

Preface: Hyperbolic System of Conservation Laws and Related Topics

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The history of hyperbolic system of conservation laws can date back to the eighteenth century, after several works of the natural philosophers, most notably L. Euler. The compressible Euler system, consisting of mass, momentum and energy in the divergence form, is the prototypical model of the hyperbolic system of conservation laws, and its main feature is the shock formation in a finite time no matter how the initial values smooth. This poses huge challenges in the mathematical analysis.

It is now well known from the pioneer works of B. Riemann in 1860 and P.D. Lax in 1957 that for the system of Euler equations, there are three basic wave patterns, that is, shock wave, rarefaction wave, and contact discontinuity. These three types of waves have essential differences: shock is compressive, rarefaction is expansive, and contact discontinuity has some diffusive structure, and all of their linear combinations are called Riemann solutions. Since the breakthrough work by J. Glimm in 1965, the 1-d well-posedness theory of small BV solution was established, mainly due to A. Bressan and his collaborators, T.P. Liu-T. Yang and P. Le Floch around 2000. The global existence of L^∞ entropy solution of γ -law gas dynamics with large initial data was built by R. Diperna, X.Q. Ding-G.Q. Chen-P.Z. Luo, P.L. Lions and his collaborators in the framework of compensated compactness during 1983-1996, and the uniqueness was not known yet. Nevertheless, the well-posedness works on the Cauchy problem in multi-spatial dimensions are few except some special problems which have strong engineering background.

The study of hyperbolic system of conservation laws has long history in Institute of Mathematics, Chinese Academy of Sciences (named as Academy of Mathematics and Systems Science (AMSS), Chinese Academy of Sciences in 1998) since 1952. The well reputational experts include Xiaqi Ding, Banghe Li, Tong Chang, Ling Hsiao, Jinghua Wang and Caizhong Li (who left Institute of Mathematics in early 1980's), and some of their outstanding students are now in oversea China, such as Guiqiang Chen, Zhouping Xin and Yuxi Zheng. Several significant contributions such as the global existence of large entropy solution for γ -law gas dynamics ($1 < \gamma \leq \frac{5}{3}$) and important developments of multi-dimensional Riemann solutions were achieved. One motivation for the studies on the hyperbolic system of conservation laws is the mathematical interest related to geometric measure, functional analysis and numerical analysis. The other motivations are important engineering problems arising in fluid mechanics, aerodynamics and so on. Due to these reasons, studies on the hyperbolic conservation laws and related topics in China develop very quickly, with some fundamental influences domestically and internationally.

In this special issue, we are happy to collect 10 papers from Chinese scholars (most of them graduated from AMSS). Roughly speaking, these papers cover multi-dimensional subsonic and subsonic-sonic Euler flows with general conservative forces (paper 1), smooth solutions of the multi-dimensional scalar conservation law with source term and their precise blow-up (paper 2),

stability of viscous shock waves for viscous conservation laws under periodic perturbation (paper 3 and paper 4), steady boundary layer solution of Boltzmann equation with specular boundary condition in the half space (paper 5), time-asymptotic expansion of bipolar hydrodynamic model for semiconductors (paper 6), entropy solution of ideal reaction chromatography system (paper 7), incompressible flow (paper 8 and paper 9) and related equation (paper 10). We expect these works are helpful for further researches for those working on the hyperbolic conservation laws and related topics, especially for younger scholars.

Finally, we would like to express our deep thanks to Academician Zhiming Ma, the Editor-in-Chief of *Acta Mathematicae Applicatae Sinica (English Series)* for the invitation to organize this special issue and to *Acta Mathematicae Applicatae Sinica (English Series)* for providing the forum for presenting the recent works in the field of hyperbolic conservation laws by Chinese scholars.