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Differential instructional effectiveness: overcoming the challenge of learning to solve trigonometry problems that involved algebraic transformation skills

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Abstract

The design principles of cognitive load theory and learning by analogy has independently contributed to our understanding why an instruction will or will not work. In an experimental study involving 97 Year 9 Australian students conducted in regular classrooms, we evaluated the effect of the unguided problem-solving approach, worked examples approach and analogy approach on learning to solve two types of trigonometry problem. These trigonometry problems ($\sin 40° = \frac{x}{6}$ vs. $\cos 50° = \frac{14}{x}$) exhibited two levels of complexity owing to the location of the pronumeral (numerator vs. denominator). The solution procedure of worked examples provided guidance, whereas the unguided problem-solving was without any guidance. Analogical learning placed emphasis on comparing a pair of isomorphic examples to facilitate transfer. Across the three approaches, solving practice problems contributed towards performance on the post-test. However, the worked examples approach and analogy approach were more efficient than the unguided problem-solving approach for acquiring skills to solve practice problems regardless of their complexity. Therefore, the worked examples approach and analogy approach that emphasizes algebraic transformation skills have the potential to reform instructional efficiency for learning to solve trigonometry problems.

Keywords Design principles · Instructional efficiency · Trigonometry problems · Cognitive load · Analogical learning

Introduction

Research in cognitive science has advanced our understanding of different design principles for the purpose of designing effective instructional methods to enhance learning. Advancement in cognitive load theory has provided guidance to design instructions that promote effective learning across different domains (Sweller et al., 2011, 2019). The cognitive load

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researchers advocate the design of instructions to minimize overloading the working memory load, which is constrained by duration (Peterson & Peterson, 1959) and capacity (Cowan, 2001; Miller, 1956) to process novel information. The advancement of learning by analogy, which extends to learning by comparison has made notable impact upon maths education (Alfieri et al., 2013; Gentner, 1983; Richland & McDonough, 2010; Rittle-Johnson & Star, 2007; Rittle-Johnson et al., 2017). The analogical comparison of a pair of isomorphic examples is expected to assist learners to draw inferences of a shared schema between them. In spite of the advancement in cognitive load theory and learning by analogy to generate design principles for instructional practices, regrettably, there is a gap between theory and practice.

While teaching maths in Australia, the lead author noted that maths teachers tend to emphasize the use of 'rote learning', which may fall short of addressing the conceptual understanding of solving two types of trigonometry problem that involve algebraic transformation skills (Ngu & Phan, 2020). For example, regarding the trigonometry problem with a pronumeral as a numerator (e.g. $\sin 40° = \frac{x}{6}$), students are taught to multiply sin40° and 6. On the other hand, for the trigonometry problem with a pronumeral as a denominator (e.g. $\cos 50° = \frac{14}{x}$), students are taught to swap cos50° with x. We used type I problem and type II problem to stand for these two types of trigonometry problem. Mathematics education researchers have raised concern in regard to 'rote learning' of mathematical procedural (Kamii & Dominick, 1997), which limits opportunities to explore the nature of mathematics problems (Hiebert et al., 1996).

A personal conversation with a Malaysian teacher reveals that he taught students to memorize and use, for example, $3 = \frac{x}{2}$ as a point of reference for solving trigonometry problems such as $\sin 40° = \frac{x}{6}$ (type I). Students are advised to modify $3 = \frac{x}{2}$ to become $3 = \frac{6}{x}$ for solving trigonometry problems such as $\cos 50° = \frac{14}{x}$ (type II). Apparently, this Malaysian teacher is aware that students can draw on prior knowledge of the algebraic transformation skills needed to solve linear equations with fractions (e.g. $3 = \frac{x}{2}$ vs. $3 = \frac{6}{x}$), and use this to solve the two types of trigonometry problem ($\sin 40° = \frac{x}{6}$ vs. $\cos 50° = \frac{14}{x}$). Nonetheless, it is unclear whether this is a common classroom instructional practice adopted by Malaysian teachers for teaching trigonometry problems. Moreover, we acknowledge that it is necessary to conduct research to ascertain such claim.

The empirical studies have supported the efficacy of the design principles generated by cognitive load theory (Sweller et al., 2011, 2019) and learning by analogy (Alfieri et al., 2013), which could provide guidance to improve instructional practices in maths classrooms. A review by Renkl (2014) supports a similarity between learning from worked examples and analogical reasoning of worked examples, both of which highlight the acquisition of generalized principles in the form of a schema. Moreover, a review on example-based learning by Hoogerheide and Roelle (2020) reveals a similarity between the use of worked examples based on cognitive load theory, and analogical comparison of worked examples based on analogical learning theory.

As far as we are aware, there is one study that directly compared worked examples and analogical learning. Specifically, Timothy et al. (2013) compared three worked example conditions (reading, self-explanation, analogy) in the domain of physics. The reading group read worked examples and solved practice problems. Although the design of the reading group is similar to the use of worked examples from a cognitive load perspective, the authors did not use the framework of cognitive load to justify the design of the reading group. The self-explaining group explained worked examples, and then generated explanation for subsequent worked examples. The analogy group compared pairs of worked examples, and then generated explanation for these pairs of worked examples. The results confirmed the hypotheses that the self-explaining group and analogy group outperformed the reading group for principle knowledge but not procedural knowledge. Presumably, not having opportunity to solve practice problems may have hampered the self-explanation group and analogy group to acquire procedural knowledge.

Given that researchers normally investigate the efficacy of the instructional design generated by cognitive load theory and analogical comparison framework separately, it is our purpose to fill this gap in cognitive science literature. We conducted an experimental study involving 97 Year 9 students in regular classrooms to examine the relative efficiency of the unguided problem-solving approach, worked examples approach and analogy approach for learning to solve trigonometry problems. The design of the unguided problem-solving approach and worked examples approach will be based on the design principles recommended by cognitive load theory, whereas the design of the analogy approach will be based on the framework of learning by analogy.

Cognitive load theory

Cognitive load theory is an instructional theory that generates design principles to minimize the negative impact on working memory (Sweller et al., 2011, Sweller et al., 2019). The working memory is restricted in processing novel information which contains materials that are new to a learner (Cowan, 2001; Peterson & Peterson, 1959). The working memory not only processes individual elements but it also processes the interaction between elements within the materials. Anything that needs to be learnt constitutes an element (e.g. symbol, number, procedure, etc.) (Chen et al., 2017). Once a learner has successfully processed and understood the materials, it will be stored in the long-term memory as generalized knowledge structure in the form of a schema. The working memory is not limited in processing a schema; rather, it can process a schema as a single unit of element instead of multiple elements. This is because a schema encapsulates interacting elements into a single unit of element. Accordingly, a learner can retrieve a schema from the long-term memory to solve a familiar problem with minimum effort. In fact, the vast number of schemas in varying degrees of specification in the long-term memory enables us to function efficiently in a variety of contexts. Therefore, the main aim of instructional design is to ensure that the generation of schema will occur in working memory, and subsequently store in the long-term memory.

There are three types of cognitive load that influences the design of instruction (Sweller, 2010). First, the intrinsic cognitive load is determined by the level of element interactivity relative to a learner's expertise in a domain. The interaction between elements within materials constitutes element interactivity. The level of element interactivity is proportionate to the level of the complexity of materials. Once a learner has acquired expertise, the level of element interactivity and thus the imposition of intrinsic cognitive load will decrease and vice versa (Kalyuga et al., 1998). Second, the extraneous cognitive load is determined by the level of element interactivity arises from inappropriate design of instructional materials, which interferes with learning. For example, the generation and testing of different solution steps via mean-end analysis in the case of problem-solving without any guidance is not helpful for the acquisition of schema (e.g. Sweller & Cooper, 1985).

Third, the germane cognitive load is determined by the level of element interactivity that is essential for learning and thus is intrinsic to the learning materials. For example, the variability practice problems increase the level of element interactivity (Likourezos et al., 2019), but are helpful for schema acquisition. The variability practice problems require a learner to invest cognitive resources to distinguish a similar solution procedure across variable contexts, which is helpful for the acquisition of the schema for a particular category of problems (Quilici & Mayer, 1996).

Unguided problem-solving approach and worked examples approach

The design principles of cognitive load theory have mainly been applied in the context of well-structured domains such as mathematics that has a specific solution procedure (Sweller et al., 2011). One of the cognitive load effects is the worked examples effect. A worked example consists of a problem and solution steps. Those solution steps are considered problem-solving schemas. The worked examples effect occurs when studying worked examples mixes with solving practice problems imposes lower cognitive load but yields higher learning outcomes than solving equivalent practice problems (e.g. Renkl, 2017; van Gog et al., 2011, 2019). Students are likely to use weak problem-solving strategies such as trial-and-error or means-ends analysis to solve practice problems without any guidance. For example, a level of element interactivity may arise from using means-ends-analysis to reduce the difference between the goal and sub-goal, which is irrelevant to schemas acquisition and thus imposes extraneous cognitive load. It should be noted that the unguided problem-solving approach for solving a well-structured domain such as trigonometry problems differs from the notion of exploring different solution paths mainly in ill-structured domains for gaining problem-solving competence. In the current study, the implementation of the unguided problem-solving approach for learning to solve trigonometry problems requires students to solve a set of practice problems without any guidance.

To promote learning of trigonometry problems, the design of a worked example encompasses both conceptual knowledge and procedural knowledge (Fig. 1). According to Rittle-Johnson et al. (2015), conceptual knowledge refers to the underlying principle knowledge in a domain, whereas procedural knowledge refers to a sequence of actions to solve a problem. As shown in Fig. 1, the worked example includes a prompt (i.e. ÷ 26 becomes × 26) to help a learner to conceptualize ÷ 26 as an inverse operation to × 26 (conceptual knowledge). Moreover, the arrow aims to direct a learner's attention to move ÷ 26 from the left-hand side to the right-hand side of the trigonometry problem (procedural knowledge). Therefore,

$$\tan 75^\circ = \frac{x}{26} \qquad [\div 26 \text{ becomes } \times 26]$$

$$x = \tan 75^\circ \times 26$$
$$x = 97.03$$

Fig. 1 A worked example of a trigonometry problem with a pronumeral as a numerator the worked example highlights the interplay between conceptual knowledge and procedural knowledge for learning to solve trigonometry problems that involve algebraic transformation skills. In the acquisition phase, students needed to complete multiple worked example–practice problem pairs (Sweller, et al., 2011; van Gog et al., 2011). They were expected to study and understand the solution procedure of worked examples, and then to transfer their understanding to solve practice problems that share similar problem structure. We will discuss learning by analogy in the next section.

The analogy approach

Multiple empirical studies in cognitive science research have affirmed the merit of learning by analogy to foster analogical transfer (e.g. Alfieri et al., 2013). A central tenet of learning by analogy is the use of prior knowledge of a learned problem to assist learning of a new problem that shares a similar problem structure and thus a similar solution procedure. In essence, the learned problem and new problem constitutes a pair of isomorphic problems. Successful one-to-one mapping of elements and their relationship between a pair of isomorphic problems would enable a learner to make inferences of a shared problem structure and thus a shared solution procedure between them.

Figure 2 shows the design of the analogy approach. The linear equation with a fraction ($\frac{m}{7}=2$) serves as a source example for learning to solve type I problem ($\tan 75° = \frac{x}{26}$). This pair of isomorphic examples has a pronumeral that acts as a numerator and they share similar design feature, which is the same as the worked example in the worked examples approach. Thus, we will not describe their design feature separately here. Researchers have incorporated supporting cues to promote active comparison of a pair of isomorphic examples to facilitate analogical transfer (Alfieri et al., 2013; Richland & Simms, 2015). Prior research provides prompts (e.g. why might it be helpful to compare a pair of isomorphic examples?) to encourage deep thinking about the structural features of a pair of isomorphic examples (e.g. Rittle-Johnson & Star, 2007). In contrast, we provided explanatory prompts to assist students in conceptualizing the inverse operation, which is critical for solving the trigonometry problem ($\div 26$ becomes $\times 26$), and the linear equation with a fraction ($\div 7$ becomes $\times 7$). In accordance with prior research (Richland et al., 2017), we placed a pair of isomorphic examples side-by-side to eliminate the cognitive load imposed in retrieving a source example. Furthermore, we juxtaposed a pair of isomorphic examples to encourage students to engage in one-to-one mapping of the relational elements (e.g. map *m* to *x*), which would expect to facilitate the identification of a shared solution procedure between

A linear equation with a fraction	A trigonometry problem with pronumeral a as a numerator
$\frac{m}{7}=2$ [÷7 becomes ×7]	$\tan 75° = \frac{x}{26}$ [÷ 26 becomes × 26]
$m = 2 \times 7$ $m = 14$	$x = \tan 75° \times 26$ $x = 97.03$

Fig. 2 A pair of isomorphic examples

them. Note that we expected students to have prior knowledge of the concept of pronumeral in which both m and x stand for a pronumeral.

In the acquisition phase, students were specifically instructed to study and compare such a pair of isomorphic examples. Having done that, they solved a practice problem. Prior work on analogical learning mainly focuses on conceptual understanding, and pay limited attention to the acquisition of procedural knowledge (e.g. Alfieri et al., 2013; Timothy et al., 2013). Conversely, research has affirmed the importance of including practice problems to promote procedural fluency (Loibl et al., 2020). Therefore, to facilitate the acquisition of procedural knowledge, we innovated the design of analogical learning by requiring students to solve a practice problem after they had compared a pair of isomorphic examples.

Target domain

Research has investigated the effects of different approaches (geometric approach vs. ratio approach) on learning trigonometric functions in the domain of Trigonometry (Kendal & Stacey, 1998; Weber, 2005). The geometric approach emphasizes the use of a unit circle to illustrate the geometric process associated with the trigonometric functions. The ratio approach, however, uses the ratio of two sides of a right-angled triangle to define trigonometric functions (e.g. $\sin 30° = \frac{1}{2}$). Apparently, the geometric approach was effective in assisting students to acquire conceptual understanding of the trigonometric functions in different quadrants (e.g. $\sin 270° = -1$) (Weber et al., 2008). However, Kendal and Stacey (1998) reported that students who were exposed to the ratio approach outperformed those students who learned via the unit circle model.

The focus of our enquiry is to assist students acquire the algebraic transformation skills involved in solving trigonometry problems (e.g. $\sin 30° = \frac{x}{2}$, solve for x). There is a similarity between type I problems (e.g. $\sin 30° = \frac{x}{2}$) and linear equations with fractions (e.g. $20 = \frac{x}{8}$), in which both share a similar solution procedure. In the domain of linear equations, Rittle-Johnson and Star (2009) assessed students' conceptual understanding of the '=' sign concept in linear equations. For example, students were required to indicate whether (i) $2x = 30$ and (ii) $15 + 2x = 30 + 15$ are equivalent, and explained their reasoning. In the present study, we assessed students' conceptual knowledge of performing inverse operations, and then recognizing that such action would maintain the equality of the equations. As will be discussed later, in the concept test, for example, students demonstrated their conceptual understanding if they could indicate that (i) $\frac{x}{10} = \sin 53°$ and (ii) $x = \sin 53° \times 10$ are equivalent.

The complexity of trigonometry problems

In the domain of trigonometry, the trigonometric ratios (e.g. $\sin\varnothing = \frac{\text{opposite side}}{\text{hypotensue}}$) are defined as the ratios of two sides of a right-angled triangle. In the present study, our focus is to assist students to acquire algebraic transformation skills required to calculate an unknown side length of a right-angled triangle whereby one angle and one known side length are given. Such a definition of trigonometric ratios results in two types of trigonometry problem ($\tan 75° = \frac{x}{26}$ vs. $\sin 32° = \frac{50}{x}$), one of which has a pronumeral that acts as a numerator (type I), and the other has a pronumeral that acts as a numerator (type II). Given that a trigonometry problem ($\tan 75° = \frac{x}{26}$) and a linear equation with a fraction ($\frac{m}{7} = 2$) shares a similar solution procedure (Fig. 2), we unpack the complexity of the two types of trigonometry problem in line with prior research in the domain of linear equations (e.g. Nhu & Phan, 2016) from a cognitive load perspective.

Element interactivity, according to Sweller (2010), is a basic concept of cognitive load theory. Anything (e.g. a number, a symbol, a concept, a procedure) that requires learning is regarded as an element (Chen et al., 2017). Element interactivity refers to the interaction between elements in learning materials. The level of element interactivity acts as an index to determine the complexity of materials (Sweller, 2010). Using the concept of element interactivity, Chen et al. (2015) explained the complexity of the solution procedure of a linear equation by estimating the number of interacting elements in its solution procedure. Likewise, building on prior work of Sweller and his colleagues (e.g. Chen et al., 2015), we have classified the complexity of linear equations using the number of relational line and operational line (Ngu & Phan, 2015, 2016). For a linear equation such as $2x+7=23$, a relational line refers to the quantitative relationship in which the left-hand side of the equation equals to its right-hand side. An operational line refers to the use of an operation (i.e. $+7$ becomes -7, inverse operation) to alter the state of the problem, and such an action preserves its equality. The number of relational line and operational line determines the level of element interactivity, which reflects the complexity of the linear equations. As shown in Fig. 3, the type II problem with a pronumeral as a denominator ($\sin 32° = \frac{50}{x}$) has more operational lines (2 vs. 1) and relational line (3 vs. 2) than the type I problem with a pronumeral as a numerator ($\tan 75° = \frac{x}{26}$). Furthermore, the nature of the elements also influences the complexity of linear equations (e.g. fraction, decimals, the pronumeral serves as a denominator) (Ngu & Phan, 2016). Accordingly, we distinguish two types of trigonometry problem that exhibits two levels of complexity where $\sin 32° = \frac{50}{x}$ (type II) is more complex than $\tan 75° = \frac{x}{26}$ (type I).

The current study

The objective of this current study is to compare the relative efficiency of the unguided problem-solving approach, worked examples approach and analogy approach for learning to solve trigonometry problems that exhibit two levels of complexity (type I vs. type II) due to the location of the pronumeral ($\tan 75° = \frac{x}{26}$ vs. $\sin 32° = \frac{50}{x}$).

The first research question examined whether performance outcomes will favour type II problems across the three approaches. Given that the type II problems are more complex

$\tan 75° = \frac{x}{26}$ [÷ 26 becomes × 26]	$\sin 32° = \frac{50}{x}$ [÷x becomes × x]
$x = \tan 75° \times 26$	$x \times \sin 32° = 50$
$x = 97.03$	$x \sin 32° = 50$ [× sin 32° becomes ÷ sin 32°]
	$x = \frac{50}{\sin 32°}$
	$x = 95.35$

Fig. 3 Two types of trigonometry problem that differ due to the location of the pronumerals (numerator vs. denominator)

than type I problems, we hypothesize that students will perform better for the type I problems than type II problems irrespective of the approach (hypothesis 1).

The second research question examined the relative efficiency of the three approaches on learning two types of trigonometry problem that exhibit two levels of complexity. On learning to solve trigonometry problems, an emphasis on analogical comparison activities would be more effective than the problem-solving without guidance. However, processing two worked examples (i.e., a pair of isomorphic examples) concurrently would impose higher cognitive load than processing one worked examples in the worked examples approach, especially for students whose prior knowledge of linear equations with fractions were fragmented. We expect that the worked examples effect will occur in which the guidance provided by the worked examples approach would be more efficient than the unguided problem-solving approach. Research has indicated that the impact of instructional design would be more pronounced for the complex problems rather than simple problems (Ngu et al., 2018; Chen et al., 2015; Sweller et al., 2011). Therefore, considering the design of the three approaches in the light of the aforementioned review, we hypothesize that performance will not differ across the three approaches for the type I problems (hypothesis 2a); however, performance on the type II problem will follow the order: worked examples > analogy > unguided problem-solving (hypothesis 2b).

The third research question tested whether performance on the concept test will differ across the three approaches. We predict that active analogical comparison of a pair of isomorphic examples would facilitate in-depth understanding of the structural features of the trigonometry problems (Rittle-Johnson & Star, 2007; Timothy et al., 2013). Moreover, the worked examples that show solution steps would facilitate the acquisition of problem-solving schemas for trigonometry problems better than unguided problem-solving. Accordingly, we hypothesize that performance on the concept test would follow the order: analogy > worked examples > unguided problem-solving (hypothesis 3).

The fourth research question examined whether accuracy in solving practice problems would reflect in success for the post-test and concept test. Research has affirmed the facilitative effect of solving practice problems upon learning outcomes (e.g. Ngu et al., 2018). We hypothesize association between scores on the practice problems and the post-test (hypothesis 4a), as well as scores on the practice problems and the concept test (hypothesis 4b).

Design of the experiment

We used a 3 (approach: unguided problem-solving vs. worked examples vs. analogy) × 2 (problem type: type I vs. type II) design to examine the impact of the three approaches on the pre-test, practice problems and post-test. The independent variable was the approach and the dependent variables were mean proportion scores for the pre-test, practice problems, and post-test. The approach was a between-subjects factor, whereas the problem type was a within-subjects factor. We conducted one-way ANOVA to examine the difference between the three instructional approaches for the concept test. We calculated the correlation coefficient for the practice problems and post-test, and the practice problems and concept test to examine the impact of solving practice problems upon performance for the post-test and concept test.

Participants

Ninety-seven Australian Year 9 students (boys = 49, girls = 48) whose mean age was 15.26 ($SD = 0.28$) consented to participate in the study. They were drawn from two schools and two classes of each participating school in a regional area of Australia. According to the Head teacher, students had received lessons about the topic of trigonometry in Term 1 and thus they had basic knowledge of trigonometry problems. Data collection occurred in Term 2.

Materials

The pre-test was identical to the post-test (Appendix) in terms of the number and types of trigonometry problem. It had a total of 18 problems with an equal number of type I problems (e.g. $\sin 30° = \frac{x}{4}$) and type II problems ($\cos 39° = \frac{5.5}{x}$). The instruction sheet included the definition of trigonometric ratios, and solution steps of solving trigonometry problems (e.g. $\cos 55\,° \times 9.23 = 5.29$). The students had studied the definition of trigonometric ratio prior to participating in this study. Using a calculator, we expected the students to have learned, for example, $\cos 55^0 = 0.573576…$. In the instruction sheet, students were instructed to use a calculator to check the answer, for example, $\cos 55^0 \times 9.23 = 5.29$ (correct to two decimal places). In other words, using a calculator, the students were expected to calculate $\cos 55^0 \times 9.23$, and then express the answer in two decimal places such as 5.29. This instruction sheet was common across the three approaches. All students attended a teaching phase, which was based on the material in the instruction sheet. Accordingly, the material in the instruction sheet aimed to assist students revise what they had been taught.

In relation to acquisition problems, there was an equal distribution of 12 type I problems (e.g. $\sin 30° = \frac{x}{4}$) and 12 type II problems (e.g. $\cos 39° = \frac{5.5}{x}$) across the three approaches. Students in the unguided problem-solving approach solved 24 problems. Students in the worked examples approach completed 12 worked example–practice problem pairs. For each pair, students studied a worked example (Fig. 1) and solved a practice problem that shared a similar problem structure. We graded the 12 practice problems only. The 12 worked example–problem pairs were similar to the 24 problems in the unguided problem-solving approach except half of these were presented as worked examples that showed solution steps.

For the analogy approach, there were 12 pairs of isomorphic worked examples. Each pair (e.g. $\tan 75° = \frac{x}{26}$ paired with $\frac{m}{7} = 2$) shared a similar solution procedure (Fig. 2). Students were required to compare the solution procedure of each pair and then solve a similar practice problem. Again, we graded the 12 practice problems only. Overall, students in the analogy approach were presented with similar acquisition problems as those students in the worked examples approach except for those additional 12 linear equations with fractions.

Lastly, the concept test (Appendix) aimed to evaluate students' understanding of the algebraic skills involved in solving trigonometry problems. Students needed to indicate, for example, whether (i) $\frac{x}{14} = \tan 63°$ and (ii) $x = \tan 63° \times 14$ were equivalent and justify their answers.

Procedure

The procedure was identical for students across the two schools. We obtained ethics clearance prior to data collections. We administered group-testing - in regular classrooms. Two researchers and a maths teacher supervised the group-testing for each class across the two schools. We distributed the written task for each phase and collected the task after the allocated time had elapsed. We randomly allocated 97 Year 9 students to the unguided problem-solving approach, worked examples approach, and analogy approach, resulting in 31 students in each approach. Two students who did not complete all tasks were eliminated in the final analysis of data.

The experimental procedure comprised: (i) a pre-test (10 min), (ii) a teaching phase (10 min), (iii) an acquisition phase that consisted of an instruction sheet (5 min), acquisition problems (15 min), (iv) a post-test (10 min), and (v) a concept test (10 min). In previous cognitive load research, the length of the experimental procedure to complete various tasks differs across studies, which may take about an hour (Ngu et al., 2018; Carroll, 1994; Chen et al., 2015; Cooper & Sweller, 1987; Kalyuga et al., 2001). We postulated that an hour intervention would be sufficient to assess the impact of different instructional approaches to assist students acquire the algebraic skills required to solve two types of trigonometry problem. We assessed students' acquisition of procedural knowledge (post-test) and conceptual knowledge (concept test) to evaluate the effects the instructional approaches upon their performance outcomes.

We provided a briefing in regard to the aim of the experiment and the allocated time to complete each task. Specifically, we informed students that they were going to learn how to solve trigonometry problems. They should try their best to complete the written tasks, which did not constitute *a test* of mathematics performance. Furthermore, they could raise their hands and seek help if they could not understand the materials in the acquisition phase. For example, students may seek clarification in relation to the concept of the inverse operation (e.g. $\div x$ becomes $\times x$). Because only a few students asked for help, we did not document the data.

They were told to complete each task individually. The instruction to complete each task was presented on the first page of each task. We informed the students to read the instruction before they worked on each task. They did not receive assistance for solving practice problems.

The procedure began by having all students sat for a pre-test. Next, a researcher introduced the trigonometric ratios using the material in the instruction sheet as a guide. Then,

Table 1 Scoring: samples of students' solutions

Sample solutions	Score	Explanation
$\tan 35^\circ = \frac{x}{18}$ $x = \tan 35^\circ \times 18$ $x = 14.74$ (wrong)	One point	Applied an inverse operation correctly but made a computation error
$\sin 50^\circ = \frac{x}{4}$ $x = \sin 50^\circ \div 4$ (*wrong*) $x = 3.06$	Zero point	Applied an inverse operation incorrectly though the answer was correct

students proceeded to complete their respective acquisition phases. Once students had completed the acquisition phase, they undertook a post-test. Lastly, students completed a concept test. Across the three approaches, students were matched with similar time allocation and tasks. The only difference between the three approaches was the design of the acquisition problems. We expected learning to occur predominately in the acquisition phase, though students may also learn from the teaching phase.

Coding

The respective Cronbach's alpha for the pre-test, post-test and concept test were 0.96, 0.96 and 0.81. Concerning the practice problems, the Cronbach's alpha for the unguided problem-solving approach, worked examples approach, and analogy approach were 0.98, 0.96 and 0.93 respectively. The two researchers graded the pre-test, practice problems, post-test and concept test.

We allocated one point for each test item in the pre-test and post-test, and each practice problem that solved accurately regardless whether students showed solution steps or not. As shown in Table 1, we disregarded computational errors; however, we allocated zero point if students made procedural errors. For the concept test, we did not grade the explanations because only a few students provided explanations. We assigned one point if students indicated, for example, (i) $\frac{x}{18} = \cos 43°$ and (ii) $x = \cos 43° \times 18$ were equivalent. The interscorer reliability was 100% for the concept test. However, the interscorer reliability for the pre-test, post-test and practice problems was about 90%

Table 2 Means and standard deviations of proportion correct scores on the pre-test, practice problems, post-test, and concept test

	Unguided problem-solving approach	Worked examples approach	Analogy approach
Scores (proportion)	$n=31$	$n=31$	$n=31$
	M (SD)	M (SD)	M (SD)
Pre-test			
Type I problems	0.18 (0.33)	0.39 (0.47)	0.36 (0.45)
Type II problems	0.06 (0.20)	0.14 (0.29)	0.18 (0.33)
Practice problems			
Type I problems	0.50 (0.45)	0.76 (0.41)	0.84 (0.32)
Type II problems	0.24 (0.40)	0.56 (0.46)	0.55 (0.48)
Post-test			
Type I problems	0.71 (0.44)	0.76 (0.41)	0.85 (0.33)
Type II problems	0.39 (0.47)	0.60 (0.44)	0.58 (0.47)
Concept test	0.59 (0.32)	0.54 (0.27)	0.52 (0.37)

The pre-test was the same as the post-test (type I problems = 9, type II problems = 9). For the practice problems, the worked examples approach and the analogy approach had six type I problems and six type II problems, respectively. The unguided problem-solving approach had 12 type I problems and 12 type II problems. The concept test had four questions and there were two pairs of trigonometric equations for each question owing to some ambiguous cases: for example, a student wrote $x = \sin(40 \times 15)$ instead of $x = \sin 40° \times 15$ despite providing a correct answer such as 9.64. We resolved any discrepancy in the scores through discussion.

Results

The means and standards deviation of the pre-test, practice problems, post-test and concept test are presented in Table 2.

Pre-test. The main effect of problem type was significant, $F(1, 90) = 20.81$, $p < 0.001$, $\eta^2 = 0.19$, indicating higher mean score for the type I problems than type II problems. The approach × problem type interaction was nonsignificant, $F(2, 90) = 0.90$, $p = 0.41$, $\eta^2 = 0.02$. The main effect of approach was also nonsignificant, $F(2, 90) = 2.40$, $p = 0.10$, $\eta^2 = 0.05$. We performed pairwise comparisons to further examine the main effect of problem type. Students in the worked examples approach ($p < 0.001$) as well as the analogy approach ($p = 0.01$) performed better for the type I problems than type II problems. However, for the unguided problem-solving approach, no difference between the type I problems and type II problems was observed ($p = 0.09$). This result was a surprise given that we randomly assigned students to three approaches. In fact, students in the unguided problem-solving approach scored lower than the other two approaches for both type I and type II problems. The univariate test confirmed that students across the three approaches neither differed on type I problems, $F(2, 90) = 2.23$, $p = 0.11$, $\eta^2 = 0.05$ nor on type II problems, $F(2, 90) = 1.34$, $p = 0.27$, $\eta^2 = 0.03$.

Across the three approaches, it appears that prior exposure to trigonometry problems assisted students to gain basic knowledge about type I problems, but their knowledge of the type II problems was poor. In other words, most students had almost completely forgotten how to solve trigonometry problem when the pronumeral acted as a denominator, pointing to a lack of conceptual understanding of solving trigonometry problems that involved algebraic transformation skills. It reveals that students had fragmented knowledge regarding the application of the inverse operation to solve type II problems. Indirectly, the results indicated the challenge posed to students on learning to solve type II problems. Differential performance between the three approaches was not observed for either the type I problems or the type II problems. Thus, we had established a base-line performance in which students across the three approaches neither differed for the type I problems nor the type II problems.

Practice problems. A significant main effect for the problems type was found, $F(1, 90) = 32.36$, $p < 0.001$, $\eta^2 = 0.27$, revealing higher performance for the type I problems than type II problems, a nonsignificant approach × approach interaction was observed, $F(2, 90) = 0.38$, $p = 0.69$, $\eta^2 = 0.01$, and a significant main effect of approach was found, $F(2, 90) = 7.38$, $p < 0.001$, $\eta^2 = 0.14$. For the main effect of problem type, a follow-up pairwise comparisons revealed that students performed better for the type I problems than type II problems across the unguided problem-solving approach ($p = 0.00$), worked examples approach ($p = 0.01$) as well as analogy approach ($p < 0.001$). As hypothesized, students across the three approaches were more competent to solve the type I problems rather than type II problems.

For the main effect of approach, the univariate test confirmed differential performance on the practice problems among the three approaches for the type I problems,

$F(2, 90)=6.37$, $p<0.001$, $\eta^2=0.13$ as well as type II problems, $F(2, 90)=5.08$, $p=0.01$, $\eta^2=0.10$. For the type I problems, pairwise comparisons indicated that students in both the worked examples approach ($p=0.03$) and analogy approaches ($p<0.001$) outperformed those students in the unguided problem-solving approach. Such results did not support hypothesis 2a. However, students in the worked examples approach and analogy approach did not differ ($p>0.05$), thus supporting hypothesis 2a. A similar pattern of results emerged for the type II problems. The results supported hypothesis 2b in that both the worked examples approach ($p=0.02$) and analogy approach ($p=0.02$) performed better than the unguided problem-solving approach. However, the results did not support hypothesis 2b in that the worked examples approach was not better than the analogy approach.

In sum, for the practice problems, as hypothesized, the type II problems posed greater challenge to learn than the type I problems across the three approaches. It is interesting to note that the beneficial effects of the worked examples approach and analogy approach not only were observed for the type II problems but also for the type I problems. However, the worked examples approach was not better than the analogy approach irrespective of the problem type. Presumably, prior knowledge of students pertaining to linear equations with fractions may have reduced cognitive load demand on processing two worked examples concurrently in the analogy approach.

Post-test. Similar to the results obtained for the pre-test and practice problems, the main effect of problem type was significant, $F(1, 90)=25.60$, $p<0.001$, $\eta^2=0.22$, indicating that the mean score for the type I problems was higher than type II problems. Neither the approach × problem type interaction effect, $F(2, 90)=0.91$, $p=0.41$, $\eta^2=0.02$, nor the main effect of approach, $F(2, 90)=1.86$, $p=0.16$, $\eta^2=0.04$ was significant. For the main effect of the problem type, in line with hypothesis 1, students in the unguided problem-solving approach ($p<0.001$), as well as in the analogy approach ($p<0.001$) performed better for the type I problems than type II problems. Contrary to hypothesis 1, no difference between type I problems and type II problems was observed for the worked examples approach ($p=0.06$).

For the nonsignificant main effect on the approach, the univariate test indicated neither differential performance for the type I problems, $F(2, 90)=0.98$, $p=0.38$, $\eta^2=0.02$ nor the type II problems, $F(2, 90)=2.04$, $p=0.14$, $\eta^2=0.04$. Thus, the results supported hypothesis 2a, but not hypothesis 2b.

Overall, the results for the post-test partially supported hypotheses. Only two approaches (unguided problem-solving approach and analogy approach) instead of all three approaches performed better for type I problems than type II problems. Neither did all three approached differ on type I problems (in line with hypothesis 2a), nor did they differ on type II problems (contrary to hypothesis 2b).

Concept test. Contrary to the hypothesis, one way ANOVA performed on the three approaches for the concept test revealed nonsignificant result, $F(2, 90)=0.41$, $p=0.66$, $\eta^2=0.01$. It appears that the guidance provided by the worked examples approach as well as the analogy approach was not more beneficial than the unguided problem-solving approach in regard to the acquisition of conceptual knowledge for the trigonometry problems.

Correlation coefficient test. We conducted correlation coefficient test to examine the extent to which performance on practice problems was reflected in the accuracy of the post-test and concept test. In support of hypothesis 4a, for the post-test, significant and positive correlation coefficients were found for the unguided problem-solving approach ($r=0.89$, $n=31$, $p<0.001$), worked examples approach ($r=0.84$, $n=31$, $p<0.001$) and analogy approach $r=0.50$, $n=31$, $p=0.005$). Thus, the results indicated positive effects of solving practice problems upon accuracy of solving test problems in the post-test.

For the concept test, significant and positive correlation coefficients were found for the unguided problem-solving approach ($r=0.44$, $n=31$, $p=0.01$). However, positive correlation but not significant were found for the worked examples approach ($r=0.29$, $n=31$, $p=0.11$) and the analogy approach ($r=0.31$, $n=31$, $p=0.09$). Such results only partially supported hypothesis 4b. Notably, neither the worked examples approach nor the analogy approach assisted students to acquire better conceptual understanding of the algebraic transformation skills involved in solving trigonometry problems. In short, solving practice problems had significant positive effects on the performance for the post-test across all the three approaches, but it only limited to the unguided problem-solving approach for the concept test. According to Rittle-Johnson et al. (2015), maths educators in Asian countries have the view that an emphasis on solving practice problems leads to the development of conceptual understanding. Thus, it appears that the developing of procedural knowledge through solving many practice problems in the unguided problem-solving approach had facilitated the development of conceptual knowledge.

Discussion

The objective of this current study was to compare the relative efficiency of the unguided problem-solving approach, worked examples approach and analogy approach for learning to solve trigonometry problems that exhibit two levels of complexity (type I vs. type II) due to the location of the pronumeral (numerator vs. denominator). Consistent with prior studies, the findings indicated that students across the three approaches performed better for the type I problems than type II problems (Chen et al., 2015; Sweller et al., 2011). In line with the hypothesis, the design of the worked examples approach (e.g. van Gog et al., 2019) and analogy approach (e.g. Rittle-Johnson et al., 2017) that highlighted conceptual knowledge and procedural knowledge were more efficient than the unguided problem-solving approach without any guidance for acquiring skills to solve practice problems regardless of their complexity. That said, their advantage did not extend to solve the test problems.

Processing a pair of isomorphic worked examples simultaneously (analogy approach) would impose approximately twice the level of element interactivity as compared to the processing of a single worked example (worked examples approach) (Chen et al., 2017). Nonetheless, students in the analogy approach may have sufficient prior knowledge of linear equations with fractions, which may reduce the level of element interactivity and thus cognitive load imposition when they processed a pair of isomorphic worked examples concurrently. Thus, contrary to hypothesis, the worked examples approach was not better than the analogy approach.

As revealed by the correlation coefficient test, across the three approaches, accuracy in solving practice problems led to accuracy in the post-test. In contrast, significant correlation between accuracy for the practice problems was reflected in the concept test for the unguided problem-solving approach only. A review by Rittle-Johnson et al. (2015) indicates that the acquisition of conceptual knowledge facilitates the acquisition of procedural knowledge and vice versa. Accordingly, the acquisition of procedural knowledge through the completion of multiple practice problems in the unguided problem-solving approach may have contributed towards the acquisition of conceptual knowledge.

From the theoretical point of view, the findings have supported an advantage of providing instructional guidance. Apparently, students could scrutinize the solution procedure of worked examples (worked examples approach) and a pair of isomorphic examples (analogy approach) that shows the interplay of conceptual knowledge and procedural knowledge, and transfer their understanding to solve similar practice problems. We have innovated the design of analogical learning by requiring students to solve a practice problem after they had performed analogical comparison for a pair of isomorphic examples. Therefore, building on prior research on analogical learning that emphasizes the identification of the underlying problem structure (Alfieri et al., 2013; Rittle-Johnson et al., 2017; Timothy et al., 2013), we have also highlighted the importance of acquiring procedural knowledge through solving practice problems. There is evidence that accuracy in solving practice problems was associated with accuracy in the post-test for the analogy approach.

Considering the practical implication in maths classroom, it is important to make a distinction between two types of trigonometry problem that exhibit two levels of complexity. Differential levels of complexity reflect differential levels of intrinsic cognitive load associated with two types of trigonometry problem. Thus, it would be helpful to incorporate the relational line and operational line (Ngu & Phan, 2016) to describe the solution steps of trigonometry problems in mathematics curriculum so that mathematics educators can assist students to differentiate two types of trigonometry problem that exhibit two levels of complexity.

Contrary to prior research that indicates instructional consequences for complex problems rather than simple problems (Ngu et al., 2018; Chen et al., 2015), the results point to the efficacy of the worked examples approach and analogy approach to enhance learning of the trigonometry problems irrespective of their complexity. Accordingly, we recommend the incorporation of the worked examples approach and analogy approach in maths curriculum for learning to solve trigonometry problems.

Limitations and future directions

In terms of the research design, cognitive load researchers normally do not measure students' motivation during the learning phase. The students in the present study may not share a similar level of motivation during the acquisition phase. Given that we randomly assigned students to three groups, the percentage of students who may not be motivated to complete the written tasks may be similar across the three groups and thus would not have affected the overall learning outcomes. Nonetheless, we could incorporate a measure of students' motivation in future study to tease out such issue.

Prior research has used a pre-test to control prior knowledge between groups (Rittle-Johnson, 2006). While we randomly assigned students to three groups and used a pre-test to establish a base-line performance on the trigonometry problems, we will also include mathematics test scores that consist of various topics to establish group equivalence in future enquiry. Moreover, to improve the reliability of the results, we will conduct a large-scale investigation across multiple schools in future research.

Prior studies have revealed that the inverse method is more efficient than the balance method for learning to solve linear equations especially for complex linear equations (Ngu et al., 2018). The main difference between the inverse method and balance method lies in the operational line (e.g. $+2$ becomes -2 vs. -2 on both sides). Thus, familiarity with the use of the inverse method would facilitate understanding of not only the solution procedure of the worked examples, but also one-to-one mapping of a pair of isomorphic examples. Importantly, it should assist students to improve their performance on the concept test, which essentially evaluated the algebraic transformation skills related to the inverse operation. Therefore, the design of the intervention programme in future research should include an opportunity to practise the test items in the concept test.

Research has indicated the benefit of placing a pair of isomorphic examples in the same orientation to facilitate one-to-one mapping task between a pair of isomorphic examples (Kurtz & Gentner, 2013; Matlen et al., 2020). Instead, as shown in Fig. 2, we placed the fraction of the linear equation (i. e. $\frac{m}{7}$). and the fraction of the trigonometry problem (i.e. $\frac{x}{26}$) in opposite direction (right-hand side vs. left-hand side). It would be worthwhile to tease out this issue in additional research. We speculated that the processing of a pair of isomorphic examples simultaneously would incur higher cognitive load than processing one worked example as in the case of the worked examples approach. It is important to include a measure of cognitive load in future enquiry to validate this prediction.

The mathematics educators tend to view problem-solving in the context of solving ill-structured mathematical problems that have multiple solutions (Hong & Kim, 2016). Thus, a problem-solver will explore and test different solution paths to arrive at multiple ways to solve an ill-structured problem. However, the cognitive science researchers tend to view problem-solving mostly in well-structured problems that have fixed schemas and thus specific solution procedures, and to lesser extent, ill-structured problems (Sweller et al., 2011). In their seminal work, Sweller and Cooper (1985) compared worked examples and conventional solving problems to facilitate learning of algebraic transformation problems (e.g. $a-c=(d+e)/c$, solve for d). Other cognitive science researchers have documented a comparison between worked examples and unaided problem-solving (Hoogerheide & Roelle, 2020) or studying worked examples and solving practice problems (van Gog, et al., 2019) to enhance problem-solving skills mainly in well-structured problems.

The theoretical rationale of the present study is based on cognitive science literature. The target domain of the present study is trigonometry problems that involve specific algebraic transformation skills. Accordingly, we named one of the compared instructional approaches as an unguided problem-solving approach. Nonetheless, additional research could replace ‘unguided problem-solving approach’ with ‘unguided solving task approach’ in line with the view of mathematics educators who tend to use problem-solving in the context of ill-structured problems rather than well-structured problems.

Conclusion

To conclude, it is timely to explore alternative instructional practices to enhance learning of trigonometry problems that posed a challenge to students. The facilitative effect of solving practice problems upon the post-test was evidence across the three approaches (unguided problem-solving, worked examples, analogy); moreover, it extends to the concept test for the unguided problem-solving approach. Importantly, the inclusion of practice problems for the analogy approach had assisted students to acquire procedural knowledge.

The advantage of the worked examples approach and analogy approach over the unguided problem-solving approach was manifested for the practice problems only. Nevertheless, we contend that the worked examples approach and analogy approach have the potential to reform instructional practices for learning to solve trigonometry problems — if we strengthen students' prior knowledge of the inverse operation concept, which is central to the algebraic transformation skills involved in solving trigonometry problems.

Appendix

Pre-test or Post-test

You have 10 minutes to solve as many questions as you can. Show working in the space provided. Give the answers correct to two decimal places.

Section 1

1. $\sin 30° = \frac{x}{4}$

2. $\tan 15° = \frac{x}{2.2}$

3. $\frac{x}{32} = \cos 46°$

Section 2

1. $\sin 75° = \frac{84}{x}$

2. $\tan 50° = \frac{71}{x}$

3. $\cos 39° = \frac{5.5}{x}$

Concept test

Are two equation pairs (i) & (ii) equivalent (have the same answer)? Circle 'Yes' or 'No' and give a reason (a short sentence). Are two equation pairs (a) & (b) equivalent (have the same answer)? Circle 'Yes' or 'No' and give a reason (a short sentence). You have 10 minutes to complete this section.

Equation	Equation	Circle 'Yes 'or 'No'	Reason
(i) $x\cos 27° = 18$	(ii) $\cos 27° = \frac{18}{x}$	Yes No	
(a) $x\cos 27° = 18$	(b) $x = \frac{18}{\cos 27°}$	Yes No	

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Declarations

Competing Interests The authors declare no competing interests.

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Current themes of research:

Cognitive processes (e.g. cognitive load). Instructional designs. Learning by analogy and learning by comparison. The self-systems (e.g. self-esteem, self-efficacy). Human optimization and achievement bests. Student well-beings.

Most relevant publications in the field of Psychology of Education:

Ngu, B. H., Phan, H. P. (2022). Advancing the study of solving linear equations with negative pronumerals: A smarter way from a cognitive load perspective. *PLoS ONE*, 17(3), e0265547.

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Current themes of research:

Mindfulness. Positive psychology. The self-systems (e.g. self-esteem, self-efficacy). Human optimization and achievement bests. Academic optimism. Student well-beings. Cognitive processes of learning (e.g. cognitive load).

Most relevant publications in the field of Psychology of Education:

Ngu, B. H., Phan, H. P. (2022). Advancing the study of solving linear equations with negative pronumerals: A smarter way from a cognitive load perspective. *PLoS ONE*, 17(3), e0265547.

Ngu, B. H., Phan, H. P. (2022). Developing problem-solving expertise for word problems. *Frontiers in Psychology*, 13. doi:10.3389/fpsyg.2022.725280.

Phan, H. P., Ngu, B. H. (2022). Advancing the study of 'Goals of Best Practice' (GsBP): Towards achieving optimal best – Educational implications to developments in flow research and positive optimal psychology. *Frontiers in Psychology*, 13, Article 838,560. doi:10.3389/fpsyg.2022.838560.

Phan, H. P., Ngu, B. H. (2021). Perceived 'optimal efficiency': Theorization and conceptualization for development and implementation. *Heliyon*, 7(1), e06042. https://doi.org/10.1016/j.heliyon.2021.e06042.

Phan, H. P., Ngu, B. H., Chen, S.-C., Wu, L., Shih, J.-H., & Shi, S.-Y. (2021). Life, death, and spirituality: A conceptual analysis for educational research development. *Heliyon*, 7(5), e06971. https://doi.org/10.1016/j.heliyon.2021.e06971.

Phan, H. P., Ngu, B. H., Shih, J.-H., Lin, R.-Y., Shi, S.-Y., & Wang, H.-W. (2020). Validating 'optimizing' concepts: The importance of personal resolve, effective functioning, and academic striving. *Educational Psychology*, 40(4), 448–472. https://doi.org/10.1080/01443410.2019.1693507.

Phan, H. P., & Ngu, B. H. (2020). Schooling experience and academic performance of Taiwanese students: The importance of psychosocial effects, positive emotions, levels of best practice, and personal well-being. *Social Psychology of Education*, 23, 1073–1101. https://doi.org/10.1007/s11218-020-09569-9.

Phan, H. P., Ngu, B. H., & Yeung, A. S. (2019). Optimization: In-depth examination and proposition. *Frontiers in psychology*, 10, 1398–1398. https://doi.org/10.3389/fpsyg.2019.01398.

Ngu, B. H., Phan, H. P., Wang, H.-W., Shih, J.-H., Shi, S.-Y., & Lin, R.-Y. (2019). Best practice in mathematics learning: A theoretical discussion for consideration. In R. V. Nata (Ed.), *Progress in education* (Vol. 55, pp. 79—112). New York: Nova Science Publishers.