



## Special Issue: Topics in Stochastic Programming

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Stochastic programming has seen recent advances with far-reaching impact involving risk measures, distributionally robust optimization, and applications in areas ranging from energy and natural resources to economics and finance to statistics and machine learning. This special issue on stochastic programming includes papers in: (i) risk and distributionally robust optimization; (ii) scenario generation, reduction, and analysis; (iii) asymptotic analysis, including consistency and rates of convergence; and (iv) algorithms, computation, and applications.

This special issue is dedicated to Jitka Dupačová (1939–2016), one of the founders of stochastic programming. Her deep contributions continue to influence the state of the field today, as suggested by the four areas just sketched. Dupačová's early work in minimax solutions of stochastic programs [6,24] predated by half a century a recent surge of work in distributionally robust optimization. Minimax formulations are a primary means of studying stability in stochastic optimization [7]. Dupačová also pioneered a complementary approach via her contamination technique [8], which she extended in multiple directions (e.g., [4,9]). Her minimax and contamination work, along with further work focused on risk aversion (e.g., [17,18]), are practically useful and well-grounded in theory.

Two others major lines of Dupačová's research contributions concern scenario generation and scenario reduction [15], with precursor work including [5,21], and her work on asymptotics [10,23]. Dupačová continually nudged herself and colleagues to pursue ideas that generalized to multi-stage problems [11,13,22], foreseeing the growing

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prominence of time-dynamic stochastic optimization. Dupačová applied stochastic programming to real-world problems in water management [14], melt control [20], and especially finance [2,12,16,19].

Jitka Dupačová was the first woman vice dean at the Faculty of Mathematics and Physics at Charles University in 1979. She became the Faculty's first female full professor in 1986. That same year Dupačová organized the fourth International Conference on Stochastic Programming in Prague, a significant and very successful undertaking. She established a new PhD program in Econometrics and Operations Research at Charles University. Dupačová supervised 15 PhD students, among them Miloš Kopa, Petr Lachout, and Pavel Popela, all active in the stochastic programming community, and Petr Dobiáš, Václav Kozmík, Jana Čerbáková, and Jan Polívka, all active in the financial industry.

This special issue contains 13 excellent papers, which we group and summarize as follows:

- *Risk and distributionally robust optimization*
  - Nilay Noyan, Merve Meraklı and Simge Küçükyavuz [31] formulate two-stage stochastic programs that seek to outperform a multivariate benchmark using CVaR constraints for second-stage metrics beyond cost. Novel effective cut-generation algorithms are applied to a multi-criteria network design problem for disaster relief.
  - Alois Pichler and Huifu Xu [32] study stability of a distributionally robust, risk-averse two-stage stochastic program. Here, stability concerns how the optimal value and optimal solution change as the ambiguity set, feasible region, and underlying support set of the random vector vary.
  - Joint chance-constraints (CCs) can be approximated using single CCs via a Bonferroni bound. Under certain conditions, a distributionally robust (DR) single CC permits a convex reformulation. Weijun Xie, Shabbir Ahmed and Ruiwei Jiang [35] find conditions under which an optimized union bound of a DR joint CC yields a convex problem.
  - Jingnan Fan and Andrzej Ruszczyński [26] introduce a new class of dynamic risk measures, called process-based risk measures, and develop theoretical foundations for it. Their approach is designed to handle decision-dependent probability distributions, including those that arise in Markov decision processes, while ensuring time consistency.
- *Scenario generation, reduction, and analysis*
  - While most scenario-generation approaches in stochastic programming focus on deviations from a nominal distribution, Jamie Fairbrother, Amanda Turner and Stein Wallace [25] develop a problem-driven approach, focusing on model formulations involving tail risk, such as conditional value at risk.
  - Scenario-generation and reduction schemes often hinge only on distances between probability measures. Motivated by stability results, René Henrion and Werner Römisch [28] instead develop problem-specific methods that use semi-infinite programs for classes of two-stage and chance-constrained models.

- Napat Rujeerapaiboon, Kilian Schindler, Daniel Kuhn and Wolfram Wiesemann [33] find  $n$ -point distributions with maximum Wasserstein distance to their closest  $m$ -point distributions,  $m < n$ , i.e., distributions that do not lend themselves to scenario reduction. They also bound benefits of continuous over discrete scenario reduction.
  - Simone Garatti and Marco Campi [27] optimize over an intersection of scenario-based constraint sets. They study risk, i.e., the probability a scenario-based solution is infeasible to constraints drawn from the same distribution, complexity, i.e., the number of supporting constraint sets, and characterize the dependence of risk and complexity.
- *Asymptotic analysis*
    - Arnab Sur and John Birge [34] develop consistency and large-deviation results for nonlinear stochastic optimization models with expected-value constraints. The analysis allows for sequences of approximating measures beyond empirical measures and extends to DRO models using a minimax formulation.
    - Using the law of the iterated logarithm, Dirk Banholzer, Jörg Fliege and Ralf Werner [1] derive rates of convergence for sample average approximation estimators, which hold almost surely and in mean, rather than in distribution. As a consequence, they characterize the asymptotic bias of optimal estimators and optimality gap estimators.
  - *Algorithms, computation, and applications*
    - Merve Bodur and Jim Luedtke [3] approximate multi-stage stochastic linear programs using linear decision rules, in the primal and dual. Distinguishing state and control variables yields more flexible policies and tighter bounds than those from static linear decision rules. Applications are shown in inventory planning and capacity expansion.
    - Leo Lozano and Cole Smith [30] solve a class of two-stage stochastic programs with only binary decisions by reformulating the second stage using binary decision diagrams, parameterized by first-stage decisions. The resulting reformulation is solved using an enhanced Benders' decomposition algorithm with application to a stochastic TSP.
    - Andrea Lodi, Enrico Malaguti, Giacomo Nannicini and Dimitri Thomopulos [29] propose a branch-and-cut algorithm for solving convex nonlinear chance-constrained optimization problems—involving recovery for scenarios violating the chance constraint—with an application in hydro scheduling.

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