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PREFACE

Series B

Special Issue: Global Solution of Integer, Stochastic and Nonconvex Optimization Problems

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Integer, stochastic, and nonconvex optimization problems enable modeling of an immense range of important and realistic settings. Integer decision variables are used to model discrete decisions and logic, stochastic optimization models support decision-making in the face of uncertain outcomes that can be modeled with random variables, and nonconvex optimization allows modeling extremely flexible relationships between quantities in a model. While possessing tremendous modeling power, these problem classes are also among the most challenging optimization problems to solve. Indeed, most problems in these classes are at least $\mathcal{NP}$-hard, and hence theoretically efficient algorithms that work for any instance in these classes are likely not possible. Despite this challenge, there has been tremendous advances in computational methods that can provide exact or provably approximate solutions to these problems at scales of practical interest. This special issue contains 29 articles that contribute to this line of research.

Dedication to Shabbir Ahmed

This special issue is dedicated to Shabbir Ahmed (1969–2019) who made massive contributions to the fields of integer, stochastic, and nonconvex optimization in his

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career. A common theme in Ahmed's work is his invention of methods that enable practical computational solution of theoretically intractable problems. His collection of works is the inspiration for the theme of this special issue. We highlight some of his contributions in the areas of *Mixed-Integer Linear and Nonlinear Optimization*; *Stochastic, Robust, and Distributionally Robust Optimization*; and *Multi-stage Optimization*. We also use this list of topics to provide a rough categorization of the papers in this special issue.

Mixed-Integer Linear and Nonlinear Optimization Ahmed made enormous contributions to the field of integer programming. Some of this work was motivated by the need to solve challenging stochastic integer programs. Earliest in this line of work, Ahmed together with co-authors proposed the so-called *pairing inequalities*—a class of cutting-planes implied by two or more knapsack constraints [36]. This work predated the work on multi-row cutting-plane in the last two decades [29]. In [15], Ahmed and co-authors introduced and studied the notion of forbidden vertices, that is the study of convex hulls of polyhedral sets where a few extreme points are *forbidden* or removed. This study was directly motivated by the need to improve the classical integer L-shaped method [47]. Not only were the results successful in improving the L-shaped method [14], these ideas were relevant to the study of other combinatorial optimization problems such as [1, 39]. In [56], Ahmed and co-authors showed the critical importance of improving formulations by improving so-called *Big-M* coefficients for solving chance constrained problems, and developed the notion of quantile cuts [74]. With regards to general MILP formulations, Ahmed and his collaborators analyzed different logarithmic-sized models for non-separable piece-wise linear optimization [68], which have fundamentally changed the landscape of MILP formulations especially for approximating mixed-integer nonlinear programs (MINLPs).

Ahmed made significant contributions to the field of MINLPs, writing influential papers in various topics such as formulations, outer-approximations, inner-approximations, cutting-plane theory, duality theory, exact algorithms and approximation guarantees. In [66], Ahmed and co-authors develop various mixed-integer convex programming reformulations for binary hyperbolic programs. In [69], Ahmed and his co-authors exploiting the results in [19] developed extremely efficient branch-and-bound algorithms using polyhedral approximations to solve mixed-integer second-order conic programs. Motivated by the need to find good feasible solutions by inner-approximating MINLPs like the pooling problem [41] (via discretization), in [40], Ahmed and co-authors analysed various MILP reformulations and approximations of mixed-integer bilinear sets. In [43], Ahmed and co-authors studied various MINLP generalizations of fixed charge network flow problems. In [31, 34], Ahmed and co-authors developed a strong duality theory for augumented Lagrangian relaxations of MILPs and MINLPs with application to multi-stage programming [6]. Finally, in [5, 77, 78] Ahmed and co-authors explored various connections between structured nonlinear integer programs with submodular optimization in order to convexify, obtain cutting-planes and provide algorithmic results.

Stochastic, Robust, and Distributionally Robust Optimization Ahmed was a pioneer in the study of methods for solving stochastic integer programs (SIPs), including designing convergent algorithms for problems with integer recourse [11], studying classes of valid inequalities to improve the continuous relaxation [14, 35, 37], and deriving a new decomposition algorithm for SIPs with pure binary first-stage variables [3, 28]. Ahmed also made significant contributions to the study of sample average approximation for a variety of stochastic programming models, particularly those involving some sort of nonconvexity [10, 48, 55, 67]. Another area of interest for Ahmed was the study of models and methods for controlling risk, including extensive contributions to methods and approximations for problems with chance constraints [4, 8, 12, 49, 62, 74, 75] and problems seeking to optimize a (convex) risk measure [2, 13, 61]. In the area of distributionally robust optimization (DRO), Ahmed's work is differentiated from most other work in DRO by its emphasis on the study of practical methods for cases where the DRO problem may not have convex reformulation [18, 71–73]. Finally, Ahmed made significant advances in the use of stochastic optimization in applications [17, 59, 60].

Multi-stage Optimization Ahmed also made significant contributions to the study of multi-stage optimization problems, which are complicated by the need to anticipate future optimal decisions when making current decisions. Ahmed's earliest contributions in this area consisted of approximations and algorithms for solving capacity expansion problems [7, 9]. More recently, Ahmed and co-authors invented the stochastic dual dynamic integer programming (SDDIP) algorithm [82], which extends the stochastic dual dynamic programming algorithm, a leading algorithm for solving multi-stage stochastic *linear* programs, to the mixed-integer setting. Ahmed demonstrated the usefulness of SDDIP in solving problems in unit commitment [51, 81] and hydropower [44] applications. Ahmed generalized the SDDIP algorithm to more general cases [6]. This special issue contains several articles further extending this algorithm, demonstrating the influence these developments have had.

Overview of the Special Issue

This special issue contains 29 papers which we categorize and very briefly introduce as follows:

Mixed-Integer Linear and Nonlinear Optimization Coniglio, Furini, and Ljubíc [26] study a family of discrete optimization problems requiring the maximization of the expected value of a concave, strictly increasing, and differentiable function composed with a set-union operator. This problem generalizes the problem introduced by Ahmed and Atamtürk [5]. In [16], Atamtürk and Narayanan use polarity to give an outer polyhedral approximation for the epigraph of set functions, which gives an alternative proof for the convex hull description of the epigraph of submodular functions. Using lifting, Shi, Prokopyev, and Zeng [63] develop facet-defining inequalities for a mixed binary set modelling a structured submodular maximization problem. In [22], Castro, Cire, and Beck, propose a general lifting procedure that exploits the binary decision diagram encoding of binary integer programs. Cheon [25] analyses an outer-approximation based method for solving an optimization problem related to finding the best set of input values of a given trained neural network in the presence of constraints on input values. In [54], Nohra, Raghunathan, and Sahinidis present a new class of convex quadratic relaxations for quadratic programs with binary and continuous variables via the use of quadratic cuts. These cuts lead to an an outer-approximation of a semi-infinite convex relaxation which under certain conditions is equivalent to a well-known semidefinite relaxation. By exploiting augmented Lagrangian duals, Cordova, Oliveira and Sagastizábal [27] develop a convergent primal-dual algorithm for mixed-integer nonconvex optimization problems. By generalizing the Motzkin-Straus formulation, Stozhkov et al. [65] present a family of formulations for the maximum s-defective clique problem and the maximum s-plex problem. In [23], Chandra and Tawarmalani consider a probability quantification problem, which is the problem of estimating the probability of the optimal value of a structured convex program exceeding a given threshold value. Hazimeh, Mazumder and Saab [42] design a specialized nonlinear branch-and-bound algorithm capable of delivering certified optimality for large-scale $\ell_0\ell_2$-regularized regression problems. Bolusani and Ralphs [21] extend the well-known Benders decomposition framework to multilevel/multistage mixed-integer linear optimization problems. Bodur, Ahmed, Boland and Nemhauser [20] exploit the idea of resource-directive decomposition to reformulate loosely coupled integer programs so that they can be decomposed into a resource-directive master problem and a set of multiobjective programming subproblems. Gupta et al. [38] give novel dual bounds and the first provable approximation guarantees for the network reconfiguration problem. Song and Khan [64] develop novel relaxations for the solutions of parametric ordinary differential equations and prove the theoretical tightness of their relaxations in comparison to prior results.

Stochastic, Robust, and Distributionally Robust (DR) Optimization Malaguti, Monaci and Pruente [52] study the use of K-adaptable policies in stochastic optimization. In [76], Xie, Zhang, and Ahmed apply DRO in bottleneck combinatorial problems. Ho, Kılınç-Karzan, Küçükyavuz, and Lee [45] study the problem of DR chance-constrained programs under Wasserstein ambiguity, while Luo and Mehrotra [50] introduce a decomposition method for DR two-stage stochastic mixed-integer conic programs. In [24], Chen and Luedtke investigate theory of sample average approximation for two-stage stochastic LP when the relatively complete recourse assumption does not hold. Ramirez-Pico and Moreno [58] introduce a generalized adaptive-partition based method for two-stage stochastic LPs. In [57], Ragavan, Hunter, Pasupathy, and Taaffe explore an adaptive sampling line search method for stochastic integer programming. Wiebe, Cecílio, Dunlop, and Misener [70] study a robust optimization approach to warped Gaussion process-constrained optimization. Finally, Koçyiğit, Rujeerapaiboon, and Kuhn [46] explore the problem of robust multidimensional pricing.

Multi-stage Optimization Muir and Toriello [53] investigate a multi-stage extension of the classical node packing problem. In [30], Dowson, Morton, and Downward propose an algorithm for solving a class of bi-objective multi-stage stochastic linear programs. Zhang and Sun [80] present an extension of the stochastic dual dynamic integer programming algorithm for multi-stage stochastic mixed-integer nonlinear setting. In [32], Füllner and Rebennack study an extension of nested Benders decomposition to the non-convex setting. Yu and Shen [79] investigate solution methods for multi-stage distributionally robust mixed-integer programming with moment-based ambiguity sets. Finally, Gonzalez Grandon, Henrion, and Pérez-Aros [33] study a multi-stage problem with probabilistic constraints in the case of continuous distributions of the random variables.

Review Process All the papers in this special issue were reviewed according to the strict refereeing standards of *Mathematical Programming*. One paper in this special issue was co-authored by J. Luedtke and one paper was co-authored by N. Sahinidis. The paper that was co-authored by Luedtke was handled by Mathematical Programming, Series B editor-in-chief Sven Leyffer. The paper that was co-authored by Sahinidis was handled by co-editor Luedtke. Neither Luedtke nor Sahinidis had any information about the review of their papers.

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References

1. Abdi, A., Cornuéjols, G., Lee, D.: Resistant sets in the unit hypercube. Math. Oper. Res. **46**(1), 82–114 (2021)
2. Ahmed, S.: Convexity and decomposition of mean-risk stochastic programs. Math. Program. **106**(3), 433–446 (2006)
3. Ahmed, S.: A scenario decomposition algorithm for 0–1 stochastic programs. Oper. Res. Lett. **41**(6), 565–569 (2013)
4. Ahmed, S.: Convex relaxations of chance constrained optimization problems. Optim. Lett. **8**(1), 1–12 (2014)
5. Ahmed, S., Atamtürk, A.: Maximizing a class of submodular utility functions. Math. Program. **128**(1), 149–169 (2011)
6. Ahmed, S., Cabral, F.G., Freitas Paulo da Costa, B.: Stochastic Lipschitz dynamic programming. In: Math. Program. pp. 1–39 (2020)
7. Ahmed, S., King, A.J., Parija, G.: A multi-stage stochastic integer programming approach for capacity expansion under uncertainty. J. Glob. Optim. **26**(1), 3–24 (2003)
8. Ahmed, S., Luedtke, J., Song, Y., Xie, W.: Nonanticipative duality, relaxations, and formulations for chance-constrained stochastic programs. Math. Program. **162**(1–2), 51–81 (2017)
9. Ahmed, S., Sahinidis, N.V.: An approximation scheme for stochastic integer programs arising in capacity expansion. Oper. Res. **51**(3), 461–471 (2003)
10. Ahmed, S., Shapiro, A.: The sample average approximation method for stochastic programs with integer recourse (2002). http://www.optimization-online.org/DB_FILE/2002/02/440.pdf
11. Ahmed, S., Tawarmalani, M., Sahinidis, N.V.: A finite branch-and-bound algorithm for two-stage stochastic integer programs. Math. Program. **100**(2), 355–377 (2004)
12. Ahmed, S., Xie, W.: Relaxations and approximations of chance constraints under finite distributions. Math. Program. **170**(1), 43–65 (2018)
13. Ahmed, S., Çakmak, U., Shapiro, A.: Coherent risk measures in inventory problems. Eur. J. Oper. Res. **182**(1), 226–238 (2007)
14. Angulo, G., Ahmed, S., Dey, S.S.: Improving the integer L-shaped method. INFORMS J. Comput. **28**(3), 483–499 (2016)
15. Angulo, G., Ahmed, S., Dey, S.S., Kaibel, V.: Forbidden vertices. Math. Oper. Res. **40**(2), 350–360 (2015)
16. Atamtürk, A., Narayanan, V.: Submodular function minimization and polarity. Math. Program. (2022). https://doi.org/10.1007/s10107-020-01607-w
17. Basciftci, B., Ahmed, S., Gebraeel, N.Z., Yildirim, M.: Stochastic optimization of maintenance and operations schedules under unexpected failures. IEEE Trans. Power Syst. **33**(6), 6755–6765 (2018). https://doi.org/10.1109/TPWRS.2018.2829175

18. Basciftci, B., Ahmed, S., Shen, S.: Distributionally robust facility location problem under decision-dependent stochastic demand. Eur. J. Oper. Res. **292**(2), 548–561 (2021)
19. Ben-Tal, A., Nemirovski, A.: On polyhedral approximations of the second-order cone. Math. Oper. Res. **26**(2), 193–205 (2001)
20. Bodur, M., Ahmed, S., Boland, N., Nemhauser, G.L.: Decomposition of loosely coupled integer programs: a multiobjective perspective. Math. Program. (2022). https://doi.org/10.1007/s10107-021-01765-5
21. Bolusani, S., Ralphs, T.K.: A framework for generalized Benders' decomposition and its application to multilevel optimization. Math. Program. (2022). https://doi.org/10.1007/s10107-021-01763-7
22. Castro, M.P., Cire, A.A., Beck, J.C.: A combinatorial cut-and-lift procedure with an application to 0–1 second-order conic programming. Math. Program. (2022). https://doi.org/10.1007/s10107-021-01699-y
23. Chandra, A., Tawarmalani, M.: Probability estimation via policy restrictions, convexification, and approximate sampling. Math. Program. (2022). https://doi.org/10.1007/s10107-022-01823-6
24. Chen, R., Luedtke, J.: On sample average approximation for two-stage stochastic programs without relatively complete recourse. Math. Program. (2022). https://doi.org/10.1007/s10107-021-01753-9
25. Cheon, M.S.: An outer-approximation guided optimization approach for constrained neural network inverse problems. Math. Program. (2022). https://doi.org/10.1007/s10107-021-01653-y
26. Coniglio, S., Furini, F., Ljubić, I.: Submodular maximization of concave utility functions composed with a set-union operator with applications to maximal covering location problems Math. Program. (2022). https://doi.org/10.1007/s10107-022-01864-x
27. Cordova, M., Oliveira, W.d., Sagastizábal, C.: Revisiting augmented Lagrangian duals. Math. Program. (2022). https://doi.org/10.1007/s10107-021-01703-5
28. Deng, Y., Ahmed, S., Shen, S.: Parallel scenario decomposition of risk-averse 0–1 stochastic programs. INFORMS J. Comput. **30**(1), 90–105 (2018)
29. Dey, S.S., Molinaro, M.: Theoretical challenges towards cutting-plane selection. Math. Program. **170**(1), 237–266 (2018)
30. Dowson, O., Morton, D.P., Downward, A.: Bi-objective multistage stochastic linear programming. Math. Program. (2022). https://doi.org/10.1007/s10107-022-01872-x
31. Feizollahi, M.J., Ahmed, S., Sun, A.: Exact augmented Lagrangian duality for mixed integer linear programming. Math. Program. **161**(1), 365–387 (2017)
32. Füllner, C., Rebennack, S.: Non-convex nested Benders decomposition. Math. Program. (2022). https://doi.org/10.1007/s10107-021-01740-0
33. Gonzalez Grandon, T., Henrion, R., Pérez-Aros, P.: Dynamic probabilistic constraints under continuous random distributions. Math. Program. (2022). https://doi.org/10.1007/s10107-020-01593-z
34. Gu, X., Ahmed, S., Dey, S.S.: Exact augmented Lagrangian duality for mixed integer quadratic programming. SIAM J. Optim. **30**(1), 781–797 (2020)
35. Guan, Y., Ahmed, S., Nemhauser, G.L.: Sequential pairing of mixed integer inequalities. Discret. Optim. **4**(1), 21–39 (2007)
36. Guan, Y., Ahmed, S., Nemhauser, G.L.: Cutting planes for multistage stochastic integer programs. Oper. Res. **57**(2), 287–298 (2009)
37. Guan, Y., Ahmed, S., Nemhauser, G.L., Miller, A.J.: A branch-and-cut algorithm for the stochastic uncapacitated lot-sizing problem. Math. Program. **105**(1), 55–84 (2006)
38. Gupta, S., Khodabakhsh, A., Mortagy, H., Nikolova, E.: Electrical flows over spanning trees. Math. Program. (2022). https://doi.org/10.1007/s10107-020-01614-x
39. Gupte, A.: Convex hulls of superincreasing knapsacks and lexicographic orderings. Discret. Appl. Math. **201**, 150–163 (2016)
40. Gupte, A., Ahmed, S., Cheon, M.S., Dey, S.: Solving mixed integer bilinear problems using MILP formulations. SIAM J. Optim. **23**(2), 721–744 (2013)
41. Gupte, A., Ahmed, S., Dey, S.S., Cheon, M.S.: Relaxations and discretizations for the pooling problem. J. Global Optim. **67**(3), 631–669 (2017)
42. Hazimeh, H., Mazumder, R., Saab, A.: Sparse regression at scale: branch-and-bound rooted in first-order optimization. Math. Program. (2022). https://doi.org/10.1007/s10107-021-01712-4
43. He, Q., Ahmed, S., Nemhauser, G.L.: Minimum concave cost flow over a grid network. Math. Program. **150**(1), 79–98 (2015)
44. Hjelmeland, M.N., Zou, J., Helseth, A., Ahmed, S.: Nonconvex medium-term hydropower scheduling by stochastic dual dynamic integer programming. IEEE Trans. Sustain. Energy **10**(1), 481–490 (2018)

45. Ho-Nguyen, N., Kılınç-Karzan, F., Küçükyavuz, S., Lee, D.: Distributionally robust chance-constrained programs with right-hand side uncertainty under wasserstein ambiguity. Math. Program. (2022). https://doi.org/10.1007/s10107-020-01605-y
46. Koçyiğit, Ç., Rujeerapaiboon, N., Kuhn, D.: Robust multidimensional pricing: Separation without regret. Math. Program. (2022). https://doi.org/10.1007/s10107-021-01615-4
47. Laporte, G., Louveaux, F.V.: The integer L-shaped method for stochastic integer programs with complete recourse. Oper. Res. Lett. **13**(3), 133–142 (1993)
48. Luedtke, J., Ahmed, S.: A sample approximation approach for optimization with probabilistic constraints. SIAM J. Optim. **19**(2), 674–699 (2008)
49. Luedtke, J., Ahmed, S., Nemhauser, G.L.: An integer programming approach for linear programs with probabilistic constraints. Math. Program. **122**(2), 247–272 (2010)
50. Luo, F., Mehrotra, S.: A decomposition method for distributionally-robust two-stage stochastic mixed-integer conic programs. Math. Program. (2022). https://doi.org/10.1007/s10107-021-01641-2
51. Mahmutoğulları, A.I., Ahmed, S., Çavuş, O., Aktürk, M.S.: The value of multi-stage stochastic programming in risk-averse unit commitment under uncertainty. IEEE Trans. Power Syst. **34**(5), 3667–3676 (2019). https://doi.org/10.1109/TPWRS.2019.2902511
52. Malaguti, E., Monaci, M., Pruente, J.: K-adaptability in stochastic optimization. Math. Program. (2022). https://doi.org/10.1007/s10107-021-01767-3
53. Muir, C., Toriello, A.: Dynamic node packing. Math. Program. (2022). https://doi.org/10.1007/s10107-021-01624-3
54. Nohra, C.J., Raghunathan, A.U., Sahinidis, N.V.: SDP-quality bounds via convex quadratic relaxations for global optimization of mixed-integer quadratic programs. Math. Program. (2022). https://doi.org/10.1007/s10107-021-01680-9
55. Pagnoncelli, B.K., Ahmed, S., Shapiro, A.: Sample average approximation method for chance constrained programming: theory and applications. J. Optim. Theory Appl. **142**(2), 399–416 (2009)
56. Qiu, F., Ahmed, S., Dey, S.S., Wolsey, L.A.: Covering linear programming with violations. INFORMS J. Comput. **26**(3), 531–546 (2014)
57. Ragavan, P.K., Hunter, S.R., Pasupathy, R., Taaffe, M.R.: Adaptive sampling line search for local stochastic optimization with integer variables. Math. Program. (2022). https://doi.org/10.1007/s10107-021-01667-6
58. Ramirez-Pico, C., Moreno, E.: Generalized adaptive partition-based method for two-stage stochastic linear programs with fixed recourse. Math. Program. (2022). https://doi.org/10.1007/s10107-020-01609-8
59. Santoso, T., Ahmed, S., Goetschalckx, M., Shapiro, A.: A stochastic programming approach for supply chain network design under uncertainty. Eur. J. Oper. Res. **167**(1), 96–115 (2005)
60. Schütz, P., Tomasgard, A., Ahmed, S.: Supply chain design under uncertainty using sample average approximation and dual decomposition. Eur. J. Oper. Res. **199**(2), 409–419 (2009)
61. Shapiro, A., Ahmed, S.: On a class of minimax stochastic programs. SIAM J. Optim. **14**(4), 1237–1249 (2004)
62. Shen, S., Smith, J., Ahmed, S.: Expectation and chance-constrained models and algorithms for insuring critical paths. Manag. Sci. **56**, 1794–1814 (2010)
63. Shi, X., Prokopyev, O.A., Zeng, B.: Sequence independent lifting for a set of submodular maximization problems. Math. Program. (2022). https://doi.org/10.1007/s10107-022-01801-y
64. Song, Y., Khan, K.A.: Optimization-based convex relaxations for nonconvex parametric systems of ordinary differential equations. Math. Program. (2022). https://doi.org/10.1007/s10107-021-01654-x
65. Stozhkov, V., Buchanan, A., Butenko, S., Boginski, V.: Continuous cubic formulations for cluster detection problems in networks. Math. Program. (2022). https://doi.org/10.1007/s10107-020-01572-4
66. Tawarmalani, M., Ahmed, S., et al.: Global optimization of 0–1 hyperbolic programs. J. Glob. Optim. **24**(4), 385–416 (2002)
67. Verweij, B., Ahmed, S., Kleywegt, A.J., Nemhauser, G., Shapiro, A.: The sample average approximation method applied to stochastic routing problems: A computational study. Comput. Optim. Appl. **24**(2), 289–333 (2003)
68. Vielma, J.P., Ahmed, S., Nemhauser, G.: Mixed-integer models for nonseparable piecewise-linear optimization: Unifying framework and extensions. Oper. Res. **58**(2), 303–315 (2010)
69. Vielma, J.P., Ahmed, S., Nemhauser, G.L.: A lifted linear programming branch-and-bound algorithm for mixed-integer conic quadratic programs. INFORMS J. Comput. **20**(3), 438–450 (2008)

70. Wiebe, J., Cecílio, I., Dunlop, J., Misener, R.: A robust approach to warped Gaussian process-constrained optimization. Math. Program. (2022). https://doi.org/10.1007/s10107-021-01762-8
71. Xie, W., Ahmed, S.: Distributionally robust chance constrained optimal power flow with renewables: a conic reformulation. IEEE Trans. Power Syst. **33**(2), 1860–1867 (2017)
72. Xie, W., Ahmed, S.: Distributionally robust simple integer recourse. CMS **15**(3), 351–367 (2018)
73. Xie, W., Ahmed, S.: On deterministic reformulations of distributionally robust joint chance constrained optimization problems. SIAM J. Optim. **28**(2), 1151–1182 (2018)
74. Xie, W., Ahmed, S.: On quantile cuts and their closure for chance constrained optimization problems. Math. Program. **172**(1), 621–646 (2018)
75. Xie, W., Ahmed, S.: Bicriteria approximation of chance-constrained covering problems. Oper. Res. **68**(2), 516–533 (2020)
76. Xie, W., Zhang, J., Ahmed, S.: Distributionally robust bottleneck combinatorial problems: uncertainty quantification and robust decision making. Math. Program. (2022). https://doi.org/10.1007/s10107-021-01627-0
77. Yu, J., Ahmed, S.: Maximizing a class of submodular utility functions with constraints. Math. Program. **162**(1–2), 145–164 (2017)
78. Yu, J., Ahmed, S.: Polyhedral results for a class of cardinality constrained submodular minimization problems. Discret. Optim. **24**, 87–102 (2017)
79. Yu, X., Shen, S.: Multistage distributionally robust mixed-integer programming with decision-dependent moment-based ambiguity sets. Math. Program. (2022). https://doi.org/10.1007/s10107-020-01580-4
80. Zhang, S., Sun, X.A.: Stochastic dual dynamic programming for multistage stochastic mixed-integer nonlinear optimization. Math. Program. (2022). https://doi.org/10.1007/s10107-022-01875-8
81. Zou, J., Ahmed, S., Sun, X.A.: Multistage stochastic unit commitment using stochastic dual dynamic integer programming. IEEE Trans. Power Syst. **34**(3), 1814–1823 (2018)
82. Zou, J., Ahmed, S., Sun, X.A.: Stochastic dual dynamic integer programming. Math. Program. **175**(1), 461–502 (2019)