

The 2013 Newton Institute Programme on polynomial optimization

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The rapidly growing field of *polynomial optimisation* (PO) is concerned with optimisation problems in which the objective and constraint functions are all polynomials. There are applications of PO in a surprisingly wide variety of contexts, including, for example, operational research, statistics, applied probability, quantitative finance, theoretical computer science and various branches of engineering and the physical sciences. Not only that, but current research on PO is remarkably inter-disciplinary in nature, involving researchers from all of the above-mentioned disciplines, together with several branches of mathematics including graph theory, numerical analysis, algebraic geometry, commutative algebra and moment theory.

This special issue of *Mathematical Programming Series B* was originally conceived during a 4-week residential programme on PO which took place in July and August 2013 at the *Isaac Newton Institute for the Mathematical Sciences*, an internationally recognised research institute in Cambridge, United Kingdom. The programme

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included a summer school, a workshop, a series of presentations from key speakers, and a day devoted to interactions between academics and industry. The organizers of the event were Adam N. Letchford, Jean Bernard Lasserre, Markus Schweighofer and Jörg Fliege, with Monique Laurent and Kurt Anstreicher acting as international advisors. The event was attended by over 80 researchers from over 20 countries. Details of the programme can be found on the web at: <http://www.newton.ac.uk/event/POP>.

We received a total of twenty submissions to the special issue. All submissions were subjected to a strict refereeing process, as would be expected of a premier journal like MPB. In the end, only twelve of the submissions were accepted. A summary of these submissions, in order of acceptance, follows:

- *van Dam and Sotirov* give some new bounding procedures for a rather general graph partitioning problem, and show that the resulting bounds dominate previous known ones, yet remain tractable in practice.
- *Peña et al.* show that a wide family of PO problems can be reformulated as linear optimisation problems over completely positive cones. This extends a result of Burer, which applied only to certain mixed 0–1 quadratic programs.
- *de Klerk et al.* analyse a known polynomial-time approximation scheme for minimising a polynomial of fixed-degree over the simplex. They establish intriguing connections with Bernstein polynomials and multinomial distributions.
- *Bomze and Overton* study a trust-region-like problem in which one wishes to optimise a non-convex quadratic function over the intersection of two ellipsoids. They give conditions for this problem to be polynomial-time solvable.
- *Magron et al.* use a combination of max-plus algebra and sums-of-squares arguments to compute verifiable numerical proofs of validity for inequalities involving polynomials and/or transcendental functions.
- *Lasserre* studies certain convex sets that are defined via polynomial inequalities and quantifiers. These have applications in robust optimisation. He shows how to construct tractable inner and outer approximations of these sets.
- *de Laat and Vallentin* presents an innovative approach to packing problems in discrete geometry, based on a (highly non-trivial) combination of graph theory, topology and semidefinite programming.
- *Nie* uses a combination of convex algebraic geometry and semidefinite programming to derive local optimality conditions for PO problems. In some cases, these conditions enable one to enumerate local minima efficiently.
- *Ottaviani et al.* use convex algebraic geometry to classify all convex sets that can be obtained by intersecting the semidefinite cone of order four with a three-dimensional hyperplane.
- *Gouveia et al.* use matrix factorisations and second order cone constraints to construct convex sets which lie in a space of polynomial dimension and which, when projected into an appropriate subspace, approximate any given polytope.
- *Djeumou Fomeni et al.* present a new approach to mixed 0–1 polynomial programs, based on a combination of the classical Reformulation-Linearisation Technique and a new way of generating cutting planes.
- *Vera and Dobre* consider PO problems whose difficulty arises due to their extreme symmetry. They show that approaches based on copositive representations can be speeded up significantly by handling the symmetry explicitly.

We think that these twelve papers give a good “snapshot” of current work in PO.

We wish to thank the former and current directors of the Newton Institute (Sir David Wallace and Professor John Toland) for allowing the PO programme to take place, along with all of the administrative staff at the Institute, who enabled the event to run smoothly. We also thank the former and current editors of MPB (Professor Danny Ralph and Professor Jong-Shi Pang) for permitting and overseeing the special issue, along with all of the authors who submitted papers to the issue, and all of those who acted as anonymous referees.