Correction to: The infinitesimally bendable Euclidean hypersurfaces

M. Dajczer¹ · Th. Vlachos²

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All the results of the paper remain true as stated, but there is a serious gap in the proof of Theorem 13. Equations (26) and (27) need to be corrected as follows:

$$d\tau = -\theta \circ A$$

and

$$(\nabla_X \theta)Y = -\beta(AX, Y) + 2\tau \langle AX, Y \rangle,$$

respectively. A correct proof of the part following equation (28) goes as follows:

We claim that the system of differential equations formed by (26), (27) and (28) is completely integrable. The integrability condition for the first equation is easy to verify. For the second equation, we have to see that

$$(\nabla_X \nabla_Y \theta - \nabla_Y \nabla_X \theta - \nabla_{[X, Y]} \theta)Z = -\theta(R(X, Y)Z)$$

holds. Using (26) and (27), we obtain

$$(\nabla_X \nabla_Y \theta)Z = -(\nabla_X \beta)(AY, Z) - \beta(\nabla_X AY, Z) - 2\theta(AX) \langle AY, Z \rangle + 2\tau \langle \nabla_X AY, Z \rangle.$$

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✉ Th. Vlachos
tvlachos@uoi.gr

M. Dajczer
marcos@impa.br

¹ IMPA, Estrada Dona Castorina, 110, Rio de Janeiro 22460-320, Brazil

² Mathematics Department, University of Ioannina, 45110 Ioannina, Greece

Hence,

$$(\nabla_X \nabla_Y \theta - \nabla_Y \nabla_X \theta - \nabla_{[X,Y]} \theta)Z = -(\nabla_X \beta)(AY, Z) + (\nabla_Y \beta)(AX, Z) \\ - 2\theta(AX)\langle AY, Z \rangle + 2\theta(AY)\langle AX, Z \rangle.$$

In this note, we fix a mistake in the proof. Using (28) we obtain that

$$(\nabla_X \nabla_Y \theta - \nabla_Y \nabla_X \theta - \nabla_{[X,Y]} \theta)Z = -\theta(AX)\langle AY, Z \rangle + \theta(AY)\langle AX, Z \rangle.$$

On the other hand, we have from the Gauss equation that

$$\theta(R(X, Y)Z) = \langle AY, Z \rangle \theta(AX) - \langle AX, Z \rangle \theta(AY),$$

and the integrability condition for the second equation follows.

Finally, the integrability condition for the last equation, namely, that

$$(\nabla_X \nabla_Y \beta - \nabla_Y \nabla_X \beta - \nabla_{[X,Y]} \beta)(Z, W) = -\beta(R(X, Y)Z, W) - \beta(R(X, Y)W, Z)$$

can be verified by a similar computation, and this proves the claim.

Start with a solution L^* and $\mathcal{Y}^*$ of system (S) with corresponding tensors θ^* , β^* and function τ^* . Fix a point $p_0 \in M^n$ and let L_0 and $\mathcal{Y}_0$ be a solution of the integrable system

$$(S_0) \begin{cases} \mathcal{Y}_* X = -LAX \\ (\tilde{\nabla}_X L)Y = \langle AX, Y \rangle \mathcal{Y} \end{cases}$$

with initial conditions $\theta_0(p_0) = \theta^*(p_0)$, $\beta_0(p_0) = \beta^*(p_0)$ and $\tau_0(p_0) = \tau^*(p_0)$. Then $L = L^* - L_0$ and $\mathcal{Y} = \mathcal{Y}^* - \mathcal{Y}_0$ are a solution of (S) such that $\theta = \theta^* - \theta_0$, $\beta = \beta^* - \beta_0$ and $\tau = \tau^* - \tau_0$. Clearly $\theta(p_0) = \beta(p_0) = \tau(p_0) = 0$. Since θ , β and τ solve the homogeneous integrable system (26), (27) and (28), hence $\theta = \beta = \tau = 0$.

The rest of the proof is the same as in the last two paragraphs.

We also need to modify the proof of Proposition 14 as follows:

Proof of Proposition 14 Since M^n is simply-connected, there is a global orthonormal frame $\{X, Y\}$ of $\Delta^\perp$ as above. By Lemma 17 and Proposition 18, the Codazzi tensor B on M^n is given by $B|_{\Delta^\perp} = A|_{\Delta^\perp} D$ and $\Delta \subset \ker B$, where $D = \theta J$ and $J \in \Gamma(\text{End}(\Delta^\perp))$ is such that $JX = Y$ and $JY = 0$. Moreover, $\theta \in C^\infty(M)$ is arbitrarily prescribed along an integral curve of X and required to satisfy (30). Therefore, the one-parameter family of Codazzi tensors $A(t) = A + tB$, $t \in \mathbb{R}$, gives rise to an isometric bending of f having the infinitesimal bending determined by B as its variational vector field.