Annali di Matematica Pura ed Applicata (1923 -) (2019) 198:803–810
https://doi.org/10.1007/s10231-018-0799-8

Non-uniform dependence for a generalized Degasperis–Procesi equation

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Received: 13 June 2018 / Accepted: 4 October 2018 / Published online: 11 October 2018

Abstract
In the paper, we consider the Cauchy problem for a generalized Degasperis–Procesi equation. We prove that the data-to-solution map is not uniformly continuous.

Keywords A generalized Degasperis–Procesi equation · Non-uniform dependence

Mathematics Subject Classification 35Q53

1 Introduction

In this paper, we study a generalized Degasperis–Procesi equation introduced by Novikov in [1]:

$$(1-\partial_x^2)u_t = \partial_x(2-\partial_x)(1+\partial_x)u^2. \tag{1.1}$$

It was shown in [1] that Eq. (1.1) possesses a hierarchy of local higher symmetries and the first non-trivial one is $u_\tau = \partial_x[(1-\partial_x u)]^{-1}$.

Equation (1.1) belongs to the following class [1]:

$$(1-\partial_x^2)u_t = F(u, u_x, u_{xx}, u_{xxx}), \tag{1.2}$$

which has attracted much attention on the possible integrable members of (1.2).

The first well-known integrable member of (1.2) is the Camassa–Holm (CH) equation [2]:

$$(1-\partial_x^2)u_t = -(3uu_x - 2u_x u_{xx} - uu_{xxx}).$$

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The CH equation can be regarded as a shallow water wave equation [2–4]. It is completely integrable [2,5], has a bi-Hamiltonian structure [6,7], and admits exact peaked solitons of the form $ce^{-|x-ct|}$, $c > 0$, which are orbitally stable [8]. It is worth mentioning that the peaked solitons present the characteristic for the traveling water waves of greatest height and largest amplitude and arise as solutions to the free-boundary problem for incompressible Euler equations over a flat bed, cf. [9–12]. The local well-posedness for the Cauchy problem of the CH equation in Sobolev spaces and Besov spaces was discussed in [13–18]. It was shown that there exist global strong solutions to the CH equation [13,14,19] and finite time blow-up strong solutions to the CH equation [13,14,19,20]. The existence and uniqueness of global weak solutions to the CH equation were proved in [21,22]. The global conservative and dissipative solutions of CH equation were discussed in [23,24].

The second well-known integrable member of (1.2) is the Degasperis–Procesi (DP) equation [25]:

$$(1-\partial_x^2)u_t = -(4uu_x - 3u_x u_{xx} - uu_{xxx}).$$

The DP equation can be regarded as a model for nonlinear shallow water dynamics, and its asymptotic accuracy is the same as the CH shallow water equation [26]. The DP equation is integrable and has a bi-Hamiltonian structure [27]. An inverse scattering approach for computing n-peakon solutions to the DP equation was presented in [28]. Its traveling wave solutions were investigated in [29,30]. The local well-posedness of the Cauchy problem of the DP equation in Sobolev spaces and Besov spaces was established in [31–33]. Similar to the CH equation, the DP equation has also global strong solutions [34–36] and finite time blow-up solutions [33–40]. In addition, it has global weak solutions [36,37,40,41]. Although the DP equation is similar to the CH equation in several aspects, these two equations are truly different. One of the novel features of the DP equation different from the CH equation is that it has not only peakon solutions [27] and periodic peakon solutions [40], but also shock peakons [42] and the periodic shock waves [38].

The third well-known integrable member of (1.2) is the Novikov equation [1]

$$(1-\partial_x^2)u_t = 3uu_x u_{xx} + u^2 u_{xxx} - 4u^2 u_x.$$

The most difference between the Novikov equation and the CH and DP equations is that the former one has cubic nonlinearity and the latter ones have quadratic nonlinearity. It was shown that the Novikov equation is integrable, possesses a bi-Hamiltonian structure, and admits exact peakon solutions $u(t,x) = \pm\sqrt{c}e^{|x-ct|}$, $c > 0$ [43]. The local well-posedness for the Novikov equation in Sobolev spaces and Besov spaces was studied in [44,45,45–47]. The global existence of strong solutions was established in [44] under some sign conditions, and the blow-up phenomena of the strong solutions were shown in [47]. The global weak solutions for the Novikov equation were discussed in [48].

The local well-posedness and global existence of strong solutions for the generalized Degasperis–Procesi were studied in [49].

To our best knowledge, Eq. (1.1) can transform into the following equivalent form:

$$\begin{cases} u_t - 2uu_x = \partial_x(1-\partial_x^2)^{-1}(u^2 + (u^2)_x), \ t > 0, \ x \in \mathbb{R}, \\ u(0,x) = u_0(x), \quad x \in \mathbb{R}. \end{cases} \tag{1.3}$$

Using this result and the method of approximate solutions, we prove the following non-uniform dependence result.

Theorem 1.1 *If $s > \frac{3}{2}$, then the data-to-solution map for the generalized Degasperis–Procesi equation defined by Cauchy problem (1.3) is not uniformly continuous from any bounded subset in H^s into $\mathcal{C}([0, T]; H^s)$.*

Notations Since all spaces of functions are over $\mathbb{R}$, for simplicity, we drop $\mathbb{R}$ in our notations of function spaces if there is no ambiguity.

2 Proof of the main theorem

In this section, we will give the proof of the main theorem. Motivated by Himonas and Holliman [32], we first construct a sequence approximate solutions. Lately, we will show that the distance between approximate solutions and actual solutions is decaying. Finally, we can conclude that Cauchy problem (1.3) is not uniformly continuous.

Lemma 2.1 ([50]) *For any $s > 0$, there exists a positive constant $c = c(s)$ such that*

$$||fg||_{H^s} \le c(||f||_{H^s}||g||_{L^\infty} + ||g||_{H^s}||f||_{L^\infty}).$$

For any $s > \frac{1}{2}$, there exists a positive constant c such that

$$||f||_{L^\infty} \le c||f||_{H^s}.$$

Lemma 2.2 ([50]) *Let $s > \frac{1}{2}$. Assume that $f_0 \in H^s$, $F \in L^1_T(H^s)$ and $\partial_x v \in L^1_T(H^{s-1})$. If $f \in \mathcal{C}([0, T]; H^s)$ solves the following 1D linear transport equation:*

$$\begin{cases} \partial_t f + v\partial_x f = F, \\ f(0, x) = f_0, \end{cases}$$

then there exists a positive constant $C = C(s)$ such that

$$||f||_{H^s} \le \mathrm{e}^{CV(t)}\Big(||f_0||_{H^s} + \int_0^t \mathrm{e}^{-CV(\tau)}||F(\tau)||_{H^s}\mathrm{d}\tau\Big),$$

where

$$V(t) = \begin{cases} \int_0^t ||\partial_x v||_{H^{s-1}}\, \mathrm{d}\tau, & s > \frac{3}{2}, \\ \int_0^t ||\partial_x v||_{H^s}\, \mathrm{d}\tau, & \frac{1}{2} < s \le \frac{3}{2}. \end{cases}$$

Lemma 2.3 ([32]) *Let $\phi \in S(\mathbb{R})$, $\delta > 0$ and $\alpha \in \mathbb{R}$. Then for any $s \ge 0$ we have that*

$$\lim_{n\to\infty} n^{-\frac{1}{2}\delta-s}||\phi\left(\frac{x}{n^\delta}\right)\cos(nx-\alpha)||_{H^s} = \frac{1}{\sqrt{2}}||\phi||_{L^2},$$
$$\lim_{n\to\infty} n^{-\frac{1}{2}\delta-s}||\phi\left(\frac{x}{n^\delta}\right)\sin(nx-\alpha)||_{H^s} = \frac{1}{\sqrt{2}}||\phi||_{L^2}.$$

Lemma 2.4 ([49]) *Let $s > \frac{3}{2}$ and $u_0 \in H^s$. There exists a positive time $T = T(||u_0||_{H^s})$ such that (1.2) has a solution $u \in \mathcal{C}([0, T]; H^s)$. Moreover, there exists a constant $C = C(s) > 0$ such that*

$$||u||_{L^\infty_T(H^r)} \le C||u_0||_{H^r}, \quad \forall\, r \ge s.$$

Proof of the theorem Let $\omega \in \{0, 1\}$ and φ be a $C_0(\mathbb{R})$ such that

$$\varphi(x) = \begin{cases} 1, & |x| \le 1, \\ 0, & |x| \ge 2. \end{cases}$$

Let ϕ be a $C_0(\mathbb{R})$ such that $\phi(x)\varphi(x) = \varphi(x)$. We introduce the following sequence of high-frequency approximate solutions:

$$u_\omega^{h,n} = n^{-\frac{\delta}{2}-s}\varphi\left(\frac{x}{n^\delta}\right)\cos(nx + 2\omega t).$$

We also define the solution $u_\omega^{\ell,n}$ which satisfies the following equation with the low-frequency initial data:

$$\begin{cases} \partial_t u_\omega^{\ell,n} - 2u_\omega^{\ell,n}\partial_x u_\omega^{\ell,n} = \partial_x(1-\partial_x^2)^{-1}[(u_\omega^{\ell,n})^2 + \partial_x(u_\omega^{\ell,n})^2], \\ u_\omega^{\ell,n}(0,x) = \omega n^{-1}\phi(\frac{x}{n^\delta}). \end{cases}$$

Since

$$\begin{aligned} \partial_t u_\omega^{h,n} &= -2\omega n^{-\frac{\delta}{2}-s}\varphi\left(\frac{x}{n^\delta}\right)\sin(nx+2\omega t) \\ &= -\,2nu_\omega^{\ell,n}(0,x)n^{-\frac{\delta}{2}-s}\varphi\left(\frac{x}{n^\delta}\right)\sin(nx+2\omega t), \\ 2u_\omega^{\ell,n}\partial_x u_\omega^{h,n} &= -\,2nu_\omega^{\ell,n}n^{-\frac{\delta}{2}-s}\varphi\left(\frac{x}{n^\delta}\right)\sin(nx+2\omega t) \\ &\quad +\,2u_\omega^{\ell,n}n^{-\frac{3\delta}{2}-s}\partial_x\varphi\left(\frac{x}{n^\delta}\right)\cos(nx+2\omega t), \end{aligned}$$

then we can find that

$$\begin{aligned} \partial_t u_\omega^{h,n} - 2u_\omega^{\ell,n}\partial_x u_\omega^{h,n} =& 2n[u_\omega^{\ell,n}(t,x) - u_\omega^{\ell,n}(0,x)]n^{-\frac{\delta}{2}-s}\varphi\left(\frac{x}{n^\delta}\right)\sin(nx+2\omega t) \\ &- 2u_\omega^{\ell,n}(t,x)n^{-\frac{3\delta}{2}-s}\partial_x\varphi\left(\frac{x}{n^\delta}\right)\cos(nx+2\omega t). \end{aligned}$$

Letting $U_\omega^n = u_\omega^{h,n} + u_\omega^{\ell,n}$, we obtain U_ω^n satisfies the following equation:

$$\begin{aligned} \partial_t U_\omega^n - 2U_\omega^n\partial_x U_\omega^n =& -\,2u_\omega^{h,n}\partial_x u_\omega^{\ell,n} - 2u_\omega^{h,n}\partial_x u_\omega^{h,n} \\ &+ 2n[u_\omega^{\ell,n}(t,x) - u_\omega^{\ell,n}(0,x)]n^{-\frac{\delta}{2}-s}\varphi\left(\frac{x}{n^\delta}\right)\sin(nx+2\omega t) \\ &- 2u_\omega^{\ell,n}(t,x)n^{-\frac{3\delta}{2}-s}\partial_x\varphi\left(\frac{x}{n^\delta}\right)\cos(nx+2\omega t) \\ &+ \partial_x(1-\partial_x^2)^{-1}[(u_\omega^{\ell,n})^2 + \partial_x(u_\omega^{\ell,n})^2]. \end{aligned}$$

Now, letting V_ω^n be the solution of the Cauchy problem for the equation:

$$\begin{cases} \partial_t V_\omega^n - 2V_\omega^n\partial_x V_\omega^n = \partial_x(1-\partial_x^2)^{-1}[(V_\omega^n)^2 + \partial_x(V_\omega^n)^2], \\ V_\omega^n(0,x) = n^{-\frac{\delta}{2}-s}\varphi\left(\frac{x}{n^\delta}\right)\cos(nx) + \omega n^{-1}\phi(\frac{x}{n^\delta}). \end{cases}$$

Denoting $W_\omega^n = U_\omega^n - V_\omega^n$, it easy to show that

$$\begin{aligned}
&\partial_t W_\omega^n - 2U_\omega^n \partial_x W_\omega^n - 2W_\omega^n \partial_x V_\omega^n \\
&\quad = -2u_\omega^{h,n} \partial_x u_\omega^{\ell,n} - 2u_\omega^{h,n} \partial_x u_\omega^{h,n} \\
&\qquad + 2n[u_\omega^{\ell,n}(t,x) - u_\omega^{\ell,n}(0,x)] n^{-\frac{\delta}{2}-s} \varphi\left(\frac{x}{n^\delta}\right) \sin(nx + 2\omega t) \\
&\qquad - 2u_\omega^{\ell,n}(t,x) n^{-\frac{3\delta}{2}-s} \partial_x \varphi\left(\frac{x}{n^\delta}\right) \cos(nx + 2\omega t) \\
&\qquad + \partial_x (1-\partial_x^2)^{-1} [(u_\omega^{\ell,n})^2 + \partial_x (u_\omega^{\ell,n})^2] \\
&\qquad - \partial_x (1-\partial_x^2)^{-1} [(V_\omega^n)^2 + \partial_x (V_\omega^n)^2] := \sum_{i=1}^{5} I_i.
\end{aligned}$$

According to Lemmas 2.3, 2.4, we have for $0 \le \sigma \le s$

$$||u_\omega^{h,n}(t)||_{H^\sigma} \le Cn^{\sigma - s}, \quad ||u_\omega^{\ell,n}(t)||_{H^\sigma} \le C||u_\omega^{\ell,n}(t)||_{H^s} \le C||u_\omega^{\ell,n}(0)||_{H^s} \le Cn^{\frac{1}{2}\delta - 1}. \tag{2.1}$$

Choosing $\delta > 0$ such that $s - 1 - \delta > \frac{1}{2}$, we have $||f||_{L^\infty} \le ||f||_{H^{s-1-\delta}}$. Therefore, by Lemma 2.1 and (2.1), we have

$$\begin{aligned}
||I_1||_{H^{s-1}} &\le C(||u_\omega^{h,n}||_{H^{s-1}} ||\partial_x u_\omega^{\ell,n}||_{H^{s-1}} + ||u_\omega^{h,n}||_{H^{s-1-\delta}} ||\partial_x u_\omega^{h,n}||_{H^{s-1}} \\
&\quad + ||u_\omega^{h,n}||_{H^{s-1}} ||\partial_x u_\omega^{h,n}||_{H^{s-1-\delta}}) \\
&\le Cn^{\frac{1}{2}\delta - 2} + Cn^{-1-\delta}, \\
||I_2||_{H^{s-1}} &\le Cn \int_0^t ||\partial_\tau u_\omega^{\ell,n}(\tau)||_{H^{s-1}} \mathrm{d}\tau \cdot ||n^{-\frac{\delta}{2}-s} \varphi\left(\frac{x}{n^\delta}\right) \sin(nx + 2\omega t)||_{H^{s-1}} \\
&\le C(||u_\omega^{\ell,n}||_{H^s} ||\partial_x u_\omega^{\ell,n}||_{H^s} + ||\partial_x (1-\partial_x^2)^{-1} [(u_\omega^{\ell,n})^2 + \partial_x (u_\omega^{\ell,n})^2]||_{H^{s-1}}) \\
&\le C||u_\omega^{\ell,n}||_{H^s}^2 + ||\partial_x u_\omega^{\ell,n}||_{H^s}^2) \le Cn^{\delta - 2}, \\
||I_3||_{H^{s-1}} &\le C||u_\omega^{\ell,n}||_{H^{s-1}} ||n^{-\frac{3\delta}{2}-s} \partial_x \varphi\left(\frac{x}{n^\delta}\right) \cos(nx + 2\omega t)||_{H^{s-1}} \\
&\le Cn^{\frac{\delta}{2}-1} n^{-\delta - 1} = Cn^{-\frac{\delta}{2}-2}, \\
||I_4||_{H^{s-1}} &\le C||\partial_x (1-\partial_x^2)^{-1} [(u_\omega^{\ell,n})^2 + \partial_x (u_\omega^{\ell,n})^2]||_{H^{s-1}} \le Cn^{\delta - 2} \\
||I_5||_{H^{s-1}} &\le ||\partial_x (1-\partial_x^2)^{-1} [(V_\omega^n)^2 + \partial_x (V_\omega^n)^2]||_{H^{s-1}} \le Cn^{\delta - 2}.
\end{aligned}$$

Now, setting $\delta < \min\{s - \frac{3}{2}, 1\}$, it follows from Lemma 2.2 that

$$||W_\omega^n||_{H^{s-1}} \le Cn^{-1}(n^{-\delta} + n^{\delta - 1}).$$

According to Lemmas 2.4, we have

$$\begin{aligned}
||W_\omega^n(t)||_{H^{s+1}} &\le ||U_\omega^n(t)||_{H^{s+1}} + ||V_\omega^n(t)||_{H^{s+1}} \\
&\le ||u_\omega^{h,n}(t)||_{H^{s+1}} + C(||u_\omega^{\ell,n}(0)||_{H^{s+1}} + ||V_\omega^n(0)||_{H^{s+1}}) \\
&\le Cn,
\end{aligned}$$

which implies

$$||W_\omega^n||_{H^s} \le C||W_\omega^n||_{H^{s-1}}^{\frac{1}{2}} ||W_\omega^n||_{H^{s+1}}^{\frac{1}{2}} \le C(n^{-\delta} + n^{\delta - 1})^{\frac{1}{2}}. \tag{2.2}$$

Then, combining (2.1) and (2.2), we have

$$\begin{aligned}||V_1^n(t) - V_0^n(t)||_{H^s} &\geq ||U_1^n(t) - U_0^n(t)||_{H^s} - C\varepsilon_n \\ &\geq ||u_1^{h,n}(t) - u_0^{h,n}(t)||_{H^s} - C\varepsilon_n' \\ &\geq 2|\sin t| \cdot ||n^{-\frac{\delta}{2}-s}\varphi\left(\frac{x}{n^\delta}\right)\sin(nx+t)||_{H^s} - C\varepsilon_n',\end{aligned} \tag{2.3}$$

where

$$\varepsilon_n = (n^{-\delta} + n^{\delta-1})^{\frac{1}{2}}, \qquad \varepsilon_n' = (n^{-\delta} + n^{\delta-1})^{\frac{1}{2}} + n^{\frac{1}{2}\delta-1}.$$

Letting n go to ∞ and using Lemma 2.3, (2.2), (2.3), it follows that

$$\liminf_{n\to\infty} ||V_1^n(t) - V_0^n(t)||_{H^s} \geq \sqrt{2}||\varphi||_{L^2}|\sin t|. \tag{2.4}$$

Noticing that $|\sin t| = \sin t$ when $t \in [0, \pi]$, then $\sin t > 0$ in an interval $(0, t_0)$ for some $0 < t_0 < \pi$. This together with the fact that

$$\lim_{n\to\infty} ||V_1^n(0) - V_0^n(0)||_{H^s} \leq ||n^{-1}\phi(\frac{x}{n^\delta})||_{H^s} \leq n^{\delta-1} \to 0, \quad \text{as } n \to \infty,$$

completes the proof of Theorem 1.1. □

Acknowledgements This work was partially supported by NSFC (No. 11801090).

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