Correction to: Semi-analytical prices for lookback and barrier options under the Heston model

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Abstract

In this note, we point out a mistake in Theorem 1 of De De Gennaro Aquino and Bernard (Decis Econ Finance 42(2):715–741, 2019) and provide some missing references where the problem of pricing barrier options under the Heston model had previously been discussed.

Keywords Derivatives pricing · Lookback options · Barrier options · Path-dependent options · Heston model · Stochastic volatility

JEL Classification C65 · G13

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Discussion

There is a mistake in the proof of Theorem 1 in De Gennaro Aquino and Bernard (2019). Specifically, this mistake has roots in Subsection 2.2. Therein, we write that under the Heston stochastic volatility model, the spot price at maturity T can be written

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as

$$S_T = S_0 e^{rT} \exp \left(\sqrt{1 - \rho^2} \int_0^T \sqrt{v_s} dW_s + \mu_v \right), \quad (1)$$

where μ_v is given by

$$\mu_v := rT - \frac{1}{2} \int_0^T v_s ds + \frac{\rho}{\sigma} \left(v_T - v_0 - \kappa \theta T + \kappa \int_0^T v_s ds \right).$$

Then, we observe that after conditioning with respect to the filtration generated by the variance process $\{v_t\}_{0 \leq t \leq T}$, i.e., $\mathcal{F}_t^v = \sigma(\{v_s : 0 \leq s \leq t\})$, μ_v may be interpreted as a deterministic quantity and the stock price S_T defined by (1) has a (conditional) lognormal distribution.

While the latter observation is not inherently incorrect if one needs to examine path-independent payoffs (that is, payoffs depending only on the stock price at maturity), the same argument cannot be relied upon for path-dependent payoffs, which is the case of our paper. In essence, one can show that it leads to a description of the underlying stock price where the variance process is erroneously averaged.

This error thus invalidates the statement of Theorem 1, where we derive the joint density of the logarithmic stock price at maturity and its maximum, conditional on $\mathcal{F}^v$, and the joint density of the logarithmic stock price at maturity and its minimum, yet conditional on $\mathcal{F}^v$.¹ In light of this issue, and for Theorem 1 being a key ingredient to derive the pricing formulas for lookback, partial lookback and barrier options under the Heston model in Sections 3, 4 and 5, respectively, all semi-analytical formulas provided in our paper do not give exact prices, but only (non-convergent) approximations of them.

A final comment. Concurrently to the letter pointing out the above-mentioned mistake, we were further communicated that the problem of pricing barrier options under the Heston model had previously been discussed in Lipton (2001) and Lipton and McGhee (2002). Regrettably, we were not aware of these studies at the time and thus apologize to the authors for the omission.

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References

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¹ The theorem is flawed in the sense that it does not provide the true conditional joint density of the logarithmic spot price and its maximum, or minimum, *under the Heston model*. For reference only, the expressions in Eqs. (9), (11) and (12) remain valid if all involved coefficients are indeed (unconditionally) constant.

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