming a proxy of the lognormal return distribution. The return distributions of PI strategies, in general, have leptokurtic behaviour and in the scenarios with the highest volatility we observe higher excess kurtosis.

Table 4 presents the results for the performance ratios of Sharpe (1994) and Sortino and Van Der Meer (1991). The Sharpe ratio measures excess return per unit of risk (volatility). The higher the ratio, the better the performance adjusted by the risk. The ratio is given by

$$\text{Sharpe Ratio} = \frac{\bar{r}_p - r_f}{\sigma_p}, \quad (15)$$

where $\bar{r}_p$ is the expected return of the portfolio, σ_p is the volatility of portfolio and r_f is the risk-free interest rate of the market. The Sortino ratio measures the risk-adjusted return of an investment (Sortino and Price 1994), and is a modification of the Sharpe ratio, penalizing only those returns falling below an investor's required rate of return,

$$\begin{aligned} \text{Sortino Ratio} &= \frac{\bar{r}_p - MAR}{\sigma_d}, \\ \text{for } \sigma_d &= \sqrt{\sum_{i=1}^n \frac{\min[(r_i - MAR), 0]^2}{n}}, \end{aligned} \quad (16)$$

where $\bar{r}_p$ is the expected return of the portfolio, MAR is minimum acceptable return defined by the investor, and σ_d is the downside deviation volatility of portfolio, that can be interpreted as the annualized standard deviation of returns below the target.

In all market scenarios the best Sharpe ratio is given by the unleveraged CPPI ($m = 1$) strategy, which dominates all the other strategies. In terms of Sortino ratio, only in bullish and volatile markets, SLPI delivers the best risk-adjusted return. However, in all other scenarios, the unleveraged CPPI also presents the best performance.

4.2 Investor decision results

4.2.1 Expected Utility Theory

The results for EUT investors are depicted in Table 5 as well as in Figs. 1 and 2.

From Table 5 we can conclude that in bullish markets (Scenarios 1, 4 and 7) even the risk averse investor (Panel A), prefers the passive investment in the risky asset. The same applies to risk neutral (Panel B) and risk loving investors (Panel C). The difference in utilities is statistically significant to all investors. For risk loving investors, just investing in the risky asset beat all PI strategies, also in all other scenarios. Risk neutral investors also tend to prefer the risky asset investment, however, they opt PI strategies in market conditions when the risky asset is expected to pay the same as the riskless asset, in which case they prefer CPPIs and TIPP over OBPI and SLPI. Comparing the numbers in bold in Table 5 – best PI strategies vs best becnh-mark strategies – it is clear that, at our results are in line with the literature and that, at least for these utilities, EUT cannot explain the popularity of PI investments.

Table 5 Expected Utility Preferences – EUT. Expected utility results based upon three utility functions (U_1, U_2, U_3) representative of investors' risk aversion, risk neutrality or risk seeking attitudes. Expected utilities are computed using wealth at maturity for PI and non-PI strategies. Returns of the risky asset are generated using a GBM and the risk-free rate is set at 5%. Risky daily returns are simulated for a period of one year (252 trading days) and 100,000 paths for each of the 9 scenarios presented in Table 2. Values in bold identify the preferred strategy. Using paired t-tests, we analyse the null hypothesis that the expected utility value of a PI strategy is equal to that of the non-PI strategy with the highest expected utility value

Floor = 100%	SLPI	OBPI Synt.	CPPI m=1	CPPI m=3	TIPP	Risky Asset	50:50	Risk Free
Panel A: Risk averse $U_1(w) = \ln(w)$								
S1 $\mu = 15\% \sigma = 10\%$	4.730***	4.721***	4.660***	4.672***	4.662***	4.750	4.704	4.654
S2 $\mu = 10\% \sigma = 10\%$	4.687***	4.682***	4.658***	4.663***	4.659***	4.700	4.679	4.654
S3 $\mu = 5\% \sigma = 10\%$	4.652***	4.650***	4.653*	4.653*	4.653*	4.650	4.653	4.654
S4 $\mu = 15\% \sigma = 20\%$	4.691***	4.687***	4.660***	4.671***	4.662***	4.734	4.700	4.654
S5 $\mu = 10\% \sigma = 20\%$	4.667***	4.662***	4.658***	4.662***	4.659***	4.686	4.675	4.654
S6 $\mu = 5\% \sigma = 20\%$	4.647***	4.641***	4.653*	4.654***	4.653*	4.634	4.649	4.654
S7 $\mu = 15\% \sigma = 30\%$	4.672***	4.663***	4.660***	4.670***	4.662***	4.710	4.694	4.654
S8 $\mu = 10\% \sigma = 30\%$	4.657***	4.645***	4.658***	4.661***	4.658***	4.660	4.668	4.654
S9 $\mu = 5\% \sigma = 30\%$	4.643***	4.628***	4.653*	4.654	4.653*	4.610	4.643	4.654
Panel B: Risk neutral $U_2(w) = w$								
S1 $\mu = 15\% \sigma = 10\%$	113.96***	112.78***	105.66***	106.91***	105.90***	115.00	110.00	105.00
S2 $\mu = 10\% \sigma = 10\%$	109.00***	108.28***	105.39***	105.96***	105.52***	110.00	107.50	105.00
S3 $\mu = 5\% \sigma = 10\%$	105.08***	104.84***	105.12*	105.12*	105.12*	105.00	105.00	105.00
S4 $\mu = 15\% \sigma = 20\%$	110.49***	109.56***	105.66***	106.89***	105.89***	115.00	110.00	105.00
S5 $\mu = 10\% \sigma = 20\%$	107.51***	106.61***	105.39***	105.96***	105.52***	110.00	107.50	105.00
S6 $\mu = 5\% \sigma = 20\%$	105.09**	104.24***	105.12*	105.11*	105.12*	105.00	105.00	105.00
S7 $\mu = 15\% \sigma = 30\%$	109.09***	107.70***	105.67***	106.91***	105.87***	115.00	110.00	105.00
S8 $\mu = 10\% \sigma = 30\%$	106.98***	105.40***	105.39***	105.97***	105.51***	110.00	107.50	105.00
S9 $\mu = 5\% \sigma = 30\%$	105.11*	103.43***	105.13**	105.13**	105.12*	105.00	105.00	105.00

Table 5 (continued)

Floor = 100%	SLPI	OBPI Synt.	CPPI m=1	CPPI m=3	TIPP	Risky Asset	50:50	Risk Free
Panel C: Risk lover $U_3(w) = w^2$								
S1	13,146*** $\mu = 15\% \sigma = 10\%$	12,825***	11,165***	11,434***	11,216***	13,627	12,261	11,025
S2	11,997*** $\mu = 10\% \sigma = 10\%$	11,802***	11,107***	11,230***	11,134***	12,340	11,644	11,025
S3	11,113*** $\mu = 5\% \sigma = 10\%$	11,038***	11,051***	11,053***	11,051***	11,152	11,060	11,025
S4	12,607*** $\mu = 15\% \sigma = 20\%$	12,263***	11,165***	11,447***	11,216***	14,019	12,354	11,025
S5	11,843*** $\mu = 10\% \sigma = 20\%$	11,559***	11,109***	11,244***	11,137***	12,727	11,744	11,025
S6	11,239*** $\mu = 5\% \sigma = 20\%$	11,002***	11,051***	11,059***	11,053***	11,473	11,137	11,025
S7	12,547*** $\mu = 15\% \sigma = 30\%$	12,045***	11,168***	11,489***	11,216***	14,768	12,548	11,025
S8	11,941*** $\mu = 10\% \sigma = 30\%$	11,460***	11,110***	11,273***	11,138***	13,384	11,908	11,025
S9	11,415*** $\mu = 5\% \sigma = 30\%$	10,967***	11,054***	11,084***	11,058***	12,090	11,298	11,025

The test statistic is significant at a level ***= 1%, **= 5% or, *= 10%

Fig. 1 Risk Tolerance Function: sensitivity to RRA

Fig. 2 Risk Tolerance Function: strategies ranking

Figures 1 and 2 represent expected utilities, for various levels of relative risk aversion (RRA).³ For the levels RRA of -1, 0 and 1 the images mimic the results of Table 5, but we can now understand what would happen for significantly higher level of risk aversion. The graphs of the 9 scenarios on both figures are organised in a mean-variance logic, where vertical increase mean higher expected return and horizontal increase mean higher volatility scenarios, respectively.

Figure 1 highlights the impact that both expected return and volatility conditions have on investors' preferences across the different strategies. For low volatility scenarios, the more risk averse investor are, the more they become indifferent between the PI and non-PI strategies (notice the almost flat dark blue line in Scenarios 1 and 2). As volatility levels increase preferences' graphs start being less and less flat, varying considerably with RRA coefficients. The results from Table 5 are verified for all investors. PI strategies are dominated by either (i) the simple investment in the risky asset (for investors with negative or relatively low levels of risk aversion) or (ii) the investment in the riskless asset (for investors with higher levels of risk aversion). Comparing just across PI strategies, we notice that, no matter the levels investors' risk aversion, the naïve strategies of SLPI or the unleveraged CPPI 1 (depending on the scenarios) are preferred over the more complex CPPI 3 or TIPP strategies. From all graphs it is noticeable a flat portion across the strategies CPPI 1, CPPI 3 and TIPP, showing all investors would do equality well simply by the safety-first CPPI 1 (putting aside the present value of the future guarantee in the riskless asset and

³ The levels presented in Figs. 1 and 2 are not directly comparable with those of Table 5, as the expression in (6) is only utility *equivalent* to the 2nd order approximation of $E[U(w)]$. For further details see Levy and Markowitz (1979). Still for the levels of -1, 0 and 1 we get the same qualitative results as in Table 5.

Table 6 Monte Carlo simulation results - Prospect and Cumulative Prospect Theories. This table shows the results of (mean or cumulative) prospect value. Prospect values are calculated using the portfolio gains and losses, relative to a reference point (100 or 0% return) at maturity of the portfolio insurance and benchmark strategies. Gains and losses at maturity are the result of the accumulated daily returns of each strategy. Returns of the stock market were generated using a GBM. Returns of the risky asset are generated using a GBM and the risk-free rate is set at 5%. Risky daily returns are simulated for a period of one year (252 trading days) and 100,000 paths for each of the 9 scenarios presented in Table 2. Returns of the risky asset are generated using a GBM and the risk-free rate is set at 5%. Risky daily returns are simulated for a period of one year (252 trading days) and 100,000 paths for each of the 9 scenarios presented in Table 2. The null hypothesis in the paired t-test is that the (mean or cumulative) prospect value of a portfolio insurance strategy is equal to that of the benchmark strategy with the highest prospect value

Floor = 100%	SLPI	OBPI Synt.	CPPI m=1	CPPI m=3	TIPP	Risky Asset	50:50	Risk Free
Panel A: Mean prospect value ($\lambda = 1.0$)								
S1 $\mu = 15\% \sigma = 10\%$	14.96***	14.36***	7.79***	9.15***	8.06***	17.53	12.84	7.16
S2 $\mu = 10\% \sigma = 10\%$	9.98***	9.69***	7.47***	8.08***	7.60***	11.76	9.77	7.16
S3 $\mu = 5\% \sigma = 10\%$	5.74***	5.92***	7.16	7.12**	7.15*	5.68	6.58	7.16
S4 $\mu = 15\% \sigma = 20\%$	9.68***	9.85***	7.79***	9.02*	8.03***	15.22	11.95	7.16
S5 $\mu = 10\% \sigma = 20\%$	6.82***	6.59***	7.45***	7.92***	7.56***	9.14	8.65	7.16
S6 $\mu = 5\% \sigma = 20\%$	4.67***	4.27***	7.15*	6.99***	7.11***	3.52	5.64	7.16
S7 $\mu = 15\% \sigma = 30\%$	6.93***	6.42***	7.75***	8.65***	7.92***	11.61	10.50	7.16
S8 $\mu = 10\% \sigma = 30\%$	5.43***	4.58***	7.45**	7.73*	7.51***	6.43	7.70	7.16
S9 $\mu = 5\% \sigma = 30\%$	3.63***	2.47***	7.12**	6.73***	7.02***	0.73	4.62	7.16
Panel B: Mean prospect value ($\lambda = 2.25$)								
S1 $\mu = 15\% \sigma = 10\%$	14.69***	14.08***	7.79***	9.15***	8.06***	16.91	12.75	7.16
S2 $\mu = 10\% \sigma = 10\%$	9.61***	9.16***	7.47***	8.08***	7.60***	10.19	9.51	7.16
S3 $\mu = 5\% \sigma = 10\%$	5.21***	5.15***	7.16	7.12**	7.15*	2.07	5.81	7.16
S4 $\mu = 15\% \sigma = 20\%$	8.60***	7.43***	7.79***	9.02***	8.03***	10.44	10.43	7.16
S5 $\mu = 10\% \sigma = 20\%$	5.64***	3.65***	7.45***	7.92***	7.56***	1.96	6.27	7.16
S6 $\mu = 5\% \sigma = 20\%$	3.38***	1.09***	7.16	6.99***	7.11***	-6.44	2.23	7.16
S7 $\mu = 15\%; \sigma = 30\%$	5.10***	1.11***	7.75***	8.65***	7.92***	0.74	6.45	7.16
S8 $\mu = 10\% \sigma = 30\%$	3.52***	-0.87***	7.45***	7.73***	7.51***	-7.33	2.48	7.16
S9 $\mu = 5\% \sigma = 30\%$	1.65***	-3.28***	7.12**	6.73***	7.02***	-15.96	-1.77	7.16
Panel C: Mean cumulative prospect value ($\lambda = 2.25$)								
S1 $\mu = 15\% \sigma = 10\%$	15.50**	12.98***	2.52***	6.58***	8.01***	15.83	11.98	7.16
S2 $\mu = 10\% \sigma = 10\%$	11.25***	8.36***	7.55***	8.21***	7.53***	8.67	8.42	7.16
S3 $\mu = 5\% \sigma = 10\%$	7.56***	3.60***	7.22***	7.19**	4.63***	1.04	4.70	7.16
S4 $\mu = 15\% \sigma = 20\%$	15.65***	9.08***	4.13***	9.13***	7.71***	10.48	9.44	7.16
S5 $\mu = 10\% \sigma = 20\%$	12.53***	5.31***	7.49***	8.84***	7.26***	3.42	5.82	7.16
S6 $\mu = 5\% \sigma = 20\%$	9.65***	1.37***	3.70***	7.06**	6.80***	-4.12	1.97	7.16
S7 $\mu = 15\% \sigma = 30\%$	17.04***	7.00***	7.86***	11.61***	7.20**	6.96	7.65	7.16
S8 $\mu = 10\% \sigma = 30\%$	14.34***	3.65***	7.52***	10.25***	6.88***	-0.37	3.82	7.16
S9 $\mu = 5\% \sigma = 30\%$	11.30***	0.14***	5.21***	8.70***	6.68***	-8.06	-0.18	7.16

The test statistic is significant at a level ***= 1%, **= 5% or, *= 10%

investing only the remaining in the riskless asset). Furthermore, the expected utility of just investing in the riskless asset is also never far away the values associated with CPPI 1, CPPI 3 and TIPP. The preference values for the OBPI strategy, instead, are always placed between the two naïves PI strategies, i.e between the SLPI and CPPI 1, regardless of market conditions or investors' preferences.

Figure 2 shows the same results, emphasising the ranking of the various PI and non-PI strategies, for each level of RRA. It is clear that for bearish markets with low volatility (Scenario 3) all investors are almost indifferent across strategies, while as volatility increase the riskless investment is preferred by any risk aversion investor (in Scenarios 6 and 9 the red dots are above all other for positive RRA). In normal markets investors with levels of RRA above 2 prefer either the 50:50 strategy or the riskless one, while those with RRA up to two prefer either the risky investment or the 50:50 strategy, depending on how the volatility conditions.

4.2.2 Prospect theories

The results for the prospect theories – PT and CPT – are presented in Table 6. In Panel A ($\lambda = 1$) investor's reactions to losses and gains are equal. In Panel B, we incorporate a different loss reaction ($\lambda = 2.25$). In both situations, there is no distinct weighting of the gains, and losses; therefore, we calculate the simple mean of the simulated prospect values. In Panel C we incorporate the original cumulative probability weightings ($\delta = 0.69$, $\gamma = 0.61$ and $\alpha = 0.88$), as proposed by Tversky and Kahneman (1992). In Panel A we confirm EUT results with prospect values showing that, for bullish and normal market conditions (Scenarios 1-2, 4-5, and 7-8), simply investing in the risky asset yields the best global results. In bearish conditions (Scenarios 3, 6 and 9), the unleveraged CPPI strategy is the best choice. The leveraged CPPI and TPPI, as well as the risk-free investment have prospect values very similar to unleveraged CPPI. In Panel B we find that when incorporating investors' different reactions to losses, results become more favourable to PI strategies. In some bullish market conditions with medium or low volatility, the simple risky asset investment still delivers higher prospect values, as high negative returns are not likely. However, for normal and bearish markets with normal or high volatility, PI strategies yields higher prospect, and (in most cases) the leveraged CPPI is the best strategy. In bearish market conditions with low to medium volatility, the risk-free investment has the highest prospect value. From CPT in Panel C, we get that SLPI is the preferable strategy for almost all scenarios. In fact, there is only one scenario where a non-PI strategy dominate PI strategies – Scenario 1, bullish and low volatility markets conditions – where the best choice is the simple risky investment.

These results support a possible explanation for the popularity of portfolio insurance on a behavioural finance context, as investors' do react differently to losses and gains and have subjective assessments of low and high probabilities, which are not captured by EUT. On the contrary, as behavioural theories incorporates distinct levels of reactions to strategy outcomes, its approach may support the longevity of strategies that offer protection on the downside and, at the same time, keep a potential to benefit from the upside. It is nonetheless important to notice that at least under the more advanced CPT approach context investors' would still prefer

Table 7 Monte Carlo simulation results - Distributions of Returns. This table shows descriptive statistics on the distributions of returns at maturity for the PI strategies under analysis. Returns of the risky asset are generated using a GBM. PI strategies' returns were simulated using Monte Carlo, and the risk-free rate is set at 5%. Risky daily returns are simulated for a period of one year (252 trading days) and 100,000 paths for each of the 9 scenarios presented in Table 2

Floor = 80%	SLPI	OBPI Synt.	CPPI m=1	CPPI m=3	TIPP
Panel A: Mean return p.a. (%)					
S1 $\mu = 15\% \sigma = 10\%$	16.20	15.60	7.77	13.83	12.23
S2 $\mu = 10\% \sigma = 10\%$	10.49	9.93	6.41	9.16	8.58
S3 $\mu = 5\% \sigma = 10\%$	5.16	4.63	5.13	5.13	5.14
S4 $\mu = 15\% \sigma = 20\%$	16.02	16.02	7.79	13.56	12.00
S5 $\mu = 10\% \sigma = 20\%$	10.37	8.08	6.42	9.07	8.44
S6 $\mu = 5\% \sigma = 20\%$	5.11	3.26	5.12	5.11	5.11
S7 $\mu = 15\% \sigma = 30\%$	15.04	9.93	7.77	13.04	11.56
S8 $\mu = 10\% \sigma = 30\%$	9.86	5.50	6.41	8.84	8.22
S9 $\mu = 5\% \sigma = 30\%$	5.12	5.12	5.12	5.13	5.12
Panel B: Volatility p.a. (%)					
S1 $\mu = 15\% \sigma = 10\%$	10.03	10.02	2.58	8.66	6.63
S2 $\mu = 10\% \sigma = 10\%$	10.03	10.02	2.49	7.92	6.57
S3 $\mu = 5\% \sigma = 10\%$	10.03	9.99	2.40	2.40	6.42
S4 $\mu = 15\% \sigma = 20\%$	20.16	19.08	5.15	16.19	12.88
S5 $\mu = 10\% \sigma = 20\%$	19.72	18.43	4.95	14.99	12.50
S6 $\mu = 5\% \sigma = 20\%$	19.10	17.55	4.77	13.77	12.03
S7 $\mu = 15\% \sigma = 30\%$	28.70	24.97	7.69	22.53	18.31
S8 $\mu = 10\% \sigma = 30\%$	27.46	23.68	7.41	21.07	17.66
S9 $\mu = 5\% \sigma = 30\%$	26.09	26.09	7.15	19.58	16.96
Panel C: Skewness					
S1 $\mu = 15\% \sigma = 10\%$	0.00	0.00	0.22	0.46	0.02
S2 $\mu = 10\% \sigma = 10\%$	0.01	0.01	0.24	0.59	0.09
S3 $\mu = 5\% \sigma = 10\%$	0.01	0.03	0.22	0.22	0.18
S4 $\mu = 15\% \sigma = 20\%$	0.07	0.25	0.43	0.78	0.24
S5 $\mu = 10\% \sigma = 20\%$	0.19	0.38	0.45	0.93	0.35
S6 $\mu = 5\% \sigma = 20\%$	0.33	0.53	0.45	1.07	0.47
S7 $\mu = 15\% \sigma = 30\%$	0.47	0.68	0.67	1.09	0.51
S8 $\mu = 10\% \sigma = 30\%$	0.62	0.83	0.70	1.25	0.63
S9 $\mu = 5\% \sigma = 30\%$	0.78	0.78	0.69	1.40	0.75
Panel D: Excess Kurtosis					
S1 $\mu = 15\% \sigma = 10\%$	-0.01	-0.02	0.07	0.03	-0.11
S2 $\mu = 10\% \sigma = 10\%$	-0.03	0.00	0.12	0.36	-0.16
S3 $\mu = 5\% \sigma = 10\%$	-0.06	-0.12	0.06	0.06	-0.27
S4 $\mu = 15\% \sigma = 20\%$	-0.36	-0.37	0.29	0.26	-0.39
S5 $\mu = 10\% \sigma = 20\%$	-0.45	-0.34	0.34	0.69	-0.35
S6 $\mu = 5\% \sigma = 20\%$	-0.50	-0.26	0.33	1.14	-0.29
S7 $\mu = 15\% \sigma = 30\%$	-0.57	-0.05	0.75	0.81	-0.35
S8 $\mu = 10\% \sigma = 30\%$	-0.41	0.22	0.82	1.35	-0.20
S9 $\mu = 5\% \sigma = 30\%$	-0.20	-0.20	0.81	1.90	-0.04

Table 8 Monte Carlo simulation results - Performance ratios. This table shows the results of performance ratios (Sharpe and Sortino) for the returns at maturity for the PI strategies under analysis. Returns of the risky asset are generated using a GBM. PI strategies' returns were simulated using Monte Carlo, and the risk-free rate is set at 5%. Risky daily returns are simulated for a period of one year (252 trading days) and 100,000 paths for each of the 9 scenarios presented in Table 2

Floor = 80%	SLPI	OBPI Synt.	CPPI m=1	CPPI m=3	TIPP
Panel A: Sharpe					
S1 $\mu = 15\%$ $\sigma = 10\%$	1.61	1.55	2.99	1.59	1.84
S2 $\mu = 10\%$ $\sigma = 10\%$	1.04	0.99	2.56	1.15	1.30
S3 $\mu := 5\%$ $\sigma = 10\%$	0.51	0.46	2.12	2.12	0.79
S4 $\mu = 15\%$ $\sigma = 20\%$	0.79	0.84	1.50	0.83	0.93
S5 $\mu = 10\%$ $\sigma = 20\%$	0.52	0.44	1.29	0.60	0.67
S6 $\mu = 5\%$ $\sigma = 20\%$	0.26	0.18	1.06	0.37	0.42
S7 $\mu = 15\%$ $\sigma = 30\%$	0.52	0.40	1.00	0.58	0.63
S8 $\mu = 10\%$ $\sigma = 30\%$	0.36	0.23	0.86	0.42	0.46
S9 $\mu = 5\%$ $\sigma = 30\%$	0.19	0.19	0.71	0.26	0.30
Panel B: Sortino (MAR = 5%)					
S1 $\mu = 15\%$ $\sigma = 10\%$	3.71	3.53	7.56	5.05	4.38
S2 $\mu = 10\%$ $\sigma = 10\%$	2.08	1.96	5.41	2.97	2.72
S3 $\mu = 5\%$ $\sigma = 10\%$	0.90	0.80	3.79	3.79	1.47
S4 $\mu = 15\%$ $\sigma = 20\%$	1.79	1.79	3.52	2.73	2.35
S5 $\mu = 10\%$ $\sigma = 20\%$	1.15	1.02	2.74	1.77	1.60
S6 $\mu = 5\%$ $\sigma = 20\%$	0.57	0.41	2.05	0.97	0.94
S7 $\mu = 15\%$ $\sigma = 30\%$	1.71	1.16	2.43	2.14	1.86
S8 $\mu = 10\%$ $\sigma = 30\%$	1.14	0.66	1.94	1.45	1.31
S9 $\mu = 5\%$ $\sigma = 30\%$	0.61	0.61	1.50	0.83	0.82

naïve PI strategies such as the SLPI, over the most commonly traded OBPI, leveraged CPPI or TIPP.

4.3 Robustness tests

We perform several tests in order to assess the adherence of results. We begin by changing the floor from 100% to 80% and re-run all PI strategies. On a second analysis, we test the parameters of CPT used by Tversky and Kahneman (1992): we change these assumptions in order to increase the overweight of small probability events in negative outcomes (we change the δ from 0.69 to 0.77, and the γ from 0.61 to 0.44, while keeping the α constant at 0.88), in line with parameters defined in the experiment by Gonzalez and Wu (1999) regarding the weighting process. Final tests are related with the changes of risk-free rates (we stress the risk-free rate by 150 basis points, from 5% to 3.5% and from 5% to 6.5%) in order to check the impact on investor choices.

Table 9 Investor Preferences under EUT, Prospect and Cumulative Prospect Theories – floor 80%. This table shows investor preferences results using EUT and behavioural approaches as in Tables 5 and 6

Floor = 80%	SLPI	OBPI Synt.	CPPI m=1	CPPI m=3	TIPP	Risky Asset	50:50	Risk Free
Panel A: Risk averse $E[U_1(w)] = \ln(w)$								
S1 $\mu = 15\%$ $\sigma = 10\%$	4.750	4.745*	4.680***	4.731***	4.718***	4.750	4.704	4.654
S2 $\mu = 10\%$ $\sigma = 10\%$	4.700	4.695*	4.667**	4.690*	4.685*	4.700	4.679	4.654
S3 $\mu = 5\%$ $\sigma = 10\%$	4.650*	4.645*	4.655*	4.653*	4.653*	4.650	4.653	4.654
S4 $\mu = 15\%$ $\sigma = 20\%$	4.733*	4.712**	4.679***	4.719**	4.710**	4.734	4.700	4.654
S5 $\mu = 10\%$ $\sigma = 20\%$	4.684*	4.665***	4.666***	4.680**	4.678***	4.686	4.675	4.654
S6 $\mu = 5\%$ $\sigma = 20\%$	4.636***	4.621***	4.654***	4.645***	4.648***	4.634	4.649	4.654
S7 $\mu = 15\%$ $\sigma = 30\%$	4.702**	4.667***	4.677***	4.700**	4.697**	4.710	4.694	4.654
S8 $\mu = 10\%$ $\sigma = 30\%$	4.659**	4.629***	4.665*	4.666*	4.668	4.660	4.668	4.654
S9 $\mu = 5\%$ $\sigma = 30\%$	4.619***	4.595***	4.653	4.634***	4.640**	4.610	4.643	4.654
Panel B: Risk neutral $E[U_2(w)] = w$								
S1 $\mu = 15\%$ $\sigma = 10\%$	116.20*	115.60	107.77***	113.83***	112.23***	115.00	110.00	105.00
S2 $\mu = 10\%$ $\sigma = 10\%$	110.49	109.93	106.41***	109.16*	108.58**	110.00	107.50	105.00
S3 $\mu = 5\%$ $\sigma = 10\%$	105.16	104.63***	105.13	105.15	105.14	105.00	105.00	105.00
S4 $\mu = 15\%$ $\sigma = 20\%$	116.02*	113.38***	107.79***	113.56***	112.00***	115.00	110.00	105.00
S5 $\mu = 10\%$ $\sigma = 20\%$	110.37*	108.08***	106.42***	109.07*	108.44**	110.00	107.50	105.00
S6 $\mu = 5\%$ $\sigma = 20\%$	105.11*	103.26***	105.12*	105.11*	105.11*	105.00	105.00	105.00
S7 $\mu = 15\%$ $\sigma = 30\%$	115.04	109.93***	107.77***	113.04***	111.56***	115.00	110.00	105.00
S8 $\mu = 10\%$ $\sigma = 30\%$	109.86	105.50***	106.41***	108.84***	108.22**	110.00	107.50	105.00
S9 $\mu = 5\%$ $\sigma = 30\%$	105.12*	101.61***	105.12*	105.13*	105.12*	105.00	105.00	105.00
Panel C: Risk lover $E[U_3(w)] = w^2$								
S1 $\mu = 15\%$ $\sigma = 10\%$	13,638*	13,499***	11,623***	13,058***	12,651***	13,627	12,261	11,025
S2 $\mu = 10\%$ $\sigma = 10\%$	12,332***	12,207***	11,330***	11,994***	11,840***	12,340	11,644	11,025
S3 $\mu = 5\%$ $\sigma = 10\%$	11,170***	11,058***	11,060***	11,117***	11,101***	11,152	11,060	11,025
S4 $\mu = 15\%$ $\sigma = 20\%$	14,024***	13,350***	11,651***	13,283***	12,759***	14,019	12,354	11,025
S5 $\mu = 10\%$ $\sigma = 20\%$	12,678***	12,109***	11,354***	12,206***	11,952***	12,727	11,744	11,025
S6 $\mu = 5\%$ $\sigma = 20\%$	11,481***	11,026***	11,076***	11,293***	11,218***	11,473	11,137	11,025
S7 $\mu = 15\%$ $\sigma = 30\%$	14,494***	12,994***	11,687***	13,624***	12,907***	14,768	12,548	11,025
S8 $\mu = 10\%$ $\sigma = 30\%$	13,158***	11,905***	11,389***	12,546***	12,122***	13,384	11,908	11,025
S9 $\mu = 5\%$ $\sigma = 30\%$	11,982***	10,973***	11,111***	11,624***	11,414***	12,090	11,298	11,025
PT / CPT								
Panel D: Mean prospect value ($\lambda = 1.0$)								
S1 $\mu = 15\%$ $\sigma = 10\%$	19.58**	16.52***	10.51***	17.06*	15.43***	17.53	12.84	7.16
S2 $\mu = 10\%$ $\sigma = 10\%$	12.95***	11.23**	8.85***	11.60*	11.05***	11.76	9.77	7.16
S3 $\mu = 5\%$ $\sigma = 10\%$	6.10***	5.84***	7.15*	6.35***	6.50***	5.68	6.58	7.16
S4 $\mu = 15\%$ $\sigma = 20\%$	18.13***	13.56***	10.30***	15.63*	14.31***	15.22	11.95	7.16
S5 $\mu = 10\%$ $\sigma = 20\%$	11.06***	8.22***	8.40***	9.79***	9.63**	9.14	8.65	7.16
S6 $\mu = 5\%$ $\sigma = 20\%$	5.59***	4.19***	6.92**	5.70***	6.04***	3.52	5.64	7.16
S7 $\mu = 15\%$ $\sigma = 30\%$	15.64***	10.02**	9.87***	13.65***	12.90***	11.61	10.50	7.16

Table 9 (continued)

Floor = 80%	SLPI	OBPI Synt.	CPPI m=1	CPPI m=3	TIPP	Risky Asset	50:50	Risk Free
S8 $\mu = 10\%$ $\sigma = 30\%$	10.00 ***	5.78***	8.21***	9.03***	9.08***	6.43	7.70	7.16
S9 $\mu = 5\%$ $\sigma = 30\%$	5.38***	2.10***	6.68 **	5.28***	5.81***	0.73	4.62	7.16
Panel E: Mean prospect value ($\lambda = 2.25$)								
S1 $\mu = 15\%$ $\sigma = 10\%$	19.03 *	16.19***	10.51***	16.84***	15.23***	16.91	12.75	7.16
S2 $\mu = 10\%$ $\sigma = 10\%$	11.44 ***	10.28***	8.85***	10.94***	10.45***	10.19	9.51	7.16
S3 $\mu = 5\%$ $\sigma = 10\%$	2.63***	3.58***	7.14*	4.75***	5.02***	2.07	5.81	7.16
S4 $\mu = 15\%$ $\sigma = 20\%$	13.27 ***	10.28***	10.10***	12.87	11.90***	10.44	10.43	7.16
S5 $\mu = 10\%$ $\sigma = 20\%$	4.23***	3.49***	8.07 ***	5.86***	6.14***	1.96	6.27	7.16
S6 $\mu = 5\%$ $\sigma = 20\%$	-3.49***	-2.19***	6.38***	0.47***	1.32***	-6.44	2.23	7.16
S7 $\mu = 15\%$ $\sigma = 30\%$	5.23***	2.72***	8.98 ***	7.49***	7.56***	0.74	6.45	7.16
S8 $\mu = 10\%$ $\sigma = 30\%$	-2.19***	-3.15***	7.01*	1.59***	2.54***	-7.33	2.48.26	7.16
S9 $\mu = 5\%$ $\sigma = 30\%$	-8.62***	-8.68***	5.08***	-3.55***	-2.06***	-15.96	-1.77	7.16
Panel F: Cumulative prospect value ($\lambda = 2.25$)								
S1 $\mu = 15\%$ $\sigma = 10\%$	15.85	15.15**	9.76***	15.20**	13.00***	15.83	11.98	7.16
S2 $\mu = 10\%$ $\sigma = 10\%$	8.69*	7.98***	8.10**	9.76 ***	8.50*	8.67	8.42	7.16
S3 $\mu = 5\%$ $\sigma = 10\%$	1.18***	0.48***	6.47**	4.43***	3.87***	1.04	4.70	7.16
S4 $\mu = 15\%$ $\sigma = 20\%$	11.21***	11.21***	8.91***	13.16 ***	9.93*	10.48	9.44	7.16
S5 $\mu = 10\%$ $\sigma = 20\%$	4.66***	2.53***	7.17	7.89 **	5.65***	3.42	5.82	7.16
S6 $\mu = 5\%$ $\sigma = 20\%$	-1.82***	-3.62***	5.38***	2.73***	1.32***	-4.12	1.97	7.16
S7 $\mu = 15\%$ $\sigma = 30\%$	9.53***	-0.88***	8.15**	12.55 ***	8.10**	6.96	7.65	7.16
S8 $\mu = 10\%$ $\sigma = 30\%$	3.88***	-0.88***	6.35***	7.57 **	4.18***	-0.37	3.82	7.16
S9 $\mu = 5\%$ $\sigma = 30\%$	-1.66***	-1.66***	4.51***	2.74***	0.30***	-8.06	-0.18	7.16

The test statistic is significant at a level ***= 1%, **= 5% or, *= 10%

4.3.1 Floor at 80%

Table 7 presents the distribution properties and Table 8 the associated performance indicators, for the PI strategies. Non-PI benchmark strategies are not floor dependent.

Comparing the PI strategies' values in Tables 7 and 8 with the equivalent ones in Tables 3 and 4, respectively, we conclude that, as expected, PI for lower floor values yield wider range of returns and high volatility. The new conditions result in positive skewness for PI strategies, but increasing exposure to risky assets leads to a less positive skew distribution. For the lower floor the unleveraged CPPI continues to yield the best reward to risk. However, a greater exposure to risk, with increasing probability of losses, results on a lower Sharpe and Sortino ratios, when compared to those in Table 4. presents CPT results for lower

The results about investors' preferences under both EUT and PT/CPT approaches are presented in Table 9. Under EUT and for a risk averse investor (Panel A), results

Table 10 Cumulative Prospect Theory – different parameters. Investor preferences results using the CPT approach, as in Panel C of Table 6, but taking $\delta = 0.77$ and $\gamma = 0.44$

Floor = 100%	SLPI	OBPI Synt.	CPPI m=1	CPPI m=3	TIPP	Risky Asset	50:50	Risk Free
S1 $\mu = 15\% \sigma = 10\%$	13.25*	12.17***	7.57***	8.68***	7.41**	13.36	9.90	7.23
S2 $\mu = 10\% \sigma = 10\%$	10.28***	8.88***	7.26**	7.64*	6.94***	7.68	7.12	7.23
S3 $\mu = 5\% \sigma = 10\%$	7.63**	6.11***	6.93***	6.67***	6.44***	1.08	4.12	7.23
S4 $\mu = 15\% \sigma = 20\%$	16.64***	10.61*	3.63***	8.96***	6.66***	10.14	8.66	7.23
S5 $\mu = 10\% \sigma = 20\%$	14.36***	8.14***	7.02**	8.20**	6.23***	4.10	5.70	7.23
S6 $\mu = 5\% \sigma = 20\%$	12.13**	5.84**	6.73***	7.17*	5.84**	-2.93	2.29	7.23
S7 $\mu = 15\% \sigma = 30\%$	21.80***	10.30***	7.31**	13.29***	6.29***	8.94	8.30	7.23
S8 $\mu = 10\% \sigma = 30\%$	19.59***	7.86**	7.00*	11.92***	5.71**	2.19	4.88	7.23
S9 $\mu = 5\% \sigma = 30\%$	16.77***	5.43***	6.70**	10.47**	5.59**	-4.87	1.30	7.23

do not change much, with the non-PI strategies being still preferable to PI-strategies in most cases. However, with the new (lower) floor, in low volatility scenarios, the naive PI strategies – SLPI and unleveraged CPPI – become preferable. For a risk neutral investor (Panel B), a 80% floor makes SLPI become a possible choice in high expected market returns with low volatility (Scenario 1), as expected utility is very similar the pure risky investment, because the probability of executing the stop loss order is lower. For a risk loving investor (Panel C), the reduction of the floor does not change the results. They are very similar to results from Panel B of Table 5, and the passive stock market strategy yields the highest utility for all bullish or neutral scenarios. Overall, expected utility values associated with PI strategies do not present significant changes with the decrease in the value of the floor.

Behavioural results (Panels E and F) are similar to the ones obtained in Table 6, confirming behavioural issues may help explaining the popularity of PI strategies. The mean prospect values of Panel E, when compared with the ones in Panel B of Table 6, suggest that, in low volatility markets condition investors prefer floors lower than 100%. However, for normal or high volatility markets conditions the full guarantee is preferable. We also observe a shift from leveraged to unleveraged CPPI strategies for high and medium volatile markets (Scenarios 4, 5, 7 and 8). Results for CPT investors, in Panel F, point to a possible explanation for some flight to safety when markets are neutral (Scenarios 3, 6 and 9). In these market conditions, we can expect a change from SLPI to risk-free investments. In positive market expectations, independently of volatility, the cumulative value of leveraged CPPI strategies is higher than SLPI (Scenarios 2, 4-5, and 7-8). Portfolio insurance strategies, under these market conditions, are still providing better results than non-PI strategies. The results for positive market conditions (i.e. expected positive risk premium), strengthens the possibility that CPT can explain investors' choice for PI strategies. The specific decisions of portfolio insurance are, nonetheless, dependent on the market conditions and characteristics of each portfolio insurance strategy—e.g. the reduction of the percentage floor from 100% to 80% depicts a shift from SLPI to CPPI strategies.

Table 11 Cumulative Prospect Theory Results – different risk free rates. Investor preferences using the CPT approach, as in Panel C of Table 6, considering $R_f = 3.5\%$ and $R_f = 6.5\%$.

Floor = 100%	SLPI	OBPI Synt.	CPPI m=1	CPPI m=3	TIPP	Risky Asset	50:50	Risk Free
Panel A: Cumulative prospect value ($\lambda = 2.25$) and Risk Free = 3.5%								
S1 $\mu = 15\% \sigma = 10\%$	14.41**	12.98**	1.71**	7.08***	3.16***	15.83	10.97	5.30
S2 $\mu = 10\% \sigma = 10\%$	10.41***	8.42*	5.56***	6.21***	3.31***	8.67	7.29	5.30
S3 $\mu = 5\% \sigma = 10\%$	7.00**	4.82**	5.32*	3.63***	5.23*	1.04	3.47	5.30
S4 $\mu = 15\% \sigma = 20\%$	14.07**	8.61***	3.14***	7.60***	5.02***	10.48	8.38	5.30
S5 $\mu = 10\% \sigma = 20\%$	11.25***	5.43***	5.52**	6.70**	4.68***	3.42	4.54	5.30
S6 $\mu = 5\% \sigma = 20\%$	8.33***	2.40**	5.28***	5.80**	5.03**	-4.12	0.54	5.30
S7 $\mu = 15\% \sigma = 30\%$	14.73**	5.41**	4.19***	8.67***	5.55**	6.96	6.22	5.30
S8 $\mu = 10\% \sigma = 30\%$	12.53***	2.95***	3.80***	7.56***	5.24**	-0.37	2.52	5.30
S9 $\mu = 5\% \sigma = 30\%$	9.83***	0.19***	3.63***	6.60***	4.94***	-8.06	-1.49	5.30
Panel B: Cumulative prospect value ($\lambda = 2.25$) and Risk Free = 6.5%								
S1 $\mu = 15\% \sigma = 10\%$	16.31**	15.15**	9.86***	11.40***	9.97***	15.83	12.99	9.08
S2 $\mu = 10\% \sigma = 10\%$	11.79***	10.64**	9.44*	6.73***	5.40***	8.67	9.52	9.08
S3 $\mu = 5\% \sigma = 10\%$	7.97***	6.99***	9.03*	8.74**	8.73**	1.04	5.85	9.08
S4 $\mu = 15\% \sigma = 20\%$	16.91**	11.75***	9.79**	12.28***	9.61**	10.48	10.69	9.08
S5 $\mu = 10\% \sigma = 20\%$	13.59**	8.32**	9.37**	10.01***	9.03*	3.42	7.01	9.08
S6 $\mu = 5\% \sigma = 20\%$	10.59**	5.57***	8.97**	9.45**	8.44**	-4.12	3.21	9.08
S7 $\mu = 15\% \sigma = 30\%$	18.48***	8.70***	9.81***	13.61***	9.04*	6.96	8.70	9.08
S8 $\mu = 10\% \sigma = 30\%$	15.73**	6.19***	9.41**	12.47***	8.53**	-0.37	5.06	9.08
S9 $\mu = 5\% \sigma = 30\%$	12.77**	3.52***	8.99*	10.88***	8.30**	-8.06	1.15	9.08

4.3.2 Changing CPT parameters

The Table 10 presents the cumulative prospect value with changes on the parameters for the curvature and the elevation of the smoothing of curves. We increase the overweight of small probabilities associated with negative outcomes (changing $\delta = 0.69$ to $\delta = 0.77$ and $\gamma = 0.41$ to $\gamma = 0.44$), in line with parameters defined in Gonzalez and Wu (1999) regarding the weighting process. The results indicate PI dominates non-PI strategies in all market conditions. As in Table 6 Panel C, SLPI dominates all other PI strategies even in a more clear way when overreaction to losses increases.

4.3.3 Changing the risk-free rate

Table 11 presents CPT results for lower ($R_f = 3.5\%$) and higher ($R_f = 6.5\%$) risk free rates. Comparing the results with those in Table 6 Panel C (that considers $R_f = 5\%$) we conclude that our results are robust to changes in riskless interest rate. The SLPI is still the dominant in most scenarios, with the simple investment in the risky asset also presenting itself as an alternative in some low volatility scenarios.

5 Conclusions

By nature, investors who are averse to losses are expected to prefer strategies that limit downside risk and simultaneously do not give away upside potential.

Although portfolio insurance (PI) strategies have the necessary characteristics to attract risk averse investors, in line with the literature, we find no evidence PI investments strategies should be preferable to non-PI strategies, under EUT, even when considering higher than standard levels of relative risk aversion. In fact, we show that for low volatility scenarios, the more risk averse investors are, the more indifferent they become between PI and non-PI strategies. For medium and high volatilities, PI strategies are dominated by either (i) the simple investment in the risky asset (for investors with negative or relatively low levels of risk aversion) or (ii) the investment in the riskless asset (for investors with higher levels of risk aversion).

As we change the framework towards the behavioural PT and CPT theories by Kahneman and Tversky (1979) and Tversky and Kahneman (1992), we show that investments in PI strategies can, at least for some markets scenarios, be explained by behavioural aspects.

By incorporating investors' different reactions to losses and gains, using the PT approach, results become more favourable to PI strategies. In some bullish market conditions with medium or low volatility, the simple risky asset investment still delivers higher prospect values, as high negative returns are not likely. Likewise, in bearish market conditions with low to medium volatility, the risk-free investment is still preferable. However, for normal and bearish markets with normal or high volatility, PI strategies yields higher prospect, and (in most cases) the leveraged CPPI is the best strategy.

Under CPT, commonly traded PI strategies such as OBPI, leveraged CPPIs or TIPP do not seem to dominate the naïve SLPI, at least for a floor of 100%. SLPI dominates in all scenarios, except in bullish and low volatility markets conditions, where the best choice remains the simple risky investment. This result is new to the literature that tends to focus only on traded strategies, ignoring the naïve SLPI and CPPI 1.

While EUT results are robust to floor level changes, both PT and CPT results are more sensitive. For lower floor levels PT starts showing the previously identified tendency toward choosing naïve PI strategies. In this case for both SLPI or CPPI 1, depending on the market scenario. According to CPT the leveraged CPPI may replace SLPI as preferable, in almost all scenarios.

Unfortunately, the over-the-counter nature PI strategies do not allow us to know the actual market size (of each) PI strategy, so it is unclear "how much" can CPT really explain.

Financial markets have presented investors very volatile conditions, with exuberant depressive behaviours. After the credit crisis of 2008, the uncertainty on stock market enhanced momentary fly to safety patterns. This behaviour has been also observed since March 2020, associated with all the uncertainty resulting from the Covid 19 pandemic situation. Thus, protecting capital and returns is still quite appealing. Regardless of individual objectives or their levels of risk aversion, we find investors still investing in retail PI investment strategies.

From our results, it seems that if complexity would be set aside, and retailers would focus on naïve strategies such as SLPI or CPPI 1, investors would not worse off. This raises a broader doubt about the *added value* of (complex and costly) retail innovation strategies for portfolio insurance.

In terms of future research it would be interesting to look at empirical distributions associated with market indices, for instance, an analysis to which extent CPT and real life distribution biases could better explain the existent PI strategies in retail markets. Another area of relevance is the evaluation under EUT and CPT of structured products and their publicised embedded protection approaches.

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