Correction to: Comparison of joint control schemes for multivariate normal i.i.d. output

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Published online: 18 December 2018
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Correction to: AStA Advances in Statistical Analysis <https://doi.org/10.1007/s10182-018-00331-3>

In the original paper (Morais et al. 2018), we incorrectly stated that

$$T^{(1)} = n (\bar{\mathbf{X}} - \boldsymbol{\mu}_0)^\top \boldsymbol{\Sigma}_0^{-1} (\bar{\mathbf{X}} - \boldsymbol{\mu}_0) \sim \chi_p^2(\lambda),$$

where $\chi_p^2(\lambda)$ represents the noncentral Chi-square distribution with p degrees of freedom and noncentrality parameter $\lambda = (\boldsymbol{\Sigma}_0^{1/2} \boldsymbol{\delta})^\top \boldsymbol{\Sigma}_0^{-1} (\boldsymbol{\Sigma}_0^{1/2} \boldsymbol{\delta})$. This statement is only valid when $\boldsymbol{\Sigma} = \boldsymbol{\Sigma}_0$, according to Anderson (1958, p. 55), Muirhead (1982, p.

The original article can be found online at <https://doi.org/10.1007/s10182-018-00331-3>.

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26, Theorem 1.4.1) and Mathai and Provost (1992, p. 201, Corollary 5.1.3a). In this particular case, we have $\lambda = \delta^\top \times \delta$.

When $\Sigma \neq \Sigma_0$, we can capitalize on various series representations for the c.d.f. of $T^{(1)}$, such as the one mentioned by Mathai and Provost (1992, p. 95, Theorem 4.12b.1), truncate such series, and provide approximations to the c.d.f. of $T^{(1)}$. Instead, we favoured the representation of the quadratic form $T^{(1)}$ as in Mathai and Provost (1992, p. 29, formula (3.1a.4)) or Duchesne and Mичеаux (2010). By doing so, this control statistic can be written as a weighted sum of independent noncentral Chi-square random variables (Mathai and Provost 1992, p. 29),

$$T^{(1)} \sim \sum_{i=1}^p \lambda_i \chi_1^2(\tau_i),$$

where λ_i , $i = 1, \dots, p$, are the eigenvalues of the matrix

$$\Sigma^{1/2} \Sigma_0^{-1} \Sigma^{1/2},$$

τ_i is the square of the i th element of vector

$$U^\top \Sigma^{-1/2} \Sigma_0^{1/2} \delta;$$

U is the orthogonal matrix such that

$$U^\top \Sigma^{1/2} \Sigma_0^{-1} \Sigma^{1/2} U = \Lambda = \text{diag}(\lambda_1, \dots, \lambda_p).$$

Unfortunately, the incorrection we mentioned was only detected after the publication of Morais et al. (2018). The authors apologize for it and remind the reader that the correct marginal distribution of $T^{(1)}$, when $\Sigma \neq \Sigma_0$, ought to be considered in the statement of Theorem 1 in the original publication.

Fortunately, this had no bearing on the control limits of the joint scheme of type 1. However, the out-of-control ARL values referring to this joint scheme and to $\Sigma \neq \Sigma_0$, in tables 2–7 of Morais et al. (2018), had to be recalculated. They were obtained by taking advantage of the package *CompQuadForm* (Micheaux 2017) for the R statistical software (R Core Team 2013); the distribution function of the quadratic form in normal variables $T^{(1)}$ was computed using Imhof's method (1961), with absolute and relative accuracies equal to 10^{-6} .

Table 1 Control limits of the joint scheme of type 1 for μ and Σ , with in-control ARL equal to $ARL^{(i)}(0, \Sigma_0) = ARL^* = 500$

p	$n = p + 5$	Scheme i	$\beta^{(i)}$	$UCL_{\mu}^{(i)}$	$UCL_{\Sigma}^{(i)}$
2	7	1	0.001000501	13.814510	218.847114
3	8	1	0.001000501	16.265177	2221.257632
4	9	1	0.001000501	18.465717	23246.080493

Table 2 ARL values of the joint scheme of type 1 for μ and Σ , with in-control ARL equal to 500 (listed, for each value of δ , in order corresponding to the value in the original paper (Morais et al. 2018), the corrected value, and the Monte Carlo estimate)—(p, n) = (2, 7), scenarios 1 and 2

δ	(σ_0^2, ρ_0)		(σ^2, ρ)				Scenario
	(1, 0.3)	(1.01, 0.3)	(1.05, 0.3)	(1.2, 0.3)	(1.5, 0.3)	(2, 0.3)	
0	500.000	472.615	371.851	142.207	30.362	7.111	1
		458.104	325.220	108.945	24.010	5.989	
	499.513	457.384	325.012	108.899	24.045	5.984	
0.5	233.038	228.404	206.498	113.000	29.164	7.067	
		218.671	170.238	72.951	20.046	5.579	
	233.299	218.760	170.425	73.031	20.078	5.577	
2	4.295	4.371	4.681	5.858	7.304	5.162	
		4.265	4.147	3.762	3.128	2.298	
	4.294	4.267	4.155	3.753	3.124	2.298	
							2
0	500.000	391.981	302.119	113.203	25.239	6.305	
		322.194	233.364	82.977	19.759	5.300	
	499.673	322.206	233.731	82.877	19.734	5.307	
0.5	233.038	195.748	174.162	91.928	24.313	6.267	
		190.461	146.818	61.745	17.160	4.994	
	232.719	190.699	146.667	61.839	17.121	4.992	
2	4.295	3.675	3.926	4.884	6.096	4.521	
		4.276	4.148	3.728	3.046	2.204	
	4.295	4.276	4.151	3.729	3.048	2.201	

Table 2 continued

δ	(σ_0^2, ρ_0)		(σ^2, ρ)			Scenario	
	(1, 0.3)	(1.01, 0.303)	(1.05, 0.303)	(1.2, 0.303)	(1.5, 0.303)	(2, 0.303)	2
0	500.000	475.332	374.274	143.267	30.548	7.139	
		462.174	328.026	109.777	24.151	6.012	
	500.109	460.423	327.600	109.710	24.171	6.018	
0.5	233.038	195.748	174.162	91.928	24.313	6.267	
		219.241	170.769	73.267	20.139	5.598	
	233.022	218.933	170.810	73.240	20.163	5.593	
2.0	4.295	3.675	3.926	4.884	6.096	4.521	
		4.264	4.147	3.763	3.130	2.301	
	4.292	4.257	4.146	3.767	3.134	2.300	
	(1, 0.3)	(1.01, 0.315)	(1.05, 0.315)	(1.2, 0.315)	(1.5, 0.315)	(2, 0.315)	
0	500.000	486.581	384.357	147.718	31.331	7.258	
		478.702	339.499	113.211	24.735	6.107	
	499.871	479.215	339.559	113.126	24.755	6.120	
0.5	233.038	233.362	211.654	116.791	30.071	7.214	
		221.425	172.841	74.543	20.519	5.678	
	232.515	221.501	173.023	74.568	20.496	5.674	
2.0	4.295	4.464	4.781	5.990	7.478	5.266	
		4.264	4.147	3.767	3.140	2.313	
	4.298	4.264	4.151	3.760	3.139	2.315	

Table 3 ARL values of the joint scheme of type I for μ and Σ , with in-control ARL equal to 500 (listed, for each value of δ , in order corresponding to the value in the original paper (Morais et al. 2018), the corrected value, and the Monte Carlo estimate)—(p, n) = (2, 7), scenarios 3 and 4

δ	(σ_0^2, ρ_0)		(σ^2, ρ)				Scenario	
	(1, 0.3)	(1.01, 0)	(1.05, 0)	(1.2, 0)	(1.5, 0)	(2, 0)	(2, 0)	3
0	500.000	351.981	268.944	100.273	22.926	5.925	5.925	
		201.873	153.232	61.808	16.648	4.836	4.836	
	499.623	201.433	152.920	61.686	16.666	4.833	4.833	
0.5	233.038	166.314	148.612	80.071	21.999	5.886	5.886	
		144.557	113.341	50.264	14.947	4.604	4.604	
	232.702	144.393	113.151	50.187	14.934	4.609	4.609	
2	4.295	2.776	2.954	3.646	4.687	3.939	3.939	
		4.314	4.170	3.704	2.974	2.130	2.130	
	4.298	4.318	4.165	3.702	2.976	2.127	2.127	
0	500.000	483.532	380.878	145.232	29.386	7.140	7.140	
		453.318	323.716	109.812	24.345	6.059	6.059	
	500.118	451.532	321.468	108.046	24.532	6.011	6.011	
0.5	233.038	216.670	198.542	112.530	57.114	7.094	7.094	
		240.011	184.891	77.041	20.641	5.664	5.664	
	236.523	239.891	183.595	77.147	20.431	5.633	5.633	
2	4.295	3.703	3.985	5.092	6.678	5.048	5.048	
		4.363	4.234	3.819	3.151	2.301	2.301	
	4.292	4.407	4.178	3.845	3.183	2.258	2.258	

Table 3 continued

δ	(σ_0^2, ρ_0)	(σ^2, ρ)				Scenario
		(1, 0)	(1.01, 0.01)	(1.05, 0.01)	(1.2, 0.01)	4
0	500.000	472.748	371.961	142.244	30.366	7.111
		458.046	325.203	108.955	24.014	5.990
		454.630	321.563	110.588	23.346	6.010
0.5	499.368					
	233.038	216.670	198.542	112.530	29.386	7.094
		216.174	168.568	72.552	20.011	5.577
2	232.441	217.611	168.916	72.547	20.437	5.508
	4.295	4.450	4.762	5.947	7.376	5.175
		4.255	4.138	3.757	3.126	2.299
	4.241	4.303	4.100	3.812	3.142	2.322
	(1, 0)	(1.01, 0.1)	(1.05, 0.1)	(1.2, 0.1)	(1.5, 0.1)	(2, 0.1)
0	500.000	486.117	383.023	145.953	30.781	7.146
		452.183	323.340	110.011	24.424	6.076
		459.877	322.460	108.414	24.533	6.111
0.5	500.643					
	233.038	245.831	220.538	117.637	29.636	7.105
		193.599	153.635	69.420	19.954	5.631
2	232.188	192.446	154.350	70.510	19.651	5.578
	4.295	5.195	5.533	6.783	8.043	5.309
		4.175	4.068	3.715	3.121	2.315
	4.276	4.223	4.100	3.732	3.143	2.301

Table 4 ARL values of the joint scheme of type 1 for μ and Σ , with in-control ARL equal to 500 (listed, for each value of δ , in order corresponding to the value in the original paper (Morais et al. 2018), the corrected value, and the Monte Carlo estimate)—(p, n) = (3, 8), scenarios 1 and 2

δ	(σ_0^2, ρ_0)		(σ^2, ρ)				Scenario
	(1, 0.3)	(1.01, 0.3)	(1.05, 0.3)	(1.2, 0.3)	(1.5, 0.3)	(2, 0.3)	
0	500.000	467.837	351.115	110.813	19.156	4.239	1
		451.161	304.071	86.340	15.820	3.727	
	499.682	451.047	303.713	86.361	15.838	3.727	
0.5	207.363	203.173	182.179	89.087	18.592	4.223	
		193.297	146.773	56.875	13.401	3.517	
	207.522	193.365	147.025	56.945	13.408	3.512	
2	2.609	2.652	2.828	3.520	4.457	3.184	
		2.598	2.553	2.400	2.101	1.631	
	2.610	2.597	2.553	2.403	2.102	1.633	
	(1, 0.3)	(1.01, 0.18)	(1.05, 0.18)	(1.2, 0.18)	(1.5, 0.18)	(2, 0.18)	2
0	500.000	317.746	229.017	71.057	13.774	3.504	
		244.582	170.013	53.398	11.331	3.086	
	500.296	244.187	170.391	53.405	11.345	3.088	
0.5	207.363	150.968	130.612	59.566	13.422	3.492	
		147.492	109.853	41.210	10.122	2.954	
	207.657	147.616	109.988	41.130	10.108	2.955	
2	2.609	2.051	2.171	2.642	3.304	2.586	
		2.526	2.477	2.307	1.981	1.526	
	2.608	2.526	2.477	2.308	1.981	1.525	

Table 4 continued

δ	(σ_0^2, ρ_0) (1, 0.3)	(σ^2, ρ)				Scenario	
		(1.01, 0.303)	(1.05, 0.303)	(1.2, 0.303)	(1.5, 0.303)	(2, 0.303)	2
0	500.000	473.017	355.603	112.402	19.365	4.266	4.266
		458.692	309.012	87.573	15.985	3.750	3.750
	499.597	458.905	308.547	87.599	15.959	3.749	3.749
0.5	207.363	204.727	183.821	90.195	18.791	4.250	4.250
		194.296	147.680	57.361	13.516	3.537	3.537
	207.281	194.215	147.932	57.356	13.513	3.539	3.539
2	2.609	2.669	2.846	3.545	4.492	3.204	3.204
		2.599	2.555	2.402	2.104	1.635	1.635
	2.613	2.599	2.554	2.400	2.104	1.635	1.635
(1, 0.3)		(1.01, 0.315)	(1.05, 0.315)	(1.2, 0.315)	(1.5, 0.315)	(2, 0.315)	
0	500.000	494.460	374.403	119.179	20.253	4.379	4.379
		490.150	329.722	92.771	16.680	3.845	3.845
	499.809	489.982	330.250	92.756	16.669	3.849	3.849
0.5	207.363	211.042	190.558	94.873	19.636	4.362	4.362
		198.121	151.228	59.342	13.992	3.619	3.619
	207.283	198.142	151.276	59.223	13.995	3.618	3.618
2	2.609	2.736	2.920	3.644	4.632	3.286	3.286
		2.607	2.563	2.412	2.117	1.648	1.648
	2.604	2.605	2.566	2.412	2.119	1.648	1.648

Table 5 ARL values of the joint scheme of type I for μ and Σ , with in-control ARL equal to 500 (listed, for each value of δ , in order corresponding to the value in the original paper (Morais et al. 2018), the corrected value, and the Monte Carlo estimate)—(p, n) = (3, 8), scenarios 3 and 4

δ	(σ_0^2, ρ_0)		(σ^2, ρ)				Scenario	
	(1, 0.3)	(1.01, 0)	(1.05, 0)	(1.2, 0)	(1.5, 0)	(2, 0)	(2, 0)	3
0	500.000	247.574	176.316	55.399	11.520	3.167	3.167	
		128.517	94.872	35.122	8.829	2.718	2.718	
	500.038	128.506	94.909	35.137	8.838	2.718	2.718	
0.5	207.363	105.674	92.757	45.046	11.157	3.152	3.152	
		94.980	72.604	29.619	8.145	2.628	2.628	
	207.211	94.790	72.623	29.618	8.141	2.630	2.630	
2	2.609	1.397	1.453	1.685	2.108	2.033	2.033	
		2.428	2.374	2.188	1.857	1.439	1.439	
	2.612	2.425	2.375	2.188	1.856	1.439	1.439	
	(1, 0)	(1.01, -0.09)	(1.05, -0.09)	(1.2, -0.09)	(1.5, -0.09)	(2, -0.09)	(2, -0.09)	4
0	500.000	495.342	373.281	116.792	19.647	4.274	4.274	
		447.437	305.625	89.144	16.440	3.827	3.827	
	499.253	448.187	305.619	88.184	16.089	3.747	3.747	
0.5	207.363	176.419	164.114	88.556	18.953	4.256	4.256	
		232.951	173.260	63.558	14.250	3.627	3.627	
	206.973	232.406	172.166	62.235	13.864	3.547	3.547	
2	2.609	1.955	2.093	2.667	3.684	3.042	3.042	
		2.604	2.555	2.388	2.080	1.618	1.618	
	2.605	2.605	2.554	2.389	2.086	1.623	1.623	

Table 5 continued

δ	(σ_0^2, ρ_0)		(σ^2, ρ)				Scenario	
	(1, 0)	(1.01, 0.01)	(1.05, 0.01)	(1.2, 0.01)	(1.5, 0.01)	(2, 0.01)	4	4
0	500.000	468.149	351.365	110.880	19.162	4.239	4.239	
		451.083	304.072	86.370	15.827	3.728	3.728	
	500.360	451.202	303.275	86.450	15.828	3.726	3.726	
0.5	207.363	206.271	184.454	89.503	18.606	4.224	4.224	
		188.882	143.928	56.303	13.365	3.516	3.516	
	207.286	188.500	143.929	56.326	13.375	3.515	3.515	
2	2.609	2.740	2.920	3.625	4.547	3.201	3.201	
		2.597	2.554	2.401	2.104	1.634	1.634	
	2.610	2.598	2.554	2.398	2.100	1.633	1.633	
	(1, 0)	(1.01, 0.1)	(1.05, 0.1)	(1.2, 0.1)	(1.5, 0.1)	(2, 0.1)	(2, 0.1)	
0	500.000	497.829	375.427	117.480	19.715	4.280	4.280	
		439.085	301.424	88.852	16.472	3.835	3.835	
	500.434	438.885	302.352	88.018	16.129	3.759	3.759	
0.5	207.363	238.022	209.891	96.908	19.201	4.265	4.265	
		151.719	119.996	52.157	13.448	3.593	3.593	
	207.397	152.066	121.192	52.847	13.319	3.529	3.529	
2	2.609	3.638	3.855	4.663	5.396	3.352	3.352	
		2.600	2.560	2.421	2.140	1.670	1.670	
	2.609	2.601	2.559	2.416	2.126	1.647	1.647	

Table 6 ARL values of the joint scheme of type 1 for μ and Σ , with in-control ARL equal to 500 (listed, for each value of δ , in order corresponding to the value in the original paper (Morais et al. 2018), the corrected value, and the Monte Carlo estimate)—(p, n) = (4, 9), scenarios 1 and 2

δ	(σ_0^2, ρ_0)		(σ^2, ρ)					Scenario	
	(1, 0.3)	(1.01, 0.3)	(1.05, 0.3)	(1.2, 0.3)	(1.5, 0.3)	(2, 0.3)	1		
0	500.000	463.386	332.387	87.848	12.892	2.860			
		444.680	285.582	69.874	11.023	2.600			
	500.691	446.563	286.118	69.930	11.013	2.596			
0.5	187.575	183.761	163.557	71.725	12.604	2.853			
		173.864	129.284	45.864	9.500	2.482			
	187.590	173.998	129.456	45.861	9.517	2.478			
2	1.896	1.923	2.035	2.486	3.111	2.239			
		1.890	1.871	1.798	1.629	1.327			
	1.895	1.890	1.873	1.797	1.628	1.327			
0	(1, 0.3)	(1.01, 0.18)	(1.05, 0.18)	(1.2, 0.18)	(1.5, 0.18)	(2, 0.18)	2		
	500.000	256.507	173.950	46.421	8.334	2.295			
		191.322	129.157	36.264	7.149	2.094			
0.5	499.523	193.056	129.078	36.241	7.139	2.094			
	187.575	119.726	100.560	40.096	8.181	2.290			
		116.536	84.712	28.767	6.518	2.029			
2	187.749	117.181	84.742	28.793	6.516	2.028			
	1.896	1.466	1.531	1.793	2.172	1.773			
		1.793	1.772	1.690	1.508	1.236			
	1.895	1.791	1.772	1.690	1.508	1.236			

Table 6 continued

δ	(σ_0^2, ρ_0) (1, 0.3)	(σ^2, ρ)				Scenario	
		(1.01, 0.303)	(1.05, 0.303)	(1.2, 0.303)	(1.5, 0.303)	(2, 0.303)	2
0	500.000	470.825	338.644	89.656	13.079	2.881	2.881
		457.744	292.599	71.228	11.178	2.615	2.615
	500.037	456.821	292.922	71.382	11.184	2.617	2.617
0.5	187.575	185.670	165.616	73.011	12.784	2.874	2.874
		175.557	130.543	46.418	9.612	2.495	2.495
	187.600	175.226	130.580	46.386	9.616	2.499	2.499
2	1.896	1.936	2.049	2.506	3.141	2.255	2.255
		1.893	1.873	1.801	1.632	1.329	1.329
	1.896	1.890	1.872	1.799	1.633	1.329	1.329
(1, 0.3)		(1.01, 0.315)	(1.05, 0.315)	(1.2, 0.315)	(1.5, 0.315)	(2, 0.315)	
0	500.000	501.649	365.071	97.490	13.882	2.972	2.972
		498.745	321.287	77.290	11.837	2.693	2.693
	500.276	500.087	321.307	77.289	11.825	2.696	2.696
0.5	187.575	193.402	174.062	78.500	13.556	2.964	2.964
		180.064	135.230	48.800	10.085	2.564	2.564
	187.509	180.579	135.258	48.749	10.089	2.563	2.563
2	1.896	1.990	2.109	2.589	3.262	2.321	2.321
		1.903	1.883	1.812	1.646	1.341	1.341
	1.896	1.903	1.882	1.812	1.646	1.341	1.341

Table 7 ARL values of the joint scheme of type I for μ and Σ , with in-control ARL equal to 500 (listed, for each value of δ , in order corresponding to the value in the original paper (Morais et al. 2018), the corrected value, and the Monte Carlo estimate)—(p, n) = (4, 9), scenarios 3 and 4

δ	(σ_0^2, ρ_0)		(σ^2, ρ)		(σ^2, ρ)		Scenario	
	(1, 0.3)	(1.01, 0)	(1.05, 0)	(1.2, 0)	(1.5, 0)	(2, 0)	(2, 0)	3
0	500.000	170.359	114.706	32.154	6.550	2.042	2.042	
		89.746	64.382	21.912	5.320	1.833	1.833	
	499.763	89.918	64.331	21.932	5.323	1.834	1.834	
0.5	187.575	68.099	58.978	26.613	6.383	2.035	2.035	
		66.781	49.844	18.842	4.986	1.790	1.790	
	187.264	66.889	49.833	18.841	4.984	1.792	1.792	
2	1.896	1.071	1.088	1.163	1.329	1.354	1.354	
		1.646	1.624	1.544	1.379	1.164	1.164	
	1.895	1.647	1.624	1.544	1.379	1.165	1.165	
	(1, 0)	(1.01, -0.09)	(1.05, -0.09)	(1.2, -0.09)	(1.5, -0.09)	(2, -0.09)	(2, -0.09)	4
0	500.000	514.134	372.369	96.600	13.443	2.894	2.894	
		446.488	296.250	75.280	11.860	2.707	2.707	
	499.749	449.300	294.197	73.383	11.401	2.625	2.625	
0.5	187.575	139.927	133.658	71.334	28.775	13.037	13.037	
		228.766	166.073	54.425	10.475	2.595	2.595	
	187.682	228.900	164.145	52.586	9.989	2.507	2.507	
2	1.896	1.324	1.393	1.704	2.351	2.099	2.099	
		1.831	1.810	1.735	1.576	1.303	1.303	
	1.897	1.832	1.812	1.746	1.592	1.312	1.312	

Table 7 continued

δ	(σ_0^2, ρ_0)	(σ^2, ρ)				Scenario	
		(1, 0)	(1.01, 0.01)	(1.05, 0.01)	(1.2, 0.01)	(1.5, 0.01)	4
0	500.000	463.914	332.797	87.939	12.898	2.860	2.860
		447.056	286.646	69.928	11.039	2.600	2.600
	499.801	445.014	286.189	69.884	11.030	2.595	2.595
0.5	187.575	188.466	166.946	72.224	12.617	2.853	2.853
		168.345	125.820	45.239	9.477	2.481	2.481
	187.439	167.922	126.011	45.367	9.480	2.479	2.479
2	1.896	2.007	2.124	2.589	3.202	2.254	2.254
		1.897	1.877	1.805	1.635	1.330	1.330
	1.894	1.898	1.876	1.802	1.632	1.329	1.329
	(1, 0)	(1.01, 0.1)	(1.05, 0.1)	(1.2, 0.1)	(1.5, 0.1)	(2, 0.1)	
0	500.000	512.016	371.058	96.591	13.468	2.897	2.897
		425.455	283.917	73.883	11.785	2.701	2.701
	500.940	427.935	284.224	72.983	11.388	2.625	2.625
0.5	187.575	235.446	204.197	81.598	13.217	2.891	2.891
		122.049	96.785	41.089	9.695	2.560	2.560
	187.337	122.856	98.249	41.725	9.527	2.496	2.496
2	1.896	2.934	3.092	3.677	4.077	2.391	2.391
		1.948	1.930	1.863	1.697	1.372	1.372
	1.896	1.948	1.927	1.854	1.674	1.346	1.346

The reader must keep in mind that the out-of-control ARL values in Tables 2, 3, 4, 5, 6 and 7 refer to the joint schemes of type 1 with the parameters in Table 1 taken from Morais et al. (2018, Table 1).

Moreover, the out-of-control ARL values are listed in Tables 2, 3, 4, 5, 6 and 7 in order corresponding to the value in the original paper, the corrected value (in *italic*), and the corresponding estimate (obtained by Monte Carlo simulation with 10^6 replications), for each value of δ .

The correct out-of-control ARL values when $\Sigma \neq \Sigma_0$ in Tables 2, 3, 4, 5, 6 and 7 are rarely larger than the ones in the original paper. In this particular case, the values are set in bold, but they are still smaller than the ones of the two competing joint schemes described in Morais et al. (2018). Furthermore, the ARL reductions are in some cases substantial.

Consequently, the correct ARL values we obtained serve to increase rather than to lessen our belief that the joint scheme 1 has the best overall ARL performance, as we mentioned in the abstract and the conclusions and recommendations of Morais et al. (2018). Unsurprisingly, the ARL profiles of figures 1–4 of the original paper were not redrawn.

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