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ORIGINAL PAPER

Political and non-political side activities in an agency framework

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Abstract
This paper studies side activities, including political activities, in the context of a hidden action agency problem. Given increases in the number of employees working from home and increases in managerial political engagement, such activities have become more prevalent. We examine the impact of these activities on the optimal contact, the agent's welfare, the firm's profit, and total welfare. For the case of political activities, we study the impact of external negative and positive feedback as the result of these activities on the optimal contract and all equilibrium variables. We ask whether the firm should encourage or discourage these activities.

Keywords Side activities · Hidden action

JEL Classification L20 · L21

1 Introduction

Two recent trends in firms and organizations stand out. First, due to the Covid -19 pandemic, many workers now work from home. In fact, a recent survey by McKinsey, reports that 58% of the U.S. workforce works at home at least part of the time.[1] While we are not focusing on working from home, it has enabled agents to engage in non-work or side activities during the workday which are not available at the office. Second, the percentage of CEOs who engage in political activism has

[1] See McKinsey (2022).

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increased from 98% in 2011 to 37.53% in 2019.[2] While these two facts seem unrelated, each represents a growing trend where agents in organizations are exerting effort other than work effort in the course of their workday and in the context of a possible agency problem. They are exerting effort (albeit for different reasons) in "side activities" other than actual work effort while under contract for work effort. This paper will focus on and examine such side activities in a hidden action principal agent framework and ask under what conditions they may or may not lead to increased equilibrium value for the firm. Further, we will study the effect of side activities on the agent's welfare and the optimal incentive pay. A key issue to be addressed is whether the firm should encourage or discourage such activities.

To my knowledge, the theoretical economics literature has not considered the issue of CEO activism or engagement in what we call side activities in the context of an agency model. The literature does have many papers on the motivation for CEO activism and its effect on firm value. Theoretical papers by Melloni et al. (2019) and Hambrick and Wowak (2021) inspired by Chatterji and Toffel (2018, 2019) point out that CEO activism can affect firm value through responses of employees, investors and customers. The latter constituents may have a stronger identification with a company if the CEO's positions on issues align with theirs. This increased identification could lead to increased productivity and retention among employees. Customers may exhibit increased brand loyalty, and investors who share beliefs with the CEO may react in a favorable way. On the other hand, political statements could also irritate constituents, if their beliefs are counter to those of the vocal CEO. In this case, adverse effects on the firm's value could obtain. These papers treat CEO activism as costless messaging (cheap talk) by the CEO meant to signal type and thereby affect cash flow through external or internal constituents. What is missing in this literature is an economic analysis of the endogenous interaction of unobservable and costly work effort and side activity effort within the context of the firm's optimal contract. This paper will focus on this setting.

There are many empirical papers documenting both positive and negative effects on firm value of CEO activism. See Mkrtchyan et al. (2022) for an interesting discussion. With a large data set, Mkrtchyan et al. (2022) provide empirical evidence of positive effects on firm value of activism, especially in competitive and polarized environments. Gangopadhyay and HomRoy (2022) likewise report positive effects, while Bedendo and Siming (2021) and Bhagwat et al. (2020) document negative effects of activism.

Empirical work has also documented both negative and positive effects on pay due to the presence of side activities. Burbano (2016) provides evidence that side activities generate lower wages. Barrero et al. (2022) shows that the provision of more opportunity for employees to work from home allowed companies to reduce the growth of wages. On the other hand, Adrjan et al. (2023) provide evidence of rising pay induced by increased side activities.

[2] See Mkrtchyan et al. (2022).

These papers are just a sample of the empirical work on this topic. The takeaway from these studies is that side activities have had both positive and negative effects on firm value and pay. Empirical results on wages and profits are mixed.

The present paper will take a completely different approach in explaining these varied empirical observations. We think of non-political and political side activities as effort consuming tasks which are separate from work effort and which can interact with work effort in a moral hazard agency setting. Our paper will use the interaction of work effort and side activity effort within the context of an optimal contract to offer an explanation for the empirical observations of both positive and negative effects on profit and pay as a result of the presence of side activities.

In the case of non-political side activities, the principal is a manager and the agent is a worker. The principal cannot observe effort devoted to side activities or effort devoted to cash flow. Such side activities result in no external reaction.

If side activities are political, the agent is regarded as a CEO and the shareholders represent the principal. In this case, the principal cannot observe the effort devoted to side activities, but the principal can observe the political statements. Political statements can then lead to exogenous reactions from customers, employees and stakeholders. These reactions can be positive or negative. In the case of political activities, the above mentioned theoretical literature concentrates solely on the reactions of different constituents to different messages sent by the CEO. It then treats the actions within the firm as a black box. We treat the reactions as an exogenous cost or benefit per unit of the political activity, we concentrate on the interaction of work effort and political activity effort, and we study the effects of reactions on the optimal agency contract, the agent's welfare, and profit of the firm.

We formulate a hidden action agency problem in which the agent derives intrinsic value through engaging in side activities. These activities and work effort interact to generate costs and benefits for the agent. On the benefit side, we consider two scenarios. In the first scenario, we assume that the intrinsic benefit of side activities is independent of work effort, whereas in the second scenario the benefit of side activities is dependent on the level of work effort. Given these two alternative settings, we begin by analyzing the case where side activities provoke no external reactions. This case is relevant for non-political side activities but not necessarily for political ones. However, it is interesting to examine the no feedback setting for both cases, because it is important to understand the interaction between work effort and non-work effort activities with the external reaction channel closed.

We show that if the benefit of side activities is independent of work effort and there is no external response, then the firm will definitely lose value. This effect is due to the fact that extra work effort decreases the total net benefit of side activities in the agent's payoff, which is equivalent to side activities and work effort being substitutes in the independence model. The firm will also pay the agent a greater incentive share of cash flow in equilibrium, relative to the no side activities equilibrium. We show that in the independent case, side activities raise the marginal benefit of the incentive share and lower its marginal cost. The agent benefits from side activities, but the firm does not, in this case.

If the benefit of side activities in the agent's payoff is dependent on work effort and there is no external feedback, then the effect of side activities on the firm's equilibrium profit turns on whether work effort has a positive or negative marginal effect on the *total* (as opposed to marginal) net benefit of side activities, in the agent's payoff. If this effect is positive, firm value is enhanced by side activities and if the effect is negative, firm value falls with side activities. In the case where extra work effort raises the total net benefit of side activities to the agent, it is also true that the equilibrium incentive share declines. We show that side activities raise the incentive share's marginal cost, while lowering its marginal benefit. While the direct effect of side activities is to raise the agent's welfare, the indirect effect through the incentive share lowers the agent's welfare, making the overall impact indeterminate. For the case where extra work effort lowers the total net benefit of side activities to the agent, the effect on the incentive share can be positive or negative. In this case, we show that side activities lower the marginal cost of the incentive share but have an ambiguous effect on its marginal benefit, again making the effect on the agent's welfare indeterminate. However, these results do show that in cases where the incentive share rises with the presence of side activities, a necessary condition is that extra work effort lowers the total net benefit of side activities.

The results on profit and side activities in the absence of external feedback are important for organizational policy regulating side activities. From the organization's viewpoint, side activities should be encouraged if more work effort increases the agent's total net benefit of side activities, but they should be discouraged if more work effort reduces the agent's total net benefit of side activities. This aspect of the effects of side activities has been ignored by existing literature, because it has only focused on the effects of external reactions to political activism by CEO's and not the interaction of work effort and side activity effort in the context of an optimal agency contract within the organization.

The next section of the paper considers external reactions to political side activities. Feedback can be positive or negative, and we model the strength or intensity of feedback with a parameter. We look at feedback affecting the agent's intrinsic payoff and feedback affecting the firm's cash flow. The effect of a change in the intensity of feedback depends on whether political activities and work effort are complements or substitutes in the agent's payoff.

In the complements case, work effort and political activities increase with greater positive feedback and decrease with greater negative feedback. Likewise, the firm's profit rises with more positive feedback and falls with more negative feedback, as intuition would suggest.

The substitutes case can yield surprising results. When political activities and work effort are substitutes in the agent's payoff, an increase in negative feedback lowers political activity, raises work effort, and it can, under certain conditions, *raise* the firm's payoff. On the other hand, if increased feedback is positive, the agent's political activity increases, work effort decreases, and the firm's payoff can *fall*, under certain conditions. We obtain the counter intuitive results that more intense negative feedback can increase work effort and raise the firm's bottom line, and greater positive feedback can lower work effort and harm the firm's bottom line.

Thus, if political activities generate external feedback, there are conditions under which the firm would expect to see greater profit with more negative feedback if work effort and political activities are substitutes, but lesser profit if they are complements in the agent's payoff. Likewise greater positive feedback generates more profit if the two efforts are complements and less profit, under certain conditions, if they are substitutes in the agent's payoff. Without the analysis presented in this paper, casual reasoning is unlikely to arrive at these conclusions affecting firm value.

The effects of feedback on the agent's payoff come in the form of direct effects and indirect effects through the agent's endogenous optimal contract. While the direct effects are intuitive with positive feedback increasing the agent's payoff and negative feedback doing the opposite, the indirect effects operating through the optimal incentive share or pay can counter the direct effects. These mixed signed effects are present in the cases of feedback affecting the agent's intrinsic payoff and feedback affecting the firm's cash flow.

The above analysis is conducted under the assumption that the agent's participation constraint is non-binding at the principal's optimal compensation contract. The final section of the paper discusses how the results might change with a binding participation constraint.

The paper is organized as in the following description. Section 2 presents the model. Section 3 considers the independent case, and Sect. 4 examines the interdependent case. Section 5 presents the impact of external feedback in the case of political activities. Section 6 extends the model to the case of a binding participation constraint. Section 7 concludes. All relevant proofs are presented in the "Appendix".

2 The basic model

Consider an agency setting with hidden action. The agent exerts costly effort to produce cash flow for a principal. The agent is a manager or a CEO and the principal is a group of shareholders. In addition to exerting effort, the agent can engage in what we will call "side activities". We will denote the level of this action variable as a, and work effort will be denoted as e. The principal cannot observe the levels of e or a. The principal can observe cash flow.

The activity variable a can take various forms. It could represent political activities implemented through making statements on social issues. The principal would not be able to observe the effort put in to this action variable, but the political statements would be observable. Non-political side activities might include exercise, web browsing, texting with colleagues, taking a short power nap, or researching work related topics. Given the large number of work at home agents, side activities can also include doing household chores both in and outside the house. Like political activities, the principal cannot observe the effort exerted in such side activities.

The key difference between political side activities and non-political side activities is that political activities can lead to changes in the firm's and the agent's payoffs generated through external forces. A far left political statement could lead to right leaning customers decreasing their consumption of the firm's commodity and conversely for a far right political statement. Political statements could also offend employees or stakeholders of the firm or endear employees or stakeholders to the firm and the CEO, and these feelings could favorably or unfavorably affect cash flow and the CEO's payoff.

We are primarily interested in determining the effect of side activities on the firm's profit. Thus, so as to not bias our results one way or another, we will begin by assuming that if activities are political, there is no customer backlash or benefit. That is, we initially focus on the pure economic effects of side activities in the absence of external feedback. Later we will take up the issue of customer, investor, or employee response in the political case.

Cash flow is binary and given by $\hat{y} \in \{0, y\}, y > 0$. The probability of the high cash flow is $e \in [0, 1)$. In the absence of side activities, the cost of such work effort is $c(e)$. Side activities give the agent a personal payoff measured in money. We will consider two versions of this payoff. The first version is given by the increasing function $g(a)$. Under this formulation, the agent derives an increasing intrinsic payoff from engaging in side activities which is independent of the level of work effort. More or less engagement in the production of cash flow does not affect the payoff derived from side activities. In the second version, the total payoff from side activities is given by a general function $g(a, e)$. In this formulation, work effort and side activities can interact and each can impact the other action's productivity in cash flow.

In addition to providing benefits, side activities also impose an extra cost which is given by sae, where $s > 0$. When side activities are present, the total net cost of both work effort and side activity effort is $c(e) + sae - g$. The interdependent case is sufficiently general to encompass the situations where the net marginal cost of effort,

$$\partial[c(e) + sae - g(a, e)]/\partial e = c'(e) - (g_e(a, e) - sa)),$$

increases or decreases in the level of side activities. That is,

$$\partial[c'(e) - (g_e(a, e) - sa))]/\partial a \gtreqless 0 \text{ as } (g_{ea}(a, e) - s) \gtreqless 0.$$

The net marginal cost of effort decreases in side activities only if $g_{ea} > 0$ (e and a are complements if $(g_{ea}(a, e) - s) > 0$) and it increases in side activities if $g_{ea} \leqq 0$ (e and a are substitutes if $(g_{ea}(a, e) - s) < 0$).

The agent and the principal are risk neutral. The agent is subject to limited liability in the sense that only nonnegative payments can be made to the agent by the principal. The principal offers the agent a contract consisting of a possible fixed payment S and a percentage b of cash flow. Under limited liability, $S, b \geqq 0$. The agent's outside option is $u \geqq 0$.

We posit the following assumptions regarding the function c. The work effort cost function satisfies

A.1 $c(0) = 0, c', c'' > 0$, and $c''' \geqq 0$, for all $e \geqq 0$.

Effort cost is a strictly convex function of work effort with zero work effort generating zero cost. Moreover, marginal effort cost is an increasing and convex function of work effort.

3 Independent benefits of side activities and work effort

In this section, we assume that the payoffs from work effort and side activities are independent. The benefit function g satisfies

A.2 $g(0) = 0, g' > 0$, for $a \geqq 0$, $g'' < 0$, for $a \geqq 0$.

The benefit of a is increasing at a decreasing rate. More side activities increase the payoff to the agent, but there are diminishing returns to such payoffs.

The agent's objective function is

$$A^p(e, a) = S + bye + g(a) - c(e) - sae.$$

The agent's problem is to

$$\max_{\{e,a\}} S + bye + g(a) - c(e) - sae.$$

The first order conditions (FOC) for e and a are, respectively,

$$A^p_e = by - sa - c'(e) = 0, \text{ and} \tag{1}$$

$$A^p_a = g'(a) - se = 0. \tag{2}$$

Because $A^p_{ee} = -c'' < 0$ and $A^p_{aa} = g'' < 0$, the second order conditions (SOC) are met, if $A^p_{ee}A^p_{aa} - (A^p_{ae})^2 = -c''g'' - s^2 > 0$, at optimum, where $A^p_{ea} = -s$. Thus, the two actions are substitutes in the agent's payoff. We assume that the SOC are met.

Equations (1) and (2) define the agent's actions as functions of the principal's incentive share b. We write these as $e = e(b)$ and $a = a(b)$. Condition (1) says positive work effort implies that the agent's "net wage" $w = (by - sa)$ is positive. Employing the usual comparative static techniques, we have that

$$e'(b) = \frac{-yg''}{-c''g'' - s^2} > 0, \text{ and} \tag{3}$$

$$a'(b) = \frac{-sy}{-c''g'' - s^2} < 0. \tag{4}$$

Increases in the principal's incentive share induce greater work effort. Such increases reduce side activities, because work effort and side activities are equilibrium substitutes in the agent's payoff.

The solution values can be rewritten using (1) and (2). Equation (1) implies that $e = h(by - sa(b))$, where the inverse of c' is denoted as $h \equiv c'^{-1}(\cdot)$. If we let the inverse of g' be denoted as $v \equiv g'^{-1}(\cdot)$, we can write $a = v(se(b)) = a(b)$. Under our assumptions, h is increasing in its argument, and v is non-increasing in its argument. Further, h is concave with $h'' \leqq 0$, by $c''' \geqq 0$. The net wage w is increasing in the principal's incentive share

$$\partial w/\partial b = y - sa'(b) > 0.$$

We assume

A.3 There exists a $b' \in [0, 1)$ such that $w(b') = 0$.

Assumption A.3 allows the principal to incentivize positive effort in a profitable manner, by raising b above $b' \in [0, 1)$.

Generally, the principal's problem would be to choose S and b so as to maximize profit. The principal's profit function is

$$\Pi^p = -S + (1-b)yh(by - sa(b)),$$

and the constraints facing the principal's choice are limited liability

$$S, b \geqq 0,$$

and participation by the agent and the principal

$$S + byh(by - sa(b)) + g(a(b)) - c(h(by - sa(b))) - sa(b)h(by - sa(b)) \geqq u, \text{ and}$$
$$\Pi^p \geqq 0.$$

We can simplify this problem, if we assume that the agent's participation constraint is non-binding. If this is true, it is easy to show and well known that the optimal $S = 0$. A proof in the context of our model is presented for completeness. In Sect. 6, the binding case is analyzed.

Lemma 1 *Let the participation constraint be non-binding*, *then the optimal* $S = 0$.

Under the assumption of a non-binding participation constraint, the principal's problem simplifies to

$$\max_{\{b\}}(1-b)yh(by - sa(b)),$$

with FOC

$$\Pi^p_b = -yh(by - sa(b)) + (1-b)yh'(by - sa(b))(y - sa'(b)) = 0. \tag{5}$$

Note that to satisfy (5) the optimal b must be such that $e > 0$. Thus, the net wage $w = by - sa(b)$ must be positive.

The SOC to the principal's problem is met if Π^p_{bb} is negative. This derivative is given by

$$\begin{aligned} &-2yh'(by - sa(b))(y - sa'(b)) + (1-b)yh''(by - sa(b))(y - sa'(b))^2 \\ &+ (1-b)yh'(by - sa(b))(-sa''(b)). \end{aligned} \tag{6}$$

The first term of this expression is negative and the second is non-positive, because $c''' \geqq 0$ implies that $c'^{-1} = h$ is concave, making $h'' \leqq 0$. The third term takes on the sign of $-a''$. This derivative is given by

$$-a'' = sy(-c''g'' - s^2)^{-2}(c'''e'(b)g'' + c''g'''a'(b)), \tag{7}$$

which takes on the sign of

$$(c'''e'(b)g'' + c''g'''a'(b)). \tag{8}$$

If (8) is non-positive, the SOC will be met. The first term of (8) is non-positive, but the second is non-positive if $g''' \geqq 0$ and non-negative otherwise. We have that

$g'' \leqq 0$, so that if $g''' \geqq 0$ the SOC is met. If $g''' < 0$, then we assume that the first two terms of (6) dominate the third.

Our analysis relies on the existence of a $b \in (0, 1)$ such that (5) is met. The following result shows that this is the case.

Lemma 2 *Let assumptions A.1–A.3 hold and let* $\Pi^p_{bb} < 0$, *for* $b \in (0, 1)$. *There exists a unique* $b^p \in (0, 1)$ *which solves the principal's problem* (5).

Let us contrast this independent solution to the case where the agent does not engage in side activities. Define the principal's profit without such activities as Π^o. Using (1) with $a = 0$, equilibrium e, denoted e^o, is defined by

$$e^o = h(b^o y), \tag{9}$$

and the principal's optimal incentive share, b^o, is given by

$$\Pi^o_b = -yh(b^o y) + (1 - b^o)yh'(b^o y)y = 0, \tag{10}$$

assuming that the participation constraint is non-binding. As in the problem with side actions, the proof of Lemma 1 can be used to show that $S = 0$, if the participation constraint is non-binding.

The SOC for the principal's problem is met if $c''' \geqq 0$. Differentiating (10),

$$\Pi^o_{bb} = -2yh'(b^o y)y + (1 - b^o)yh''(b^o y)y^2 < 0,$$

by $c''' \geqq 0$. The convexity of marginal effort cost guarantees that the SOC to the principal's solution is satisfied. Using the intermediate value theorem as in Lemma 2, we see that $\underset{b \to 0}{Lim} \Pi^o_b = -y \cdot 0 + (1)yh'(0)y > 0$ and $\underset{b \to 1}{Lim} \Pi^o_b = -yh(y) < 0$. These facts along with $\Pi^o_{bb} < 0$ imply that there exists a unique $b^o \in (0, 1)$ solving (10).

The non-side activities solution is what would obtain if the principal were to ban all side activities. Note that as long as the agent's side activities problem is not a corner solution in a, the agent has a direct gain from engaging in side activities, $g(a) - sae > 0$, for a given b. If b rises, then this creates an indirect gain which reinforces the direct gain of side activities, and the agent's welfare rises. If b falls, then a negative indirect effect is created and side activities result in an indeterminate effect on the agent's welfare. The effect of side activities on the principal's profit remains to be determined.

We want to compare the equilibrium incentive share and profit between the side and non-side activities solutions. The following result makes this comparison.

Proposition 1 *Assume that the principal's participation constraint is non-binding, that assumptions A.1–A.3 hold, and that* $\Pi^p_{bb} < 0$, *for* $b \in (0, 1)$. *We have that* $b^p > b^o$ *and* $\Pi^o > \Pi^p$.

In the presence of side activities, the firm offers a greater incentive share than in the situation where there are no such activities. Intuitively, effort and side activities are substitutes in the agent's objective function. Side activities then decrease the marginal profitability of work effort, by raising its marginal cost. Symmetrically, work effort raises the marginal cost of side activities. Other things equal, this decreases work effort. The agent's net wage which incentivizes work effort is $by - sa$ instead of by. To counter this lowering effect, the principal raises the incentive share above that which would obtain without side activities. First order condition (5) tells the whole story. Side activities lower the marginal cost of raising b

$$|-yh(by - sa(b))| < |-yh(by)|,$$

and at the same time raise the marginal benefit of increasing b

$$(1-b)yh'(by - sa(b))(y - sa'(b)) > (1-b)yh'(by)y.$$

Thus, the optimal incentive share is greater with side activities.

The key factor generating the negative effect on profit is the marginal effect of a change in work effort on the net benefit of side activities, $g(a) - sae$. That is, the negative effect on profit of side activities is driven by the fact that

$$\frac{\partial}{\partial e}(g(a) - sae) = -sa < 0.$$

To see this, place a parameter α, representing the intensity of the presence of side activities, multiplicatively, on the net benefit of side activities: $\alpha(g(a) - sea)$. In this case, each of e and a are functions of (b, α), with $\frac{\partial e}{\partial \alpha} = \frac{sa\alpha g''}{(-\alpha c'' g'' - \alpha^2 s^2)} < 0$, if the SOC are met. The sign of $\partial e/\partial \alpha$ is then that of $\frac{\partial}{\partial e}(g(a) - sae) = -sa < 0$. Using the envelope theorem, the effect of a change in α on equilibrium profit is

$$\frac{\partial}{\partial \alpha}\Pi^p(b^p) = (1-b)y\frac{\partial e}{\partial \alpha} < 0.$$

It is the sign of the marginal effect of effort on the net benefit of side activities that determines this result. It is not the the sign of the second order cross effect A^p_{ea} (which determines whether e and a are substitutes or complements) that generates this result. In the independent case, these two signs are coincidentally the same.

Given $b^p > b^o$ and the fact that a is not a corner solution when side activities are available $(g(a(b^p)) - sa(b^p)e(b^p) > 0)$, the agent gains from side activities in equilibrium. That is, under the envelope theorem,

$$\frac{\partial}{\partial \alpha}A^p = \frac{\partial b}{\partial \alpha}ye + (g - sae) > 0,$$

in equilibrium.[3] However, the firm does not benefit from the agent engaging in side activities, if these activities provide benefits which do not interact in a positive way with work effort. While the agent is privately better off from such engagement,

[3] If we insert the parameter α into the difference in the gradients of Π^p and Π^o in b, we have $\Pi^p_b - \Pi^o_b = y[h(by) - h(by - \alpha sa(b))] + (1-b)y[h'(by - \alpha sa(b))(y - \alpha sa'(b)) \quad -h'(by)y] > 0$, for any $\alpha > 0$. Whence, under our assumptions that Π^p_b, Π^o_b are decreasing, $\partial b/\partial \alpha > 0$.

the firm is not. If the side activities are political statements, such statements can be observed and would quite likely be banned. If the side activities were enabled through the agent working from home, then the firm might prohibit working from home.

4 Interdependent benefits of side activities and work effort

If the agent's net marginal payoff from side activities interacts with the agent's work effort and vice versa, quite different results can obtain. In this section, the net benefit of side activities is generally written as $\alpha(g(a,e) - sea)$. The parameter $\alpha \geqq 0$ represents an intensity of the presence of the side activity.

We assume that the function $G(a,e) \equiv (g(a,e) - sea)$ is strictly concave in (a, e) on its domain $[0,+\infty) \times [0,1) \equiv \Gamma$ through the sufficiency condition

A.4 $g_{ee}, g_{aa} < 0$, and $|H_G| \equiv g_{ee}g_{aa} - (g_{ae} - s)^2 > 0$, for all $(a,e) \in \Gamma$.

The gross marginal benefit functions, g_i, are subject to diminishing returns to each action variable.

Let us use the same notation for side and non-side action solutions. Using the proof of Lemma 1, it is again true that if the participation constraint is non-binding, the principal's non-contingent payment is zero. We can then write the agent's payoff as

$$A^p(e,a) = bye - c(e) + \alpha(g(a,e) - sae). \tag{11}$$

The agent chooses e and a so as to maximize (11), and the FOC are

$$A_e^p = by - c'(e) + \alpha(g_e(a,e) - sa) = 0, \text{ and} \tag{12}$$

$$A_a^p = \alpha(g_a(a,e) - se) = 0. \tag{13}$$

For this case, $A_{ee}^p = -c'' + \alpha g_{ee}$, $A_{aa}^p = \alpha g_{aa}$, and $A_{ae}^p = \alpha(g_{ea} - s)$. The SOC are met if $(-c'' + \alpha g_{ee}), \alpha g_{aa} < 0$ and if the Hessian determinate of A^p, denoted $|H_p|$, is positive, $|H_p| = (-c'' + \alpha g_{ee})(\alpha g_{aa}) - \alpha^2(g_{ea} - s)^2 > 0$. Assumptions A.1 and A.4 imply that $(-c'' + \alpha g_{ee}), \alpha g_{aa} < 0$. Moreover,

$$|H_p| = -\alpha c'' g_{aa} + \alpha^2 |H_G| > 0.$$

Thus, the SOC for the agent's problem are satisfied. Let us use the more compact notation $G_{ee} = g_{ee} < 0, G_{aa} = g_{aa} < 0, G_{ea} = g_{ea} - s$, and $G_i = g_i - sj, i \neq j = a, e$.

Conditions (12) and (13) define functions $e = e(b,\alpha)$ and $a = a(b,\alpha)$. Comparative statics reveal

$$\frac{\partial e}{\partial b} = \frac{-y\alpha G_{aa}}{|H_p(b,\alpha)|} > 0, \ \frac{\partial a}{\partial b} = \frac{y\alpha G_{ea}}{|H_p(b,\alpha)|} \gtreqless 0, \tag{14}$$

$$\frac{\partial e}{\partial \alpha} = \frac{-G_{aa}G_e}{|H_p(b,\alpha)|} \gtreqless 0, \text{ and } \frac{\partial a}{\partial \alpha} = \frac{G_e G_{ea}}{|H_p(b,\alpha)|} \gtreqless 0. \tag{15}$$

In (14), the principal's incentive share raises work effort, but its effect on the level of side activities depends on the sign of the cross effect G_{ea}. If the two actions are independents or substitutes in G, $(G_{ea} \leqq 0)$ then $A^p_{ae} \leqq 0$, and they are independents or substitutes in payoff. In this case, a rise in b decreases or does not change a. If they are complements in G, $(G_{ae} > 0)$, then they are complements in payoff to the agent. In this case, a rise in b increases a. From (15), if side activities become more intense (α rises), the effect on work effort depends on the sign of the first order term G_e measuring the marginal effect of work effort on the net benefit of side activities. When this is positive, work effort increases with the intensity of side activities and conversely when it is negative. Side activity rises with more intensity if both G_e and G_{ea} are of the same sign and it falls when they are of opposite signs. Thus, in the complements case, an increase in α raises both e and a if $G_e > 0$ and it lowers both if $G_e < 0$. In the substitutes case, an increase in α raises e and lowers a if $G_e > 0$, and it lowers e and raises a if $G_e < 0$.

The solution value for e can be expressed as $e = h(by + \alpha(g_e(a(b,\alpha), e(b,\alpha)) - sa(b,\alpha)))$. For work effort to be positive, the principal's net wage $w = (by + \alpha(g_e(a,e) - sa))$ must be positive. We can show that the net wage is increasing in the principal's incentive share

$$\partial w/\partial b = y + \alpha(g_{ee}(\partial e/\partial b) + (g_{ea} + s)(\partial a/\partial b)) = y\frac{-\alpha c'' g_{aa}}{|H_p(b,\alpha)|} > 0.$$

As in Assumption A.3, we wish to guarantee that the principal can elicit positive effort in a profitable manner. We assume

A.5 There exists a $\hat{b} \in [0,1)$ such that $w(\hat{b}) = 0$.

Using the analysis of Lemma 1, the principal sets $S = 0$, if the participation constraint is non-binding. The principal's problem is to maximize

$$\Pi^p = (1-b)yh(by + \alpha(g_e(a(b,\alpha), e(b,\alpha)) - sa(b,\alpha))) \tag{16}$$

over a choice of b. The FOC is given by

$$\begin{aligned} 0 = \Pi^p_b &= -yh(by + \alpha(g_e(a(b,\alpha), e(b,\alpha)) - sa(b,\alpha))) \\ &+ (1-b)yh'(by + \alpha(g_e(a(b,\alpha), e(b,\alpha)) \\ &- sa(b,\alpha)))(y + \alpha(g_{ee}(\partial e(b,\alpha)/\partial b) - (g_{ea} - s)(\partial a(b,\alpha)/\partial b)). \end{aligned} \tag{17}$$

Substituting from (14), we can rewrite (17) as

$$\begin{aligned} 0 = \Pi^p_b &= -yh(by + \alpha(g_e(a(b,\alpha), e(b,\alpha)) - sa(b,\alpha))) \\ &+ (1-b)yh'(by + \alpha(g_e(a(b,\alpha), e(b,\alpha)) \\ &- sa(b,\alpha)))(y - \frac{\alpha^2 y}{|H_p(b,\alpha)|}(|H_G(b,\alpha)|)). \end{aligned} \tag{18}$$

We assume that the SOC is met, $\Pi^p_{bb} < 0$, for $b \in (0, 1)$.[4]

Existence of a solution to the principal's problem is considered in

Lemma 3 *Let assumptions* $A.1$, $A.4$, $A.5$, *and* $\Pi^p_{bb} < 0$, *for* $b \in (0, 1)$, *hold. There exists a unique* $b^p \in (0, 1)$ *which solves the principal's problem* (16).

The solutions for the non-side activities agent and the associated principal are again described by (9) and (10). As in the previous section, we wish to compare profit with no side activities to profit without side activities. Moreover, we would like to compare the principal's optimal incentive share across these solutions. The following proposition sheds light on both issues.

Proposition 2 *Assume that the principal's participation constraint is non-binding, that assumptions* $A.1$, $A.4$ *and* $A.5$ *hold, and that* $\Pi^p_{bb} < 0$, *for* $b \in (0, 1)$. *If* $G_e > 0$, *for all* $(a, e) \in \Gamma$, *we have that* $b^o > b^p$ *and* $\Pi^o < \Pi^p$. *If* $G_e < 0$, *for all* $(a, e) \in \Gamma$, *then* $\Pi^o > \Pi^p$, *but* b^o *and* b^p *cannot be ranked.*

When the marginal effect of work effort on the net benefit of side activities, G_e, is positive, then this will result in greater profit for the firm, in equilibrium, as compared to the situation where no side activities are present. The opposite is true, if an increase in work effort lowers the net benefit of side activities, $G_e < 0$. Thus, as in the independent case, the key to the effect on welfare is the impact of work effort on the total net benefit of side activities.

We can write

$$\frac{\partial \Pi^p}{\partial \alpha} = (1 - b)y\frac{\partial e}{\partial \alpha} \text{ and } \frac{\partial A^p}{\partial \alpha} = ye\frac{\partial b}{\partial \alpha} + G.$$

If $G_e > 0$, then $\frac{\partial \Pi^p}{\partial a} > 0$, however the change in the agent's welfare is uncertain. While $G > 0$, we have that $ye\frac{\partial b}{\partial \alpha} < 0$, so that $\frac{\partial A^p}{\partial \alpha}$ cannot be signed. The direct effect of side activities, G, is positive, but the indirect effect through the incentive share decreasing counters this positive effect. When work effort has a positive effect on the net benefit of side activities, the firm benefits but the effect on the agent is indeterminate. When $G_e < 0, \frac{\partial e}{\partial \alpha} < 0$ and the firm's profit decreases in side activities. The agent's welfare change is again indeterminate, because while the direct effect is positive, $G > 0$, the term $ye\frac{\partial b}{\partial \alpha}$ cannot be signed. Our incorporation of the endogenously optimal contract then uncovers indirect effects through the agent's pay which can possibly overturn the direct effects of side activities.

In the case where $G_e > 0$, the reasoning behind the fall in the optimal incentive share can be explained as in the following. Intuitively, the higher net wage generated

[4] We can write $\Pi^p_{bb} = -2y\partial e/\partial b + (1 - b)\partial^2 e/\partial b^2$. The first term is negative and the term $\partial^2 e/\partial b^2$ contains all third order derivatives some of which can be positive. We assume the first term dominates any positive terms in the second third order expression.

by side activities allows the firm to lower the incentive share in equilibrium. Side activities raise the marginal cost of raising b

$$|-yh(by+\alpha G_e(b,\alpha))| > |-yh(by)|, \tag{19}$$

and they lower the marginal benefit of raising b

$$[(1-b)y[h'(by+\alpha G_e(b,\alpha))(y-\frac{\alpha^2 y}{|H_p(b,\alpha)|}(|H_G(b,\alpha)|))] < (1-b)yh'(by)y. \tag{20}$$

It then follows that the equilibrium b with side activities will be less than without.

If $G_e < 0$, the effect on the optimal incentive share is indeterminate. The lowering of the net wage makes the marginal cost of raising b go down (reverse inequality in (19)), and it has an ambiguous effect on the marginal benefit of raising b (ambiguous inequality in (20)). That is, in (20) we have that

$$(1-b)yh'(by+\alpha G_e(b,\alpha)) > (1-b)yh'(by), \tag{21}$$

but

$$(y-\frac{\alpha^2 y}{|H_p(b,\alpha)|}(|H_G(b,\alpha)|) < y. \tag{22}$$

Thus, while profit goes down with side activities, in this case, the effect on the incentive share is uncertain. In the proof of Proposition 2, we give numerical examples of cases where the incentive share is higher or lower with side activities, in equilibrium.[5]

Once again, the key factor in the effects of side activities on profit is not the sign of the second order term G_{ea} which determines whether e and a are substitutes or complements. It is the sign of first order term G_e which determines the effect of e on the net benefit of side activities. When $G_e > 0$, this says that the agent appreciates the net benefits of side activities more, the harder that agent works. That is, the more involved is the agent in the cash flow activities of the firm, the greater the net benefit of side activities. This case might occur in situations where engaging in side activities is part of corporate culture. For example, it might be that $G_e > 0$ for an agent at Ben and Jerry's, given that their mission statement encourages activism. For this case, side activities increase profit and reduce the incentive payment in equilibrium. Burbano (2016) provides evidence from a field experiment that workers who received information about the employer's social responsibility were willing to accept a lower wage. Barrero et al. (2022) show that firms recently used increased desirable work from home time to decrease wage growth pressures by about 2 percentage points over 2 years. These studies then support a lower or subdued

[5] One special case where the incentive share rises with side activities is the independent case studied in the last section. Here, we have substitutes with $G_e = -sa < 0$ and a rising incentive share with side activities.

remuneration being associated with the provision of an environment amenable to side activities and the situation where $G_e > 0$.

If $G_e < 0$, then an agent more involved in exerting effort to produce cash flow derives a smaller marginal benefit from engaging in side activities. Side activities are not part of corporate culture or belonging to the firm. The culture is one of a straight work ethic. This might be the case at Strive, where the mission is to eliminate ESG criteria from investment decisions. It is precisely this situation that produces a deterioration of profit when side activities are present. However, the impact on remuneration is not certain. Our results do indicate that if the incentive share rises with the presence of side activities, then a necessary condition is that $G_e < 0$. Adrjan et al. (2023) document rising wages with side activities in cases where employee sentiment toward that side activity declined. We could take this as a situation where our condition $G_e < 0$ is met in the interdependent model. Note that this condition is always met in the independent model of the previous section, where side activities increase the incentive share. Either attitude, i.e., $G_e < 0$ or $G_e > 0$, on the part of the agent or either situation seems reasonable.

Real world conditions which would make G_e positive or negative are similar to those which would make G_{ea} positive or negative which in turn determines whether e and a are complements or substitutes, respectively. The former condition asks how more work effort impacts the total net benefit of side activities, whereas the latter looks at the effect of more work effort on the net marginal benefit of side activities (or, equivalently, the effect of more side activity on the net marginal benefit of work effort). If $G_{ea} > 0$, then work effort raises the marginal net benefit of side activity. As in the situation above where $G_e > 0$, the agent who works more enjoys a greater marginal net benefit of side activities. Conversely, if $G_{ea} < 0$, then more work lowers the marginal net benefit of side activities. This scenario would be analogous to the above cases where work effort lowers the total net benefit of side activity, $G_e < 0$.

5 External responses to political side activities

With political side activities, customers, stakeholders or employees could agree or disagree with statements made by the manager. We will model this feature in reduced form as a linear shift parameter in cash flow or the agent's payoff which can be positive or negative per unit of side activities a. The reaction to political side activities is a random variable $\mu > 0$ which takes on the value $+\mu$ with probability q and $-\mu$ with probability $(1-q)$, so that the (prior) expected value of μ shared by all is $\bar{\mu} = \mu(2q-1)$. We will be interested in the two cases where $q > 1/2$ and $\bar{\mu}$ is positive and where $q < 1/2$ and $\bar{\mu}$ is negative. The shift parameter is then given by

$$\bar{\mu}a. \tag{23}$$

If constituents are irritated by a political statement, then $q < 1/2$ and $\bar{\mu}a < 0$ enters cash flow or the agent's payoff, and if political statements endear the CEO to constituents, then $q > 1/2$ and $\bar{\mu}a > 0$ enters cash flow or the agent's payoff. This formulation asserts that a constant marginal cost or benefit ($\bar{\mu} \gtrless 0$) is internalized by the agent per unit of the political activity variable. Such costs or benefits are solely a function of a, are, then, independent of work effort, and are imposed or bestowed on the agent or firm externally as result of political activity. Changes in the shift parameter can be generated through the derivative of $\bar{\mu}$ in μ, $\bar{\mu}'(\mu) = 2q - 1 \gtrless 0$.

We want to consider two versions of external feedback. In the first version, the feedback is internalized only by the CEO as an increase or reduction in the intrinsic payoff generated by political statements. In this case, the external response does not directly impact the firm's cash flow. Examples of this type of feedback might occur when the CEO makes political statements to employees internal to the firm and not to the outside market or when it is employees who are most affected by the CEO's activism.[6] For example, GrubHub CEO Matt Maloney emailed employees his opinion on one of the candidates in the 2016 election and, in doing so, he alienated those employees who supported that candidate and endeared those who did not.[7] Randy Edeker CEO of Hy-Vee distributed videos to employees in 2020 which implicitly supported republican tax policies.[8] Those against such policies could have been offended and those who supported them would be incentivized. Levi Strauss CEO Chip Bergh received both positive and negative responses from employees because he made a corporate donation toward preventing gun violence.[9] Recently, CEO's have also internally communicated to employees their opinions on the controversial Supreme Court decisions on affirmative action and the federal right to abortion.[10] It is of theoretical interest to separate this channel of feedback, because it has non-contingent counter intuitive effects in the case where e and a are substitutes in the CEO's payoff.

The second version assumes that the external feedback enters through the firm's cash flow only. Of course, this type of feedback affects the agent's payment through the incentive contract. Given these two separate channels, it is then possible to discuss the effects of both types of feedback operating simultaneously. Famous examples of the second type of feedback would include the recent Bud Light backlash regarding using Dylan Mulvaney in marketing, the backlash against Target for showcasing LBGTQ+ merchandise, and the negative feedback on Disney when Bob Chapek took a contrary stand on legislation in Florida. Each of these firms experienced negative shocks to cash flows.[11] A recent example of a positive shock was the increase in downloads of Jason Aldean's song "Try That in a Small Town".[12] The lyrics were dismissive of recent protests and crime, and this induced conservatives to increase their demand for the song and his music in general.

If the feedback enters only through the CEO's intrinsic payoff, then we have that the CEO's objective function is

[6] See Manno (2023).

[7] See Cain (2016).

[8] See Lenz (2020).

[9] See Noguchi (2018).

[10] See Mcglauflin and Williams (2023) on affirmative action and Adrjan et al. (2023) for Dobbs V. Jackson.

[11] See Thaler (2023), Gregg and Bogage (2023), and Bricker (2023).

[12] See Olsen (2023).

$$A^p(e,a) = bye - c(e) + g(a,e) - sae + \bar{\mu}a. \tag{24}$$

Equation (24) couches the feedback in terms of the interdependent model with our intensity parameter α set at unity, but the independent model is just a special case where $g(e, a)$ is replaced by $g(a)$. The agent's FOC are

$$A^p_e = bye - c(e) + g_e(a,e) - sa = 0, \text{ and} \tag{25}$$

$$A^p_a = g_a(a,e) - se + \bar{\mu} = 0. \tag{26}$$

The SOC are identical to the problem without feedback.

Employing the usual comparative static techniques,

$$\frac{\partial e}{\partial \mu} = \frac{\bar{\mu}' G_{ea}}{|H_p|} \text{ and } \frac{\partial a}{\partial \mu} = \frac{-\bar{\mu}'(-c'' + G_{ee})}{|H_p|}, \tag{27}$$

where $|H_p| = (-c'' + G_{ee})(G_{aa}) - (G_{ea})^2 > 0$. If feedback is a cost (negative feedback), then $\bar{\mu}' = 2q - 1 < 0$, and if it is a benefit (positive feedback), then $\bar{\mu}' = 2q - 1 > 0$. Thus, if feedback is negative, it results in a decrease in a, and conversely, if it is positive. The effect of feedback on e depends on whether e and a are complements or substitutes in the agent's payoff. In the complements case, work effort increases with more positive feedback and decreases with more negative feedback. In the substitutes case, work effort decreases with more positive feedback and actually increases with more negative feedback. Interestingly, more positive feedback encourages political activity which substitutes for work effort and work effort declines. Moreover, extra negative feedback decreases political activity, but increases work effort.

We can use (27) to analyze the effect of this type of feedback on the agent's and the firm's welfare levels. In equilibrium, a change in μ affects the agent's payoff and the firm's payoff as follows:

$$\frac{\partial A^p}{\partial \mu} = \bar{\mu}' a + ye\frac{\partial b}{\partial \mu}, \text{ and } \frac{\partial \Pi^p}{\partial \mu} = (1-b)y\frac{\partial e}{\partial \mu}. \tag{28}$$

Consider the effect of feedback on the firm's profit. If feedback is negative (positive) and e and a are complements, then increases in μ will reduce (increase) the firm's payoff as well as the agent's two action variables. On the other hand, if e and a are substitutes, we obtain some surprising results. When the two action variables are substitutes, increased negative feedback results in more work effort, less political action, and a greater profit for the firm. Conversely, if increased feedback is positive, the agent generates less work effort, more political activity, and less profit for the firm. Thus, in the substitute case, there are opposing feedback effects of political activities on profit.

The substitute case then provides some unexpected results. With respect to the firm's payoff, we obtain the counter intuitive result that more intense negative feedback can increase work effort and improve the firm's bottom line. Equally surprising is the converse statement that more intense positive feedback can lower work effort and hurt the firm's bottom line. These unconditional results are relevant for the case where only the CEO's intrinsic payoff is impacted by the external response.

The effect of intrinsic feedback on the agent's payoff contains two terms. The first term in (28), $\bar{\mu}'a$, is positive with positive feedback and negative with negative feedback. This is the direct effect which gives us what our intuition would suggest. The second term is the indirect effect whose sign depends on the sign of $\frac{\partial b}{\partial \mu}$. Assuming that the firm's second order condition for b is met, $\Pi^p_{bb} = -2y\partial e/\partial b + (1-b)y\partial^2 e/\partial b^2 < 0$, the sign of $\frac{\partial b}{\partial \mu}$ is that of $\Pi^p_{b\mu} = -y\frac{\partial e}{\partial \mu} + (1-b)\frac{\partial^2 e}{\partial b \partial \mu}$. The second term $(1-b)\frac{\partial^2 e}{\partial b \partial \mu}$ depends on third order partial derivatives which would be impossible to sign through economic intuition. Let us treat these third order effects as negligible (zero). Under a quadratic scenario, (27) implies, in the case of complements, that the sign of $-y\frac{\partial e}{\partial \mu}$ is negative with positive feedback and positive with negative feedback, and, in the case of substitutes the converse is true. Thus, the sign of $-y\frac{\partial e}{\partial \mu}$ is opposite to that of $\bar{\mu}'\alpha a$ in the complements case, but the same in the substitutes case. Thus, if we concentrate on the quadratic case, the agent's welfare is increased with positive feedback and decreased with negative feedback in the substitutes case, but, in the complements case, the effect of feedback on the agent's welfare is indeterminate. In the complements case, positive feedback results in a decrease in pay and conversely for negative feedback. The effect on pay counters the direct effect on cash flow. While quadratic functions and the assumption of substitutes does lead to some determinate results, third order effects could make even these results indeterminate. The indirect effect working through the optimal pay, $ye\frac{\partial b}{\partial \mu}$, can confound what our intuition would suggest with regard to how feedback affects the agent welfare. This analysis then highlights the point that the effect of feedback on the agent's welfare depends on the endogenously optimal incentive contract. This issue has been ignored by previous research.

Next, consider the case where feedback affects the firm's cash flow and thereby indirectly affects the agent's payoff through the incentive contract. Let the feedback term be given by $\bar{\delta}a$, where $\bar{\delta} = \delta(2q-1)$. For this case, the agent's objective function becomes

$$A^p(e,a) = bye + b\bar{\delta}a - c(e) + g(a,e) - sae, \tag{29}$$

and firm's profit is

$$\Pi^p = (1-b)(ye + \bar{\delta}a). \tag{30}$$

Comparative statics conducted on the agent's problem yield (The SOC are identical to the problem without feedback and with $\alpha = 1$.)

$$\frac{\partial e}{\partial \delta} = \frac{\bar{\delta}' b G_{ea}}{|H_p|} \text{ and } \frac{\partial a}{\partial \delta} = \frac{-\bar{\delta}' b(-c'' + G_{ee})}{|H_p|}. \tag{31}$$

This form of feedback has the same qualitative effects on e and a. If e and a are substitutes in the agent's payoff, then positive feedback increases a but decreases e, while negative feedback does the opposite and increases e but decreases a. However, if e and a are complements, then negative feedback decreases both activities and positive feedback increases both activities.

In equilibrium, a change in δ affects the agent's payoff and the firm' payoff as follows:

$$\frac{\partial A^p}{\partial \delta} = +b\bar{\delta}'a + (ye + \bar{\delta}a)\frac{\partial b}{\partial \delta} \text{ and } \frac{\partial \Pi^p}{\partial \delta} = (1-b)\left(y\frac{\partial e}{\partial \delta} + \bar{\delta}'a + \bar{\delta}\frac{\partial a}{\partial \delta}\right). \quad (32)$$

The effect on the change in profit now contains three terms, $(y\frac{\partial e}{\partial \delta} + \bar{\delta}'a + \bar{\delta}\frac{\partial a}{\partial \delta})$, determining its sign. In the complements case, each of these terms is positive (negative) with positive (negative) feedback, so that positive feedback increases profit while negative feedback decreases profit.

In the substitutes case, we have mixed signs in the expression $(y\frac{\partial e}{\partial \delta} + \bar{\delta}'a + \bar{\delta}\frac{\partial a}{\partial \delta})$, so that the effect on profit depends on magnitudes. Let a and e be substitutes. If feedback is negative, $(2q-1) < 0$, we have $sign\frac{\partial \Pi^p}{\partial \delta} = sign(y\frac{\partial e}{\partial \delta} + (2q-1)a + \delta(2q-1)\frac{\partial a}{\partial \delta})$, with $y\frac{\partial e}{\partial \delta} > 0$, $(2q-1)a < 0$, and $\delta(2q-1)\frac{\partial a}{\partial \delta} > 0$. The sum of the last two terms, $(2q-1)a + \delta(2q-1)\frac{\partial a}{\partial \delta} = (2q-1)a(1 + \frac{\partial a}{\partial \delta}\frac{\delta}{a})$ is nonnegative if the elasticity of a with respect to $\delta$is greater than or equal to one in absolute value. Thus, in the case where e and a are substitutes,

$$\frac{\partial \Pi^p}{\partial \delta} > 0, \text{ with negative feedback, if } -\frac{\partial a}{\partial \delta}\frac{\delta}{a} \geqq 1.$$

However, if a is inelastic in $\delta, -\frac{\partial a}{\partial \delta}\frac{\delta}{a} \in (0,1)$, then

$$\frac{\partial \Pi^p}{\partial \delta} \gtreqless 0 \text{ with negative feedback, if } y\frac{\partial e}{\partial \delta} \gtreqless (1-2q)(a + \delta\frac{\partial a}{\partial \delta}) > 0.$$

If feedback is negative and the downward response in a is large in a percentage sense (a is elastic in δ), then negative feedback leads to more profit, as in the case of a shift in the CEO's intrinsic utility. This is also the case if a is inelastic in δ and the upward shift in effort is great relative to the shift downward shift in $\bar{\delta}a$. Unlike the case of intrinsic shifts, profit can be decreased with a negative response and a inelastic in δ, if the upward shift in effort is small in relation to the downward shift in $\bar{\delta}a$. In sum, the counter intuitive results with cash flow shifts in the substitute case are still present in cases where there is negative feedback and the elasticity of a in δ is greater than one in absolute value or it is less than one in absolute value and feedback has a large positive effect on effort.

Again let a and e be substitutes and let feedback be positive. In this case, $sign\frac{\partial \Pi^p}{\partial \delta} = sign(y\frac{\partial e}{\partial \delta} + (2q-1)a + \delta(2q-1)\frac{\partial a}{\partial \delta}y)$, with $y\frac{\partial e}{\partial \delta} < 0$, $(2q-1)a > 0$,and $\delta(2q-1)\frac{\partial a}{\partial \delta} > 0$. For this case, we have

$$\frac{\partial \Pi^p}{\partial \delta} \gtreqless 0 \text{ with positive feedback if } (2q-1)a(1 + \frac{\partial a}{\partial \delta}\frac{\delta}{a}) \gtreqless -y\frac{\partial e}{\partial \delta} > 0.$$

Positive feedback pushes work effort down and political activity up. The former effect lowers profit and the latter effect raises it. The latter effect is not present in the intrinsic shift case where profit definitely shifts down with positive feedback. With a cash flow shift, profit can go up or down with positive feedback. It decreases (increases) if the absolute value of the effect on work effort is large (small) relative to the effect on political activity. The effect on political activity is small (large) when the elasticity of a in δ is small (large).

Thus with substitutes, whether cash flow feedback is positive or negative, there can be a countervailing push in profit in the opposite direction of the feedback just as in the intrinsic feedback shift. In the negative feedback case, this occurs when the absolute value of the elasticity of a in δ is large relative to the changes in work effort and in the positive feedback case it occurs when this elasticity is relatively small relative to the changes in work effort.

Finally, if feedback takes place through both firm cash flow and intrinsic payoff to the CEO, the change in profit is just the addition of the two sets of changes in (28) and (32). The analysis is the same as in the above cash flow feedback discussion with the possible opposing forces in the substitutes case.

The qualitative effects of feedback on A^p are again indeterminate, with the direct term, $+\bar{\delta}'a$, generating increases with positive feedback and decreases with negative feedback. The indirect affect through the incentive share has mixed signs in all cases. Assuming that the firm's second order condition for b is met,

$$\Pi^p_{bb} = -2y\partial e/\partial b - 2\bar{\delta}\partial a/\partial b + (1-b)[y\partial^2 e/\partial b^2 + \bar{\delta}\partial^2 a/\partial b^2] < 0,$$

the sign of $\frac{\partial b}{\partial \mu}$ is that of

$$\Pi^p_{b\delta} = -y\partial e/\partial \delta - \bar{\delta}'a - \bar{\delta}\partial a/\partial \delta + (1-b)\bar{\delta}'\partial a/\partial b + (1-b)[y\partial^2 e/\partial b\partial \delta + \bar{\delta}\partial^2 a/\partial b\partial \delta]. \tag{33}$$

In addition to (31), we have, from the agent's problem,

$$\frac{\partial e}{\partial b} = \frac{-yG_{aa} + \bar{\delta}G_{ea}}{|H_p|} \text{ and } \frac{\partial a}{\partial b} = \frac{-(-c'' + G_{ee})\bar{\delta} + yG_{ea}}{|H_p|}. \tag{34}$$

In (33), the expression $(1-b)[y\partial^2 e/\partial b\partial \delta + \bar{\delta}\partial^2 a/\partial b\partial \delta]$ contains third order derivatives which would be arbitrary to sign. As above, let us take these as negligible and concentrate on the quadratic case, which entails the first four terms of (33). First let e and a be complements with positive feedback ($\bar{\delta}, \bar{\delta}' > 0$), (31) and (34) imply that the first three terms of (33) are negative, but the fourth is positive. If feedback is negative the first three terms are positive and the fourth is indeterminate. In the complements case under a quadratic scenario, feedback leads to mixed signs of $\frac{\partial b}{\partial \mu}$. Moving to the substitutes case with positive feedback, we have that the first term of (33) is positive, the second is negative, the third is negative, and the fourth is indeterminate. Finally, with substitutes and negative feedback, the first term of (33) is negative, the second is positive, the third is positive and the fourth is negative. The sign of $\frac{\partial b}{\partial \delta}$ is mixed in all cases, even when we ignore the third order derivatives in the last of the 5 terms of (33). The term $(ye + \bar{\delta}a)$ is mixed in the negative feedback case and positive in the positive feedback case. Thus, the product $(ye + \bar{\delta}a)\frac{\partial b}{\partial \mu}$ has mixed sign in all cases, making the indirect effect of cash flow feedback on the agent's welfare indeterminate.

In sum, the impact of cash flow feedback on the agent's welfare has complex indirect effects on the agent's optimal payment resulting in indeterminate welfare changes for the agent regardless of whether the feedback is positive or negative. Once again, the indirect effects of feedback through the endogenous incentive contract can lead to unpredictable changes in the agent's welfare.

6 Notes on the case of a binding participation constraint

Our results so far have relied on the assumption that the participation constraint is non-binding. How might the results change with a binding participation constraint? In the present model, the participation constraint turns out to be a lower bound for the incentive share b. To see this, first note that whether the participation constraint is binding or not, the principal does not make a positive non-contingent payment S. We again will formulate the problem in the context of the interdependent activity variables with the independent case as a special case (where a and e are substitutes). We have

Lemma 4 *Let assumptions A.*1 *and A.*4 *hold. In the equilibrium of the interdependent model, the principal optimally sets* $S = 0$.

The agent's objective function is then

$$A^p(b, \alpha) = bye(b, \alpha) - c(e(b, \alpha)) + \alpha(g(a(b, \alpha), e(b, \alpha)) - se(b, \alpha)a(b, \alpha)),$$

with

$$\frac{\partial A^p}{\partial b} = ye(b, \alpha) > 0.$$

Thus, the participation constraint is just

$$b \geqq A^{p-1}(u, \alpha),$$

where $A^{p-1}(u, \alpha)$ is the inverse of $A^p(b, \alpha)$ in b given α.

Think of the equilibrium as the solution to a two stage problem. In the first stage the firm sets the incentive share, knowing that the agent will optimize over e and a in the second stage, given that share. A binding participation constraint yields an incentive share implicitly defined by $A^p(b, \alpha) - u = 0$, with $b = b(\alpha)$ and

$$\frac{\partial b}{\partial \alpha} = \frac{-G(a(b, \alpha), e(b, \alpha))}{ye(b, \alpha)}. \tag{35}$$

At the agent's optimum, the intrinsic net utility G is positive, so that a rise in the intensity of the side activity will decrease the optimal (constrained) incentive share with a binding participation constraint. Agent's second stage equilibrium payoff is

$$\begin{aligned} A^p(b(\alpha),\alpha) =& bye(b(\alpha),\alpha) - c(e(b(\alpha),\alpha)) + \alpha(g(a(b(\alpha),\alpha), e(b(\alpha),\alpha)) - se(b(\alpha),\alpha)a(b(\alpha),\alpha)) \\ =& u \end{aligned} \tag{36}$$

We now wish to compare the results of Proposition 2 with the present case of a binding participation constraint. An increase in α will decrease the equilibrium incentive share in both the substitute and complement case, whereas this decrease was only guaranteed in the complement case under non-binding participation. The agent's welfare is constant with an increase in α, due to binding participation. That is, $\partial A^p/\partial\alpha = G + ye\frac{\partial b}{\partial \alpha} = 0$, by (35) and (36). For the agent, the rise in payoff due to the direct effect of an increase in α, G, is cancelled by the decrease in payoff due to the fact that greater α decreases the incentive share. The effect of a rise in α on the firm's equilibrium profit with binding participation is

$$\frac{\partial \Pi^p}{\partial \alpha} = -\frac{\partial b}{\partial \alpha}ye + (1-b)y\frac{\partial e}{\partial \alpha}. \tag{37}$$

The binding participation constraint brings us a new positive term, $-\frac{\partial b}{\partial \alpha}ye > 0$, which is not present in the non-binding case. Only the second term, $(1-b)y\frac{\partial e}{\partial \alpha}$, is present in the non-binding case. The second is negative if $G_e < 0$ and positive if $G_e > 0$. Thus, we obtain the result that more intense side activity raises profit in the case where $G_e > 0$, in both the binding and non-binding cases. However, if $G_e < 0$, we have a decrease in profit with an increase in α for the non-binding case, but an ambiguous change, if the participation constraint is binding. The two terms of (37) are opposite in sign when $G_e < 0$ and relative magnitudes determine the impact on profit. More intense political activity lowers the incentive share so as to raise profit but at the same time lowers profit because it decreases work effort. Only the latter effect is present in the non-binding case.

How do the effects of feedback change with a binding participation constraint? Let feedback enter only through the CEO's intrinsic payoff and again set $\alpha = 1$. Using the same analysis leading to (35) and (36), we have that the effect of a change in μ on the firm's incentive share is

$$\frac{\partial b}{\partial \mu} = \frac{-1(\bar{\mu}')a}{ye}, \tag{38}$$

and we know that the effect of a change in μ on the agent's payoff is zero, because $\frac{\partial A^p}{\partial \mu} = \frac{\partial b}{\partial \mu}ye + \bar{\mu}'a = \frac{-1(\bar{\mu}')a}{ye} + \bar{\mu}'a = 0$. (i.e., the participation constraint is binding.) Thus, positive feedback reduces the incentive share, while negative feedback increases the incentive share. Considering the change in the firm's profit, we have the following analog to (28)

$$\frac{\partial \Pi^p}{\partial \mu} = -\frac{\partial b}{\partial \mu} ye + (1-b)y\frac{\partial e}{\partial \mu}. \tag{39}$$

The effect on the firm's profit has a new term,$-\frac{\partial b}{\partial \mu}ye$, which is positive with positive feedback and negative with negative feedback. It follows that nothing changes in the complements case, as positive feedback results in the positivity of both terms and greater profit while negative feedback results in the negativity of both terms and less profit. However, in the substitutes case, where $\frac{\partial e}{\partial \mu}$ is positive with negative feedback and negative with positive feedback, we get two opposing terms even with feedback only affecting the CEO's intrinsic payoff. This is the same sort of mixed sign condition that we obtained in the case of feedback affecting the firm's cash flow only, in the non-binding case.

If the feedback is through cash flow, then again the agent's equilibrium payoff is unaffected but the firm's profit is impacted as follows

$$\frac{\partial \Pi^p}{\partial \delta} = -\frac{\partial b}{\partial \delta} ye + (1-b)(y\frac{\partial e}{\partial \delta} + \bar{\delta}'a + \bar{\delta}\frac{\partial a}{\partial \delta}), \tag{40}$$

where

$$\frac{\partial b}{\partial \delta} = \frac{-1(\bar{\delta}')ab}{ye}.$$

In this case, the firm's payoff has changed from the non-binding case with the addition of the term $-\frac{\partial b}{\partial \delta}ye$ which is positive with positive feedback and negative with negative feedback. This term reinforces the second term, $(1-b)(y\frac{\partial e}{\partial \delta} + \bar{\delta}'a + \bar{\delta}\frac{\partial a}{\partial \delta})$, if e and a are complements, making profit go up even more in the positive feedback case and making it go down more in the negative feedback situation. If e and a are substitutes, the second term of (40) is the same as in the nonbinding case. Under certain conditions outlined above in the nonbinding case, it can change profit in the reverse direction to the sign of the feedback. That is, it can work against the term $-\frac{\partial b}{\partial \delta}ye$, which moves with the direction of the feedback. Thus, whether the participation constraint is binding or not, the case where e and a are substitutes can produce situations where the direction of the external feedback is reverse to the direction of the change in profit. That is, ignoring the relationship between work effort and effort devoted to political activity, under an optimal contract, can lead to inaccurate predictions.

7 Conclusion

We examine side activities on the job in the context of a hidden action agency problem. If these activities have benefits which are independent of work effort and generate no external feedback, then they decrease profit, but they increase the agent's incentive payment and welfare. If side activities have benefits which depend on work effort and there is no external feedback, the impact of side activities depends on work effort's marginal impact on the total net benefit of side activities. When this effect is positive, the firm's value is increased with side activities, and the equilibrium incentive share is reduced. The agent's attitude is such that greater work effort results in more enjoyment of the net benefits of side activities. When this effect is negative, the firm's value is decreased, and the incentive share can rise or fall with side activities. This attitude is indicative of an agent whose increased work effort diminishes the net benefit of side activities. These results tell us that in cases where the incentive share rises with the presence of side activities, it must be that extra work effort lowers the total net benefit of side activities. Moreover, in the interdependent case, the impact of side activities on the agent's welfare contains a positive direct term and an indirect term through the incentive share which can counter the positive direct term. In all cases of interdependent work and side activities, the indirect term makes the change in the agent's welfare indeterminate.

If side activities are political and they provoke external feedback, then the impact of feedback depends on whether these activities are substitutes or complements in the agent's payoff, in both the intrinsic and cash flow feedback cases. More intense positive feedback raises the firm's payoff, if work effort and political activities are complements. In this case, greater negative feedback has the opposite effect. However, in the substitute case, more positive feedback can, under certain conditions, reduce the agent's work effort, increase political activity, but decrease the firm's profit. More negative feedback can, under certain conditions, increase the agent's work effort, decrease political activity, but increase the firm's bottom line. These counter intuitive results are uncovered as a result of this paper examining the interaction of side activity effort and work effort in the context of an optimal agency contract. Finally, positive feedback has positive direct effects on the agent's payoff, and negative feedback has negative direct effects on the agent's payoff. However, in both the cases of intrinsic and cash flow feedback there can be indirect effects which counter the direct effects through the optimal contract.

The above results shed light on the effects of side activities on the firm's equilibrium. It is important for the firm to detect how side activities and work effort interact in the agent's payoff, before it designs regulatory policy concerning these activities.

Appendix

Proof of Lemma 1 The principal's Lagrangian is

$$\begin{aligned}L = &-S + (1-b)yh(by - sa(b)) + \lambda(S + byh(by - sa(b)) + g(a(b)) - c(h(by - sa(b)))\\ &- sa(b)h(by - sa(b)) - u) + \gamma_S S + \gamma_b b + \rho[-S + (1-b)yh(by - sa(b))].\end{aligned}$$

The FOC for S is

$$-1 + \lambda + \gamma_S - \rho = 0,$$

with $\lambda, \gamma_S, \rho \geqq 0$. If the participation constraint is non-binding, $\lambda = 0$ and $\gamma_S = 1 + \rho > 0$. Thus, $S = 0$. ∎

Proof of Lemma 2 Utilizing A.3, if $b \to b'$, then from (1), $e \to 0$. We have

$$\underset{b \to b'}{Lim} \Pi_b^p = -y \cdot 0 + (1-0)yh'(0)(y - sa'(0)) > 0.$$

Next, take $b \to 1$. We have

$$\underset{b \to 1}{Lim} \Pi_b^p = -yh(y - sa(1)) < 0.$$

These two limits, continuity, and $\Pi_{bb}^p < 0$ imply that there is a unique $b^p \in (0, 1)$ which solves the principal's problem (5). ∎

Proof of Proposition 1 For each b, consider the difference

$$\Pi_b^p - \Pi_b^o = y[h(by) - h(by - sa(b))] + (1-b)y[h'(by - sa(b))(y - sa'(b)) - h'(by)y].$$

Because $sa(b) > 0$, for each b, $h(by) - h(by - sa(b)) > 0$, for each b. Because $h' > 0$ is nonincreasing, $h'(by - sa(b)) \geqq h'(by)$. Further, $(y - sa'(b)) > y$, by $a'(b) < 0$. Whence, for each b, $\Pi_b^p - \Pi_b^o > 0$. Given that each of these functions is decreasing in b, $b^p > b^o$.

The equilibrium profit difference is

$$\Pi^o - \Pi^p = [(1-b)yh(by)] - [(1-b)yh(by - sa(b))], \text{ for each } b.$$

Because $sa(b) > 0$, for each b, $\Pi^o - \Pi^p > 0$, for all b. It follows that at optimum, $\Pi^o(b^o) - \Pi^p(b^p) > 0$. ∎

Proof of Lemma 3 If $b \to \hat{b}$, then from (1), $e \to 0$, by A.5. We have

$$\underset{b \to \hat{b}}{Lim} \Pi_b^p = -y \cdot 0 + (1-0)yh'(0)(y\frac{-\alpha c''(e(1,\alpha))g_{aa}(a(1,\alpha), e(1,\alpha))}{|H_p(a(1,\alpha), e(1,\alpha))|}) > 0.$$

Next, take $b \to 1$.

$$\underset{b \to 1}{Lim} \Pi_b^p = -yh(y + \alpha(g_e(a(1,\alpha), e(1,\alpha)) - sa(1,\alpha))) < 0.$$

These two limits, continuity and $\Pi_{bb}^p < 0$ imply that there is a unique $b^p \in (0, 1)$ which solves the principal's problem (15). ∎

Proof of Proposition 2 We can write

$$\begin{aligned}\Pi_b^p(b,\alpha) - \Pi_b^o(b) =& [yh(by) - yh(by + \alpha G_e(b,\alpha))] \\ &+ [(1-b)y[h'(by + \alpha G_e(b,\alpha))(y - \frac{\alpha^2 y}{|H_p(b,\alpha)|}(|H_G(b,\alpha)|)) - h'(by)y].\end{aligned}$$

If $G_e > 0$, then the first term is negative and the second is also negative at each b. Given $\Pi_{bb} < 0$, $b^o > b^p$. These results hold for any $\alpha > 0$. If $G_e < 0$, then the first term of this expression is positive and the second is of either sign. We cannot rank b^o and b^p. An example will make this point. Let $G = \gamma(e - e^2) + (a - a^2) - sae$, and

$c = e^2/2$. The principal's optimal $b = \frac{2(y-\gamma\alpha)+s\alpha}{4y}$ and optimal profit is $\Pi^p = \frac{[2(y+\gamma\alpha)+s\alpha]^2}{8(2+4\gamma\alpha-s^2\alpha)}$. If $y = 0.9, \gamma = .3$, and $s = .48$, we have that $b^o > b^p$, for $\alpha \in (0, 10)$. If we keep the same parameters but raise s to $s = .67$, we have $b^o < b^p$, for $\alpha \in (0, 10)$.

Comparing profit, in equilibrium, we have, using (15),

$$\frac{\partial \Pi^p(b,\alpha)}{\partial \alpha} = (1-b)y\frac{\partial e}{\partial \alpha} = (1-b)y\frac{-\alpha G_{aa}(b,\alpha)G_e(b,\alpha)}{|H_p(b,\alpha)|}.$$

If $G_e > 0$, then as we move from $\alpha = 0$ to positive α, equilibrium profit increases. It follows that the solution with side activities dominates. If $G_e < 0$, the opposite obtains. ∎

Proof of Lemma 4: The principal's problem has the Lagrangian

$$\begin{aligned} L = &- S + (1-b)ye(b,\alpha) + \lambda[S + bye(b,\alpha) + \alpha(g(e(b,\alpha), a(b,\alpha)) - sa(b,\alpha)e(b,\alpha)) \\ &- c(e(b,\alpha)) - u] + \rho[-S + (1-b)ye(b,\alpha)] + \gamma_S S + \gamma_b b, \end{aligned}$$

where $\lambda \geqq 0$ is the multiplier on the agent's participation constraint, $\rho \geqq 0$ is the multiplier on the firm's participation constraint, and γ_i are the non-negativity multipliers for the two choice variables $i = S, b$. The FOC include

$$\begin{aligned} &S : -1 + \lambda - \rho + \gamma_S = 0, \text{ and} \\ &b : [-ye + (1-b)y\frac{\partial e}{\partial b}] + \lambda ye + \rho[-ye + (1-b)y\frac{\partial e}{\partial b}] + \gamma_b = 0. \end{aligned}$$

Assume to the contrary that $S > 0$, so that $\gamma_S = 0$ and $\lambda = (1+\rho) > 0$. The FOC for b can be rewritten

$$\lambda(1-b)y\frac{\partial e}{\partial b} = -\gamma_b \leqq 0.$$

Because $\lambda, y\frac{\partial e}{\partial b} > 0$, we have $(1-b) \leqq 0$. Whence, $-S + (1-b)ye(b,\alpha) < 0$ and we have a contradiction of firm participation. ∎

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