CORRECTION

Correction to: Beating the market? A mathematical puzzle for market efficiency

Michael Heinrich Baumann[1]

Abstract

In this note, we point out a missing assumption for 'Michael Heinrich Baumann, Beating the market? A mathematical puzzle for market efficiency, Decis Econ Finance 45: 279–325, 2022.' In detail, we have to assume the (almost sure) survival of the controllers. Further, we discuss this assumption concerning relevance for theory and implementations and how it may alter the results and we give directions for future research.

Correction to: Decisions in Economics and Finance (2022) 45:279–325
https://doi.org/10.1007/s10203-021-00361-8

1 Assumption and results

In the whole article, we have to assume that investments, e.g., I^L, I^S, are almost surely nonzero, i.e., that almost surely all controllers survive. In the proofs, whenever there is an investment level in the denominator, these calculations are only true for the almost-sure nonzero case. Calculations with expected investments stay the same. Results may change from surely to almost surely.

The original article can be found online at https://doi.org/10.1007/s10203-021-00361-8.

✉ Michael Heinrich Baumann
michael.baumann@uni-bayreuth.de

[1] University of Bayreuth, Universitätsstraße 30, 95447 Bayreuth, Germany

2 Discussion

Under this new assumption, the calculations in the proofs basically stay the same except for the null sets. In the discrete-time case, if, e.g., $I^L(t_{n-1}) \neq 0$, but $I^L(t_n) = 0$, it follows that

$$
\begin{aligned}
0 = {} & I^L(t_n) \\
= {} & I_0^* + K g^L(t_n) \\
= {} & I_0^* + K g^L(t_{n-1}) + K I^L(t_{n-1}) \cdot \frac{p(t_n) - p(t_{n-1})}{p(t_{n-1})} \\
= {} & I^L(t_{n-1}) + K I^L(t_{n-1}) \cdot \frac{p(t_n) - p(t_{n-1})}{p(t_{n-1})} \\
= {} & I^L(t_{n-1}) \left(1 + K \cdot \frac{p(t_n) - p(t_{n-1})}{p(t_{n-1})} \right) \\
\Rightarrow \quad & \frac{p(t_n)}{p(t_{n-1})} = 1 - \frac{1}{K}.
\end{aligned}
$$

If the price changes $\frac{p(t_n)}{p(t_m)}$ between all different points in time $t_m < t_n$ are distributed with any distribution that is absolutely continuous to the Lebesgue measure, the probability of this controller to die out is zero. For other controllers, a similar reasoning applies. Thus, under this relatively weak absolute continuity assumption, the mentioned puzzle for market efficiency remains intact. The definition of the 'robust positive expectation property' (RPEP) is stated and under specific assumptions also proven under our new assumption.

In tree models, the probability of a controller to die out can be nonzero. However, e.g., in the space of all recombinant binomial tree models with constant multiplicative growth rates with given trend but varying variance, models where a controller dies might be called degenerated. A detailed analysis of necessary and other sufficient conditions for avoiding such 'degenerate' scenarios is an interesting topic for future work. The same is true for an investigation of cases when the assumption is not true. Does the robust positive expectation property still hold without the new assumption? If not: Are there any bounds for (expected) gains/losses?

For implementations, a nonzero investment can, e.g., be achieved by defining some minimal (positive or negative) investment. However, this would (slightly) change the strategy and, thus, lead to different results, which again are worth further investigation. For example, it would be interesting whether one can prove robust positive expectation property results also for a 'minimal investment simultaneously long short strategy.'

3 Further remarks

We know that the RPEP does not hold in discrete time, i.e., whenever trading on data. Positive expected gains for "arbitrary short, small, and fully unknown trends" are in the article only proven in continuous time under our assumptions if the average trend

is nonzero. In discrete time, it is not clear how intervals with either positive or negative trends can be detected to "reset" a controller. Thus, a detailed analysis of cases where the trend often changes its sign is important. It is an open question whether a higher trading frequency is "good" because of the limits to continuous trading or whether it is "bad" since more often trend changes can occur between two trades.

Publisher's Note Springer Nature remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.